Ch10 Q12. Why does a stream of water from a faucet become narrower as it falls?

The further from the faucet, the faster it moves due to gravity. The flow rate Q defined by

$$Q = \frac{\Delta V}{\Delta t}$$

which is the flow of volume through a point over an amount of time. It turns out that at a point where the fluid has a cross-sectional area of A_1 the fluid moves with speed v_1,

$$Q = v_1 A_1.$$

Continuity, says that Q is the same along the flow of the fluid. Thus, the speeds and areas at different points, say point 2, are related by the fact that they have the same Q:

$$\begin{aligned} v_1 A_1 = \; Q & = v_2 A_2 \\ \Rightarrow v_1 A_1 & = v_2 A_2. \end{aligned}$$

For the current problem, as the stream falls, it speeds up due to gravity (of course, by $v = \sqrt{2gh}$ for a fall by height h). Thus, as v increases, to keep $vA = Q$ constant, A must decrease. This is why a stream tapers as it falls.

Ch10 Q19. Roofs of house are sometimes "blown" off (or are they pushed off?) during a tornado or hurricane. Explain using Bernoulli's principle.

The pressure outside of the house is lower than that inside the house, since due to Beroulli's principle, a faster fluid has a lower pressure than a slower fluid. So the roofs are pushed out from the inside.

Ch10 Q21. When blood pressure is measured, why must the arm cuff be held the level of the heart?

So it can measure pressure without including increases or decreases with respect to height. Pressure changes like

$$P(h) = P_0 + \rho g h$$

so, to measure pressure at exit vessels of the heart (forgive me biologists for not knowing the correct term), you need the cuff to be at the same level.

Ch10 P43. How fast does water flow from a hole at the bottom of very wide, 4.7 m deep storage tank filled with water? Ignore viscosity.

From Bernoulli, we have that

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

where $h_2 = h_1 = 4.7$m, and 1 refers to a point just outside the hole and 2 refers to a point inside. If you compare v_1 to v_2, v_2 is a lot slower than v_1, so we ignore it (mathematically set it to zero). Thus,

$$101325\,\text{Pa} + \frac{1}{2} 1000\text{kg/m}^3\, v_1^2 = P_2 + 0$$

where we simplified things. It turns out the pressure at 2 is

$$\begin{aligned} P_2 &= P_0 + \rho g h \\ &= 101325\,\text{Pa} + 1000\text{kg/m}^3 g h \\ \Rightarrow 101325\,\text{Pa} + \frac{1}{2} 1000\text{kg/m}^3\, v_1^2 &= 101325\,\text{Pa} + 1000\text{kg/m}^3 g h \end{aligned}$$

$$
\begin{aligned}
\Rightarrow \frac{1}{2} 1000\mathrm{kg/m^3}\, v_1^2 &= 1000\mathrm{kg/m^3} gh \\
\Rightarrow v_1^2 &= 2gh \\
&= \sqrt{2gh} \\
&= \sqrt{2(9.8)(4.7)}\frac{\mathrm{m}}{\mathrm{s}} \\
&= 9.6\,\mathrm{m/s}.
\end{aligned}
$$

Ch10 P50. A 6.0-cm diameter horizontal pipe gradually narrows to 4.5-cm. When water flows through the pipe at a certain rate, the gauge pressure in these two sections is 33.5 kPa and 22.6 kPa, respectively. What is the volume rate of flow?

The first area is $\pi(.030\mathrm{m})^2 = 0.0028\mathrm{m}^2$, and the second is $0.0016\mathrm{m}^2$. We have that

$$
\begin{aligned}
2.8 \times 10^{-3}\mathrm{m}^2 v_1 &= 1.6 \times 10^{-3}\mathrm{m}^2 v_2 \\
\Rightarrow v_2 &= \frac{28}{16} v_1.
\end{aligned}
$$

From Bernoulli,

$$
\begin{aligned}
\frac{1}{2}\rho v_1^2 + P_1 &= \frac{1}{2}\rho v_2^2 + P_2 \\
\Rightarrow v_1^2 &= v_2^2 - 2\frac{\Delta P}{\rho} \\
&= \left(\frac{28}{16}\right)^2 v_1^2 - 2\frac{\Delta P}{\rho} \\
\Rightarrow v_1 &= \sqrt{\frac{2\Delta P/\rho}{\left(\frac{28}{16}\right)^2 - 1}} \\
&= \sqrt{\frac{2\frac{10.9\mathrm{kPa}}{1000\mathrm{kg/m^3}}}{\left(\frac{28}{16}\right)^2 - 1}} \\
&= 3.17\mathrm{m/s}.
\end{aligned}
$$

Thus, the volume flow is $Q = v_1 A_1 = 3.17\mathrm{m/s} \cdot 2.8 \times 10^{-3}\mathrm{m}^2 = 9.0 \times 10^{-3}\mathrm{m}^3/\mathrm{s}$.

Ch11 Q3. How would you double the maximum speed of a simple harmonic oscillator (SHO)?

Typically, the frequency of an oscillator is fixed. For example, a mass and spring's angular frequency is

$$
\omega = \sqrt{k/m}
$$

and k is a constant for a spring as is m usually. Since $v_{\max} = A\omega = A\sqrt{k/m}$, a good way to double the max speed is by doubling the amplitude.

If we could find a spring with the same equlibrium length but a different k value, another way to double the max-speed is to quadruple k, for $k_{\mathrm{double}} = 4k_{\mathrm{old}}$

$$
\begin{aligned}
v_{\max-\mathrm{old}} &= A\sqrt{\frac{k_{\mathrm{old}}}{m}} \\
\Rightarrow v_{\max-\mathrm{double}} &= A\sqrt{\frac{k_{\mathrm{double}}}{m}} \\
&= A\sqrt{\frac{4k_{\mathrm{old}}}{m}} \\
&= 2A\sqrt{\frac{k_{\mathrm{old}}}{m}} \\
&= 2v_{\max-\mathrm{old}}.
\end{aligned}
$$

Similarly, if replace the mass with a mass which is a fourth as massive,

$$\begin{aligned}
\Rightarrow v_{\max-\text{double}} &= A\sqrt{\frac{k}{m_{\text{double}}}} \\
&= A\sqrt{\frac{k}{m_{\text{old}}/4}} \\
&= \frac{A}{1/2}\sqrt{\frac{k}{m_{\text{old}}}} \\
&= 2v_{\max-\text{old}}.
\end{aligned}$$

Ch11 Q11. Is the frequency of a simple periodic wave equal to its source? Why or why not?

When something waves to create waves, the source moves with the medium, so it makes sense that they move in tandem.

Ch11 P16. It takes a force of 91.0 N to compress the spring of a toy popgun 0.175 m to "load" a 0.160-kg ball. With what speed will the ball leave the gun if fired horizontally?

We have that

$$\begin{aligned}
91.0\text{N} &= k\,(0.175\text{m}) \\
\Rightarrow k &= \frac{91.0}{0.175}\text{N/m} \\
&= 520.\,\text{N/m}.
\end{aligned}$$

Thus, we have that

$$\begin{aligned}
\omega &= \sqrt{k/m} \\
&= \sqrt{520.\text{N/m}/0.160\text{kg}} \\
&= 57.0\,\text{rad/s}.
\end{aligned}$$

The maximum speed is $A\omega$, so since $A = 0.175\text{m}$,

$$v_{\max} = 9.98\text{m/s}.$$

Ch11 P21. At $t = 0$, an 885-g mass at rest on the end of a horizontal spring ($k = 184\text{N/m}$) is struck by a hammer which gives it an initial speed of 2.26 m/s. Determine (a) the period and frequency of the motion (b) the amplitude, (c) the maximim acceleration, (d) the total energy, and (e) the kinetic energy when $x = 0.40A$ where A is the amplitude.

a) We have that

$$\begin{aligned}
\omega &= \sqrt{k/m} \\
&= \sqrt{\frac{184\text{N/m}}{0.885\text{kg}}} \\
&= 14.4\text{rad/s} \\
&= 2\pi f \\
&= 2\pi/T \\
\Rightarrow T &= \frac{2\pi}{\omega} \\
&= 0.436\text{s}.
\end{aligned}$$

The frequency is then

$$\begin{aligned}
f &= 1/0.436\text{s} \\
&= 2.29\,\text{Hz}.
\end{aligned}$$

b) We have that

$$\begin{aligned} v &= 2.26\text{m/s} \\ &= A\omega \\ \Rightarrow A &= v/\omega \\ &= 2.26\text{m/s}/14.4\text{rad/s} \\ &= 0.157\text{m}. \end{aligned}$$

c) The max a is $A\omega^2 = v_{\text{max}}\omega$. So,

$$\begin{aligned} a_{\max} &= 2.26\text{m/s}\cdot 14.4\text{rad/s} \\ &= 32.6\,\text{m/s}^2. \end{aligned}$$

d) The total energy is the energy at any time. Choosing the center, we have that the potential is zero and the kinetic energy is

$$\begin{aligned} \frac{1}{2}mv_0^2 &= \frac{1}{2}0.885\text{kg}(2.26\text{m/s})^2 \\ &= 2.26\text{J}. \end{aligned}$$

To check, consider at the maximum ampltitude, then the energy is

$$\begin{aligned} \frac{1}{2}kx^2 &= \frac{1}{2}184\text{N/m}(0.1567\text{m})^2 \\ &= 2.26\text{J}, \end{aligned}$$

which is consistent.

e) We have that

$$\begin{aligned} 2.26\,\text{J} &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}184\text{N/m}(0.4\cdot 0.157\text{m})^2 \\ &= \text{KE} + 0.363\text{J} \\ \Rightarrow \text{KE} &= 2.26\,\text{J} - 0.363\text{J} \\ &= 1.90\text{J}. \end{aligned}$$