Chapter 2.

Q1. Does a car speedometer measure speed, velocity, or both? Explain.

Speed.

P7. You are driving home from school steadily at 95 km/h for 180 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 4.5 hours. a) How far is your hometown from school? b) What was your average speed?

For a), draw a picture.

Write down what you know. Calling the distance travel along the first part of the trip (we denote A) as d_A and the distance along the second part d_B,

$$\begin{aligned} d_A &= 180\,\mathrm{km} \\ d_B &= \,? \end{aligned}$$

We have the speeds

$$\begin{aligned} v_A &= 95\mathrm{km/h} \\ v_B &= 65\mathrm{km/h}. \end{aligned}$$

Finally, we have that the total time t is 4.5 h, and obviously,

$$t = 4.5\mathrm{h} \;=\; t_A + t_B.$$

Now, write down possibly relevant equations. One obvious one is possible is the definition of speed

$$v = \frac{\Delta x}{\Delta t}.$$

Putting in the labels for the corresponding legs of the trip,

$$\begin{aligned} v_A &= \frac{d_A}{t_A}, \\ v_B &= \frac{d_B}{t_B}. \end{aligned}$$

We are looking eventually for d_B, so

$$\begin{aligned} t_B v_B &= d_B \\ = t_B(65\mathrm{km/h}) &= d_B \end{aligned}$$

We have v_B, so we need t_B, the time of the second leg B. To find this, we can solve for

$$\begin{aligned} 4.5\mathrm{h} &= t_A + t_B \\ \Rightarrow t_B &= 4.5\mathrm{h} - t_A \end{aligned}$$

Thus,

$$d_B = (4.5\mathrm{h} - t_A) 65\mathrm{km/h}.$$

Now, we need t_A! However, we can use that

$$\begin{aligned} v_A &= \frac{d_A}{t_A} \\ =95\,\mathrm{km/h} &= \frac{180\mathrm{km}}{t_A} \end{aligned}$$

to solve for t_A:

$$\begin{aligned} =95\,\mathrm{km/h} &= \frac{180\mathrm{km}}{t_A} \\ \Rightarrow t_A &= \frac{180\mathrm{km}}{95\,\mathrm{km/h}}. \end{aligned}$$

This implies that

$$d_B = \left(4.5\mathrm{h} - \frac{180\mathrm{km}}{95\mathrm{km/h}}\right) 65\mathrm{km/h}.$$

This is completely solved in one expression. I get a result of 170km. This is the same result as the more stepwise derivation we did in recitation.

For a review of these steps, note that we can identify the parts of the above equation:

$$d_B = \underbrace{\left(\underbrace{4.5\mathrm{h}}_{t} - \underbrace{\frac{180\mathrm{km}}{95\mathrm{km/h}}}_{t_A}\right)}_{t_B} \underbrace{65\mathrm{km/h}.}_{v_B}$$

We solved for t_A, the time for leg A:

$$180\,\mathrm{km}/95\mathrm{km/h} = 1.89\mathrm{h}.$$

So, I drove 65 km/h for 4.5h−1.89h=2.61h. This is t_B. Thus, the distance travelled was 2.61h·65km=170 km. This is d_B.

Therefore, the *total* distance is

$$170\,\mathrm{km} + 180\mathrm{km} = 350\mathrm{km}.$$

For b), the average speed is the total distance divided by the time, so

$$350\,\mathrm{km}/4.5\mathrm{h} = 78\mathrm{km/h}.$$

It is *not* the average of the two speeds.

Q5. Can an object have a northward velocity and a southward acceleration? Explain.

Yes. Acceleration is rate of change of velocity, and can be in the opposite direction to motion. Example of coasting up a hill that slopes up northward. Your acceleration will be down the hill (southward), but your instantaneous velocity will be northward.

P26. A world-class sprinter can reach a top speed (of about 11.5 m/s) in the first 18.0 m of a race. What is the average acceleration of this printer and how long does it take her to reach that speed?

Assume acceleration is constant. Write down what you know:

$$\begin{aligned} v_f &= 11.5\,\mathrm{m/s} \\ a &= ? \\ x_f &= 18.0\,\mathrm{m}. \end{aligned}$$

We are only considering the first 18 meters of the run, and we assume she's starting from rest, so

$$\begin{aligned} v_f &= 11.5\,\mathrm{m/s} \\ a &= ? \\ x_f &= 18.0\,\mathrm{m}. \\ v_i &= 0 \\ x_i &= 0. \\ t &= ? \end{aligned}$$

So, now, let's write down our 1D constant acceleration equations:

$$\begin{aligned} x_f &= x_i + v_i t + \frac{1}{2} a t^2 \\ v_f &= v_i + a t \\ v_f^2 &= v_i^2 + 2a\,(x_f - x_i). \end{aligned}$$

Which equation do we use? We choose the equation with only one unknown given what we have. For a), since we just need a, the acceleration, given what we already have, the third makes the most sense.

Thus,

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(x_f - x_i) \\ =(11.5\mathrm{m/s})^2 &= 0 + 2a\,(18.0\mathrm{m}) \\ &= 2a\,(18.0\mathrm{m}) \\ \Rightarrow \frac{(11.5\,\mathrm{m/s})^2}{(18.0\,\mathrm{m})} &= 2a \\ \Rightarrow a &= \frac{(11.5\,\mathrm{m/s})^2}{2(18.0\,\mathrm{m})} \\ &= 3.67\,\mathrm{m/s}^2. \end{aligned}$$

For b), the easiest equation to use is the second, since we have a now. Thus,

$$\begin{aligned} 11.5\,\mathrm{m/s} &= 3.67\,\mathrm{m/s}^2 t \\ \Rightarrow t &= \frac{11.5\,\mathrm{m/s}}{3.67\,\mathrm{m/s}^2} \\ &= 3.13\,\mathrm{s}. \end{aligned}$$

Q11. As a free-falling object speeds up, what is happening to its acceleration—does it increase, descrease, or stay the same? (a) Ignore air resistance (b) Consider air resistance.

For a), it is constant. For b), it decreases until it is zero. Air resistance is a velocity dependent force, so the force is stronger when an object is faster, but decreases as the object slows. Eventually, the object will reach a final speed and no longer speed up. This speed is called the “terminal velocity.”

Q13. Can an object have zero velocity and non-zero acceleration at the same time? Give examples.

Yes, momentarily. A ball at the top of its height is an example of this. If an object isn't moving, for it to starting moving again, it has to have a non-zero acceleration, otherwise it will stay still.

P42. A baseball is hit almost straight up into the air with a speed of 25 m/s. Estimate (a) how high it goes, (b) how long it is in the air. (c) What factors make this an estimate?

I'll assume there is no air resistance, thus, this will be an estimate. (Answer to c). Also, assume that it was released close to the ground. Moreover, assume it is thrown very close to straight up, otherwise, we'd need to consider it's 2D motion (left and right).

For a), what do we know? We only have

$$\begin{aligned} v_i &= 25\text{m/s}. \\ a &= -9.8\,\text{m/s}^2 \\ &= -g. \end{aligned}$$

We have no more information...

Well, we can glean more information from what we know about physics. We know that it will be at rest at the top of it's hieght, so we can guess that

$$v_f = 0,$$

As long as we consider only the motion until it reaches its peak. Also, call $x=0$ the height it is thrown from. Therefore, we have that

$$\begin{aligned} x_i &= 0\,\text{m} \\ x_f &= ? \\ v_i &= 25\,\text{m/s} \\ v_f &= 0\,\text{m/s} \\ a &= -9.8\,\text{m/s}^2 \\ t &= ? \end{aligned}$$

Notice, these would change if we consider motion *after* it reaches its peak. The initial conditions, (x_i and v_i) do not change, and a is a constant, so it doesn't change, but v_f and x_f are functions of t, and thus change when we consider different times.

However, we are only concerned with the ball at the top of it's height. Therefore, looking at our equations again:

$$\begin{aligned} x_f &= x_i + v_i t + \frac{1}{2} a t^2 \\ v_f &= v_i + a t \\ v_f^2 &= v_i^2 + 2a\,(x_f - x_i). \end{aligned}$$

We easily see that, once again the third equation is best since we don't have a time:

$$\begin{aligned}
0 &= (25\,\mathrm{m/s})^2+2(-9.8\mathrm{m/s}^2)(x_f-0\mathrm{m})\\
&= (25\,\mathrm{m/s})^2-19.6\mathrm{m/s}^2x_f\\
\Rightarrow x_f 19.6\mathrm{m/s}^2 &= (25\mathrm{m/s})^2\\
\Rightarrow x_f &= \frac{(25\mathrm{m/s})^2}{19.6\mathrm{m/s}^2}\\
&= 32\,\mathrm{m}
\end{aligned}$$

which is pretty high, actually.

For b), how long it is in the air, this is a little trickier, because we are *not* considering the motion of the last problem. We are considering when the motion of the ball falls back down to where we threw it.

So, its starting x and final x are the same, 0. This is extremely easy mathematically, but hard conceptually to figure out. However, this means that

$$\begin{aligned}
x_i &= 0\,\mathrm{m}\\
x_f &= 0\,\mathrm{m}\\
v_i &= 25\mathrm{m/s}\\
v_f &= ?\\
a &= -9.8\mathrm{m/s}^2\\
t &= ?
\end{aligned}$$

And we are looking for t. From our equations:

$$\begin{aligned}
x_f &= x_i+v_it+\frac{1}{2}at^2\\
v_f &= v_i+at\\
v_f^2 &= v_i^2+2a\,(x_f-x_i).
\end{aligned}$$

The easiest one to use looks like it is the first one:

$$\begin{aligned}
0\mathrm{m} &= 0\mathrm{m}+(25\mathrm{m/s})t+\frac{1}{2}(-9.8\mathrm{m/s}^2)t^2\\
=0\mathrm{m} &= (25\mathrm{m/s})t-(4.9\mathrm{m/s}^2)t^2\\
\Rightarrow 0\mathrm{m/s} &= (25\mathrm{m/s})-(4.9\mathrm{m/s}^2)t\\
\Rightarrow (4.9\mathrm{m/s}^2)t &= (25\mathrm{m/s})\\
\Rightarrow t &= \frac{(25\mathrm{m/s})}{(4.9\mathrm{m/s}^2)}\\
&= 5.1\mathrm{s}.
\end{aligned}$$

Chapter 3.

Q9. Can the magnitude of a vector ever (a) equal, or (b), be less than one of its components?

(a) Yes, in the case when a vector lies along $\hat{x}$, $\hat{y}$, $\hat{z}$, any of the axis directions. By the pythogorean theorem, it cannot be less, though.

P8. An airplane is traveling 835 km/h in a direction 41.5° west of north. (a) find the components of the velocity in the north direction and west. (b) find the displacement components.

(a) The components in the north direction is the cosine of 41.5° by inspecting the right triangle.

$$\begin{aligned} v_{\text{north}} &= 835\,\text{km/h}\cdot\cos(41.5°) \\ &= 625\,\text{km/h}. \end{aligned}$$

The west component is the sine of that angle. Given the direction, this is a positive component.

$$\begin{aligned} v_{\text{west}} &= 835\,\text{km/h}\cdot\sin(41.5°) \\ &= 553\text{km/h}. \end{aligned}$$

We can multiply each component separately to obtain the displacement along each direction. It turns out that the definition we had for velocity

$$v = \frac{\Delta x}{\Delta t}$$

works for vectors too:

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}.$$

Of course, time is not a vector, but $\Delta\vec{x}$ is. Specifically, this one equation says that

$$\begin{aligned} v_x &= \frac{\Delta x}{\Delta t} \\ v_y &= \frac{\Delta y}{\Delta t}. \end{aligned}$$

Therefore, for a constant $\vec{v}$,

$$\Delta\vec{x} = \vec{v}\,\Delta t,$$

which is scalar multiplication of the left-side. This just says that

$$\begin{aligned} \Delta x &= v_x\,\Delta t, \\ \Delta y &= v_y\Delta t, \end{aligned}$$

it is just written in one expression. Thus, using this,

$$\begin{aligned} d_{\text{north}} &= 625\text{km/h}\cdot 1.75\text{h} \\ &= 1090\text{km}, \\ d_{\text{west}} &= 553\text{km/h}\cdot 1.75\text{h} \\ &= 968\text{km}. \end{aligned}$$