Chapter 3.

Q12. In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?

It should be obvious at this point that it be slightly above. Note that we have that

$$\begin{aligned} v\cos\theta t &= d \\ v\sin\theta t - \frac{1}{2}gt^2 &= \Delta h \end{aligned}$$

where Δh is the vertical distance from you to the target. If the target is about your height,$\Delta h = 0$, so

$$\begin{aligned} v\sin\theta t - \frac{1}{2}gt^2 &= 0 \\ \Rightarrow v\sin\theta t &= \frac{1}{2}gt^2 \\ \Rightarrow v\sin\theta &= \frac{1}{2}gt \\ \Rightarrow \mathrm{v}\sin\theta &= \frac{1}{2}g\frac{d}{v\cos\theta} \\ \Rightarrow v^2 2\sin\theta\cos\theta &= gd \\ \Rightarrow \frac{v^2}{g}2\sin\theta\cos\theta &= d. \end{aligned}$$

Thus, since $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\sin 2\theta = \frac{dg}{v^2}.$$

So, as the distance changes, the angle has to increase (since sine starts at 2θ). Conceptually, what is occuring is the angle has to give the arrow some vertical velocity to counteract the distance it would fall.

P24. You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 4.0s for the dark to land back on the barrel. What is the maximum range of your gun?

Our assumption here is that the velocity of the gun is what determines how high it travels.

Knowns: $\Delta y = 0, t = 4.0, a = -9.8, v_{yi} = v\sin 90 = v = ?$.

We have that

$$\begin{aligned} 0 &= vt - \frac{1}{2}gt^2 \\ \Rightarrow \frac{1}{2}gt &= v. \end{aligned}$$

Putting this speed into the range equation, which has a maximum when $\theta = 45°$

$$\begin{aligned} R_{\max} &= v^2/g \\ &= \frac{\left(\frac{1}{2}gt\right)^2}{g} \\ &= \frac{1}{4}gt^2 \\ &= 9.8\mathrm{m/s^2} \cdot 4.0\mathrm{s}^2 \\ &= 39.2\,\mathrm{m}. \end{aligned}$$

Q13. It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bllet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.

Assume the bullet was shot straight up. Given the air resistance the bullet would feel over a 2 km trip up to the plane, it is not unlikely that the bullet would be considerable slowed over its trip up.

P31. A rescue plane wants to drop supplies to isolated mountatin climbers on a rocky ridge 235m below. If the plane is traveling horizontally with a speed of 250 km/h (69.4 m/s), how far in advance of the recipients (horizontal distance) must the goods be dropped?

The key here is the horizontal and vertical components of motion will have the same time.

Quantities:

$$\Delta x = ?, \Delta y = -235\text{m}, v_x = 69.4, v_y = 0, t = ? a_y = -g$$

Thus,

$$-235\,\text{m} \;=\; -\frac{1}{2}gt^2$$

For horizontal, since the plan is moving, it will travel initially at the horizontal speed of the plane:

$$(69.4\,\text{m/s})t \;=\; x$$

We want to solve for this distance. Solving for t,

$$t \;=\; \frac{x}{69.4\,\text{m/s}}$$

Thus,

$$\begin{aligned} 235\,\text{m} &= \frac{1}{2}g\left(\frac{x}{69.4\text{m/s}}\right)^2 \\ \Rightarrow \frac{2(235\,\text{m})}{(69.4\text{m/s})^2 g} &= x^2 \\ \Rightarrow x &= (69.4\text{m/s})\sqrt{\frac{2(235\,\text{m})}{g}} \\ &= 480.\text{m}. \end{aligned}$$

Q16. A projective has the least speed at what point in its path?

At its height.

P56. Romeo is throwing pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of the rose garden 8.0 m below her window and 8.5 m from the base of the wall. How fast are the pebbles going when they hit her window?

So, we don't know the initial velocity, but just that they should graze the window.

Only equation for x motion is $v_{xi}t = \Delta x$,we have Δx, so we need t. Again, use that both dimensions use the same time.

How to find the time? If fall down time,

$$
\begin{aligned}
-8.0\text{m} &= -\frac{1}{2}9.8\text{m/s}^2 t^2. \\
\Rightarrow t^2 &= \frac{2\,(8.0)}{9.8} \\
\Rightarrow t &= \sqrt{\frac{2\,(8.0)}{9.8}} \\
&= 1.28\,\text{s}.
\end{aligned}
$$

Thus, $v = 8.5\text{m}/1.28\text{s}{=}6.65\text{m/s}$.

If the other way, use

$$
\begin{aligned}
0 &= v_{iy}^2 - 19.6\text{m/s}^2\,8.0\,\text{m} \\
\Rightarrow v_y^2 &= 2\,g\,8.0\text{m} \\
\Rightarrow v_y &= \sqrt{2\,g\,8.0\text{m}}
\end{aligned}
$$

Also,

$$
\begin{aligned}
0 &= v_y - gt \\
\Rightarrow \frac{v_y}{g} &= t
\end{aligned}
$$

and

$$
v_x t = 8.5\,\text{m}
$$

Since v_x is constant, we know v_x will be the speed when it his the window

$$
\begin{aligned}
v_x &= \frac{8.5\text{m}}{t} \\
&= \frac{8.5\text{m}}{\frac{v_y}{g}} \\
&= \frac{8.5\text{m}}{v_y} g \\
&= \frac{8.5\text{m}}{\sqrt{2\,g\,8.0\text{m}}} g \\
&= \frac{8.5\text{m}}{\sqrt{2\,g\,8.0\text{m}}} g \\
&= 6.7\,\text{m/s}
\end{aligned}
$$

P58. a) A long jumper leave the ground at 45° above the horizontal and lands 8.0m away. What is her "takeoff" speed v_0? b) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5m vertically below. If she long jumpys fromt he edge of the left bank at 45° with the speed calculated in a), how long, or short of the opposite bank will she land?

For a), we can use the maximum range formula:

$$
\begin{aligned}
8.0\,\text{m} &= v^2/g \\
\Rightarrow v &= \sqrt{g 8.0} \\
&= 8.85\,\text{m/s}.
\end{aligned}
$$

For b), we again use the fact that the times are the same.

We know that

$$\begin{aligned}
\Delta y &= -2.5\text{m} \\
x_i &= 0\text{m} \\
\Delta x &= ? \\
v_x &= 8.85\sin 45 \\
v_y &= 8.85\sin 45 \\
a_y &= -9.8 \\
t &= ?
\end{aligned}$$

First, find the time to land from the y-component:

$$-2.5\text{m} = v\sin 45\, t - \frac{1}{2}gt^2$$

Then, use the x-component:

$$\Delta x = v\cos 45\, t$$

But, since $v\sin 45 = v\cos 45 = 6.257\,\text{m/s}$, we have that, suppressing units

$$0 = 6.257t - 4.9t^2 + 2.5.$$

And

$$\Delta x = 6.257t.$$

Therefore, we can solve for t:

$$\begin{aligned}
\frac{\Delta x}{6.257} &= t \\
\Rightarrow 0 &= 6.257\frac{\Delta x}{6.257} - 4.9\left(\frac{\Delta x}{6.257}\right)^2 + 2.5. \\
\Rightarrow 0 &= \Delta x + 2.5\text{m} - 4.9\text{m/s}^2\left(\frac{\Delta x}{6.257\text{m/s}}\right)^2
\end{aligned}$$

where I reinserted units in the last line. We can now solve for Δx directly using the quadratic formula:

$$\begin{aligned}
\Delta x &= \frac{-1 \pm \sqrt{1^2 - 4\frac{-4.9\text{m/s}^2}{(6.257\text{m/s})^2}2.5\text{m}}}{-2(4.9\text{m/s}^2)/(6.257\text{m/s})^2} \\
&= \frac{-1 \pm \sqrt{1 + 1.252\frac{\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}^2}}}{-0.2503\,\text{m}^{-1}} \\
&= \frac{1\text{m} \pm 1.501\text{m}}{0.2503} \\
&= 3.995\text{m} \pm 5.997\text{m} \\
&= 9.992, -2.002.
\end{aligned}$$

Where we took the positive root, since she is jumping forward, not backwards. This is less than a cm short, so she essentially makes it.