MRF and CRF based Image Denoising and Segmentation

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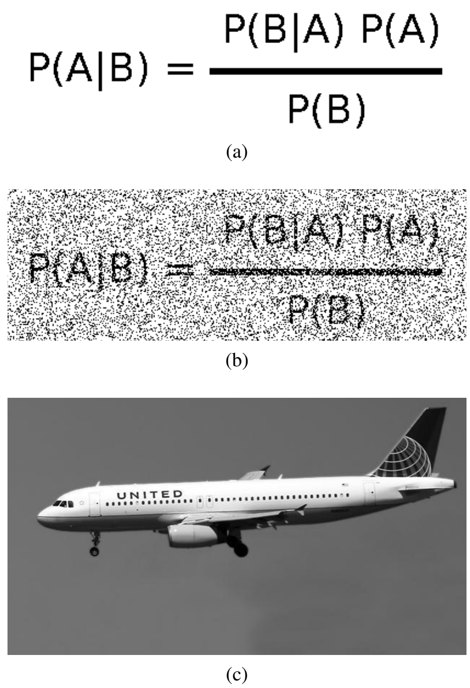
***Abstract*—In this work, we employ a pairwise Markov Random Field (MRF) and a Conditional Random Field (CRF) for bi-level image segmentation and denoising. For both tasks, the Ising pairwise model and the Iterative Conditional Mode (ICM) inference method are implemented, assuming the parameters of the unary and pairwise potentials are known. Experimental results demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

The tasks of this work are bi-level image segmentation and denoising using Markov Random Field (MRF) and Conditional Random Field (CRF). Specifically, Ising pairwise MRF is employed and the Iterative Conditional Model (ICM) is used as the inference method. For image denoising, salt and pepper noise is added to the original noiseless image (Fig. 1 (a)). The goal is to get rid of the noise and recover the original image as much as possible. For bi-level segmentation, the pixels of the original image have the intensity from 0 to 255, (Fig. 1 (c)) and the goal is assign each pixel a label, 0 or 255, corresponding to background or foreground, respectively.

This paper is structured as follows. In Section II we give a bried introduction of the existing work. In Section III and IV we discuss the MRF and CRF models. The inference method for both models are introduced in Section V. Experimental result are given in Section VI. The paper is concluded in Section VII.



(a) (b) (c)

Fig. 1: (a) noiseless image, (b) noisy image and (c) image for segmentation.

II. Related Work

Images taken with both digital cameras and conventional film cameras will pick up noise from a variety of sources. Many further uses of these images require that the noise will be (partially) removed - for aesthetic purposes as in artistic work or marketing, or for practical purposes such as computer vision.

There are a variety of kinds of noise in the images. In this work, we focus on the salt and pepper noise. In salt and pepper noise, pixels in the image are very different in color or intensity from their surrounding pixels. Generally this type of noise will only affect a small number of image pixels. When viewed, the image contains dark and white dots, hence the term salt and pepper noise. Typical sources include flecks of dust inside the camera and overheated or faulty CCD elements.

Many approaches have been proposed for image denoising. Buades et al. [2] give a thorough review of the existing methods. Typical approach is to consider the relationship between each variable and its neighbor to determin whether it is a noise pixel or not. Chen et al. [3] use a global method to find the minimizer for both image segmentation and denoising. Starck et al. [11] used a curvelet transform to address the problem. Malfait and Roos [7] used a Markov Random Field as a prior model for image denoising. Buades et al. [1] proposed a non-local algorithm for image denoising.

In computer vision, image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels, also known as superpixels). The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyze [5]. Typically, image segmentation is used to locate objects and boundaries (lines, curves, etc.) in images. More precisely,

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image segmentation is the process of assigning a label to every pixel in an image such that pixels with the same label share certain characteristics.

Research on the topic of image segmentation have been active. For a detailed review, pleae see Pal et al. [9]. Shi et al. [10] proposed to use normalized cut for image segmentation. Felzenszwalb et al. [4] proposed to segment the image using a graph-based algorithm.

In this work, we propose to use Markov Random Field (MRF) for the task of image denoising and segmentation. MRF [6] is a probabilistic graphical model widely used to model various low-level to mid-level tasks in image processing and computer vision. MRF characterize the each pixel and the relationship among neighbors using unary function, and pairwise potential functions, respectively. Therefore, the tasks are transformed into an optimization problem, where an objective function representing the total energy is to be minimized.

III. Pairwise MRF and Ising Model

In this work, suppose X is the label of a pixel. In the segmentation task, X represents background or foreground. In the denoising task, X represents actual image or noise. Y is the value of observation. The relationship of X is described by the clique potential function Ψ. The relationship of Y and S is described by the unary potential function Φ. The joint distribution function of X and Y equals is proportional to the product of clique potential functions and unary potential functions.

Pairwise MRF only considers pairwise neighborhood, which means the local interactions of the nodes in MRF is defined by pairwise potential $\Psi(x_i, x_j)$, where x_i and x_j are neighboring nodes. The joint distribution of X and Y is given by:

$$P(X,Y) = \frac{1}{Z}\prod_{i,j}\Psi(x_i,x_j)\prod_i \Phi(x_i,y_i) \tag{1}$$

Ising model is an example that arose from statistical physics. Each node in the MRF only has two states. In particular, $x_i \in \{0.255\}$. According to the Ising model, the pairwise potential function can be written as

$$\Psi(x_i,x_j) = \exp(-V(x_i,x_j)) \tag{2}$$

$$V(x_i,x_j) = 1-\delta(x_i,x_j) \tag{3}$$

where δ is the delta impulse function.

For the denoising task, the unary potential function can be defined as follows

$$\Phi(x_i,y_i) = \exp(-V(x_i,y_i)) \tag{4}$$

$$V(x_i,y_i) = 1-\delta(x_i-y_i) \tag{5}$$

For the bi-level segmentation task, the unary potential function can be defined as follows

$$\Phi(x_i,y_i) = \exp(-V(x_i,y_i)) \tag{6}$$

$$V(x_i,y_i) = \log\left(\frac{1}{\sigma_x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{y_i-\mu_x}{\sigma_x}\right)^2\right)\right) \tag{7}$$

where μ_x and σ_x are the mean and standard deviation of the Gaussian distribution of Y for background and foreground respectively.

The denoising and segmentation problems can be regarded as an optimization function. We want to find x^* that maximizes the objective function

$$x^* = \arg\max p(x|y) \tag{8}$$

$$\begin{aligned} p(x|y) &\propto p(x,y) \\ &= \frac{1}{Z}\exp\left(-\sum_{x_i\in V}V_i(x_i,y_i) - \beta\sum_{x_i\in V}\sum_{x_j\in N_{x_i}}V_2(x_i,x_j)\right) \end{aligned} \tag{9}$$

where β is the weight for pairwise potential and N_{x_i} is the neighbor of x_i.

In most cases, directly maximizing $P(x|y)$ is intractable, so the pseudo-likelihood is an alternative way of ML estimation. Pseudo-likelihood is defined as follows

$$p(x|y) \approx \prod_i p(x_i|y, x_{N_{x_i}}) = \prod_i p(x_i|y_i)p(x_i, x_{N_{x_i}}) \tag{10}$$

Given this approximation, we can maximize each pixel individually, i.e., finding x_i by maximizing $p(x_i|y, x_{N_{x_i}})$, where

$$p(x_i|y, x_{N_{x_i}}) = \exp\left(-V_i(x_i,y_i) - \beta\sum_{x_j\in N_{x_i}}V_2(x_i,x_j)\right) \tag{11}$$

IV. Conditional Random Field

In addition to MRF, we also implemented CRF for image denoising. The difference in CRF is that it involves the observation in the pairwise potential function. CRF is a discriminative undirected probabilistic graphical model which models the conditional label distribution.

The unary and pairwise potential functions become vectors:

$$f(x_i,x_j,y) = x_i x_j \begin{pmatrix} 1 \\ |y_i-y_j| \end{pmatrix} \tag{12}$$

$$g(x_i,y_i) = x_i \begin{pmatrix} 1 \\ y_i \end{pmatrix} \tag{13}$$

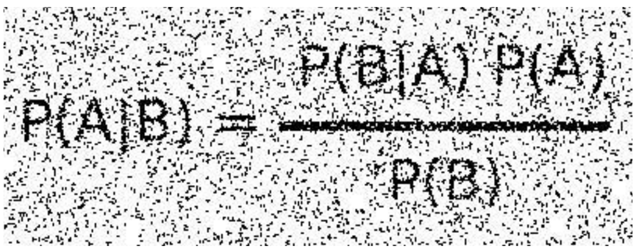
V. Iterated Conditional Model

The idea of Iterated Condional Model (ICM) is to iteratively maximize the probability of each variable conditioned on the rest, because generally the exact inference on MRF or CRF is intractable. Several algorithms are used to obtain approaximate solutions, such as loopy belief propagation [8], mean field [13], and linear programming relaxation [12].

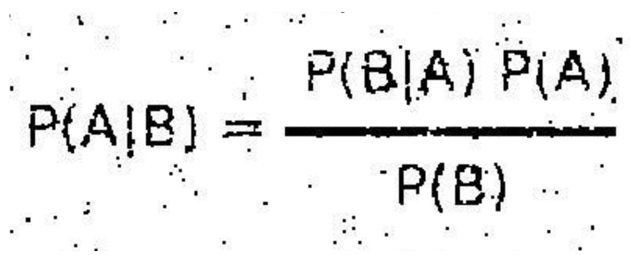
The pseudo code for ICM method is as follows:

Algorithm 1 Iterated Conditional Model

1 Initialize the label X. One way is to set X the same as Y; the other way is to randomly choose from $\{0, 255\}$;
2 Scan over the whole image (excluding the boundary pixels). For each x_i, keep all the other nodes fixed, choose the state of x_i according to
$$\arg\max_{x_i} p(x_i|y, x_{N_{x_i}}) \tag{14}$$
3 3. Repeat step 2 until a stopping criterion is met, i.e., the total number of changed is less than a threshold or a fixed number of iterations is exceeded.



(a)



(b)

Fig. 2: Image denoising results. (a) $\beta = 0.3$ and (b) $\beta = 1$. respectively.

Fig. 3: A close look at the result image. The dots at top are formed by 4 pixels.

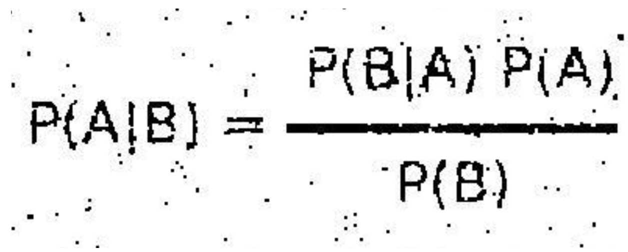


Fig. 4: Image denoising result using CRF.

VI. Experiments

A. Image Denoising

For the denoising task, since we have the ground truth image, the label of each pixel is already known. By comparing each predicted label with the true label, we are able to use the accuracy rate to evaluate the model. The more pixels they have in common, the better the model is.

We use the observation Y as the initialization of X. To evaluate the effect of the parameter β, we test the algorithm on different values. The accuracy corresponding to different β value are given in Table I. We can find out with the value of β increasing, the accuracy increased initially. After a threshold, the accuracy stays the same.

The denoised images are given in Fig. 2. It is visually clear that $\beta = 1$ outperforms $\beta = 0.3$.

TABLE I: Accuracy given different β value

β	0.1	0.2	0.5	1	2	10
Accuracy (%)	80.03	87.83	98.24	98.46	98.46	98.46

As we initialize the label using the noisy image, it takes into account the unary potential already. Therefore, as the weight of the pairwise potential goes larger, we focus more on the pairwise potential. The pixels tend to have the same label with its neighborhood.

In Fig. 3, we can take a close look at the result image. Every noise spot in the image consists of more than one pixel. Instead, they form a big cluster. This is because of the pairwise potential function. The energy function of the pairwise potential give a penalty if the neighboring pixels don't have the same states, which makes the pixels in the result image tend to form big spots.

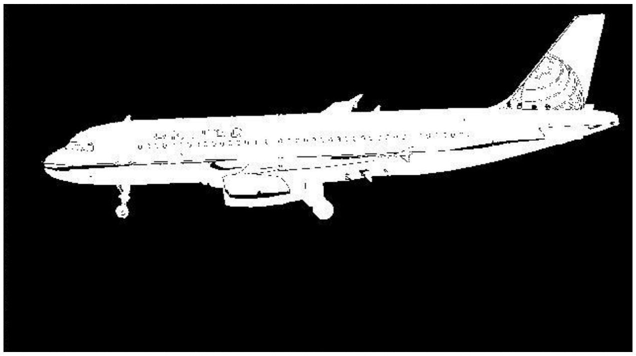
The result of CRF is also given in Fig. 4. Since the result of MRF is pretty good with respect to accuracy, the CRF does not improve the result very much.

B. Image Segmentation

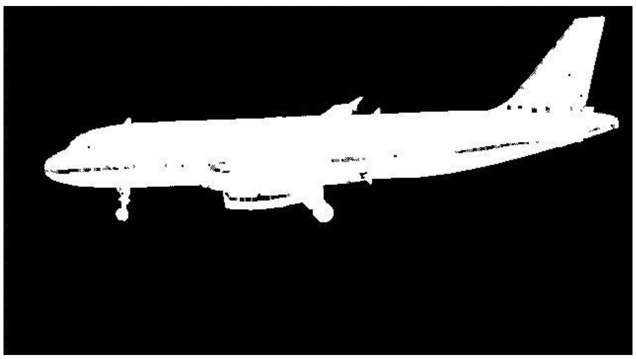
To perform the image segmentation task, we randomly initialize the labels. Fig. 5 shows the results generated by a relatively small and big β.

From the images, we can see that if the weight is small, there are some details on the body of the airplane that is treated as the background. This is because the intensity of these details is close to the background. The method to fix this is to enlarge the weight to make it emphasis more on the pairwise potential functions. In this way, the pixels tend to have the same value with their neighbors. However, if we apply a very large weight, every pixel in the image tends to have the same value, which disagrees with the purpose of image segmentation.

The weight is a trade-off between the unary potential and the pairwise potential, depending on different applications.



(a)



(b)

Fig. 5: Image segmentation results. (a) $\beta = 0.1$ and (b) $\beta = 10$. respectively.

VII. Conclusion

In this work, we employed the pairwise MRF and CRF to solve the image denoising and segmentation tasks, using Ising model to build potential function. ICM method is employed as the inference method. The results resonably meet the goal of the tasks. For the future work, we may explore differnet potential functions, i.e., high order, larger neighbors, or multi-class segmentation.

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