Conversions Float

$.1_2 = .100_2 = .4_8$

$= .1000_2 = .8_{16}$

$= 1 \times 2^{-1} = .5_{10}$

$.1_8 = \underline{.001_2} = 1 \times 2^{-3} = 1/8$

$.001_2 = .0010_2 = \underline{.2_{16}} = 2 \times 16^{-1} = 2/16 = 1/8$

$= 1/8 = \underline{.125_{10}}$

$.1_{16} = \underline{.0001_2} = 1 \times 2^{-4} = 1/16 = \underline{.0625_{10}}$

$= .000100_2 = \underline{.04_8} = 4 \times 8^{-2} = 4/64 = 1/16$

$.1_{10} \longrightarrow$

$.1 \times 16 = 1.6$

$.6 \times 16 = 9.6$

$.6 \times 16 = 9.6$ repeats

$= .1999\ldots = .1\underline{9}_{16} \leftarrow$ repeat to infinity

Note: $.19 < .1\underline{9} < .19A$ Bounded

$$.1\underline{9} = 1/16 + \sum_{i=2}^{\infty} 9/16^i = 1/16 + 9/16^2 \sum_{i=0}^{\infty} 16^{-i}$$

Using
$$S = \sum_{i=0}^{N} r^i = 1 + r + r^2 + \cdots r^N$$
$$-\, rS = r + r^2 + \cdots r^N + r^{N+1}$$
$$S(1-r) = 1 - r^{N+1}$$
$$\lim_{N \to \infty} S = \frac{1 - r^{N+1}}{1-r} = \frac{1}{1-r}$$

MEK 1/10

Then

$$.1\overline{9}_{16} = \frac{1}{16} + \frac{9}{16^2}\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{1}{16} + \frac{9}{16^2}\left(\frac{16}{15}\right)$$

$$= \frac{1}{16} + \frac{9}{240}$$

$$= \frac{15+9}{240} = \frac{24}{240}$$

$$= .1_{10}$$

Proof that $.1\overline{9}_{16} = .1_{10}$

$.1 \quad 9 \quad 9 \quad 9 \quad 9 \quad 9_{16} \ldots$

Every 4 bits

$.0001 \quad 1001 \quad 1001 \quad 1001 \quad 1001 \quad 1001_2 \ldots$

Can be written as

$.0\overline{0011}_2$ where 0011 repeats forever

Now for base 8, group every 3 bits

$.\underline{000} \ \underline{110} \ \underline{011} \ \underline{001} \ \underline{100} \ \underline{110}_2, \ldots$

$.0 \quad 6 \quad 3 \quad 1 \quad 4 \quad 6, \ldots$

$.0\overline{6314}_8$

Note! The sequence repeats but looks very different depending on the base