Code Jam to I/O for Women 2022 - Code Jam to I/O for Women 2022
Inventor Outlasting

PROBLEM ANALYSIS

Analysis

Since the available actions of any game state don't depend on whose turn it is, we can classify each game state into two outcomes: The next player (the one whose turn it is) wins (denoted as an N-position), or the previous player wins (denoted as a P-position). For example:

- N-position means: If it's Izabella's turn, then Izabella is going to win; if it's Olga's turn, then Olga is going to win.
- P-position means: If it's Izabella's turn, then Izabella is going to lose; if it's Olga's turn, then Olga is going to lose.

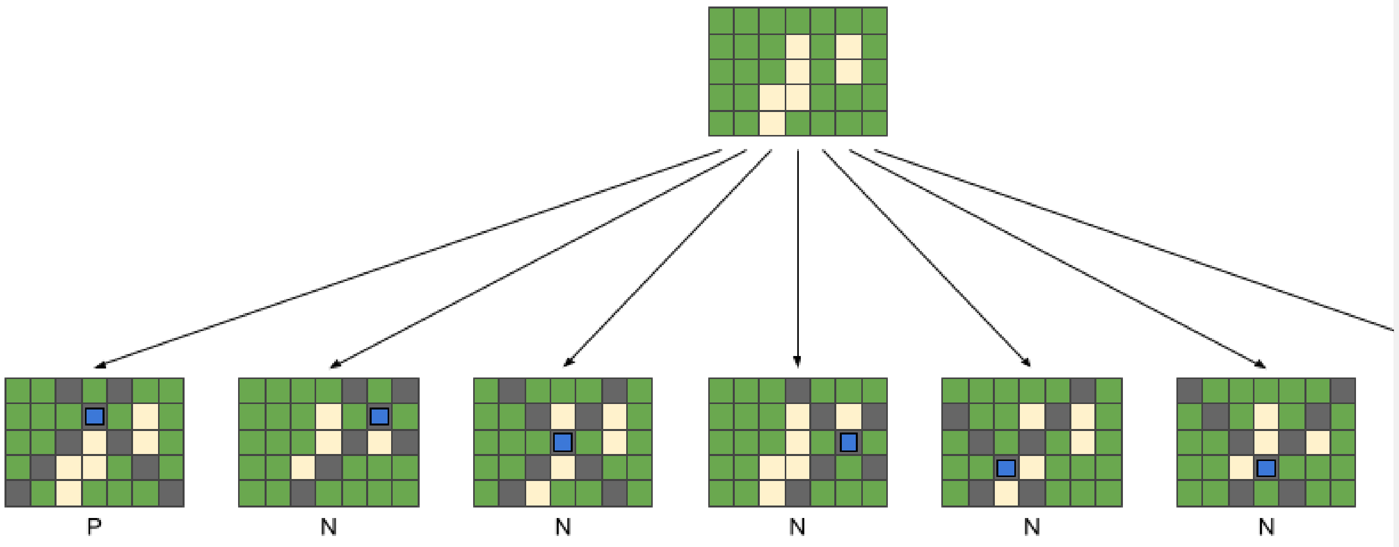
This kind of game is called an impartial game.

With the game rules from the problem statement, to decide if a given game state s is an N-position or a P-position, we have the following recursion rules:

- If there exists a spot where building an attraction on it can turn the current game state s into a P-position. In other words, forcing the opponent to lose. Then s is classified as an N-position.
- Otherwise, s is classified as a P-position because s doesn't have any available spots to build an attraction, or building an attraction on any spot will always turn the current game state into an N-position. In other words, letting the opponent win.

The goal of this problem is to find out how many spots there are on the given initial game state that the player can build an attraction on it and turn the game state into a P-position state.

Take Sample Case #1 as example, the seven different next states and their position types are



Since only one of those states is a P-position, the answer to Case #1 is 1.

Test Set 1

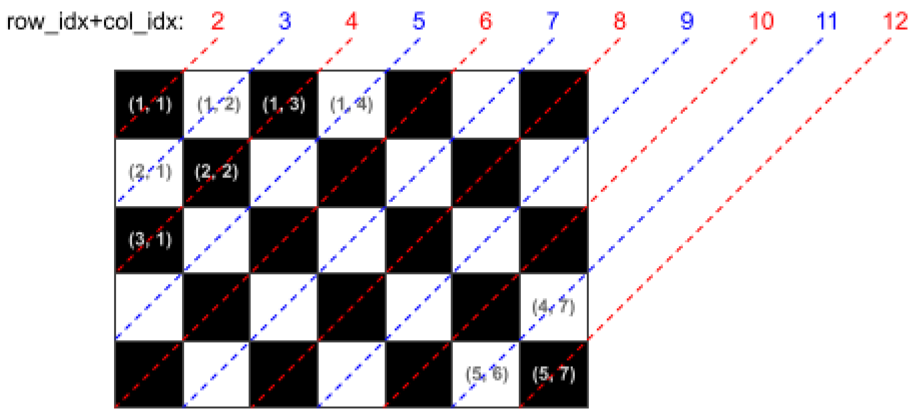
For this test set, since we only have at most 10 spots available for new attractions, we can simply use brute force search with the above recursion rules to decide whether a given game state is N-position or P-position.

The game tree size and the state-space complexity of this game both have size $O(S!)$, where S represents the number of spots available to build an attraction on the initial game state. For each state, in addition to the recursion for next states, we also need $O(\min(\mathbf{R}, \mathbf{C}))$ time to mark the new advertisement signs on the board. Therefore, the brute force search solution to this problem has time complexity $O(S! \times \min(\mathbf{R}, \mathbf{C}))$.

In this brute force search, using dynamic programming to store every seen state won't help decrease the time complexity because the size of the dynamic programming table we will end up with is still up to $O(S!)$.

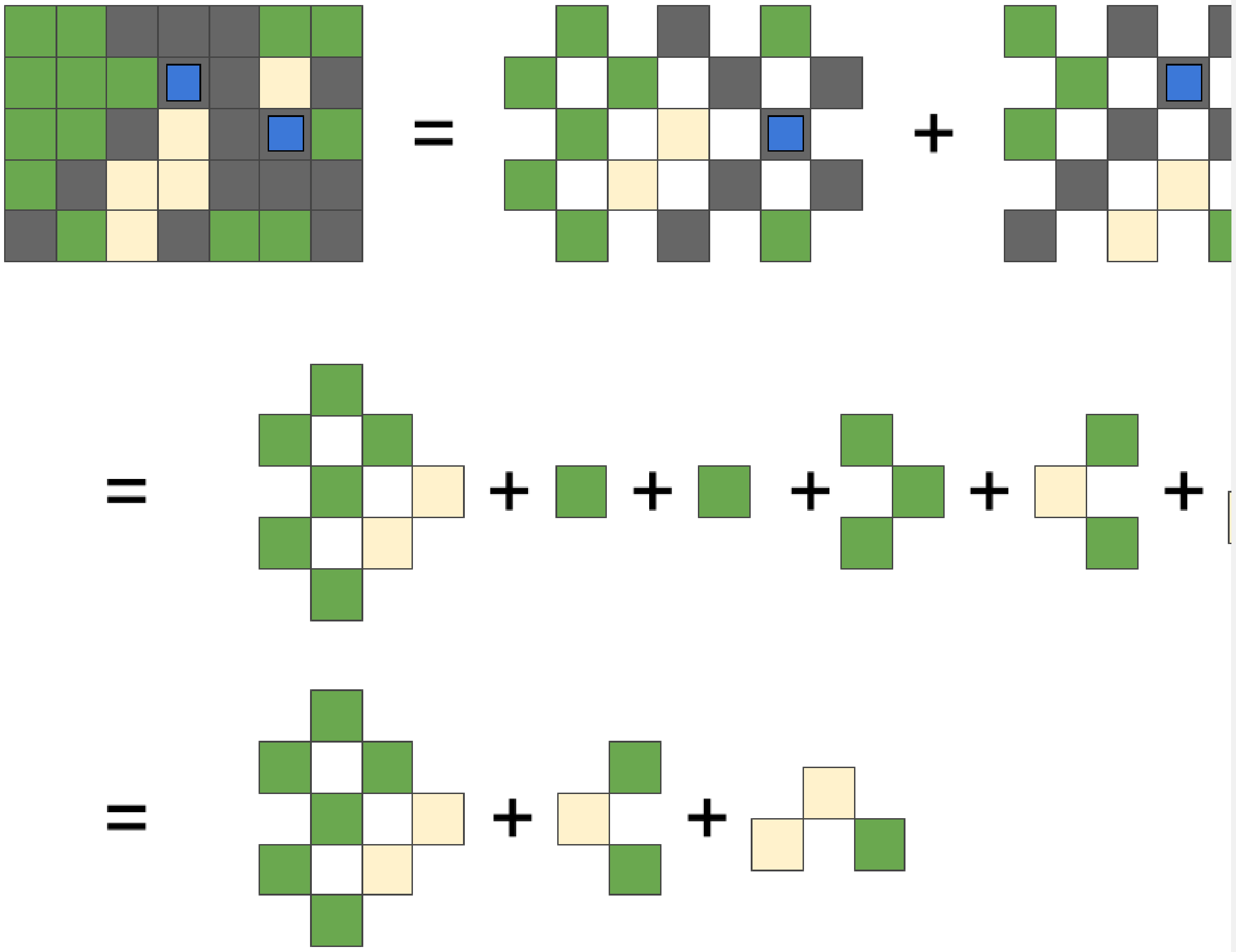
Test Set 2

Since the sign builders always extend in the diagonal direction like chess bishops, the cells on the game board can actually be seen as two independent parts: the black cells and the white cells in a chessboard shown below.



Formally, from each attraction, only cells that have the same `row_idx+col_idx` parity can be affected. See the chess board above, the black cells have even `row_idx+col_idx`, while the white cells have odd `row_idx+col_idx`.

Furthermore, since the sign builders only extend until they reach an occupied cell, the areas that were previously divided by other extending signs are also independent. And the independent areas with no available spot to build can simply be ignored.



For example, the top left is a board with 2 attractions built on $(2, 4)$ and $(3, 6)$. This board is equal to 6 independent "sub-games" shown above. And from observation, we can see only 3 of them have available spots, and each of them will be completely occupied in one move. Therefore, there are exactly 3 moves remaining to finish the game no matter how the players play. So we can easily figure out that this example is an N-position (the next player wins).

To formalize the calculation steps we demonstrated, we need the Sprague-Grundy theorem. According to the Sprague-Grundy theorem, if we have an impartial game in which the players take turns to play exactly one move and the player who cannot move loses, we can assign each game state a value (called the SG-value). Each game-state's SG-value only depends on the SG-value of the game-states we can reach with exactly one move.

Specifically, for any game state s, we have:

- $SG(s) = 0$ if s doesn't have any available spots to build an attraction.
- Otherwise, let $\{p_1, p_2, \ldots, p_n\}$ be all possible game states after building exactly one attraction at each of the available spots on s. Then we have that $SG(s) = \text{mex}(\{SG(p_1), SG(p_2), \ldots, SG(p_n)\})$, where $\text{mex}(X)$ means the minimum non-negative integer that doesn't exist in the set X.

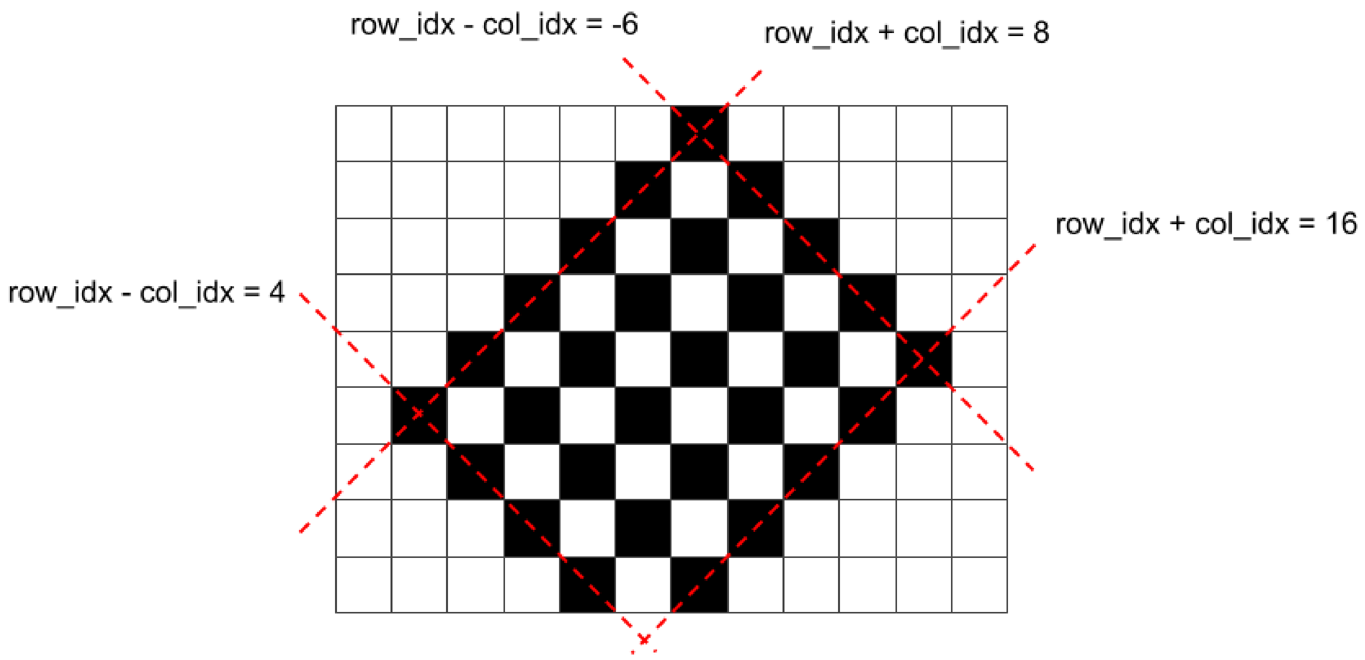
If a state s is split into several independent sub-games $\{q_1, q_2, \ldots, q_k\}$, the SG-value of s can be calculated as

$$SG(s) = SG(\{q_1, q_2, \ldots, q_k\}) = SG(q_1) \oplus SG(q_2) \oplus \cdots \oplus SG(q_k),$$

where $\oplus$ is the binary xor function.

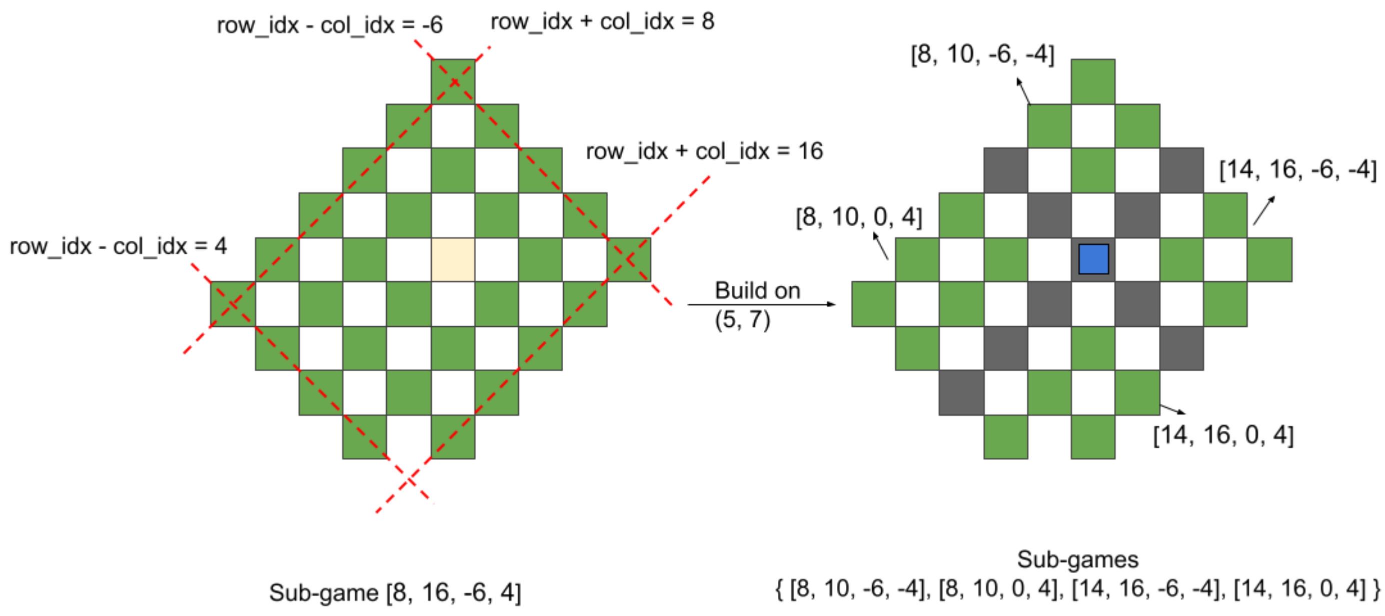
Finally, if $SG(s) = 0$, then s is a P-position; otherwise s is an N-position. So the goal of this problem is to find out how many spots on the given initial game state the player can build an attraction on and turn the game state into SG-value = 0.

The recursive function to calculate the SG-value of a given sub-game can be implemented as:



The sub-game marked as black cells is represented as [8, 16, -6, 4].

- a sub-game can be represented by a tuple `[row_idx_plus_col_idx_begin, row_idx_plus_col_idx_end, row_idx_minus_col_idx_begin, row_idx_minus_col_idx_end]`, like the example shown above,
- for each available spots in this range, put an attraction on it and calculate the new SG-value. Then the SG-value of this sub-game is calculated by applying mex to them. And

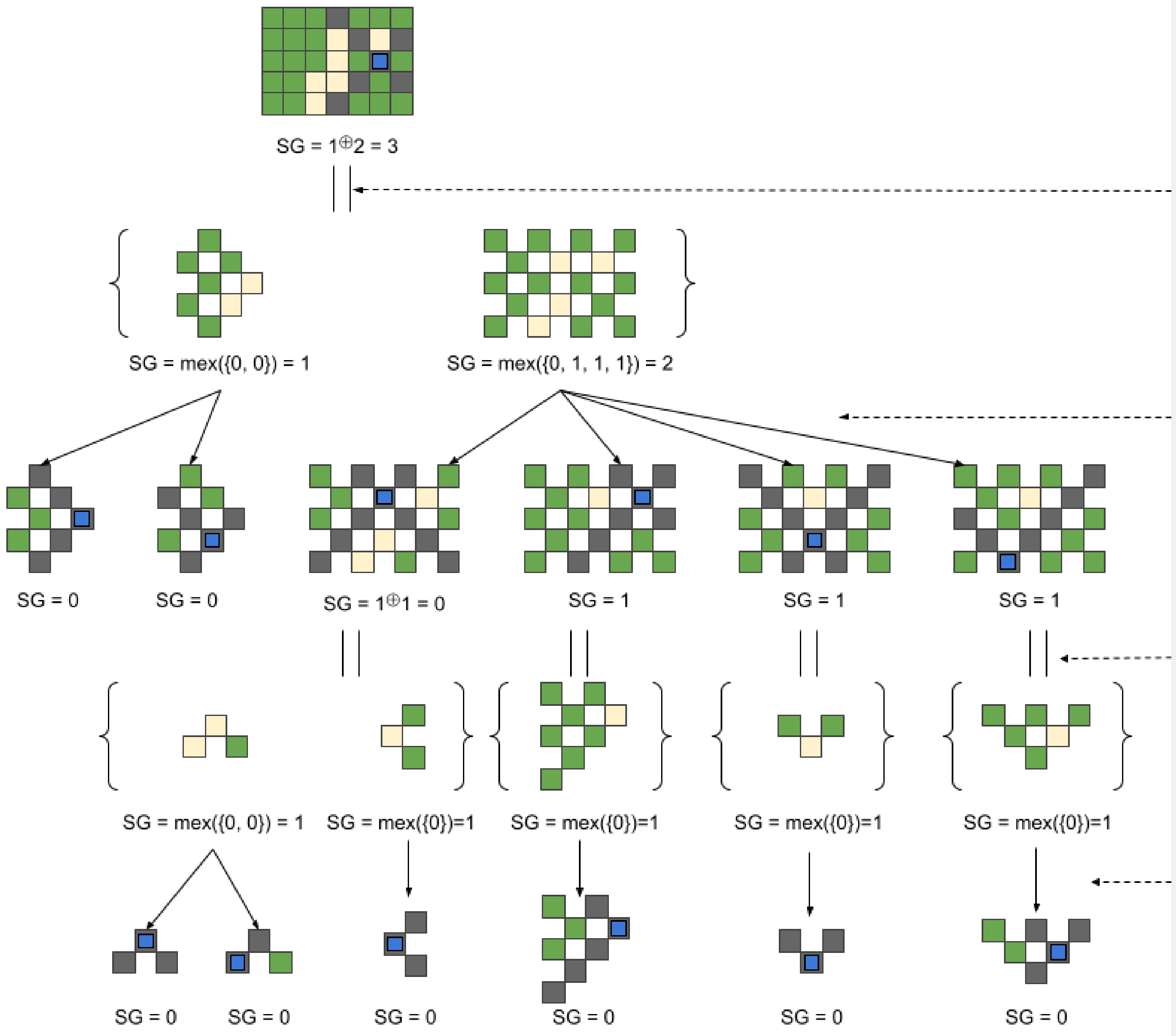


- when an attraction is put on a spot `[row_idx, col_idx]`, the current sub-game `[row_idx_plus_col_idx_begin, row_idx_plus_col_idx_end, row_idx_minus_col_idx_begin, row_idx_minus_col_idx_end]` is divided into four new sub-games
    - `[row_idx_plus_col_idx_begin, row_idx + col_idx - 2    , row_idx_minus_col_idx_begin, row_idx - col_idx - 2    ],`

- `[row_idx_plus_col_idx_begin, row_idx + col_idx - 2    , row_idx - col_idx + 2      , row_idx_minus_col_idx_end]`,
- `[row_idx + col_idx + 2      , row_idx_plus_col_idx_end, row_idx_minus_col_idx_begin, row_idx - col_idx - 2    ]`,
- `[row_idx + col_idx + 2      , row_idx_plus_col_idx_end, row_idx - col_idx + 2      , row_idx_minus_col_idx_end]`,

as the example above, and the SG-value of the original sub-game with the new attraction on it is calculated as XOR of the SG-values of those four new sub-games

Now we know how to use the Sprague-Grundy theorem. Take Sample Case #1 as an example, what will happen if Izabella puts her attraction at third row and sixth column?



We can see that putting the first attraction on this cell will give an SG-value of 3. This means that Olga will having a winning move from this position, which is not good for Izabella!

Note that we can use dynamic programming to store the SG-value of each seen sub-game. In this way, the state-space complexity of this search is $O(\mathbf{R}^2 \times \mathbf{C}^2)$. And when calculating the SG-value of each sub-game, we need to visit all $O(S)$ available spots in it to enumerate the next states. When adding an attraction, we don't need to actually mark those $O(\min(\mathbf{R}, \mathbf{C}))$ advertisement signs, we only need to use constant time to split the sub-games and calculate XOR from their results. Therefore, the total time complexity of this solution is $O(S \times \mathbf{R}^2 \times \mathbf{C}^2)$. And since S is $O(\mathbf{R} \times \mathbf{C})$, the time complexity can also be written as $O(\mathbf{R}^3 \times \mathbf{C}^3)$.

Test Data

We recommend that you practice debugging solutions without looking at the test data.