Implementation of Floyd's algoritm :

To find the shortest paths for all veritces in a graph

Problem:

- Given a weighted connected graph (directed or undirected), the **all-pairs shortest-paths problem** asks to find the distances (the length of the shortest paths) from each vertex to all other vertices.

- it is convenient to record the lengths of shortest paths in an n-by-n matrix D called the **distance matrix**: the element d_{ij} in the ith row and jth column of this matrix indicates the length of shortest path from the ith vertex to the jth vertex.

- Floyd's algorithm computes the distance matrix of a weighted graph with n vertices through a series of n-by-n matrices: $\mathcal{D}^0, \ldots, D^k, \ldots, D^n$ (k=0,1,....,n)

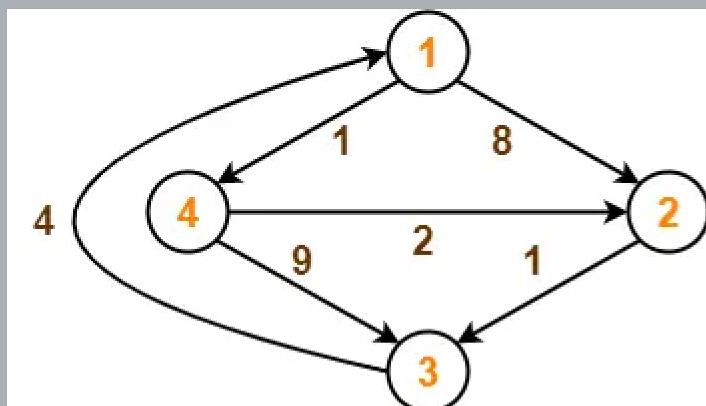
Algorithm :

```
//implements floyd's algorithm for all-pair shortest-paths problem
//input: the weight matrix W of a graph
//output: the distance matrix of the shortest paths lengths

D = W
for k = 1 to n do
    for i = 1 to n do
        for j= 1 to n do
            D[i,j] = min{ D[i,j] , D[i,j] + D[k,j] }
return D
```

Example:

- consider the weighted graph and its adjacency matrix



$$D_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & \infty & 0 & \infty \\ \infty & 2 & 9 & 0 \end{bmatrix} \end{array}$$

$$k = 1$$

$$D^0[2][2] = 0$$

$$D^1[2][2] => D^0[2][1] + D^0[1][1] \qquad = infinity$$

$$D^0[2][2] < infinity.$$

$$D^1 = 0$$

$$D_1 = \begin{array}{cc} \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 0 & 8 & \infty & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 9 & 0 \end{bmatrix} \end{array}$$

$$D_2 = \begin{array}{cc} & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 0 & 8 & 9 & 1 \\ \infty & 0 & 1 & \infty \\ 4 & 12 & 0 & 5 \\ \infty & 2 & 3 & 0 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{cc} & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 0 & 8 & 9 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 12 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix} \end{array}$$

$$D_4 = \begin{array}{cc} & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 0 & 3 & 4 & 1 \\ 5 & 0 & 1 & 6 \\ 4 & 7 & 0 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix} \end{array}$$

C code:

```
#include<stdio.h>
#include<stdlib.h>
#include<time.h>
#define INF 99999
int min(int,int);
void floyds(int p[10][10],int n) {
        int i,j,k;
        for (k=1;k<=n;k++)
          for (i=1;i<=n;i++)
           for (j=1;j<=n;j++)
            if(i==j)
             p[i][j]=0; else
             p[i][j]=min(p[i][j],p[i][k]+p[k][j]);
}
int min(int a,int b) {
        if(a<b)
          return(a); else
          return(b);
}
```

```c
void main() {
       clock_t start,end;
       int p[10][10],w,n,e,u,v,i,j;
       printf("\n Enter the number of vertices: ");
       scanf("%d",&n);
       printf("\n Enter the number of edges: ");
       scanf("%d",&e);
       for (i=1;i<=n;i++) {
               for (j=1;j<=n;j++){
                   if(i==j)
                       p[i][j]=0;
                   else
                       p[i][j]=INF;
               }
       }
       printf("\n Enter the edges with corresponding weights (source : destinatin  : weight) : \n");
       for (i=1;i<=e;i++) {
               scanf("%d %d %d",&u,&v,&w);
               p[u][v]=w;
       }
       printf("\n Matrix of input data:\n");
       for (i=1;i<=n;i++) {
               for (j=1;j<=n;j++){
                   if(p[i][j]==INF)
                       printf("INF\t");
                   else
                       printf("%d \t",p[i][j]);
               }
               printf("\n");
        }
```

```c
start=clock();
floyds(p,n);
end=clock();
        printf("\n Transitive closure:\n");
        for (i=1;i<=n;i++) {
                for (j=1;j<=n;j++)
                   printf("%d \t",p[i][j]);
                printf("\n");
        }
        printf("\n The shortest paths are:\n");
        for (i=1;i<=n;i++)
           for (j=1;j<=n;j++) {
                  if(i!=j)
                     printf("\n %d---->%d = %d",i,j,p[i][j]);
        }
        printf("\n Execution Time = %f",(((double)(end-start))/CLOCKS_PER_SEC));

}
```

Output:

```
PS C:\Users\user pc\Documents\ada lab> gcc floyd.c
PS C:\Users\user pc\Documents\ada lab> ./a.exe

 Enter the number of vertices: 4
 Enter the number of edges: 6

 Enter the edges with corresponding weights (source : destinatin  : weight) :
 1 2 8
 1 4 1
 2 3 1
 3 1 4
 4 2 2
 4 3 9

 Matrix of input data:
 0       8       INF     1
 INF     0       1       INF
 4       INF     0       INF
 INF     2       9       0

 Transitive closure:
 0       3       4       1
 5       0       1       6
 4       7       0       5
 7       2       3       0

 The shortest paths are:
 1---->2 = 3
 1---->3 = 4
 1---->4 = 1
 2---->1 = 5
 2---->3 = 1
 2---->4 = 6
 3---->1 = 4
 3---->2 = 7
 3---->4 = 5
 4---->1 = 7
 4---->2 = 2
 4---->3 = 3
Execution Time = 0.000002
```

Time complexity:

- The time complexity of Floyd's algorithm is:

$$\mathcal{O}(\mathcal{V}^3)$$

where V is the number of vertices in the graph. This means that the time it takes to run the algorithm grows cubically with the number of vertices in the graph.

- The algorithm achieves this time complexity by performing three nested loops over all the vertices in the graph to update the shortest path distances between each pair of vertices.

Python Code:

```
import time
INF = 99999

def min(a, b):
    return a if a < b else b

def floyds(p, n):
    for k in range(1, n+1):
        for i in range(1, n+1):
            for j in range(1, n+1):
                if i == j:
                    p[i][j] = 0
                else:
                    p[i][j] = min(p[i][j], p[i][k] + p[k][j])
```

```python
def main():
    n = int(input("Enter the number of vertices: "))
    e = int(input("Enter the number of edges: "))

    p = [[INF for _ in range(n+1)] for _ in range(n+1)]

    for i in range(1, n+1):
        for j in range(1, n+1):
            if i == j:
                p[i][j] = 0

    print("Enter the edges with corresponding weights (source : destination : weight):")
    for _ in range(e):
        u, v, w = map(int, input().split())
        p[u][v] = w

    print("\nMatrix of input data:")
    for i in range(1, n+1):
        for j in range(1, n+1):
            if p[i][j] == INF:
                print("INF", end="\t")
            else:
                print(p[i][j], end="\t")
        print()
```

```python
    start=time.time();
    floyds(p, n)
    end=time.time();
    print("\nTransitive closure:")
    for i in range(1, n+1):
        for j in range(1, n+1):
            print(p[i][j], end="\t")
        print()

    print("\nThe shortest paths are:")
    for i in range(1, n+1):
        for j in range(1, n+1):
            if i != j:
                print(f"\n {i}---->{j}={p[i][j]}", end="")
    print()
    print("execution time = ",(end-start))
    print()

if __name__ == "__main__":
    main()
```

Output:

```
Enter the number of vertices:  4
Enter the number of edges:  6
Enter the edges with corresponding weights (source : destination : weight):
 1 2 8
 1 4 1
 2 3 1
 3 1 4
 4 2 2
 4 3 9

Matrix of input data:
0       8       INF     1
INF     0       1       INF
4       INF     0       INF
INF     2       9       0

Transitive closure:
0       3       4       1
5       0       1       6
4       7       0       5
7       2       3       0

The shortest paths are:

 1----->2=3
 1----->3=4
 1----->4=1
 2----->1=5
 2----->3=1
 2----->4=6
 3----->1=4
 3----->2=7
 3----->4=5
 4----->1=7
 4----->2=2
 4----->3=3
execution time =  4.0531158447265625e-05
```

Difference between C and Python:

- **C is faster than Python due to its code being directly converted into machine code through compilation, while Python relies on interpretation at runtime.**
- **C has a static type system, leading to more efficient memory allocation and manipulation of data.**
- **Python's built-in abstractions and features can introduce overhead compared to C**
- **Python is chosen for its ease of use, readability, and rapid development, while C is preferred for performance-critical tasks.**

Python can be combined with C or C++ using interfaces like ctypes or Cython to leverage both language's strengths

THANK YOU