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Modeling Cross-blockchain Process Using Queueing Theory: The Case of Cosmos

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***Abstract*—In order to solve the interconnection and intercommunication problem of a large number of co-existing blockchains, such as public chains, private chains, and consortium chains, cross-chain technology has recently become a hot research topic among scholars years. Due to the limited processing speed of cross-chain transactions, too many cross-chain transactions in the short term may cause network congestion and negatively affect cross-chain performance. Therefore, it is necessary to evaluate and optimize the performance of the cross-blockchain transaction process. This paper takes a typical cross-blockchain model Cosmos as an example and proposes a queuing theoretical model based on limited space. The difference equation is established through the three-dimensional continuous-time Markov process, and performance metrics such as average queue length, transaction execution time, and transaction response time are obtained. Finally, we experimentally simulate the analytical solutions of the relevant performance metrics to verify the effectiveness of the proposed model. We believe this analytical approach can be generalized to other cross-blockchain systems.**

***Index Terms*—Blockchain; Cross-blockchain; Relays; Performance modeling; Queueing theory; Simulation**

I. Introduction

As a bridge connecting blockchains, cross-chain technology realizes the interoperability problem between different blockchains and saves blockchains from scattered islands. [1], [2] proposes four blockchain interoperability models: notary, sidechain/relay, hash lock, and distributed private key. Among them, the sidechain/relay model is widely used by many representative solutions [3] in asset transfer, atomic swap or any other more complex use case [4], [5]. In terms of specific technologies using the sidechain/relay model, Cosmos is a highly scalable, robust, and easily upgradeable blockchain cross-chain network architecture. Each blockchain is supported by the BFT algorithm, providing reliable underlying technical support for forming the Internet.

Performance is a major constraint for blockchain systems. This is especially true for systems with high-performance requirements, such as online transaction processing and real-time payment systems [6]. Performance evaluation and analysis of blockchain systems can identify bottlenecks and inspire the scalability of blockchain systems. Motivated by these reasons, many studies have proposed performance evaluation solutions for distributed ledger systems based on analytical modeling methods, such as Markov chains [7], queuing models [8], [9], Random Petri Net [10], [11], or other models [12]. In addition, there is some research on blockchain performance scaling and optimization [13]–[15].

Due to the different consensus algorithms and communication protocols of different blockchains, the cross-blockchain process will also be different [6], [16]. Therefore, the biggest obstacle to the expansion of cross-chain technology is still its performance issue. To date, no research has focused on modeling and analysis of system performance across blockchains.

This paper addresses the above challenges by modeling the cross-chain process based on queuing theory, aiming to understand the cross-blockchain systems' performance better.

The main contributions of this paper are as follows.

- We take the typical cross-chain case Cosmos as an example to model its cross-chain process. Considering the impact of transaction arrival rate, block size, block consensus time, and other factors on its performance metric indicators, a batch service model based on queuing theory is proposed.
- By analyzing the three-dimensional continuous-time Markov process of the model, when the service stage is divided into two processes of block generation and block consensus verification, the difference equation is established, and the sub-rate matrix is constructed to solve these equations.
- We build the Cosmos test tool $tm - load - test$ on the cloud platform to test the applicability of different parameters and simulate the established model. We obtained the influence of parameters such as system capacity, transaction arrival rate, block size, etc., on system indicators to verify the model's effectiveness.

II. Cosmos Cross-blockchain Architecture

Cosmos [1] is one of the most representative cross-blockchain solutions implemented by the side chain/relay model.

A. Cosmos architecture

Cosmos is a scalable, easy-to-use, interoperable decentralized network composed of multiple independent parallel blockchains. There are three critical components: The hub is a relay chain maintained by the government and used as a trust center for cross-chain messages. The zone is a parachain

[1]https://github.com/cosmos

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that participates in the Cosmos network. To support cross-chain interoperability between parachains, Cosmos proposed an Inter-Blockchain Communication Protocol (IBC) to perform cross-chain operations with the hub [17]. Cosmos hub is developed based on the Tendermint consensus and is more scalable. In theory, it can connect to any type of blockchain. The consensus of Cosmos hub is completed through the tendermint core (blockchain consensus engine, responsible for data transmission between nodes and Byzantine consensus).

We take Cosmos as a case for performance modeling and analysis. A series of experiments are then carried out to validate the effectiveness of our proposed simulation model. Although different cross-chain technologies may have different implementations when dealing with cross-blockchain transactions. However, as long as the core processes are similar to those of Cosmos and are dependent on cross-blockchain protocols, gateways, or other intermediaries, the model can be borrowed from our study for performance analysis.

B. Consensus process

In a single hub system, cross-chain requests are queued in the mempool of the proposer node through different zones, waiting for the tendermint core to perform consensus verification on them.

Tendermint's consensus mechanism is based on the Byzantine fault-tolerant algorithm. According to the rules, the validators must reach a consensus on each block in rounds. Each round consists of three basic steps:

- Proposal stage. A designated validator proposes a block. The proposer in each round is selected deterministically from an ordered list in proportion to the voting weight. The voting ratio in the process is calculated based on the Stake ratio. Each validator node has a different voting weight according to the number of tokens pledged by each validator node.
- Prevoting stage. Each validator broadcasts its own prevote. When a block in the round receives more than $2/3$ of the prevote, it enters the next stage.
- Precommit phase. Each validator broadcasts its precommitted vote. When the vote exceeds $2/3$, it enters the next stage.

When the block reaches the agreement of $2/3$ of the adivators in the prevoting and precommit stage, the block enters the submission stage and the new height stage. Otherwise, it means that the block submission failed. In this case, the Tendermint protocol will choose the next validator to propose a new block at the same height and start voting again. At this time, the consensus time for this block is much longer than that of other blocks, but there is no rollback phenomenon. The process is shown in Fig.1.

III. System Model Using Queuing Theory

Queuing theory, also known as stochastic service system theory, is a mathematical method to solve different types of service quality and performance of the queueing system. In this paper, we model and test the core cross-blockchain

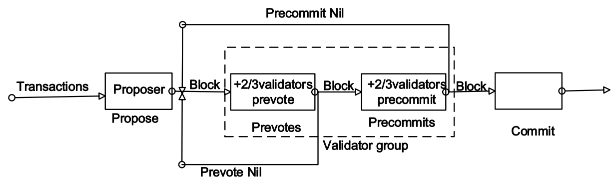

Fig. 1. Cosmos consensus process

processes of Cosmos and use the batch service queuing theory model to evaluate the performance of the Cosmos hub system.

A. Cosmos cross-blockchain queuing system

In the cross-chain activities of Cosmos, multiple Hub (relay)-centric blockchain alliances will appear in the cross-chain network, so cross-chain transactions in the whole cross-chain ecosystem will form a vast network. Due to the complexity of the blockchain alliance, we first solve the performance problem of the transaction cross-chain system of the single-relay system.

In the Cosmos hub, the validator is the consensus node of the hub, and the Tendermint algorithm randomly selects some validator nodes to form a validator group. Here, we use n validator nodes to illustrate the consensus process of cross-chain transactions in the validator node group. We first select one of the validators as the proposer node (generated by the validators in turn). Then the proposer node starts to monitor and collect all transactions of the entire network and store them in the mempool to wait for consensus. Second, the proposer node will assemble a new block of cross-chain transactions, the proposal block, and broadcast it to other validators. When all validator nodes in the entire network receive this proposal block, they start to read all transactions in this block. After verifying that there is no problem, vote a positive vote and broadcast the voting message to all validator nodes to complete the consensus. However, when the prevote or precommit fails to pass the vote, the block will be sent back for verification, and this process loops until the block are successfully verified. Finally, the successfully verified block is submitted to the chain. The process is shown in Fig.2.

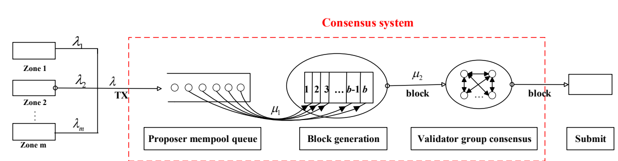

Fig. 2. Cross-chain process of the Cosmos hub system.

To verify the performance of the cross-chain system, we try to find a replicable and quantifiable modeling method to evaluate the system's performance. Due to the randomness of the arrival of cross-chain transactions, we apply the transaction batch model of queuing theory to calculate the average metric of the system and identify performance bottlenecks in the system's current state.

Next, we build a queueing system — Cosmos hub system:

Arrival process: Assuming there are m zones in the system, the ith zone sends λ_i cross-chain transactions per second on average, which is equivalent to randomly receiving $\lambda = \sum_{i=1}^{m} \lambda_i$ cross-chain transactions per second in the mempool. In Cosmos hub system, the number of cross-chain transactions that arrive is a Poisson flow with parameter λ (the interval between two adjacent cross-chain transactions obeys the exponential distribution of parameter λ).

Service rules: In the Cosmos cross-chain process, the arriving cross-chain transactions are processed in chronological order, so we set the service rule as First Come First Serve (FCFS).

Service process: Cross-chain transactions arrive at the proposer node's transaction pool randomly, waiting to be packaged to generate blocks, which is considered the first phase of service and is called block generation. Here, we assume that the transaction generation block time follows an exponential distribution with parameter μ_1. After that, the leader sends the packaged block to the validator node group to verify the transactions, which is regarded as the second stage of the service. We assume that the transaction verification time follows an exponential distribution with parameter μ_2.

The maximum system capacity: There are at most N transactions in the Cosmos hub system. Therefore, the memory pool can receive at most N transactions when no blocks are generated. When the system performs consensus verification on a block, there are at most $(N-b)$ transactions in the queue.

Independence: We assume that all the random variables defined above are independent of each other.

The validator node group is attacked: We assume that the operational status of the validator node group is divided into two types, the normal service status is considered 1, and the out-of-service status is considered as 0. The failure and repair times of the validator node group obey the exponential distribution with parameters α, β, respectively.

B. A continuous-time Markov process

We regard the validator node group as a service station. The service time includes block generation time and block verification time. Assume transactions arrive in a Poisson process and the mempool is limited. We establish a continuous-time Markov process for the Cosmos hub system and derive the stable probability vector of the system by the matrix analysis.

C. A continuous-time Markov process

At time slot t, let $I(t)$ represents the number of cross-chain transactions in the queue, $J(t)$ represents the number of cross-chain transactions in a block, $K(t)$ represents the service state of the validator node group. Then, $(I(t), J(t), K(t))$ can be considered the Cosmos hub system state at t. Here $i = 0, 1, \cdots, N$, $j = 0, 1, 2, \cdots, b$,$k = 0, 1$. Where $(N-b+1, 0, 0), (N-b+2, 0, 0), \cdots, (N, 0, 0)$ and $(N-b+1, 0, 1), (N-b+2, 0, 1), \cdots, (N, 0, 1)$ indicate that the system does not generate any block, the number of cross-chain transactions be accommodated in the queue can reach N. $(N-b, 0, 0), (N-b, 1, 0), \cdots, (N-b, b, 0)$and $(N-b, 0, 1), (N-b, 1, 1), \cdots, (N-b, b, 1)$ indicate that when the last block is generated, the number of cross-chain transactions be accumulated in the queue can reach$(N-b)$.

So $(I(t), J(t), K(t))$ is a continuous-time Markov process with the state space Ω. Fig.3 denotes the state transition relation of the Markov process $\{(I(t), J(t), K(t)) : t \geq 0\}$.

According to the state transition diagram, the stationary state equation of this system is expressed as follows:

•$State\,\{(0,0,0)\}$:

$$-(\lambda+\beta)p(0,0,0)+\alpha p(0,0,1)=0 \quad (1)$$

•$State\,\{(0,0,1)\}$:

$$-(\lambda+\alpha)p(0,0,1)+\beta p(0,0,0)+\mu_2\sum_{j=1}^{b}p(0,j,1)=0 \quad (2)$$

•$State\,\{(0,j,0), j=1,2,\cdots,b\}$:

$$-(\lambda+\beta)p(0,j,0)+\alpha p(0,j,1)=0 \quad (3)$$

•$State\,\{(0,j,1), j=1,2,\cdots,b\}$:

$$-(\lambda+\alpha+\mu_2)p(0,j,1)+\beta p(0,0,1)+\mu_1 p(j,0,1)=0 \quad (4)$$

•$State\,\{(i,0,0), i=1,2,\cdots,N-b\}$:

$$-(\lambda+\beta)p(i,0,0)+\lambda p(i-1,0,0)+\alpha p(i,0,1)=0 \quad (5)$$

•$State\,\{(i,0,1), i=1,2,\cdots,N-b\}$:

$$\begin{aligned}-(\lambda+\alpha+\mu_1)p(i,0,1)+\beta p(i,0,0)+p(i-1,0,1)\\ +\mu_2\sum_{j=1}^{b}p(i,j,1)=0\end{aligned} \quad (6)$$

•$State\,\{(N-b,j,0), j=1,2,\cdots,b-1\}$:

$$-\beta p(N-b,j,0)+\lambda p(N-b-1,j,0)+\alpha p(N-b,j,1)=0 \quad (7)$$

•$State\,\{(N-b,j,1), j=1,2,\cdots,b-1\}$:

$$-(\alpha+\mu_2)p(N-b,j,1)+\lambda p(N-b-1,j,1)+\beta p(N-b,j,0)=0 \quad (8)$$

•$State\,\{(i,b,0), i=1,2,\cdots,N-b-1\}$:

$$-(\lambda+\beta)p(i,b,0)+\lambda p(i-1,b,0)+\alpha p(i,b,1)=0 \quad (9)$$

•$State\,\{(i,b,1), i=1,2,\cdots,N-b-1\}$:

$$-(\lambda+\alpha+\mu_2)p(i,b,1)+\lambda p(i-1,b,1)+\beta p(i,b,0)+\mu_1 p(b+i,0,1)=0 \quad (10)$$

•$State\,\{(N-b,b,0)\}$:

$$-\beta p(N-b,b,0)+\lambda p(N-b-1,b,0)+\alpha p(N-b,b,1)=0 \quad (11)$$

•$State\,\{(N-b,b,1)\}$:

$$\begin{aligned}-(\alpha+\mu_2)p(N-b,b,1)+\lambda p(N-b-1,b,1)+\beta p(N-b,b,0)\\ +\mu_1 p(N,0,1)=0\end{aligned} \quad (12)$$

•$State\,\{(i,0,0), i=N-b+1,N-b+2,\cdots,N-1\}$:

$$-(\lambda+\beta)p(i,0,0)+\lambda p(i-1,0,0)+\alpha p(i,0,1)=0 \quad (13)$$

•$State\,\{(i,0,1), i=N-b+1,N-b+2,\cdots,N-1\}$:

$$-(\lambda+\alpha+\mu_1)p(i,0,1)+\lambda p(i-1,0,1)+\beta p(i,0,0)=0 \quad (14)$$

•$State\,\{(N,0,0)\}$:

$$-\beta p(N,0,0)+\lambda p(N-1,0,0)+\alpha p(N,0,1)=0 \quad (15)$$

•$State\,\{(N,0,1), i=N-b+1,N-b+2,\cdots,N-1\}$:

$$-(\alpha+\mu_1)p(N,0,1)+\lambda p(N-1,0,1)+\beta p(N,0,0)=0 \quad (16)$$

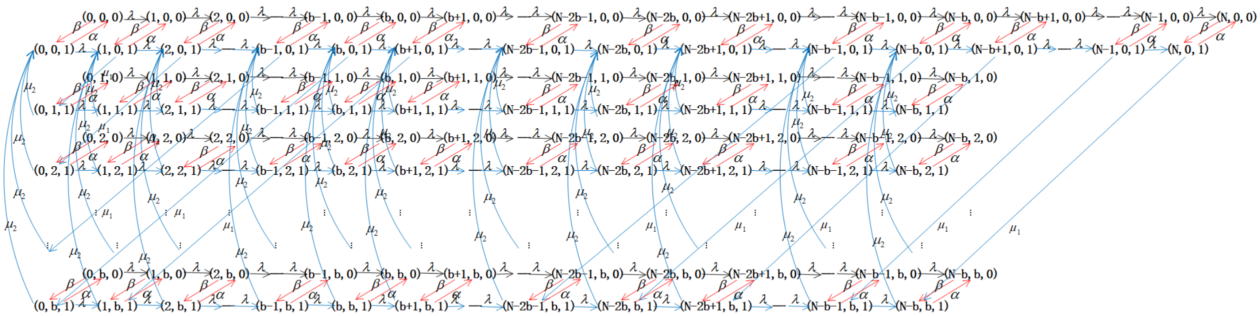

Fig. 3. State transition diagram.

$\bullet State\,\{(i,j,0), i=1,2,\cdots,N-b-1; j=1,2,\cdots,b-1\}:$

$$-(\lambda+\beta)p(i,j,0)+\lambda p(i-1,j,0)+\alpha p(i,j,1)=0 \tag{17}$$

$\bullet State\,\{(i,j,1), i=1,2,\cdots,N-b-1; j=1,2,\cdots,b-1\}:$

$$-(\lambda+\alpha+\mu_2)p(i,j,1)+\lambda p(i-1,j,1)+\beta p(i,j,0)=0 \tag{18}$$

State

$$\{(i,0,k), i=N-b+1,N-b+2,\cdots,N-1; k=0,1\}$$

means that the system does not generate any blocks and transactions continue to accumulate until the maximum capacity of the system is reached, so we analyze this situation separately. we substitute the equation (13)-(16) into (9)-(12) and combine the remaining state equations to obtain the $2(N-b+1)\times 2(N-b+1)$-order minimum generator of this system, as follows:

$$Q=\begin{bmatrix}
B_0 & A_0 \\
B_1 & A_1 & A_0 \\
B_2 & & A_1 & A_0 \\
\vdots & & & A_1 & A_0 \\
\vdots & & & & \ddots & \ddots \\
B_b & & & & & A_1 & A_0 \\
 & B_b & & & & & A_1 & A_0 \\
 & & B_b & & & & & \ddots & \ddots \\
 & & & & & & & & A_1 & A_0 \\
 & & & \ddots & B_b & C_1 & C_2 & \cdots & C_{b-1} & A_M
\end{bmatrix}$$

Where $A_0,A_1,B_0,B_i(i=1,2,\cdots,b),C_j(j=1,2\cdots,b-1),A_M$ are $2(b+1)\times 2(b+1)$-order matrices. Let $\pi=(\pi_0,\pi_1,\pi_2,\cdots,\pi_{N-b})$ be the stationary probability vector of this system, and each sub-vector $\pi_i=(\pi_{i00},\pi_{i01},\pi_{i10},\pi_{i11},\pi_{i20},\pi_{i21},\cdots,\pi_{ib0}),\pi_{ib1}), i=0,1,2,\cdots,N-b$ is a $2(b+1)$ dimensional row vector, then the system of stationary equations is

$$\begin{cases}\pi Q=0\\ \pi e=1\end{cases} \tag{19}$$

Where e is a column vector of ones with proper dimension. Then, we have

$$\pi_0 B_0+\pi_1 B_1+\pi_2 B_2+\cdots+\pi_b B_b=0 \tag{20}$$

$$\pi_0 A_0+\pi_1 A_1+\pi_{b+1} B_b=0 \tag{21}$$

$$\pi_{i-1}A_0+\pi_i A_1+\pi_{i+b}B_b=0, i=2,3,\cdots,N-2b \tag{22}$$

$$\begin{aligned}&\pi_{i-1}A_0+\pi_i A_1+\pi_{N-b}C_{i-(N-2b)}=0,\\ &i=N-2b+1,N-2b+2,\cdots,N-b-1\end{aligned} \tag{23}$$

$$\pi_{N-b-1}A_0+\pi_{N-b}A_M=0 \tag{24}$$

$$\pi e=1 \tag{25}$$

Since the matrix A_0 is a diagonal matrix, we use the matrix analysis method to solve the steady state probability vector. The diagonal matrix A_0 is expressed as $A_0=\lambda I$ (I is the $2(b+1)$ order identity matrix). Let $R_{N-b}=I$, then

$$\pi_{N-b}=\pi_{N-b}R_{N-b} \tag{26}$$

According to Equation (24), we have

$$\pi_{N-b-1}=\pi_{N-b}(-\frac{1}{\lambda}A_M)=\pi_{N-b}R_{N-b-1} \tag{27}$$

Here $R_{N-b-1}=-\frac{1}{\lambda}A_M$ is called the sub-rate matrix. Substitute equation (27) into equation (23), we get

$$\begin{aligned}\pi_{N-b-(i+1)}&=\pi_{N-b}[-\frac{1}{\lambda}(R_{N-b-i}A_1+C_{b-i})]\\ &=\pi_{N-b}R_{N-b-(i+1)},\\ &i=1,2,\cdots,b-1\end{aligned} \tag{28}$$

Here

$$R_{N-b-(i+1)}=-\frac{1}{\lambda}(R_{N-b-i}A_1+C_{b-i}), i=1,2,\cdots,b-1.$$

Substitute equation (28) into equation (22), we get

$$\begin{aligned}\pi_{N-b-(i+1)}&=\pi_{N-b}[-\frac{1}{\lambda}(R_{N-b-i}A_1+R_{N-i}B_b)]\\ &=\pi_{N-b}R_{N-b-(i+1)},\\ &i=b,b+1,\cdots,N-b-1\end{aligned} \tag{29}$$

Here

$$\begin{aligned}R_{N-b-(i+1)}&=-\frac{1}{\lambda}(R_{N-b-i}A_1+R_{N-i}B_b),\\ &i=b,b+1,\cdots,N-b-1.\end{aligned}$$

According to equation (20), we have

$$\begin{aligned}\pi_0 &= -\pi_{N-b}(R_1B_1 + R_2B_2 + \cdots + R_bB_b)B_0^{-1} \\ &= \pi_{N-b}R_0,\end{aligned} \tag{30}$$

Here $R_0 = -(R_1B_1+R_2B_2+\cdots+R_bB_b)B_0^{-1}$, the process of solving $R_i, i = 0, 1, 2, \cdots, N-b$ is shown in Algorithm 1.

Algorithm 1 R_i algorithm

Require: $I, \lambda, A_M, N, b, A_1, C_{1,\cdots,b-1}, B_b, B_0$
Ensure: $R_{0,\cdots,N-b}$

```
R_{N-b} ← I
R_{N-b-1} ← -(1/λ) A_M
for i ← 1 to b - 1 by 1 do
    R_{N-b-(i+1)} ← -(1/λ)(R_{N-b-i} A_1 + C_{b-i})
end for
for i ← b to N - b - 2 by 1 do
    R_{N-b-(i+1)} ← -(1/λ)(R_{N-b-i} A_1 + R_{N-i} B_b)
end for
R_0 ← -(R_1 B_1 + R_2 B_2 + R_b B_b) B_0^{-1}
```

Combine equations (21) and (25), we have

$$\begin{cases} \pi_{N-b}(R_0A_0 + R_1A_1 + R_{b+1}B_b) = 0 \\ \pi_{N-b}(R_0 + R_1 + R_2 + \cdots + R_{N-b-1} + I)e = 1 \end{cases} \tag{31}$$

For solve π_{N-b}, from equations (26)-(30), the steady-state probability vector π can be solved.

1) Performance analysis:

In this section, we provide the performance metric indicator of the Cosmos hub system, and its expression is given by π and R. Analyze the influence of parameters on system performance indicators through numerical calculations.

When the Cosmos hub system is stable, we write

$$\lim_{t\to+\infty} I(t) = L_q, \lim_{t\to+\infty} J(t) = J_b, \lim_{t\to+\infty} K(t) = K_b \tag{32}$$

(a) Average number of transactions in a block.

$$E(J_b) = \sum_{j=0}^{b}(j\sum_{j=0}^{b}\sum_{k=0}^{1}\pi_{ijk}) \tag{33}$$

(b) Average queue length of the Cosmos hub system

$$E(L_q) = \sum_{i=0}^{N-b}(i\sum_{j=0}^{b}\sum_{k=0}^{1}\pi_{ijk}) \tag{34}$$

(c) Probability of transaction rejection of the Cosmos hub system

$$P_{rjc} = \sum_{j=0}^{b}\sum_{k=0}^{1}\pi_{N-b,j,k} \tag{35}$$

(d) Average transaction execution time of the Cosmos hub system

$$\begin{aligned} E(T_{exe}) &= \sum_{l=0}^{b-1}\sum_{h=0}^{[\frac{N-b-l}{b}]}\pi_{hb+l,0,1}(h+1)(\frac{1}{\mu_1}+\frac{1}{\mu_2}) \\ &+\sum_{l=0}^{b-1}\sum_{h=0}^{[\frac{N-b-l}{b}]}\sum_{j=1}^{b}\pi_{hb+l,j,1}[\frac{1}{\mu_2}+(h+1)(\frac{1}{\mu_1}+\frac{1}{\mu_1})] \end{aligned} \tag{36}$$

The proof process is shown in [18], where $[\frac{N-b-l}{b}]$ is the rounding function.

(e) Average transaction response time of the Cosmos hub system

$$E(T_{resp}) = \frac{E(L_q)}{\lambda(1-P_{rjc})} \tag{37}$$

(f) Throughput of the Cosmos hub system

$$E(TPS) = \lambda(1 - p_{rjc}) \tag{38}$$

IV. Model Simulation and Performance Evaluation

We install the distributed load testing tool $tm-load-test$ on the 4C8G cloud host to perform load testing on the Cosmos blockchain network to determine the magnitude of the influencing factors in the model. Therefore, $MATLABR2016a$ is used to simulate the impact of several vital factors λ, N, $\mu_1(\mu_2)$ and $\alpha(\beta)$ on performance metric indicators and to verify the approximate precision of the model.

A. Influence of transaction arrival rate (λ)

We set the block size at $10-50$, the block generation rate and the block consensus rate to be $\mu_1 = 200, \mu_2 = 100$ (transactions/s), respectively, the system failure rate and the repair rate are $\alpha = \beta = 100$ (transactions/s), respectively, and the capacity of the system transaction pool is $N = 1000$. By randomly sending different arrival rates for cross-chain transactions, λ ranges from 100-2000 (transactions/s).

Fig.4 shows the impact of the transaction arrival rate λ on various performance metric indicators of the Cosmos hub system. When the block size b is determined, the transaction execution time and response time are inversely proportional to the transaction arrival rate λ. System throughput is proportional to λ, and the probability of transaction rejection is almost unaffected by λ. The average number of transactions in a block and the average queue length in the system are inversely proportional to λ and the block size. Therefore, the smaller the block size is set to $b = 10$, the smaller the system queue stacking will be, and the shorter the execution and response time.

B. Influence of Cosmos hub system capacity (N)

We set the range of N to be 200 to 4000 transactions, $\mu_1 = \mu_2 = 100$(transactions/s), $\alpha = \beta = 100$ (transactions/s), and the transaction arrival rate $\lambda = 2000$. Fig. 5 shows the variation of performance metrics under different block sizes b.

We can conclude from Fig. 5 that when the Cosmos hub system capacity N is larger, the queue length is longer, and the transaction response time is longer. However, the metrics are unstable when the block size b is small. When $b = 40, 50$, the changes in the system queue length, the transaction execution time, and the transaction response time are stable, and the moderate setting of N can meet the performance metrics that restrict each other.

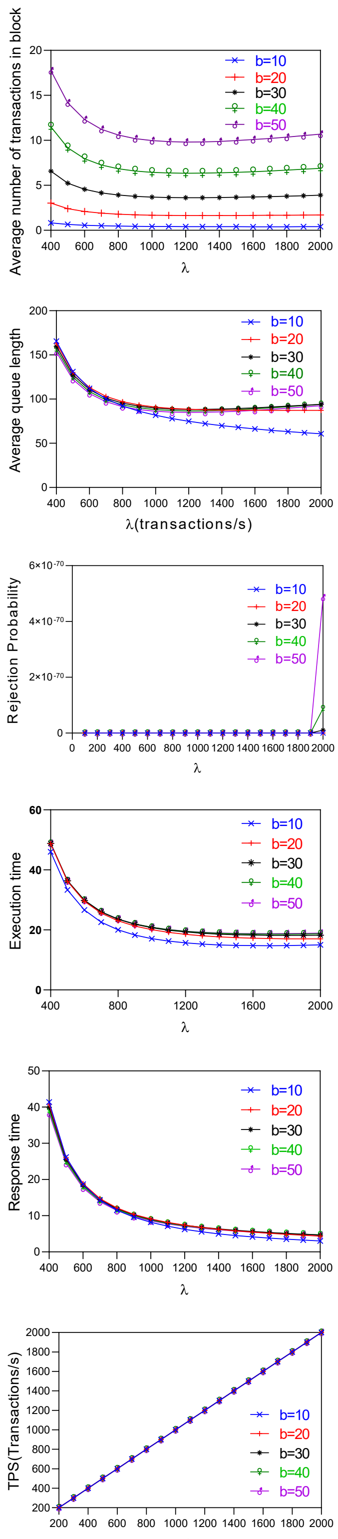

Fig. 4. Performance indicators with λ

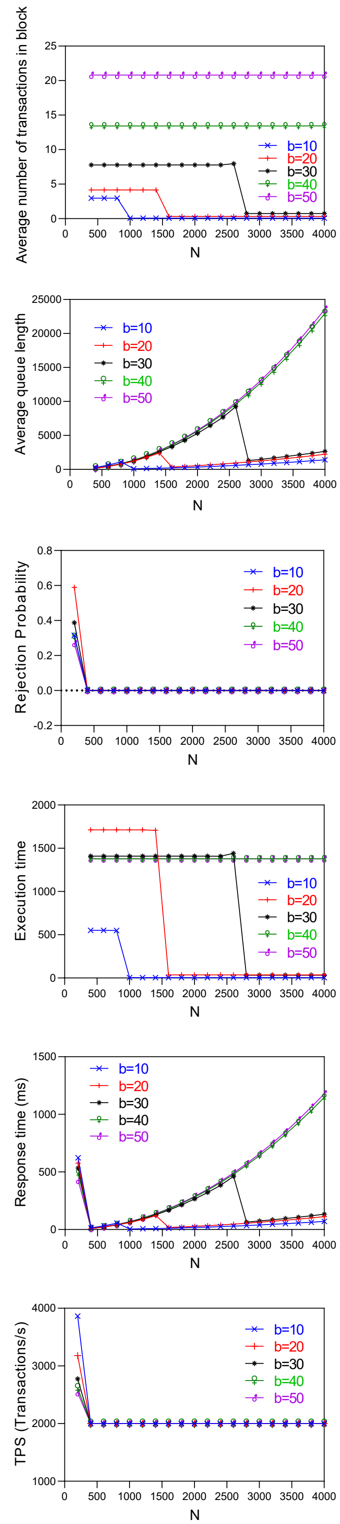

Fig. 5. performance indicators with N

C. Influence of transaction block generate rate (μ_1)

We set the range of N to be 10 to 500 transactions, $\lambda = 1000$ (transactions/s), $\alpha = \beta = 100$ (transactions/s), and the transaction arrival rate $\lambda = 2000$, the transaction pool capacity of the system is $N = 1000$. Fig. 6 shows the variation of performance metric indicators in different block sizes b.

D. Influence of transaction block generate rate (α)

We set the range of α to be 20 to 4000 transactions, $\lambda = 1000$(transactions/s), $\mu 1 = \mu 2 = 100$ (transactions/s), and the system transaction pool capacity is $N = 1000$. Fig. 7 shows the variation of performance metric indicators under different block sizes b.

It can be seen from Fig. 7 that when the block generation rate is $\mu_1 < 50$, it is evident that the system's performance indicators are unstable. The block generation time is too long, the system accumulates seriously, and the performance indicators change enormously. Similarly, a similar conclusion can be drawn from Fig. 7. When the system failure rate is $\alpha < 100$, that is, the failure time is too long, and the system is in an unstable state. Since μ_2 and μ_1 (α and β) have the exact change rule for various performance indexes of the system, the influence of the parameter μ_2 (β) on the performance index is no longer described here.

V. Conclusion

Cross-chain technology is the key to developing the blockchain field and the core technology to realize cross-chain technology. Therefore, it is necessary to study the theory and performance of cross-chain technology. This paper first selects the representative cross-chain technology cosmos to model the core cross-chain process. Second, we apply the queue theory model to establish a differential equation for the consensus system with the help of three Markov chains and obtain the performance evaluation index expression of the Cosmos system. Finally, we tested the performance of the Cosmos hub system to obtain the system stability conditions. We also tested the consensus time, latency, response time, and the number of transactions in the Cosmos system for cross-chain transactions to verify the model's validity. The model proposed and validated in this paper can still be used to evaluate the performance of other cross-chain technologies using similar patterns.

Future directions for this research are threefold: 1) Optimize the existing queueing theory model to improve the utilization of node group consensus in the Cosmos Hub. 2) Improve the equipment configuration used to build the Cosmos cross-chain platform and improve the performance bottleneck of the relay system. 3) Compare other cross-chain systems with the relay model to expand the scope of application of the model.

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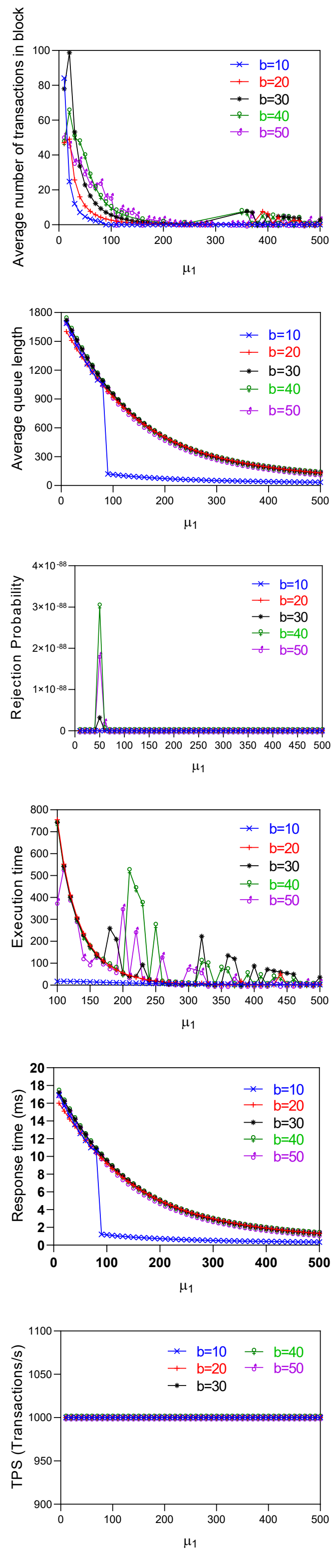

Fig. 6. Performance indicators with μ_1

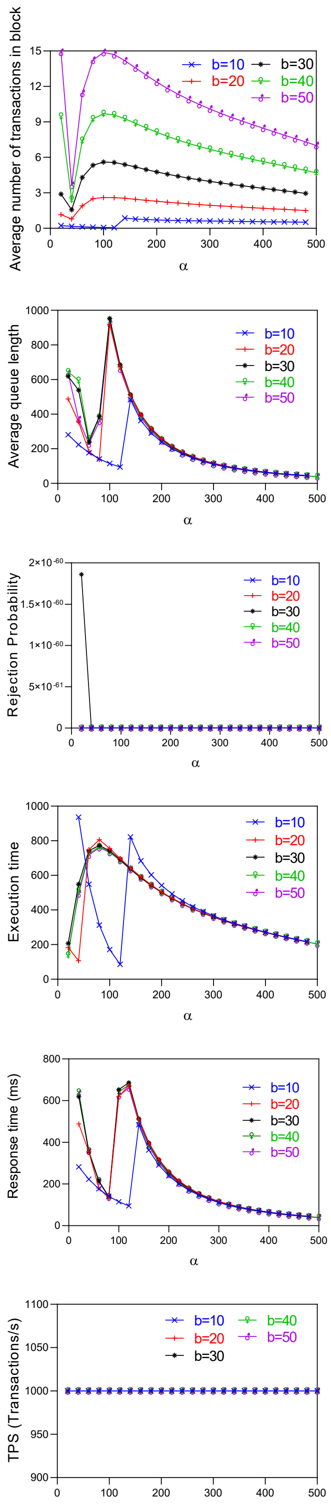

Fig. 7. Performance indicators with α

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