2. System model

Our research explores the combination of WSN and energy harvesting, which allows sensors to receive energy from both renewable sources and reliable energy sources like regular batteries. We divide packets into two groups based on urgency: emergent, and non-emergent. Emergent packets contain time-critical information, like alerts for fires, earthquakes, or foreign objects, and must be delivered immediately. In contrast, non-emergent packets contain non-real time information, such as weather forecasts and smart meter data. Both emergent and non-emergent groups can transmit one packet or more packets at a time. Each packet is time-sensitive and may be discarded if it fails to meet a certain deadline, indicating that it is no longer useful. There are two possible energy sources, either harvested from the environment or supplied by the regular battery, which can be used to transmit each data packet. The energy requirement of each packet is the same.

With the information provided above, we explore two scenarios in our study: (1) only one sensor node is considered and the packets comes in batches with different priority where each batch consists of one packet or two packets, (2) a simplified WSN comprising three interconnected nodes is considered and the way packets come at node 1 is same as scenario 1.

To provide further explanation, we model each node in the WSN as a variation of the M/M/1/K system, which includes a finite packet queue, a finite energy queue, and a regular battery. Both packets and energy units arrive according to a Poisson process, and the time that each packet can wait in the queue and the time required for its service are defined as exponential distributions. Depending on their application, the arrived packets are roughly classified into two categories: high priority (HP) and low priority (LP). The way that packets come can be divided into two groups: HP and LP. If there is only one seat and here comes two HP or LP packets at once, one of them will be blocked and the other will enter the packet queue followed the rule below. HP packets have a non-preemptive priority over LP packets, and packets with the same priority follow a first-come, first-served (FCFS) approach. Specifically, when a new HP packet arrives, it is placed in front of any LP packets in the queue, pushing them to the back. However, if an LP packet is already being served, the HP packet at the head of the queue must wait its turn due to the non-preemption policy.

When a node receives a packet, it is either added to the packet queue or rejected and discarded due to queue overflow. If there is enough energy available in the energy queue, the packet at the head of the packet queue will use the corresponding harvested energy

to start processing. However, if there is insufficient energy available, a probability value is used to determine if the regular battery can be used as an alternative energy source. If not, the packet will remain in the queue until the next state transition.

In our network setup, we have three interconnected nodes. The first node acts as the entry node, the second node acts as the exit node, and the third node acts as the control node. After completing its service at each node, a packet is directed to the next node based on a predetermined routing probability. It's important to note that packets originating from the entry and exit nodes are only allowed to pass through the control node once before being sent back to their respective previous nodes. Additionally, incoming packets are only permitted to enter the system through the entry node and exit after being serviced at the exit node or due to impatience.

2.1 Scenario 1

Within this scenario of study, we focus our attention on a single sensor node. For the purposes of our investigation, we will assume that each packet - regardless of its priority level - has an energy requirement of one unit. Subsequently, we analyze the influence of different system parameters, e.g., energy usage on the overall performance of the node.

2.2 Scenario 2

Within this unit of study, we examine a network that consists of three interconnected nodes, and this network is an extension of the one considered in Scenario 1. It is assumed that both high priority and low priority packets have an energy requirement of one unit. Subsequently, we investigate different system parameters, e.g., how the energy consumption affects not only the overall network but also each individual node within the network.

3. Analytical model

In this section, we outline and clarify the analytical models in a step-by-step manner for two situations: (1) there is only a single node with single and batch arrival, and each packet consumes the same amount of energy, and (2) there are three interconnected nodes with single and batch arrival in the network, and each packet consumes the same amount of energy.

3.1 Scenario 1

In this section, we focus on a scenario where packets coming with single or batch arrival can be divided into two priorities: high priority (HP) and low priority (LP), and both types of packets require the same amount of energy. Within each priority level, the packets are serviced in a first-come, first-served (FCFS) order. Once a packet enters the queue, it cannot be preempted, which means that an HP packet can always overtake an LP packet in the queue, but once an LP packet is in service, it cannot be interrupted. Additionally, there is a chance that a packet waiting in the queue may leave the system due to impatience. It is worth noting that when a packet is ready to be serviced, it first checks if there is enough energy in the energy queue. If there isn't enough energy, the packet may use a regular battery with a given probability. The model diagram, state balance equations, iterative algorithm, and performance metrics can be found below.

3.1.1 Model diagram

Fig. 3 - 1 illustrates the components of the model used in scenario 1, which include a finite packet queue, a finite energy queue, a regular battery, and a single server. The size of the packet queue is denoted by $N \ \ where \ \ 0 < N < \infty$, while the energy queue size is represented by $K \ \ (where \ \ 0 < K < \infty)$. The regular battery has an infinite supply of energy, and each of HP and LP packets requires one energy unit. The arrivals of single HP batches (one packet per batch), single LP batches (one packet per batch), double HP batches (two packets per batch), double LP batches (two packets per batch), and energy units are governed by Poisson processes, with respective arrival rates λ_H, λ_L, λ_{H2}, λ_{L2} and β. The impatient time for each HP and LP packet waiting in the queue is determined by an exponential distribution, with corresponding rates α_H and α_L. The service time for each of HP and LP packets in the server is exponentially distributed, with associated rates μ_H and μ_L, respectively. Additionally, when the amount of harvested energy available in the energy queue is insufficient to support an HP or LP packet, the regular battery will be used based on probabilities θ_H and θ_L, respectively.

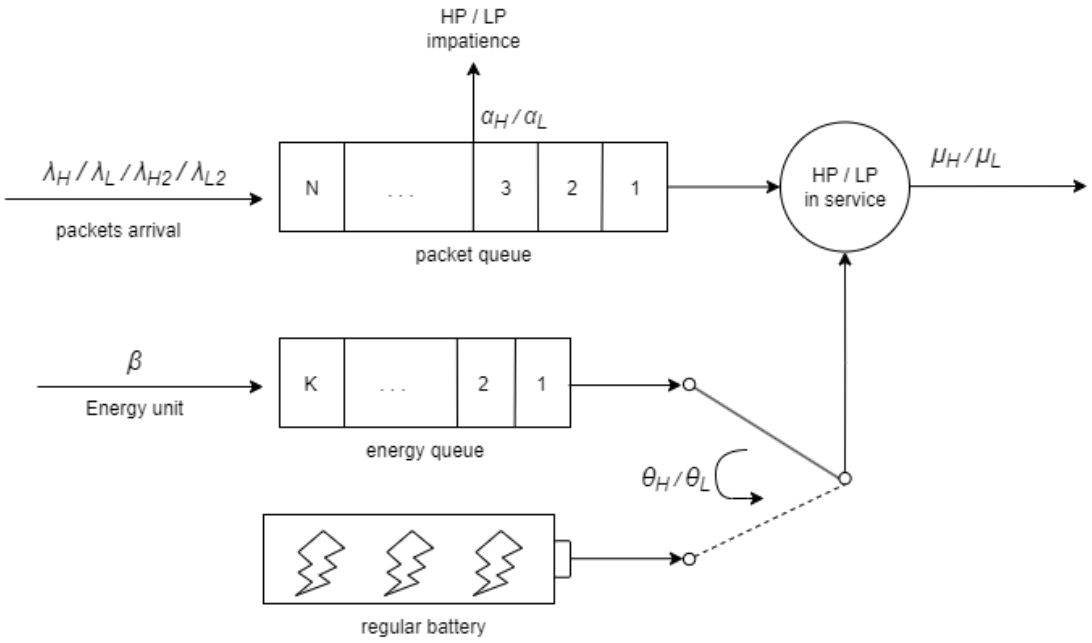

Fig. 3 - 1: The model diagram for scenario 1

3.1.2 State balance equations

The system is modeled as a Markov chain with four dimensions: (i, j, x, y), where i represents the number of high-priority (HP) packets in the system, j represents the number of low-priority (LP) packets in the system, x represents the number of energy units in the energy queue, and y represents the server status and the energy source used. The value of y can take on five different values: (1) "0" indicates that the server is idle; (2) "1" indicates that an LP packet has entered the server and consumed one energy unit from the energy queue; (3) "2" indicates that an HP packet has entered the server and consumed one energy unit from the energy queue; (4) "3" indicates that an LP packet has entered the server and consumed one energy unit from the regular battery; (5) "4" indicates that an HP packet has entered the server and consumed one energy unit from the regular battery. The steady state probability of the system is represented by $\pi(i, j, x, y)$, and the state space is defined as follows:

$$\begin{aligned} S = \{(i,j,x,y) |\ & 0 \le i + j \le N, x = 0, y = 0; \\ & i + j = 0, 1 \le x \le K, y = 0; \\ & 1 \le i + j \le N + 1, j \ge 1, 0 \le x \le K, y = 1、3; \\ & 1 \le i + j \le N + 1, i \ge 1, 0 \le x \le K, y = 2、4\} \end{aligned} \tag{3-1}$$

As a result, we can calculate the total count of possible states

$$|S| = \left(2K + \frac{5}{2}\right) N^2 + \left(6\mathrm{K} + \frac{15}{2}\right) N + 5K + 5.$$

Based on the model description, there are 190 possible cases for the total system states. The balance equations for each of these cases are presented below.

Case 1: For $i = 0,\ j = 0,\ x = 0$, and $y = 0$,

$$\begin{aligned}&[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_L)\lambda_L + \theta_L\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} \\ &\qquad + (1-\theta_L)\lambda_{L2} + \theta_L\lambda_{L2} + \beta]\pi(0,0,0,0) \\ &= \alpha_H\pi(1,0,0,0) + \alpha_L\pi(0,1,0,0) + \mu_H\pi(1,0,0,2) + \mu_L\pi(0,1,0,1) + \\ &\quad \mu_H\pi(1,0,0,4) + \mu_L\pi(0,1,0,3).\end{aligned}$$

Case 2: For $i = 0,\ j = 1,\ x = 0$, and $y = 0$,

$$\begin{aligned}&[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_L)\lambda_L + \theta_L\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} \\ &\qquad + (1-\theta_L)\lambda_{L2} + \theta_L\lambda_{L2} + \beta + \alpha_L]\pi(0,1,0,0) \\ &= (1-\theta_L)\lambda_L\pi(0,0,0,0) + (1-\theta_L)\alpha_H\pi(1,1,0,0) + \\ &\quad (1-\theta_L)2\alpha_L\pi(0,2,0,0) + (1-\theta_L)\mu_H\pi(1,1,0,2) + \\ &\quad (1-\theta_L)\mu_L\pi(0,2,0,1) + (1-\theta_L)\mu_H\pi(1,1,0,4) + \\ &\quad (1-\theta_L)\mu_L\pi(0,2,0,3).\end{aligned}$$

Case 3: For $i = 0,\ 2 \le j \le N-1,\ x = 0$, and $y = 0$,

$$\begin{aligned}&[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_L)\lambda_L + \theta_L\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} \\ &\qquad + (1-\theta_L)\lambda_{L2} + \theta_L\lambda_{L2} + \beta + (1-\theta_L)j\alpha_L + \theta_L j\alpha_L] \\ &\pi(0,j,0,0) \\ &= (1-\theta_L)\lambda_L\pi(0,j-1,0,0) + (1-\theta_L)\lambda_{L2}\pi(0,j-2,0,0) \\ &\qquad + (1-\theta_L)\alpha_H\pi(1,j,0,0) + \\ &\quad (1-\theta_L)(j+1)\alpha_L\pi(0,j+1,0,0) + (1-\theta_L)\mu_H\pi(1,j,0,2) + \\ &\quad (1-\theta_L)\mu_L\pi(0,j+1,0,1) + (1-\theta_L)\mu_H\pi(1,j,0,4) + \\ &\quad (1-\theta_L)\mu_L\pi(0,j+1,0,3).\end{aligned}$$

Case 4: For $i = 0,\ j = N,\ x = 0$, and $y = 0$,

$$\begin{aligned}&[\beta + (1-\theta_L)N\alpha_L + \theta_L N\alpha_L]\pi(0,N,0,0) \\ &= (1-\theta_L)\lambda_L\pi(0,N-1,0,0) + (1-\theta_L)\lambda_{L2}\pi(0,N-2,0,0) \\ &\qquad + (1-\theta_L)\lambda_{L2}\pi(0,N-1,0,0) + (1-\theta_L)\mu_H\pi(1,N,0,2) + \\ &\quad (1-\theta_L)\mu_L\pi(0,N+1,0,1) + (1-\theta_L)\mu_H\pi(1,N,0,4) + \\ &\quad (1-\theta_L)\mu_L\pi(0,N+1,0,3).\end{aligned}$$

Case 5: For $i = 1,\ j = 0,\ x = 0$, and $y = 0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1-\theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + \alpha_H]\pi(1,0,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(0,0,0,0) + (1-\theta_H)2\alpha_H\pi(2,0,0,0) +$$
$$(1-\theta_H)\alpha_L\pi(1,1,0,0) + (1-\theta_H)\mu_H\pi(2,0,0,2) +$$
$$(1-\theta_H)\mu_L\pi(1,1,0,1) + (1-\theta_H)\mu_H\pi(2,0,0,4) +$$
$$(1-\theta_H)\mu_L\pi(1,1,0,3).$$

Case 6: For $i = 1,\ j = 1,\ x = 0$, and $y = 0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1-\theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + (1-\theta_L)\alpha_H + \theta_L\alpha_H + (1-\theta_H)\alpha_L + \theta_H\alpha_L]\pi(1,1,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(0,1,0,0) + (1-\theta_H)\lambda_L\pi(1,0,0,0) +$$
$$(1-\theta_H)2\alpha_H\pi(2,1,0,0) + (1-\theta_H)2\alpha_L\pi(1,2,0,0) +$$
$$(1-\theta_H)\mu_H\pi(2,1,0,2) + (1-\theta_H)\mu_L\pi(1,2,0,1)$$
$$(1-\theta_H)\mu_H\pi(2,1,0,4) + (1-\theta_H)\mu_L\pi(1,2,0,3).$$

Case 7: For $i = 1,\ 2 \le j \le N-2,\ x = 0$, and $y = 0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1-\theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + (1-\theta_L)\alpha_H + \theta_L\alpha_H + (1-\theta_H)j\alpha_L + \theta_H j\alpha_L]\pi(1,j,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(0,j,0,0) + (1-\theta_H)\lambda_L\pi(1,j-1,0,0)$$
$$+ (1-\theta_H)\lambda_{L2}\pi(1,j-2,0,0) +$$
$$(1-\theta_H)2\alpha_H\pi(2,j,0,0) + (1-\theta_H)(j+1)\alpha_L\pi(1,j+1,0,0) +$$
$$(1-\theta_H)\mu_H\pi(2,j,0,2) + (1-\theta_H)\mu_L\pi(1,j+1,0,1)$$
$$(1-\theta_H)\mu_H\pi(2,j,0,4) + (1-\theta_H)\mu_L\pi(1,j+1,0,3).$$

Case 8: For $i = 1,\ j = N-1,\ x = 0$, and $y = 0$,

$$[\beta + (1-\theta_L)\alpha_H + \theta_L\alpha_H + (1-\theta_H)(N-1)\alpha_L + \theta_H(N-1)\alpha_L]$$
$$\pi(1,N-1,0,0)$$

$$= (1-\theta_H)\lambda_H\pi(0,N-1,0,0) + (1-\theta_H)\lambda_L\pi(1,N-2,0,0)$$
$$+ (1-\theta_H)\lambda_{H2}\pi(0,N-1,0,0) + (1-\theta_H)\lambda_{L2}\pi(1,N-3,0,0)$$
$$+ (1-\theta_H)\lambda_{L2}\pi(1,N-2,0,0) +$$
$$(1-\theta_H)\mu_H\pi(2,N-1,0,2) + (1-\theta_H)\mu_L\pi(1,N,0,1) +$$
$$(1-\theta_H)\mu_H\pi(2,N-1,0,4) + (1-\theta_H)\mu_L\pi(1,N,0,3).$$

Case 9: For $2 \le i \le N-1,\ j=0,\ x=0$, and $y=0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1$$
$$- \theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + (1-\theta_H)i\alpha_H + \theta_H i\alpha_H]$$
$$\pi(i,0,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(i-1,0,0,0) + (1-\theta_H)\lambda_{H2}\pi(i-2,0,0,0)$$
$$+ (1-\theta_H)(i+1)\alpha_H\pi(i+1,0,0,0) +$$
$$(1-\theta_H)\alpha_L\pi(i,1,0,0) + (1-\theta_H)\mu_H\pi(i+1,0,0,2) +$$
$$(1-\theta_H)\mu_L\pi(i,1,0,1) + (1-\theta_H)\mu_H\pi(i+1,0,0,4) +$$
$$(1-\theta_H)\mu_L\pi(i,1,0,3).$$

Case 10: For $2 \le i \le N-2,\ j=1,\ x=0$, and $y=0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1$$
$$- \theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + (1-\theta_H)i\alpha_H + \theta_H i\alpha_H +$$
$$(1-\theta_H)\alpha_L + \theta_H\alpha_L]\pi(i,1,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(i-1,1,0,0) + (1-\theta_H)\lambda_L\pi(i,0,0,0)$$
$$+ (1-\theta_H)\lambda_{H2}\pi(i-2,1,0,0) +$$
$$(1-\theta_H)(i+1)\alpha_H\pi(i+1,1,0,0) + (1-\theta_H)2\alpha_L\pi(i,1,0,0) +$$
$$(1-\theta_H)\mu_H\pi(i+1,1,0,2) + (1-\theta_H)\mu_L\pi(i,2,0,1) +$$
$$(1-\theta_H)\mu_H\pi(i+1,1,0,4) + (1-\theta_H)\mu_L\pi(i,2,0,3).$$

Case 11: For $2 \le i \le N-3,\ 2 \le j \le N-i-1,\ x=0$, and $y=0$,

$$[(1-\theta_H)\lambda_H + \theta_H\lambda_H + (1-\theta_H)\lambda_L + \theta_H\lambda_L + (1-\theta_H)\lambda_{H2} + \theta_H\lambda_{H2} + (1$$
$$- \theta_H)\lambda_{L2} + \theta_H\lambda_{L2} + \beta + (1-\theta_H)i\alpha_H + \theta_H i\alpha_H +$$
$$(1-\theta_H)j\alpha_L + \theta_H j\alpha_L]\pi(i,j,0,0)$$
$$= (1-\theta_H)\lambda_H\pi(i-1,j,0,0) + (1-\theta_H)\lambda_L\pi(i,j-1,0,0)$$
$$+ (1-\theta_H)\lambda_{H2}\pi(i-2,j,0,0) + (1-\theta_H)\lambda_{L2}\pi(i,j-2,0,0) +$$
$$(1-\theta_H)(i+1)\alpha_H\pi(i+1,j,0,0) + (1-\theta_H)(j+1)\alpha_L\pi(i,j+1,0,0) +$$

$$(1-\theta_H)\mu_H\pi(i+1,j,0,2)+(1-\theta_H)\mu_L\pi(i,j+1,0,1)+$$
$$(1-\theta_H)\mu_H\pi(i+1,j,0,4)+(1-\theta_H)\mu_L\pi(i,j+1,0,3).$$

Case 12: For $2 \le i \le N-2,\ j=N-i,\ x=0$, and $y=0$,

$$[\beta+(1-\theta_H)i\alpha_H+\theta_H i\alpha_H+(1-\theta_H)(N-i)\alpha_L+\theta_H(N-i)\alpha_L]$$
$$\pi(i,N-i,0,0)$$
$$=(1-\theta_H)\lambda_H\pi(i-1,N-i,0,0)+(1-\theta_H)\lambda_L\pi(i,N-i-1,0,0)$$
$$+(1-\theta_H)\lambda_{H2}\pi(i-2,j,0,0)+(1-\theta_H)\lambda_{H2}\pi(i-1,j,0,0)$$
$$+(1-\theta_H)\lambda_{L2}\pi(i,j-2,0,0)+(1-\theta_H)\lambda_{L2}\pi(i,j-1,0,0)+$$
$$(1-\theta_H)\mu_H\pi(i+1,N-i,0,2)+(1-\theta_H)\mu_L\pi(i,N-i+1,0,1)+$$
$$(1-\theta_H)\mu_H\pi(i+1,N-i,0,4)+(1-\theta_H)\mu_L\pi(i,N-i+1,0,3).$$

Case 13: For $i=N-1,\ j=1,\ x=0$, and $y=0$,

$$[\beta+(1-\theta_H)i\alpha_H+\theta_H i\alpha_H+(1-\theta_H)(N-i)\alpha_L+\theta_H(N-i)\alpha_L]$$
$$\pi(i,N-i,0,0)$$
$$=(1-\theta_H)\lambda_H\pi(i-1,N-i,0,0)+(1-\theta_H)\lambda_L\pi(i,N-i-1,0,0)$$
$$+(1-\theta_H)\lambda_{H2}\pi(i-2,j,0,0)+(1-\theta_H)\lambda_{H2}\pi(i-1,j,0,0)$$
$$+(1-\theta_H)\lambda_{L2}\pi(N-1,0,0,0)+$$
$$(1-\theta_H)\mu_H\pi(i+1,N-i,0,2)+(1-\theta_H)\mu_L\pi(i,N-i+1,0,1)+$$
$$(1-\theta_H)\mu_H\pi(i+1,N-i,0,4)+(1-\theta_H)\mu_L\pi(i,N-i+1,0,3).$$

Case 14: For $i=N,\ j=0,\ x=0$, and $y=0$,

$$[\beta+(1-\theta_H)N\alpha_H+\theta_H N\alpha_H]\pi(N,0,0,0)$$
$$=(1-\theta_H)\lambda_H\pi(N-1,0,0,0)+(1-\theta_H)\lambda_{H2}\pi(N-2,0,0,0)$$
$$+(1-\theta_H)\lambda_{H2}\pi(N-1,0,0,0)+(1-\theta_H)\mu_H\pi(N+1,0,0,2)+$$
$$(1-\theta_H)\mu_L\pi(N,1,0,1)+(1-\theta_H)\mu_H\pi(N+1,0,0,4)+$$
$$(1-\theta_H)\mu_L\pi(N,1,0,3).$$

Case 15: For $i=0,\ j=0,\ 1 \le x \le K-1$, and $y=0$,

$$(\lambda_H+\lambda_L+\lambda_{H2}+\lambda_{L2}+\beta)\pi(0,0,x,0)$$
$$=\beta\pi(0,0,x-1,0)+\mu_H\pi(1,0,x,2)+\mu_L\pi(0,1,x,1)+\mu_H\pi(1,0,x,4)+$$
$$\mu_L\pi(0,1,x,3).$$

Case 16: For $i = 0,\ j = 0,\ x = K$, and $y = 0$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2})\pi(0,0,K,0)$$
$$= \beta\pi(0,0,K-1,0) + \mu_H\pi(1,0,K,2) + \mu_L\pi(0,1,K,1) + \mu_H\pi(1,0,K,4) + \mu_L\pi(0,1,K,3).$$

Case 17: For $i = 0,\ j = 1,\ x = 0$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_L)\pi(0,1,0,1)$$
$$= \lambda_L\pi(0,0,1,0) + \beta\pi(0,1,0,0) + \alpha_H\pi(1,1,0,1) + \alpha_L\pi(0,2,0,1) + \mu_H\pi(1,1,1,2) + \mu_L\pi(0,2,1,1) + \mu_H\pi(1,1,1,4) + \mu_L\pi(0,2,1,3).$$

Case 18: For $i = 0,\ j = 2,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(0,2,0,1)$$
$$= \lambda_L\pi(0,1,0,1) + \lambda_{L2}\pi(0,0,1,0) + \beta\pi(0,2,0,0) + \alpha_H\pi(1,2,0,1) + 2\alpha_L\pi(0,3,0,1) + \mu_H\pi(1,2,1,2) + \mu_L\pi(0,3,1,1) + \mu_H\pi(1,2,1,4) + \mu_L\pi(0,3,1,3).$$

Case 19: For $i = 0, 3 \le j \le N,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (j-1)\alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(0,j,0,1)$$
$$= \lambda_L\pi(0,j-1,0,1) + \lambda_{L2}\pi(0,j-2,0,1) + \beta\pi(0,j,0,0) + \alpha_H\pi(1,j,0,1) + j\alpha_L\pi(0,j+1,0,1) + \mu_H\pi(1,j,1,2) + \mu_L\pi(0,j+1,1,1) + \mu_H\pi(1,j,1,4) + \mu_L\pi(0,j+1,1,3).$$

Case 20: For $i = 0,\ j = N+1,\ x = 0$, and $y = 1$,

$$[\beta + N\alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(0,N+1,0,1)$$
$$= \lambda_L\pi(0,N,0,1) + \lambda_{L2}\pi(0,N-1,0,1) + \lambda_{L2}\pi(0,N,0,1).$$

Case 21: For $i = 1,\ j = 1,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,1,0,1)$$
$$= \lambda_H\pi(0,1,0,1) + 2\alpha_H\pi(2,1,0,1) + \alpha_L\pi(1,2,0,1).$$

Case 22: For $i = 1,\ j = 2,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,2,0,1)$$

$$= \lambda_H \pi(0,2,0,1) + \lambda_L \pi(1,1,0,1) + 2\alpha_H \pi(2,2,0,1) + 2\alpha_L \pi(1,3,0,1).$$

Case 23: For $i = 1,\ 3 \le j \le N-1,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (j-1)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,j,0,1)$$
$$= \lambda_H \pi(0,j,0,1) + \lambda_L \pi(1,j-1,0,1) + \lambda_{L2} \pi(1,j-2,0,1) + 2\alpha_H \pi(2,j,0,1) + j\alpha_L \pi(1,j+1,0,1).$$

Case 24: For $i = 1,\ j = N,\ x = 0$, and $y = 1$,

$$[\beta + \alpha_H + (N-1)\alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(1,N,0,1)$$
$$= \lambda_H \pi(0,N,0,1) + \lambda_L \pi(1,N-1,0,1) + \lambda_{H2} \pi(0,N,0,1) + \lambda_{L2} \pi(1,N-2,0,1) + \lambda_{L2} \pi(1,N-1,0,1) + 2\alpha_H \pi(2,j,0,1) + j\alpha_L \pi(1,j+1,0,1).$$

Case 25: For $2 \le i \le N-1,\ j = 1,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,1,0,1)$$
$$= \lambda_H \pi(i-1,1,0,1) + \lambda_{H2} \pi(i-2,1,0,1) + (i+1)\alpha_H \pi(i+1,1,0,1) + \alpha_L \pi(i,2,0,1)$$

Case 26: For $2 \le i \le N-2,\ j = 2,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,2,0,1)$$
$$= \lambda_H \pi(i-1,2,0,1) + \lambda_{H2} \pi(i-2,2,0,1) + (i+1)\alpha_H \pi(i+1,2,0,1) + 2\alpha_L \pi(i,3,0,1)$$

Case 27: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ x = 0$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (j-1)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,j,0,1)$$
$$= \lambda_H \pi(i-1,j,0,1) + \lambda_{H2} \pi(i-2,j,0,1) + \lambda_{L2} \pi(i,j-2,0,1) + (i+1)\alpha_H \pi(i+1,j,0,1) + \alpha_L \pi(i,j+1,0,1)$$

Case 28: For $2 \le i \le N-2,\ j = N-i+1,\ x = 0$, and $y = 1$,

$$[\beta + i\alpha_H + (N-i)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,N-i+1,0,1)$$

$$= \lambda_H \pi(i-1, N-i+1, 0, 1) + \lambda_L \pi(i, N-i, 0, 1)$$
$$+ \lambda_{H2} \pi(i-2, N-i+1, 0, 1) + \lambda_{H2} \pi(i-1, N-i+1, 0, 1)$$
$$+ \lambda_{L2} \pi(i, N-i-1, 0, 1) + \lambda_{L2} \pi(i, N-i, 0, 1).$$

Case 29: For $i = N-1,\ j = 2\ \ x = 0$, and $y = 1$,

$$[\beta + (N-1)\alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H \mu_L]\pi(N-1, 2, 0, 1)$$
$$= \lambda_H \pi(N-2, 2, 0, 1) + \lambda_L \pi(N-1, 1, 0, 1) + \lambda_{H2} \pi(N-3, 2, 0, 1)$$
$$+ \lambda_{H2} \pi(N-2, 2, 0, 1) + \lambda_{L2} \pi(N-1, 1, 0, 1).$$

Case 30: For $i = N,\ j = 1,\ x = 0$, and $y = 1$,

$$[\beta + N\alpha_H + (1-\theta_H)\mu_L + \theta_H \mu_L]\pi(N, 1, 0, 1)$$
$$= \lambda_H \pi(N-1, 1, 0, 1) + \lambda_{H2} \pi(N-2, 1, 0, 1) + \lambda_{H2} \pi(N-1, 1, 0, 1).$$

Case 31: For $i = 0,\ j = 1,\ 1 \le x \le K-1$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_L)\pi(0, 1, x, 1)$$
$$= \lambda_L \pi(0, 0, x+1, 0) + \beta \pi(0, 1, x-1, 1) + \alpha_H \pi(1, 1, x, 1) + \alpha_L \pi(0, 2, x, 1) +$$
$$\mu_H \pi(1, 1, x+1, 2) + \mu_L \pi(0, 2, x+1, 1) + \mu_H \pi(1, 1, x+1, 4) +$$
$$\mu_L \pi(0, 2, x+1, 3).$$

Case 32: For $i = 0,\ j = 2,\ 1 \le x \le K-1$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + \mu_L]\pi(0, 2, x, 1)$$
$$= \lambda_L \pi(0, 1, x, 1) + \lambda_{L2} \pi(0, 0, x+1, 0) + \beta \pi(0, 2, x-1, 1) + \alpha_H \pi(1, 2, x, 1) +$$
$$2\alpha_L \pi(0, 3, x, 1) + \mu_H \pi(1, 2, x+1, 2) + \mu_L \pi(0, 3, x+1, 1) +$$
$$\mu_H \pi(1, 2, x+1, 4) + \mu_L \pi(0, 3, x+1, 3).$$

Case 33: For $i = 0,\ 3 \le j \le N,\ 1 \le x \le K-1$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (j-1)\alpha_L + \mu_L]\pi(0, j, x, 1)$$
$$= \lambda_L \pi(0, j-1, x, 1) + \lambda_{L2} \pi(0, j-2, x, 1) + \beta \pi(0, j, x-1, 1) + \alpha_H \pi(1, j, x, 1)$$
$$+ j\alpha_L \pi(0, j+1, x, 1) + \mu_H \pi(1, j, x+1, 2) + \mu_L \pi(0, j+1, x+1, 1) +$$
$$\mu_H \pi(1, j, x+1, 4) + \mu_L \pi(0, j+1, x+1, 3).$$

Case 34: For $i = 0,\ j = N+1,\ 1 \le x \le K-1$, and $y = 1$,

$$(\beta + N\alpha_L + \mu_L)\pi(0, N+1, x, 1)$$
$$= \lambda_L\pi(0, N, x, 1) + \lambda_{L2}\pi(0, N-1, x, 1) + \lambda_{L2}\pi(0, N, x, 1)$$
$$+ \beta\pi(0, N+1, x-1, 1).$$

Case 35: For $i = 1,\ j = 1,\ 1 \le x \le K-1$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \mu_L)\pi(1, 1, x, 1)$$
$$= \lambda_H\pi(0,1, x, 1) + \beta\pi(1, 1, x-1, 1) + 2\alpha_H\pi(2, 1, x, 1) +$$
$$\alpha_L\pi(1, 2, x, 1).$$

Case 36: For $i = 1,\ j = 2,\ 1 \le x \le K-1$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + \mu_L)\pi(1, 2, x, 1)$$
$$= \lambda_H\pi(0, 2, x, 1) + \lambda_L\pi(1, 1, x, 1) + \beta\pi(1, 2, x-1, 1) + 2\alpha_H\pi(2, 2, x, 1) +$$
$$2\alpha_L\pi(1, 3, x, 1).$$

Case 37: For $i = 1,\ 3 \le j \le N-1,\ 1 \le x \le K-1$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (j-1)\alpha_L + \mu_L)\pi(1, j, x, 1)$$
$$= \lambda_H\pi(0, j, x, 1) + \lambda_L\pi(1, j-1, x, 1) + \lambda_{L2}\pi(1, j-2, x, 1) + \beta\pi(1, j, x-1, 1)$$
$$+ 2\alpha_H\pi(2, j, x, 1) + j\alpha_L\pi(1, j+1, x, 1).$$

Case 38: For $i = 1,\ j = N,\ 1 \le x \le K-1$, and $y = 1$,

$$(\beta + \alpha_H + (N-1)\alpha_L + \mu_L)\pi(1, N, x, 1)$$
$$= \lambda_H\pi(0, N, x, 1) + \lambda_L\pi(1, N-1, x, 1) + \lambda_{H2}\pi(0, N, x, 1)$$
$$+ \lambda_{L2}\pi(1, N-2, x, 1) + \lambda_{L2}\pi(1, N-1, x, 1) + \beta\pi(1, N, x-1, 1)$$
$$+ 2\alpha_H\pi(2, N, x, 1) + N\alpha_L\pi(1, N+1, x, 1).$$

Case 39: For $2 \le i \le N-1,\ j = 1,\ 1 \le x \le K-1$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \mu_L]\pi(i, 1, x, 1)$$
$$= \lambda_H\pi(i-1, 1, x, 1) + \lambda_{H2}\pi(i-2, 1, x, 1) + \beta\pi(i, 1, x-1, 1) +$$
$$(i+1)\alpha_H\pi(i+1, 1, x, 1) + \alpha_L\pi(i, 2, x, 1).$$

Case 40: For $2 \le i \le N-1,\ j = 2,\ 1 \le x \le K-1$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \alpha_L + \mu_L]\pi(i, 2, x, 1)$$

$$= \lambda_H \pi(i-1,2,x,1) + \lambda_{H2} \pi(i-2,2,x,1) + \lambda_L \pi(i,1,x,1) + \beta \pi(i,2,x-1,1)$$
$$+(i+1)\alpha_H \pi(i+1,2,x,1) + 2\alpha_L \pi(i,3,x,1).$$

Case 41: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ 1 \le x \le K-1$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (j-1)\alpha_L + \mu_L]\pi(i,j,x,1)$$
$$= \lambda_H \pi(i-1,j,x,1) + \lambda_L \pi(i,j-1,x,1) + \lambda_{H2} \pi(i-2,j,x,1)$$
$$+ \lambda_{L2} \pi(i,j-2,x,1) + \beta \pi(i,j,x-1,1) +$$
$$(i+1)\alpha_H \pi(i+1,j,x,1) + j\alpha_L \pi(i,j+1,x,1).$$

Case 42: For $2 \le i \le N-2,\ j = N-i+1,\ 1 \le x \le K-1$, and $y = 1$,

$$[\beta + i\alpha_H + (N-i)\alpha_L + \mu_L]\pi(i,N-i+1,x,1)$$
$$= \lambda_H \pi(i-1,N-i+1,x,1) + \lambda_L \pi(i,N-i,x,1)$$
$$+ \lambda_{H2} \pi(i-2,N-i+1,x,1) + \lambda_{H2} \pi(i-1,N-i+1,x,1)$$
$$+ \lambda_{L2} \pi(i,N-i-1,x,1) + \lambda_{L2} \pi(i,N-i,x,1)$$
$$+ \beta \pi(i,N-i+1,x-1,1).$$

Case 43: For $i = N-1,\ j = 2,\ 1 \le x \le K-1$, and $y = 1$,

$$[\beta + (N-1)\alpha_H + \alpha_L + \mu_L]\pi(N-1,2,x,1)$$
$$= \lambda_H \pi(N-2,2,x,1) + \lambda_L \pi(N-1,1,x,1) + \lambda_{H2} \pi(N-3,2,x,1)$$
$$+ \lambda_{H2} \pi(N-2,2,x,1) + \lambda_{L2} \pi(N-1,1,x,1)$$
$$+ \beta \pi(N-1,2,x-1,1).$$

Case 44: For $i = N,\ j = 1,\ 1 \le x \le K-1$, and $y = 1$,

$$(\beta + N\alpha_H + \mu_L)\pi(N,1,x,1)$$
$$= \lambda_H \pi(N-1,1,x,1) + \lambda_{H2} \pi(N-2,1,x,1) + \lambda_{H2} \pi(N-1,1,x,1)$$
$$+ \beta \pi(N,1,x-1,1).$$

Case 45: For $i = 0,\ j = 1,\ x = K$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \mu_L)\pi(0,1,K,1)$$
$$= \lambda_L \pi(0,0,K,0) + \beta \pi(0,1,K-1,1) + \alpha_H \pi(1,1,K,1) + \alpha_L \pi(0,2,K,1).$$

Case 46: For $i = 0,\ j = 2,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_L + \mu_L]\pi(0,2,K,1)$$
$$= \lambda_L\pi(0,1,K,1) + \lambda_{L2}\pi(0,0,K,0) + \beta\pi(0,2,K-1,1) + \alpha_H\pi(1,2,K,1) + 2\alpha_L\pi(0,3,K,1).$$

Case 47: For $i = 0,\ 3 \le j \le N,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (j-1)\alpha_L + \mu_L]\pi(0,j,K,1)$$
$$= \lambda_L\pi(0,j-1,K,1) + \lambda_{L2}\pi(0,j-2,K,1) + \beta\pi(0,j,K-1,1) + \alpha_H\pi(1,j,K,1) + j\alpha_L\pi(0,j+1,K,1).$$

Case 48: For $i = 0,\ j = N+1,\ x = K$, and $y = 1$,

$$(N\alpha_L + \mu_L)\pi(0,N+1,K,1)$$
$$= \lambda_L\pi(0,N,K,1) + \lambda_{L2}\pi(0,N-1,K,1) + \lambda_{L2}\pi(0,N,K,1) + \beta\pi(0,N+1,K-1,1).$$

Case 49: For $i = 1,\ j = 1,\ x = K$, and $y = 1$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \mu_L)\pi(1,1,K,1)$$
$$= \lambda_H\pi(0,1,K,1) + \beta\pi(1,1,K-1,1) + 2\alpha_H\pi(2,1,K,1) + \alpha_L\pi(1,2,K,1).$$

Case 50: For $i = 1,\ j = 2,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \alpha_L + \mu_L]\pi(1,2,K,1)$$
$$= \lambda_H\pi(0,2,K,1) + \lambda_L\pi(1,1,K,1) + \beta\pi(1,2,K-1,1) + 2\alpha_H\pi(2,2,K,1) + 2\alpha_L\pi(1,3,K,1).$$

Case 51: For $i = 1,\ 3 \le j \le N-1,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + (j-1)\alpha_L + \mu_L]\pi(1,j,K,1)$$
$$= \lambda_H\pi(0,j,K,1) + \lambda_L\pi(1,j-1,K,1) + \lambda_{L2}\pi(1,j-2,K,1) + \beta\pi(1,j,K-1,1) + 2\alpha_H\pi(2,j,K,1) + j\alpha_L\pi(1,j+1,K,1).$$

Case 52: For $i = 1,\ j = N,\ x = K$, and $y = 1$,

$$[\alpha_H + (N-1)\alpha_L + \mu_L]\pi(1,N,K,1)$$

$$= \lambda_H \pi(0, N, K, 1) + \lambda_L \pi(1, N-1, K, 1) + \lambda_{H2} \pi(0, N, K, 1)$$
$$+ \lambda_{L2} \pi(1, N-2, K, 1) + \lambda_{L2} \pi(1, N-1, K, 1) + \beta \pi(i, N-i+1, K-1, 1).$$

Case 53: For $2 \le i \le N-1,\ j = 1,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + \mu_L]\pi(i, 1, K, 1)$$
$$= \lambda_H \pi(i-1, 1, K, 1) + \lambda_{H2} \pi(i-2, 1, K, 1) + \beta \pi(i, 1, K-1, 1) +$$
$$(i+1)\alpha_H \pi(i+1, 1, K, 1) + \alpha_L \pi(i, 2, K, 1).$$

Case 54: For $2 \le i \le N-1,\ j = 2,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + \alpha_L + \mu_L]\pi(i, 2, K, 1)$$
$$= \lambda_H \pi(i-1, 2, K, 1) + \lambda_{H2} \pi(i-2, 2, K, 1) + \lambda_L \pi(i, 1, K, 1)$$
$$+ \beta \pi(i, 2, K-1, 1) + (i+1)\alpha_H \pi(i+1, 2, K, 1) + 2\alpha_L \pi(i, 3, K, 1).$$

Case 55: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ x = K$, and $y = 1$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + (j-1)\alpha_L + \mu_L]\pi(i, j, K, 1)$$
$$= \lambda_H \pi(i-1, j, K, 1) + \lambda_L \pi(i, j-1, K, 1) + \lambda_{H2} \pi(i-2, j, K, 1)$$
$$+ \lambda_{L2} \pi(i, j-2, K, 1) + \beta \pi(i, j, K-1, 1) + (i+1)\alpha_H \pi(i+1, j, K, 1)$$
$$+ j\alpha_L \pi(i, j+1, K, 1).$$

Case 56: For $2 \le i \le N-2,\ j = N-i+1,\ x = K$, and $y = 1$,

$$[i\alpha_H + (N-i)\alpha_L + \mu_L]\pi(i, N-i+1, K, 1)$$
$$= \lambda_H \pi(i-1, N-i+1, K, 1) + \lambda_L \pi(i, N-i, K, 1)$$
$$+ \lambda_{H2} \pi(i-2, N-i+1, K, 1) + \lambda_{H2} \pi(i-1, N-i+1, K, 1)$$
$$+ \lambda_{L2} \pi(i, N-i-1, K, 1) + \lambda_{L2} \pi(i, N-i, K, 1)$$
$$+ \beta \pi(i, N-i+1, K-1, 1).$$

Case 57: For $i = N-1,\ j = 2,\ x = K$, and $y = 1$,

$$[(N-1)\alpha_H + \alpha_L + \mu_L]\pi(N-1, 2, K, 1)$$
$$= \lambda_H \pi(N-2, 2, K, 1) + \lambda_L \pi(N-1, 1, K, 1) + \lambda_{H2} \pi(N-3, 2, K, 1)$$
$$+ \lambda_{H2} \pi(N-2, 2, K, 1) + +\lambda_{L2} \pi(N-1, 1, K, 1)$$
$$+ \beta \pi(N-1, 2, K-1, 1).$$

Case 58: For $i = N,\ j = 1,\ x = K$, and $y = 1$,

$$(N\alpha_H + \mu_L)\pi(N,1,K,1)$$
$$= \lambda_H\pi(N-1,1,K,1) + \lambda_{H2}\pi(N-2,1,x,1) + \lambda_{H2}\pi(N-1,1,x,1)$$
$$+ \beta\pi(N,1,K-1,1).$$

Case 59: For $i = 1,\ j = 0,\ x = 0$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_H)\pi(1,0,0,2)$$
$$= \lambda_H\pi(0,0,1,0) + \beta\pi(1,0,0,0) + \alpha_H\pi(2,0,0,2) + \alpha_L\pi(1,1,0,2) +$$
$$\mu_H\pi(2,0,1,2) + \mu_L\pi(1,1,1,1) + \mu_H\pi(2,0,1,4) + \mu_L\pi(1,1,1,3).$$

Case 60: For $i = 1,\ j = 1,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1,1,0,2)$$
$$= \lambda_L\pi(1,0,0,2) + \beta\pi(1,1,0,0) + \alpha_H\pi(2,1,0,2) +$$
$$2\alpha_L\pi(1,2,0,2) + \mu_H\pi(2,1,1,2) + \mu_L\pi(1,2,1,1) +$$
$$\mu_H\pi(2,1,1,4) + \mu_L\pi(1,2,1,3).$$

Case 61: For $i = 1,\ 2 \le j \le N-1,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + j\alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1,j,0,2)$$
$$= \lambda_L\pi(1,j-1,0,2) + \lambda_{L2}\pi(1,j-2,0,2) + \beta\pi(1,j,0,0) + \alpha_H\pi(2,j,0,2) +$$
$$(j+1)\alpha_L\pi(1,j+1,0,2) + \mu_H\pi(2,j,1,2) + \mu_L\pi(1,j+1,1,1) +$$
$$\mu_H\pi(2,j,1,4) + \mu_L\pi(1,j+1,1,3).$$

Case 62: For $i = 1,\ j = N,\ x = 0$, and $y = 2$,

$$[\beta + N\alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1,N,0,2)$$
$$= \lambda_L\pi(1,N-1,0,2) + \lambda_{L2}\pi(1,N-2,0,2) + \lambda_{L2}\pi(1,N-1,0,2).$$

Case 63: For $i = 2,\ j = 0,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2,0,0,2)$$
$$= \lambda_H\pi(1,0,0,2) + \lambda_{H2}\pi(0,0,1,0) + \beta\pi(2,0,0,0) + 2\alpha_H\pi(3,0,0,2) +$$
$$\alpha_L\pi(2,1,0,2) + \mu_H\pi(3,0,1,2) + \mu_L\pi(2,1,1,1) + \mu_H\pi(3,0,1,4) +$$
$$\mu_L\pi(2,1,1,3).$$

Case 64: For $i = 2,\ j = 1,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2,1,0,2)$$
$$= \lambda_H\pi(1,1,0,2) + \lambda_L\pi(2,0,0,2) + \beta\pi(2,1,0,0) + 2\alpha_H\pi(3,1,0,2) +$$
$$\alpha_L\pi(2,2,0,2) + \mu_H\pi(3,1,1,2) + \mu_L\pi(2,2,1,1) + \mu_H\pi(3,1,1,4) +$$
$$\mu_L\pi(2,2,1,3).$$

Case 65: For $i = 2,\ 2 \le j \le N-2,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + j\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2,j,0,2)$$
$$= \lambda_H\pi(1,j,0,2) + \lambda_L\pi(2,j-1,0,2) + \lambda_{L2}\pi(2,j-2,0,2) + \beta\pi(2,j,0,0)$$
$$+ 2\alpha_H\pi(3,j,0,2) + (j+1)\alpha_L\pi(2,j+1,0,2) + \mu_H\pi(3,j,1,2)$$
$$+ \mu_L\pi(2,j+1,1,1) + \mu_H\pi(3,j,1,4) + \mu_L\pi(2,j+1,1,3).$$

Case 66: For $i = 2,\ j = N-1,\ x = 0$, and $y = 2$,

$$[\beta + \alpha_H + (N-1)\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2,N-1,0,2)$$
$$= \lambda_H\pi(1,N-1,0,2) + \lambda_L\pi(2,N-2,0,2) + \lambda_{H2}\pi(1,N-1,0,2)$$
$$+ \lambda_{L2}\pi(2,N-3,0,2) + \lambda_{L2}\pi(2,N-2,0,2).$$

Case 67: For $3 \le i \le N,\ j = 0,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(i,0,0,2)$$
$$= \lambda_H\pi(i-1,0,0,2) + \lambda_{H2}\pi(i-2,0,0,2) + i\alpha_H\pi(i+1,0,0,2)$$
$$+ \alpha_L\pi(i,1,0,2) + \mu_H\pi(i+1,0,1,2) + \mu_L\pi(i,1,1,1) +$$
$$\mu_H\pi(i+1,0,1,4) + \mu_L\pi(i,1,1,3).$$

Case 68: For $3 \le i \le N-1,\ j = 1,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(i,1,0,2)$$
$$= \lambda_H\pi(i-1,1,0,2) + \lambda_L\pi(i,0,0,2) + \lambda_{H2}\pi(i-2,1,0,2)$$
$$+ i\alpha_H\pi(i+1,1,0,2) + 2\alpha_L\pi(i,2,0,2) + \mu_H\pi(i+1,1,1,2)$$
$$+ \mu_L\pi(i,2,1,1) + \mu_H\pi(i+1,1,1,4) + \mu_L\pi(i,2,1,3).$$

Case 69: For $3 \le i \le N-2,\ 2 \le j \le N-i,\ x = 0$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + j\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(i,j,0,2)$$

$$= \lambda_H \pi(i-1,j,0,2) + \lambda_L \pi(i,j-1,0,2) + \lambda_{H2} \pi(i-2,j,0,2) + \lambda_{L2} \pi(i,j-2,0,2) + i\alpha_H \pi(i+1,j,0,2) + (j+1)\alpha_L \pi(i,j+1,0,2) + \mu_H \pi(i+1,j,1,2) + \mu_L \pi(i,j+1,1,1) + \mu_H \pi(i+1,j,1,4) + \mu_L \pi(i,j+1,1,3).$$

Case 70: For $3 \le i \le N-1,\ j = N-i+1,\ x = 0$, and $y = 2$,

$$[\beta + (i-1)\alpha_H + j\alpha_L + (1-\theta_H)\mu_L + \theta_H \mu_L]\pi(i,j,0,2)$$
$$= \lambda_H \pi(i-1,j,0,2) + \lambda_L \pi(i,j-1,0,2) + \lambda_{H2} \pi(i-2,j,0,2) + \lambda_{H2} \pi(i-1,j,0,2) + \lambda_{L2} \pi(i,j-2,0,2) + \lambda_{L2} \pi(i,j-1,0,2).$$

Case 71: For $i = N,\ j = 1\ x = 0$, and $y = 2$,

$$[\beta + (N-1)\alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H \mu_L]\pi(N,1,0,2)$$
$$= \lambda_H \pi(N-1,1,0,2) + \lambda_L \pi(N,0,0,2) + \lambda_{H2} \pi(N-2,1,0,2) + \lambda_{H2} \pi(N-1,1,0,2) + \lambda_{L2} \pi(N,0,0,2)$$

Case 72: For $i = N+1,\ j = 0,\ x = 0$, and $y = 2$,

$$[\beta + N\alpha_H + (1-\theta_H)\mu_L + \theta_H \mu_L]\pi(N+1,0,0,2)$$
$$= \lambda_H \pi(N,0,0,2) + \lambda_{H2} \pi(N-1,0,0,2).$$

Case 73: For $i = 1,\ j = 0,\ 1 \le x \le K-1$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_H)\pi(1,0,x,2)$$
$$= \lambda_H \pi(0,0,x+1,0) + \beta \pi(1,0,x-1,2) + \alpha_H \pi(2,0,x,2) + \alpha_L \pi(1,1,x,2) + \mu_H \pi(2,0,x+1,2) + \mu_L \pi(1,1,x+1,1) + \mu_H \pi(2,0,x+1,4) + \mu_L \pi(1,1,x+1,3).$$

Case 74: For $i = 1,\ j = 1,\ 1 \le x \le K-1$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + \mu_H)\pi(1,1,x,2)$$
$$= \lambda_L \pi(1,0,x,2) + \beta \pi(1,1,x-1,2) + \alpha_H \pi(2,1,x,2) + 2\alpha_L \pi(1,2,x,2) + \mu_H \pi(2,1,x+1,2) + \mu_L \pi(1,2,x+1,1) + \mu_H \pi(2,1,x+1,4) + \mu_L \pi(1,2,x+1,3).$$

Case 75: For $i = 1,\ 2 \le j \le N-1,\ 1 \le x \le K-1$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + j\alpha_L + \mu_H)\pi(1, j, x, 2)$$
$$= \lambda_L\pi(1, j-1, x, 2) + \lambda_{L2}\pi(1, j-2, x, 2) + \beta\pi(1, j, x-1, 2) + \alpha_H\pi(2, j, x, 2)$$
$$+ (j+1)\alpha_L\pi(1, j+1, x, 2) + \mu_H\pi(2, j, x+1, 2) + \mu_L\pi(1, j+1, x+1, 1)$$
$$+ \mu_H\pi(2, j, x+1, 4) + \mu_L\pi(1, j+1, x+1, 3).$$

Case 76: For $i = 1,\ j = N,\ 1 \le x \le K-1$, and $y = 2$,

$$(\beta + N\alpha_L + \mu_H)\pi(1, N, x, 2)$$
$$= \lambda_L\pi(1, N-1, x, 2) + \lambda_{L2}\pi(1, N-2, x, 2) + \lambda_{L2}\pi(1, N-1, x, 2)$$
$$+ \beta\pi(1, N, x-1, 2).$$

Case 77: For $i = 2,\ j = 0,\ 1 \le x \le K-1$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \mu_H]\pi(2, 0, x, 2)$$
$$= \lambda_H\pi(1, 0, x, 2) + \beta\pi(2, 0, x-1, 2) + 2\alpha_H\pi(3, 0, x, 2) +$$
$$\alpha_L\pi(2, 1, x, 2) + \mu_H\pi(3, 0, x+1, 2) + \mu_L\pi(2, 1, x+1, 1) +$$
$$\mu_H\pi(2+1, 0, x+1, 4) + \mu_L\pi(2, 1, x+1, 3).$$

Case 78: For $i = 2,\ j = 1,\ 1 \le x \le K-1$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + \mu_H]\pi(2, 1, x, 2)$$
$$= \lambda_H\pi(1, 1, x, 2) + \lambda_L\pi(2, 0, x, 2) + \beta\pi(2, 1, x-1, 2) +$$
$$2\alpha_H\pi(3, 1, x, 2) + 2\alpha_L\pi(2, 2, x, 2) + \mu_H\pi(3, 1, x+1, 2) +$$
$$\mu_L\pi(2, 2, x+1, 1) + \mu_H\pi(3, 1, x+1, 4) + \mu_L\pi(2, 2, x+1, 3).$$

Case 79: For $i = 2,\ 2 \le j \le N-2,\ 1 \le x \le K-1$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + j\alpha_L + \mu_H]\pi(2, j, x, 2)$$
$$= \lambda_H\pi(1, j, x, 2) + \lambda_L\pi(2, j-1, x, 2) + \lambda_{L2}\pi(2, j-2, x, 2) + \beta\pi(2, j, x-1, 2)$$
$$+2\alpha_H\pi(3, j, x, 2) + (j+1)\alpha_L\pi(2, j+1, x, 2) + \mu_H\pi(3, j, x+1, 2) +$$
$$\mu_L\pi(2, j+1, x+1, 1) + \mu_H\pi(3, j, x+1, 4) + \mu_L\pi(2, j+1, x+1, 3).$$

Case 80: For $i = 2,\ j = N-1,\ 1 \le x \le K-1$, and $y = 2$,

$$[\beta + \alpha_H + (N-1)\alpha_L + \mu_H]\pi(2, N-1, x, 2)$$
$$= \lambda_H\pi(1, N-1, x, 2) + \lambda_L\pi(2, N-2, x, 2) + \lambda_{L2}\pi(2, N-3, x, 2)$$
$$+ \lambda_{L2}\pi(2, N-2, x, 2) + \beta\pi(2, N-1, x-1, 2).$$

Case 81: For $3 \le i \le N,\ j = 0,\ 1 \le x \le K-1$, and $y = 2$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \mu_H]\pi(i,0,x,2) \\
&= \lambda_H \pi(i-1,0,x,2) + \lambda_{H2}\pi(i-2,0,x,2) + \beta\pi(i,0,x-1,2) + \\
&\quad i\alpha_H \pi(i+1,0,x,2) + \alpha_L \pi(i,1,x,2) + \mu_H \pi(i+1,0,x+1,2) \\
&+ \mu_L \pi(i,1,x+1,1) + \mu_H \pi(i+1,0,x+1,4) + \mu_L \pi(i,1,x+1,3).
\end{aligned}
$$

Case 82: For $3 \le i \le N-1,\ j = 1,\ 1 \le x \le K-1$, and $y = 2$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \alpha_L + \mu_H]\pi(i,1,x,2) \\
&= \lambda_H \pi(i-1,1,x,2) + \lambda_{H2}\pi(i-2,1,x,2) + \lambda_L \pi(i,0,x,2) + \beta\pi(i,1,x-1,2) \\
&+ i\alpha_H \pi(i+1,1,x,2) + 2\alpha_L \pi(i,2,x,2) + \mu_H \pi(i+1,1,x+1,2) \\
&\qquad + \mu_L \pi(i,2,x+1,1) + \mu_H \pi(i+1,1,x+1,4) + \mu_L \pi(i,2,x \\
&\qquad + 1,3).
\end{aligned}
$$

Case 83: For $3 \le i \le N-2,\ 2 \le j \le N-i,\ 1 \le x \le K-1$, and $y = 2$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + j\alpha_L + \mu_H]\pi(i,j,x,2) \\
&= \lambda_H \pi(i-1,j,x,2) + \lambda_L \pi(i,j-1,x,2) + \lambda_{H2}\pi(i-2,j,x,2) \\
&\qquad + \lambda_{L2}\pi(i,j-2,x,2) + \beta\pi(i,j,x-1,2) + i\alpha_H \pi(i+1,j,x,2) \\
&\qquad + (j+1)\alpha_L \pi(i,j+1,x,2) + \mu_H \pi(i+1,j,x+1,2) \\
&\qquad + \mu_L \pi(i,j+1,x+1,1) + \mu_H \pi(i+1,j,x+1,4) + \mu_L \pi(i,j \\
&\qquad + 1,x+1,3).
\end{aligned}
$$

Case 84: For $3 \le i \le N-1,\ j = N-i+1,\ 1 \le x \le K-1$, and $y = 2$,

$$
\begin{aligned}
&[\beta + (i-1)\alpha_H + (N-i+1)\alpha_L + \mu_H]\pi(i,N-i+1,x,2) \\
&= \lambda_H \pi(i-1,N-i+1,x,2) + \lambda_L \pi(i,N-i,x,2) \\
&\qquad + \lambda_{H2}\pi(i-2,N-i+1,x,2) + \lambda_{H2}\pi(i-1,N-i+1,x,2) \\
&\qquad + \lambda_{L2}\pi(i,N-i-1,x,2) + \lambda_{L2}\pi(i,N-i,x,2) \\
&\qquad + \beta\pi(i,N-i+1,x-1,2).
\end{aligned}
$$

Case 85: For $i = N,\ j = 1,\ 1 \le x \le K-1$, and $y = 2$,

$$
\begin{aligned}
&[\beta + (N-1)\alpha_H + \alpha_L + \mu_H]\pi(N,1,x,2) \\
&= \lambda_H \pi(N-1,1,x,2) + \lambda_L \pi(N,0,x,2) + \lambda_{H2}\pi(N-2,1,x,2) \\
&\qquad + \lambda_{H2}\pi(N-1,1,x,2) + \lambda_{L2}\pi(N,0,x,2) + \beta\pi(N,1,x-1,2).
\end{aligned}
$$

Case 86: For $i = N + 1,\ j = 0,\ 1 \le x \le K - 1$, and $y = 2$,

$$(\beta + N\alpha_H + \mu_H)\pi(N+1, 0, x, 2)$$
$$= \lambda_H \pi(N, 0, x, 2) + \lambda_{H2}\pi(N-1, 0, x, 2) + \lambda_{H2}\pi(N, 0, x, 2) + \beta\pi(N+1, 0, x - 1, 2).$$

Case 87: For $i = 1,\ j = 0,\ x = K$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \mu_H)\pi(1, 0, K, 2)$$
$$= \beta\pi(1, 0, K-1, 2) + \alpha_H\pi(2, 0, K, 2) + \alpha_L\pi(1, 1, K, 2).$$

Case 88: For $i = 1,\ j = 1,\ x = K$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_L + \mu_H)\pi(1, 1, K, 2)$$
$$= \lambda_L\pi(1, 0, K, 2) + \beta\pi(1, 1, K-1, 2) + \alpha_H\pi(2, 1, K, 2) + 2\alpha_L\pi(1, 2, K, 2).$$

Case 89: For $i = 1,\ 2 \le j \le N - 1,\ x = K$, and $y = 2$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + j\alpha_L + \mu_H)\pi(1, j, K, 2)$$
$$= \lambda_L\pi(1, j-1, K, 2) + \lambda_{L2}\pi(1, j-2, K, 2) + \beta\pi(1, j, K-1, 2)$$
$$+ \alpha_H\pi(2, j, K, 2) + (j+1)\alpha_L\pi(1, j+1, K, 2).$$

Case 90: For $i = 1,\ j = N,\ x = K$, and $y = 2$,

$$(N\alpha_L + \mu_H)\pi(1, N, K, 2)$$
$$= \lambda_L\pi(1, N-1, K, 2) + \lambda_{L2}\pi(1, N-2, K, 2) + \lambda_{L2}\pi(1, N-1, K, 2) + \beta\pi(1, N, K-1, 2).$$

Case 91: For $i = 2,\ j = 0,\ x = K$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \mu_H]\pi(2, 0, K, 2)$$
$$= \lambda_H\pi(1, 0, K, 2) + \beta\pi(2, 0, K-1, 2) + 2\alpha_H\pi(3, 0, K, 2) + \alpha_L\pi(2, 1, K, 2).$$

Case 92: For $i = 2,\ j = 1,\ x = K$, and $y = 2$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \alpha_L + \mu_H]\pi(2, 1, K, 2)$$
$$= \lambda_H\pi(1, 1, K, 2) + \lambda_L\pi(2, 0, K, 2) + \beta\pi(2, 1, K-1, 2) +$$

$$2\alpha_H\pi(3,1,K,2) + 2\alpha_L\pi(2,2,K,2).$$

Case 93: For $i = 2,\ 2 \le j \le N-2,\ x = K$, and $y = 2$,

$$\begin{aligned}&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + j\alpha_L + \mu_H]\pi(2,j,K,2)\\&= \lambda_H\pi(1,j,K,2) + \lambda_L\pi(2,j-1,K,2) + \lambda_{L2}\pi(2,j-2,K,2)\\&+ \beta\pi(2,j,K-1,2) + 2\alpha_H\pi(3,j,K,2) + (j+1)\alpha_L\pi(2,j+1,K,2).\end{aligned}$$

Case 94: For $i = 2,\ j = N-1,\ x = K$, and $y = 2$,

$$\begin{aligned}&[\alpha_H + (N-1)\alpha_L + \mu_H]\pi(2,N-1,K,2)\\&= \lambda_H\pi(1,N-1,K,2) + \lambda_L\pi(2,N,K,2) + \lambda_{H2}\pi(1,N-1,K,2)\\&+ \lambda_{L2}\pi(2,N-1,K,2) + \lambda_{L2}\pi(2,N,K,2) + \beta\pi(2,N-1,K-1,2).\end{aligned}$$

Case 95: For $3 \le i \le N,\ j = 0,\ x = K$, and $y = 2$,

$$\begin{aligned}&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + \mu_H]\pi(i,0,K,2)\\&= \lambda_H\pi(i-1,0,K,2) + \lambda_{H2}\pi(i-2,0,K,2) + \beta\pi(i,0,K-1,2) +\\&\quad i\alpha_H\pi(i+1,0,K,2) + \alpha_L\pi(i,1,K,2).\end{aligned}$$

Case 96: For $3 \le i \le N-1,\ j = 1,\ x = K$, and $y = 2$,

$$\begin{aligned}&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + \alpha_L + \mu_H]\pi(i,1,K,2)\\&= \lambda_H\pi(i-1,1,K,2) + \lambda_{H2}\pi(i-2,1,K,2) + \lambda_L\pi(i,0,K,2)\\&+ \beta\pi(i,1,K-1,2) + i\alpha_H\pi(i+1,1,K,2) + 2\alpha_L\pi(i,2,K,2).\end{aligned}$$

Case 97: For $3 \le i \le N-2,\ 2 \le j \le N-i,\ x = K$, and $y = 2$,

$$\begin{aligned}&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + j\alpha_L + \mu_H]\pi(i,j,K,2)\\&= \lambda_H\pi(i-1,j,K,2) + \lambda_L\pi(i,j-1,K,2) + \lambda_{H2}\pi(i-2,j,K,2)\\&+ \lambda_{L2}\pi(i,j-2,K,2) + \beta\pi(i,j,K-1,2) +\\&\quad i\alpha_H\pi(i+1,j,K,2) + (j+1)\alpha_L\pi(i,j+1,K,2).\end{aligned}$$

Case 98: For $3 \le i \le N-1,\ j = N-i+1,\ x = K$, and $y = 2$,

$$[(i-1)\alpha_H + (N-i+1)\alpha_L + \mu_H]\pi(i,N-i+1,K,2)$$

$$= \lambda_H \pi(i-1, N-i+1, K, 2) + \lambda_L \pi(i, N-i, K, 2)$$
$$+ \lambda_{H2} \pi(i-2, N-i+1, K, 2) + \lambda_{H2} \pi(i-1, N-i+1, K, 2)$$
$$+ \lambda_{L2} \pi(i, N-i-1, K, 2) + \lambda_{L2} \pi(i, N-i, K, 2)$$
$$+ \beta \pi(i, N-i+1, K-1, 2).$$

Case 99: For $i = N,\ j = 1,\ x = K$, and $y = 2$,

$$[(N-1)\alpha_H + \alpha_L + \mu_H]\pi(N, 1, K, 2)$$
$$= \lambda_H \pi(N-1, 1, K, 2) + \lambda_L \pi(N, 0, K, 2) + \lambda_{H2} \pi(N-2, 1, K, 2)$$
$$+ \lambda_{H2} \pi(N-1, 1, K, 2) + \lambda_{L2} \pi(N, 0, K, 2) + \beta \pi(N, 1, K-1, 2).$$

Case 100: For $i = N+1,\ j = 0,\ x = K$, and $y = 2$,

$$(N\alpha_H + \mu_H)\pi(N+1, 0, K, 2)$$
$$= \lambda_H \pi(N, 0, K, 2) + \lambda_{H2} \pi(N-1, 0, K, 2) + \lambda_{H2} \pi(N, 0, K, 2) + \beta \pi(N+1, 0, K$$
$$- 1, 2).$$

Case 101: For $i = 0,\ j = 1,\ x = 0$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_L)\pi(0, 1, 0, 3)$$
$$= \theta_L \lambda_L \pi(0, 0, 0, 0) + \alpha_H \pi(1, 1, 0, 3) + \theta_L \alpha_H \pi(1, 1, 0, 0) + \alpha_L \pi(0, 2, 0, 3) +$$
$$\theta_L 2\alpha_L \pi(0, 2, 0, 0) + \theta_L \mu_H \pi(1, 1, 0, 2) + \theta_L \mu_L \pi(0, 2, 0, 1) +$$
$$\theta_L \mu_H \pi(1, 1, 0, 4) + \theta_L \mu_L \pi(0, 2, 0, 3).$$

Case 102: For $i = 0,\ j = 2,\ x = 0$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + (1-\theta_L)\mu_L + \theta_L \mu_L]\pi(0, 2, 0, 3)$$
$$= \theta_L \lambda_L \pi(0, 1, 0, 0) + \lambda_L \pi(0, 1, 0, 3) + \theta_L \lambda_{L2} \pi(0, 0, 0, 0) + \alpha_H \pi(1, 2, 0, 3) +$$
$$\theta_L \alpha_H \pi(1, 2, 0, 0) + 2\alpha_L \pi(0, 3, 0, 3) + \theta_L 3\alpha_L \pi(0, 3, 0, 0) +$$
$$\theta_L \mu_H \pi(1, 2, 0, 2) + \theta_L \mu_L \pi(0, 3, 0, 1) + \theta_L \mu_H \pi(1, 2, 0, 4) +$$
$$\theta_L \mu_L \pi(0, 3, 0, 3).$$

Case 103: For $i = 0,\ 3 \le j \le N-1,\ x = 0$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (j-1)\alpha_L + (1-\theta_L)\mu_L + \theta_L \mu_L]\pi(0, j, 0, 3)$$
$$= \theta_L \lambda_L \pi(0, j-1, 0, 0) + \lambda_L \pi(0, j-1, 0, 3) + \theta_L \lambda_{L2} \pi(0, j-2, 0, 0)$$
$$+ \lambda_{L2} \pi(0, j-2, 0, 3) + \alpha_H \pi(1, j, 0, 3) +$$
$$\theta_L \alpha_H \pi(1, j, 0, 0) + j\alpha_L \pi(0, j+1, 0, 3) + \theta_L (j+1)\alpha_L \pi(0, j+1, 0, 0) +$$

$\theta_L\mu_H\pi(1,j,0,2) + \theta_L\mu_L\pi(0,j+1,0,1) + \theta_L\mu_H\pi(1,j,0,4) +$
$\theta_L\mu_L\pi(0,j+1,0,3).$

Case 104: For $i = 0,\ j = N,\ x = 0,$ and $y = 3,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (N-1)\alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(0,N,0,3)$$
$$= \theta_L\lambda_L\pi(0,N-1,0,0) + \lambda_L\pi(0,N-1,0,3) + \theta_L\lambda_{L2}\pi(0,N-2,0,0)$$
$$+ \lambda_{L2}\pi(0,N-2,0,3) + \lambda_{L2}\pi(0,N-1,0,3) + \alpha_H\pi(1,N,0,3) +$$
$$N\alpha_L\pi(0,N+1,0,3) + \theta_L\mu_H\pi(1,N,0,2) + \theta_L\mu_L\pi(0,N+1,0,1) +$$
$$\theta_L\mu_H\pi(1,N,0,4) + \theta_L\mu_L\pi(0,N+1,0,3).$$

Case 105: For $i = 0,\ j = N+1,\ x = 0,$ and $y = 3,$

$$[\beta + N\alpha_L + (1-\theta_L)\mu_L + \theta_L\mu_L]\pi(0,N+1,0,3)$$
$$= \lambda_L\pi(0,N,0,3) + \lambda_{L2}\pi(0,N-2,0,3) + \lambda_{L2}\pi(0,N-1,0,3).$$

Case 106: For $i = 1,\ j = 1,\ x = 0,$ and $y = 3,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,1,0,3)$$
$$= \lambda_H\pi(0,1,0,3) + 2\alpha_H\pi(2,1,0,3) + \alpha_L\pi(1,2,0,3).$$

Case 107: For $i = 1,\ j = 2,\ x = 0,$ and $y = 3,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,2,0,3)$$
$$= \lambda_H\pi(0,2,0,3) + \lambda_L\pi(1,1,0,3) + 2\alpha_H\pi(2,2,0,3) +$$
$$2\alpha_L\pi(1,3,0,3).$$

Case 108: For $i = 1,\ 3 \le j \le N-1,\ x = 0,$ and $y = 3,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (j-1)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,j,0,3)$$
$$= \lambda_H\pi(0,j,0,3) + \lambda_L\pi(1,j-1,0,3) + \lambda_{L2}\pi(1,j-2,0,3) + 2\alpha_H\pi(2,j,0,3) +$$
$$j\alpha_L\pi(1,j+1,0,3).$$

Case 109: For $i = 1,\ j = N,\ x = 0,$ and $y = 3,$

$$[\beta + \alpha_H + (N-1)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(1,N,0,3)$$
$$= \lambda_H\pi(0,N,0,3) + \lambda_L\pi(1,N-1,0,3) + \lambda_{L2}\pi(1,N-2,0,3).$$

Case 110: For $2 \le i \le N-1,\ j = 1,\ x = 0,$ and $y = 3,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,1,0,3)$$
$$= \lambda_H\pi(i-1,1,0,3) + \lambda_{H2}\pi(i-2,1,0,3) + (i+1)\alpha_H\pi(i+1,1,0,3)$$
$$+ \alpha_L\pi(i,2,0,3).$$

Case 111: For $2 \le i \le N-2,\ j=2,\ x=0$, and $y=3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,2,0,3)$$
$$= \lambda_H\pi(i-1,2,0,3) + \lambda_{H2}\pi(i-2,2,0,3) + \lambda_L\pi(i,1,0,3)$$
$$+ (i+1)\alpha_H\pi(i+1,2,0,3) + 2\alpha_L\pi(i,3,0,3).$$

Case 112: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ x=0$, and $y=3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (j-1)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,j,0,3)$$
$$= \lambda_H\pi(i-1,j,0,3) + \lambda_L\pi(i,j-1,0,3) + \lambda_{H2}\pi(i-2,j,0,3)$$
$$+ \lambda_{L2}\pi(i,j-2,0,3) + (i+1)\alpha_H\pi(i+1,j,0,3) + j\alpha_L\pi(i,j+1,0,3).$$

Case 113: For $2 \le i \le N-2,\ j=N-i+1,\ x=0$, and $y=3$,

$$[\beta + i\alpha_H + (N-i)\alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(i,N-i+1,0,3)$$
$$= \lambda_H\pi(i-1,N-i+1,0,3) + \lambda_L\pi(i,N-i,0,3)$$
$$+ \lambda_{H2}\pi(i-2,N-i+1,0,3) + \lambda_{H2}\pi(i-1,N-i+1,0,3)$$
$$+ \lambda_{L2}\pi(i,N-i-1,0,3) + \lambda_{L2}\pi(i,N-i,0,3).$$

Case 114: For $i=N-1,\ j=2,\ x=0$, and $y=3$,

$$[\beta + i\alpha_H + \alpha_L + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(N-1,2,0,3)$$
$$= \lambda_H\pi(N-2,2,0,3) + \lambda_L\pi(N-1,1,0,3) + \lambda_{H2}\pi(N-3,2,0,3)$$
$$+ \lambda_{H2}\pi(N-2,2,0,3) + \lambda_{L2}\pi(N-1,1,0,3).$$

Case 115: For $i=N,\ j=1,\ x=0$, and $y=3$,

$$[\beta + N\alpha_H + (1-\theta_H)\mu_L + \theta_H\mu_L]\pi(N,1,0,3)$$
$$= \lambda_H\pi(N-1,1,0,3) + \lambda_{H2}\pi(N-2,2,0,3) + \lambda_{H2}\pi(N-1,2,0,3).$$

Case 116: For $i=0,\ j=1,\ 1 \le x \le K-1$, and $y=3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_L)\pi(0,1,x,3)$$
$$= \beta\pi(0,1,x-1,3) + \alpha_H\pi(1,1,x,3) + \alpha_L\pi(0,2,x,3).$$

Case 117: For $i = 0,\ j = 2,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + \mu_L]\pi(0,2,x,3)$$
$$= \lambda_L \pi(0,1,x,3) + \beta\pi(0,2,x-1,3) + \alpha_H \pi(1,2,x,3) + 2\alpha_L \pi(0,3,x,3).$$

Case 118: For $i = 0,\ 3 \le j \le N,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (j-1)\alpha_L + \mu_L]\pi(0,j,x,3)$$
$$= \lambda_L \pi(0,j-1,x,3) + \lambda_{L2}\pi(0,j-2,x,3) + \beta\pi(0,j,x-1,3) + \alpha_H \pi(1,j,x,3)$$
$$+ j\alpha_L \pi(0,j+1,x,3).$$

Case 119: For $i = 0,\ j = N+1,\ 1 \le x \le K-1$, and $y = 3$,

$$(\beta + N\alpha_L + \mu_L)\pi(0,N+1,x,3)$$
$$= \lambda_L \pi(0,N,x,3) + \lambda_{L2}\pi(0,N-1,x,3) + \lambda_{L2}\pi(0,N,x,3)$$
$$+ \beta\pi(0,N+1,x-1,3).$$

Case 120: For $i = 1,\ j = 1,\ 1 \le x \le K-1$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \mu_L)\pi(1,1,x,3)$$
$$= \lambda_H \pi(0,1,x,3) + \beta\pi(1,1,x-1,3) + 2\alpha_H \pi(2,1,x,3) +$$
$$\alpha_L \pi(1,2,x,3).$$

Case 121: For $i = 1,\ j = 2,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + \mu_L]\pi(1,2,x,3)$$
$$= \lambda_H \pi(0,2,x,3) + \lambda_L \pi(1,1,x,3) + \beta\pi(1,2,x-1,3) +$$
$$2\alpha_H \pi(2,2,x,3) + 2\alpha_L \pi(1,3,x,3).$$

Case 122: For $i = 1,\ 3 \le j \le N-1,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (j-1)\alpha_L + \mu_L]\pi(1,j,x,3)$$
$$= \lambda_H \pi(0,j,x,3) + \lambda_L \pi(1,j-1,x,3) + \lambda_{L2}\pi(1,j-2,x,3) + \beta\pi(1,j,x-1,3)$$
$$+ 2\alpha_H \pi(2,j,x,3) + j\alpha_L \pi(1,j+1,x,3).$$

Case 123: For $i = 1,\ j = N,\ 1 \le x \le K-1$, and $y = 3$,

$$[\beta + \alpha_H + (N-1)\alpha_L + \mu_L]\pi(1,N,x,3)$$

$$= \lambda_H \pi(0, N, x, 3) + \lambda_L \pi(1, N-1, x, 3) + \lambda_{L2} \pi(1, N-2, x, 3)$$
$$+ \lambda_{H2} \pi(0, N, x, 3) + \lambda_{L2} \pi(1, N-1, x, 3) + \beta \pi(1, N, x-1, 3).$$

Case 124: For $2 \le i \le N-1,\ j = 1,\ 1 \le x \le K-1$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \mu_L)\pi(i, 1, x, 3)$$
$$= \lambda_H \pi(i-1, 1, x, 3) + \lambda_{H2} \pi(i-2, 1, x, 3) + \beta \pi(i, 1, x-1, 3)$$
$$+ (i+1)\alpha_H \pi(i+1, 1, x, 3) + \alpha_L \pi(i, 2, x, 3).$$

Case 125: For $2 \le i \le N-2,\ j = 2,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + \alpha_L + \mu_L]\pi(i, 2, x, 3)$$
$$= \lambda_H \pi(i-1, 2, x, 3) + \lambda_L \pi(i, 1, x, 3) + \lambda_{H2} \pi(i-2, 2, x, 3) + \beta \pi(i, 2, x-1, 3)$$
$$+ (i+1)\alpha_H \pi(i+1, 2, x, 3) + 2\alpha_L \pi(i, 3, x, 3).$$

Case 126: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ 1 \le x \le K-1$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + i\alpha_H + (j-1)\alpha_L + \mu_L]\pi(i, j, x, 3)$$
$$= \lambda_H \pi(i-1, j, x, 3) + \lambda_L \pi(i, j-1, x, 3) + \lambda_{H2} \pi(i-2, j, x, 3)$$
$$+ \lambda_{L2} \pi(i, j-2, x, 3) + \beta \pi(i, j, x-1, 3) +$$
$$(i+1)\alpha_H \pi(i+1, j, x, 3) + j\alpha_L \pi(i, j+1, x, 3).$$

Case 127: For $2 \le i \le N-2,\ j = N-i+1,\ 1 \le x \le K-1$, and $y = 3$,

$$[\beta + i\alpha_H + (N-i)\alpha_L + \mu_L]\pi(i, N-i+1, x, 3)$$
$$= \lambda_H \pi(i-1, N-i+1, x, 3) + \lambda_L \pi(i, N-i, x, 3)$$
$$+ \lambda_{H2} \pi(i-2, N-i+1, x, 3) + \lambda_{L2} \pi(i, N-i-1, x, 3)$$
$$+ \lambda_{H2} \pi(i-1, N-i+1, x, 3) + \lambda_{L2} \pi(i, N-i, x, 3)$$
$$+ \beta \pi(i, N-i+1, x-1, 3).$$

Case 128: For $i = N-1,\ j = 2,\ 1 \le x \le K-1$, and $y = 3$,

$$[\beta + (N-1)\alpha_H + \alpha_L + \mu_L]\pi(N-1, 2, x, 3)$$
$$= \lambda_H \pi(N-2, 2, x, 3) + \lambda_L \pi(N-1, 1, x, 3) + \lambda_{H2} \pi(N-3, 2, x, 3)$$
$$+ \lambda_{H2} \pi(N-2, 2, x, 3) + \lambda_{L2} \pi(N-1, 1, x, 3)$$
$$+ \beta \pi(N-1, 2, x-1, 3).$$

Case 129: For $i = N,\ j = 1,\ 1 \le x \le K-1$, and $y = 3$,

$$(\beta + N\alpha_H + \mu_L)\pi(N,1,x,3)$$
$$= \lambda_H\pi(N-1,1,x,3) + \lambda_{H2}\pi(N-2,1,x,3) + \lambda_{H2}\pi(N-1,1,x,3)$$
$$+ \beta\pi(N,1,x-1,3).$$

Case 130: For $i = 0,\ j = 1,\ x = K$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \mu_L)\pi(0,1,K,3)$$
$$= \beta\pi(0,1,K-1,3) + \alpha_H\pi(1,1,K,3) + \alpha_L\pi(0,2,K,3).$$

Case 131: For $i = 0,\ j = 2,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_L + \mu_L]\pi(0,2,K,3)$$
$$= \lambda_L\pi(0,1,K,3) + \beta\pi(0,2,K-1,3) + \alpha_H\pi(1,2,K,3) +$$
$$2\alpha_L\pi(0,3,K,3).$$

Case 132: For $i = 0,\ 3 \le j \le N,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (j-1)\alpha_L + \mu_L]\pi(0,j,K,3)$$
$$= \lambda_L\pi(0,j-1,K,3) + \lambda_{L2}\pi(0,j-2,K,3) + \beta\pi(0,j,K-1,3)$$
$$+ \alpha_H\pi(1,j,K,3) + j\alpha_L\pi(0,j+1,K,3).$$

Case 133: For $i = 0,\ j = N+1,\ x = K$, and $y = 3$,

$$(N\alpha_L + \mu_L)\pi(0,N+1,K,3)$$
$$= \lambda_L\pi(0,N,K,3) + \lambda_{L2}\pi(0,N-1,K,3) + \lambda_{L2}\pi(0,N,K,3)$$
$$+ \beta\pi(0,N+1,K-1,3).$$

Case 134: For $i = 1,\ j = 1,\ x = K$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \mu_L)\pi(1,1,K,3)$$
$$= \lambda_H\pi(0,1,K,3) + \beta\pi(1,1,K-1,3) + 2\alpha_H\pi(2,1,K,3) +$$
$$\alpha_L\pi(1,2,K,3).$$

Case 135: For $i = 1,\ j = 2,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \alpha_L + \mu_L]\pi(1,2,K,3)$$
$$= \lambda_H\pi(0,2,K,3) + \lambda_L\pi(1,1,K,3) + \beta\pi(1,2,K-1,3) +$$
$$2\alpha_H\pi(2,2,K,3) + 2\alpha_L\pi(1,3,K,3).$$

Case 136: For $i = 1,\ 3 \le j \le N-1,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + (j-1)\alpha_L + \mu_L]\pi(1,j,K,3)$$
$$= \lambda_H\pi(0,j,K,3) + \lambda_L\pi(1,j-1,K,3) + \lambda_{L2}\pi(1,j-2,K,3)$$
$$+ \beta\pi(1,j,K-1,3) + 2\alpha_H\pi(2,j,K,3) + j\alpha_L\pi(1,j+1,K,3).$$

Case 137: For $i = 1,\ j = N,\ x = K$, and $y = 3$,

$$[\alpha_H + (N-1)\alpha_L + \mu_L]\pi(1,N,K,3)$$
$$= \lambda_H\pi(0,N,K,3) + \lambda_L\pi(1,N-1,K,3) + \lambda_{L2}\pi(1,N-2,K,3)$$
$$+ \lambda_{L2}\pi(1,N-1,K,3) + \beta\pi(1,N,K-1,3).$$

Case 138: For $2 \le i \le N-1,\ j = 1,\ x = K$, and $y = 3$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + \mu_L)\pi(i,1,K,3)$$
$$= \lambda_H\pi(i-1,1,K,3) + \lambda_{H2}\pi(i-2,1,K,3) + \beta\pi(i,1,K-1,3)$$
$$+ (i+1)\alpha_H\pi(i+1,1,K,3) + \alpha_L\pi(i,2,K,3).$$

Case 139: For $2 \le i \le N-2,\ j = 2,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + \alpha_L + \mu_L]\pi(i,2,K,3)$$
$$= \lambda_H\pi(i-1,2,K,3) + \lambda_L\pi(i,1,K,3) + \lambda_{H2}\pi(i-2,2,K,3)$$
$$+ \beta\pi(i,2,K-1,3) + (i+1)\alpha_H\pi(i+1,2,K,3) + 2\alpha_L\pi(i,3,K,3).$$

Case 140: For $2 \le i \le N-3,\ 3 \le j \le N-i,\ x = K$, and $y = 3$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + i\alpha_H + (j-1)\alpha_L + \mu_L]\pi(i,j,K,3)$$
$$= \lambda_H\pi(i-1,j,K,3) + \lambda_L\pi(i,j-1,K,3) + \lambda_{H2}\pi(i-2,j,K,3)$$
$$+ \lambda_{L2}\pi(i,j-2,K,3) + \beta\pi(i,j,K-1,3) +$$
$$(i+1)\alpha_H\pi(i+1,j,K,3) + j\alpha_L\pi(i,j+1,K,3).$$

Case 141: For $2 \le i \le N-2,\ j = N-i+1,\ x = K$, and $y = 3$,

$$[i\alpha_H + (N-i)\alpha_L + \mu_L]\pi(i,N-i+1,K,3)$$
$$= \lambda_H\pi(i-1,N-i+1,K,3) + \lambda_L\pi(i,N-i,K,3)$$
$$+ \lambda_{H2}\pi(i-2,N-i+1,K,3) + \lambda_{L2}\pi(i,N-i-1,K,3)$$
$$+ \lambda_{H2}\pi(i-1,N-i+1,K,3) + \lambda_{L2}\pi(i,N-i,K,3)$$
$$+ \beta\pi(i,N-i+1,K-1,3).$$

Case 142: For $i = N-1,\ j = 2,\ x = K$, and $y = 3$,

$$[(N-1)\alpha_H + \alpha_L + \mu_L]\pi(N-1,2,K,3)$$
$$= \lambda_H\pi(N-2,2,K,3) + \lambda_L\pi(N-1,1,K,3) + \lambda_{H2}\pi(N-3,2,K,3) + \lambda_{H2}\pi(N-2,2,K,3) + \lambda_{L2}\pi(N-1,1,K,3) + \beta\pi(N-1,2,K-1,3)$$

Case 143: For $i = N,\ j = 1,\ x = K$, and $y = 3$,

$$(N\alpha_H + \mu_L)\pi(N,1,K,3)$$
$$= \lambda_H\pi(N-1,1,K,3) + \lambda_{H2}\pi(N-2,1,K,3) + \lambda_{H2}\pi(N-1,1,K,3) + \beta\pi(N,1,K-1,3)$$

Case 144: For $i = 1,\ j = 0,\ x = 0$, and $y = 4$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \mu_H)\pi(1,0,0,4)$$
$$= \theta_H\lambda_H\pi(0,0,0,0) + \alpha_H\pi(2,0,0,4) + \theta_H 2\alpha_H\pi(2,0,0,0) + \alpha_L\pi(1,1,0,4) + \theta_H\alpha_L\pi(1,1,0,0) + \theta_H\mu_H\pi(2,0,0,2) + \theta_H\mu_L\pi(1,1,0,1) + \theta_H\mu_H\pi(2,0,0,4) + \theta_H\mu_L\pi(1,1,0,3).$$

Case 145: For $i = 1,\ j = 1,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1,1,0,4)$$
$$= \theta_H\lambda_H\pi(0,1,0,0) + \theta_H\lambda_L\pi(1,0,0,0) + \lambda_L\pi(1,0,0,4) + \alpha_H\pi(2,1,0,4) + \theta_H 2\alpha_H\pi(2,1,0,0) + 2\alpha_L\pi(1,2,0,4) + \theta_H 2\alpha_L\pi(1,2,0,0) + \theta_H\mu_H\pi(2,1,0,2) + \theta_H\mu_L\pi(1,2,0,1) + \theta_H\mu_H\pi(2,1,0,4) + \theta_H\mu_L\pi(1,2,0,3).$$

Case 146: For $i = 1,\ 2 \le j \le N-2,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + j\alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1,j,0,4)$$
$$= \theta_H\lambda_H\pi(0,j,0,0) + \theta_H\lambda_L\pi(1,j-1,0,0) + \lambda_L\pi(1,j-1,0,4) + \theta_H\lambda_{L2}\pi(1,j-2,0,0) + \lambda_{L2}\pi(1,j-2,0,4) + \alpha_H\pi(2,j,0,4) + \theta_H 2\alpha_H\pi(2,j,0,0) + (j+1)\alpha_L\pi(1,j+1,0,4) + \theta_H(j+1)\alpha_L\pi(1,j+1,0,0) + \theta_H\mu_H\pi(2,j,0,2) + \theta_H\mu_L\pi(1,j+1,0,1) + \theta_H\mu_H\pi(2,j,0,4) + \theta_H\mu_L\pi(1,j+1,0,3).$$

Case 147: For $i = 1,\ j = N-1,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (N-1)\alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1, N-1, 0, 4)$$
$$= \theta_H\lambda_H\pi(0, N-1, 0, 0) + \theta_H\lambda_L\pi(1, N-2, 0, 0) + \lambda_L\pi(1, N-2, 0, 4)$$
$$+ \theta_H\lambda_{L2}\pi(1, N-3, 0, 0) + \lambda_{L2}\pi(1, N-3, 0, 4) +$$
$$\alpha_H\pi(2, N-1, 0, 4) + N\alpha_L\pi(1, N, 0, 4) + \theta_H\mu_H\pi(2, N-1, 0, 2) +$$
$$\theta_H\mu_L\pi(1, N, 0, 1) + \theta_H\mu_H\pi(2, N-1, 0, 4) + \theta_H\mu_L\pi(1, N, 0, 3).$$

Case 148: For $i = 1,\ j = N,\ x = 0$, and $y = 4$,

$$[\beta + N\alpha_L + (1-\theta_L)\mu_H + \theta_L\mu_H]\pi(1, N, 0, 4)$$
$$= \lambda_L\pi(1, N-1, 0, 4) + \lambda_{L2}\pi(1, N-2, 0, 4) + \lambda_{L2}\pi(1, N-1, 0, 4).$$

Case 149: For $i = 2,\ j = 0,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2, 0, 0, 4)$$
$$= \theta_H\lambda_H\pi(1, 0, 0, 0) + \lambda_H\pi(1, 0, 0, 4) + \theta_H\lambda_{H2}\pi(0, 0, 0, 0) + 2\alpha_H\pi(3, 0, 0, 4) +$$
$$\theta_H 3\alpha_H\pi(3, 0, 0, 0) + \alpha_L\pi(2, 1, 0, 4) + \theta_H\alpha_L\pi(2, 1, 0, 0) +$$
$$\theta_H\mu_H\pi(3, 0, 0, 2) + \theta_H\mu_L\pi(2, 1, 0, 1) + \theta_H\mu_H\pi(3, 0, 0, 4) +$$
$$\theta_H\mu_L\pi(2, 1, 0, 3).$$

Case 150: For $i = 2,\ j = 1,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2, 1, 0, 4)$$
$$= \theta_H\lambda_H\pi(1, 1, 0, 0) + \lambda_H\pi(1, 1, 0, 4) + \theta_H\lambda_L\pi(2, 0, 0, 0) +$$
$$\lambda_L\pi(2, 0, 0, 4) + \theta_H\lambda_{H2}\pi(0, 1, 0, 0) + 2\alpha_H\pi(3, 1, 0, 4) + \theta_H 3\alpha_H\pi(3, 1, 0, 0) +$$
$$2\alpha_L\pi(2, 2, 0, 4) + \theta_H 2\alpha_L\pi(2, 2, 0, 0) +$$
$$\theta_H\mu_H\pi(3, 1, 0, 2) + \theta_H\mu_L\pi(2, 2, 0, 1) + \theta_H\mu_H\pi(3, 1, 0, 4) +$$
$$\theta_H\mu_L\pi(2, 2, 0, 3).$$

Case 151: For $i = 2,\ 2 \le j \le N-3,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + j\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(2, j, 0, 4)$$
$$= \theta_H\lambda_H\pi(1, j, 0, 0) + \lambda_H\pi(1, j, 0, 4) + \theta_H\lambda_L\pi(2, j-1, 0, 0) +$$
$$\lambda_L\pi(2, j-1, 0, 4) + \theta_H\lambda_{H2}\pi(0, j, 0, 0) + \theta_H\lambda_{L2}\pi(2, j-2, 0, 0)$$
$$+ \lambda_{H2}\pi(2, j-2, 0, 4) + 2\alpha_H\pi(3, j, 0, 4) + \theta_H 3\alpha_H\pi(3, j, 0, 0) +$$
$$(j+1)\alpha_L\pi(2, j+1, 0, 4) + \theta_H(j+1)\alpha_L\pi(2, j+1, 0, 0) +$$
$$\theta_H\mu_H\pi(3, j, 0, 2) + \theta_H\mu_L\pi(2, j+1, 0, 1) + \theta_H\mu_H\pi(3, j, 0, 4) +$$

$\theta_H \mu_L \pi(2, j+1, 0, 3)$.

Case 152: For $i = 2,\ j = N-2,\ x = 0$, and $y = 4$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + (N-2)\alpha_L + (1-\theta_H)\mu_H + \theta_H \mu_H] \\
&\pi(2, N-2, 0, 4) \\
&= \theta_H \lambda_H \pi(1, N-2, 0, 0) + \lambda_H \pi(1, N-2, 0, 4) + \\
&\quad \theta_H \lambda_L \pi(2, N-3, 0, 0) + \lambda_L \pi(2, N-3, 0, 4) + \theta_H \lambda_{H2} \pi(0, N-2, 0, 0) \\
&+ \theta_H \lambda_{L2} \pi(2, N-4, 0, 0) + \theta_H \lambda_{L2} \pi(2, N-4, 0, 4) + \theta_H \lambda_{L2} \pi(2, N-3, 0, 0) + \\
&\quad 2\alpha_H \pi(3, N-2, 0, 4) + (N-1)\alpha_L \pi(2, N-1, 0, 4) + \theta_H \mu_H \pi(3, N-2, 0, 2) \\
&+ \theta_H \mu_L \pi(2, N-1, 0, 1) + \theta_H \mu_H \pi(3, N-2, 0, 4) + \theta_H \mu_L \pi(2, N-1, 0, 3).
\end{aligned}
$$

Case 153: For $i = 2,\ j = N-1,\ x = 0$, and $y = 4$,

$$
\begin{aligned}
&[\beta + \alpha_H + (N-1)\alpha_L + (1-\theta_H)\mu_H + \theta_H \mu_H] \\
&\pi(2, N-1, 0, 4) \\
&= \lambda_H \pi(1, N-1, 0, 4) + \lambda_L \pi(2, N-2, 0, 4) + \lambda_{H2} \pi(1, N-1, 0, 4) \\
&\qquad + \lambda_{L2} \pi(2, N-3, 0, 4) + \lambda_{L2} \pi(2, N-2, 0, 4).
\end{aligned}
$$

Case 154: For $3 \le i \le N-1,\ j = 0,\ x = 0$, and $y = 4$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + (1-\theta_H)\mu_H + \theta_H \mu_H]\pi(i, 0, 0, 4) \\
&= \theta_H \lambda_H \pi(i-1, 0, 0, 0) + \lambda_H \pi(i-1, 0, 0, 4) + \theta_H \lambda_{H2} \pi(i-2, 0, 0, 0) \\
&\qquad + \lambda_{H2} \pi(i-2, 0, 0, 4) + i\alpha_H \pi(i+1, 0, 0, 4) + \\
&\quad \theta_H (i+1)\alpha_H \pi(i+1, 0, 0, 0) + \alpha_L \pi(i, 1, 0, 4) + \theta_H \alpha_L \pi(i, 1, 0, 0) + \\
&\quad \theta_H \mu_H \pi(i+1, 0, 0, 2) + \theta_H \mu_L \pi(i, 1, 0, 1) + \theta_H \mu_H \pi(i+1, 0, 0, 4) + \\
&\quad \theta_H \mu_L \pi(i, 1, 0, 3).
\end{aligned}
$$

Case 155: For $i = N,\ j = 0,\ x = 0$, and $y = 4$,

$$
\begin{aligned}
&[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (N-1)\alpha_H + (1-\theta_H)\mu_H + \theta_H \mu_H]\pi(N, 0, 0, 4) \\
&= \theta_H \lambda_H \pi(N-1, 0, 0, 0) + \lambda_H \pi(N-1, 0, 0, 4) + \theta_H \lambda_{H2} \pi(N-2, 0, 0, 0) \\
&+ \lambda_{H2} \pi(N-2, 0, 0, 4) + \theta_H \lambda_{H2} \pi(N-1, 0, 0, 0) + N\alpha_H \pi(N+1, 0, 0, 4) + \\
&\quad \alpha_L \pi(N, 1, 0, 4) + \theta_H \mu_H \pi(N+1, 0, 0, 2) + \theta_H \mu_L \pi(N, 1, 0, 1) + \\
&\quad \theta_H \mu_H \pi(N+1, 0, 0, 4) + \theta_H \mu_L \pi(N, 1, 0, 3).
\end{aligned}
$$

Case 156: For $3 \le i \le N-2,\ j = 1,\ x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(i,1,0,4)$$
$$= \theta_H\lambda_H\pi(i-1,1,0,0) + \lambda_H\pi(i-1,1,0,4) + \theta_H\lambda_L\pi(i,0,0,0) +$$
$$\lambda_L\pi(i,0,0,4) + \theta_H\lambda_{H2}\pi(i-2,1,0,0) + \lambda_{H2}\pi(i-2,1,0,4)$$
$$+ i\alpha_H\pi(i+1,1,0,4) + \theta_H(i+1)\alpha_H\pi(i+1,1,0,0) +$$
$$2\alpha_L\pi(i,2,0,4) + \theta_H 2\alpha_L\pi(i,2,0,0) +$$
$$\theta_H\mu_H\pi(i+1,1,0,2) + \theta_H\mu_L\pi(i,2,0,1) + \theta_H\mu_H\pi(i+1,1,0,4) +$$
$$\theta_H\mu_L\pi(i,2,0,3).$$

Case 157: For $i = N-1,\ j = 1,\ x = 0,$ and $y = 4,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (N-2)\alpha_H + \alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]$$
$$\pi(N-1,1,0,4)$$
$$= \theta_H\lambda_H\pi(N-2,1,0,0) + \lambda_H\pi(N-2,1,0,4) + \theta_H\lambda_L\pi(N-1,0,0,0)$$
$$+ \lambda_L\pi(N-1,0,0,4) + \theta_H\lambda_{H2}\pi(N-3,1,0,0) + \lambda_{H2}\pi(N-3,1,0,4)$$
$$+ \theta_H\lambda_{H2}\pi(N-2,1,0,0) + \theta_H\lambda_{L2}\pi(N-1,0,0,0) + (N-1)\alpha_H\pi(N,1,0,4)$$
$$+ 2\alpha_L\pi(N-1,2,0,4) + \theta_H\mu_H\pi(N,1,0,2) + \theta_H\mu_L\pi(N-1,2,0,1) +$$
$$\theta_H\mu_H\pi(N,1,0,4) + \theta_H\mu_L\pi(N-1,2,0,3).$$

Case 158: For $3 \le i \le N-3,\ 2 \le j \le N-i-1,\ x = 0,$ and $y = 4,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + j\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]\pi(i,j,0,4)$$
$$= \theta_H\lambda_H\pi(i-1,j,0,0) + \lambda_H\pi(i-1,j,0,4) + \theta_H\lambda_L\pi(i,j-1,0,0) +$$
$$\lambda_L\pi(i,j-1,0,4) + \theta_H\lambda_{H2}\pi(i-2,j,0,0) + \lambda_{H2}\pi(i-2,j,0,4)$$
$$+ \theta_H\lambda_{L2}\pi(i,j-2,0,0) + \lambda_{L2}\pi(i,j-2,0,4)$$
$$+ i\alpha_H\pi(i+1,j,0,4) + \theta_H(i+1)\alpha_H\pi(i+1,j,0,0) +$$
$$(j+1)\alpha_L\pi(i,j+1,0,4) + \theta_H(j+1)\alpha_L\pi(i,j+1,0,0) +$$
$$\theta_H\mu_H\pi(i+1,j,0,2) + \theta_H\mu_L\pi(i,j+1,0,1) + \theta_H\mu_H\pi(i+1,j,0,4) +$$
$$\theta_H\mu_L\pi(i,j+1,0,3).$$

Case 159: For $3 \le i \le N-3,\ j = N-i,\ x = 0,$ and $y = 4,$

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + (N-i)\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]$$
$$\pi(i,N-i,0,4)$$
$$= \theta_H\lambda_H\pi(i-1,N-i,0,0) + \lambda_H\pi(i-1,N-i,0,4) +$$

$$\theta_H\lambda_L\pi(i,N-i-1,0,0)+\lambda_L\pi(i,N-i-1,0,4)$$
$$+\theta_H\lambda_{H2}\pi(i-2,N-i,0,0)+\lambda_{H2}\pi(i-2,N-i,0,4)$$
$$+\theta_H\lambda_{H2}\pi(i-1,N-i,0,0)+\theta_H\lambda_{L2}\pi(i,N-i-2,0,0)$$
$$+\lambda_{L2}\pi(i,N-i-2,0,4)+\theta_H\lambda_{L2}\pi(i,N-i-1,0,0)+$$
$$i\alpha_H\pi(i+1,N-i,0,4)+(N-i+1)\alpha_L\pi(i,N-i+1,0,4)+$$
$$\theta_H\mu_H\pi(i+1,N-i,0,2)+\theta_H\mu_L\pi(i,N-i+1,0,1)+$$
$$\theta_H\mu_H\pi(i+1,N-i,0,4)+\theta_H\mu_L\pi(i,N-i+1,0,3).$$

Case 160: For $3\le i\le N-1,\ j=N-\mathrm{i}+1,\ x=0$, and $y=4$,

$$[\beta+(i-1)\alpha_H+(N-i+1)\alpha_L+(1-\theta_H)\mu_H+\theta_H\mu_H]$$
$$\pi(i,N-i+1,0,4)$$
$$=\lambda_H\pi(i-1,N-i+1,0,4)+\lambda_L\pi(i,N-i,0,4)$$
$$+\lambda_{H2}\pi(i-2,N-i+1,0,4)+\lambda_{H2}\pi(i-1,N-i+1,0,4)$$
$$+\lambda_{L2}\pi(i,N-i-1,0,4)+\lambda_{L2}\pi(i,N-i,0,4).$$

Case 161: For $i=N,\ j=1,\ x=0$, and $y=4$,

$$[\beta+(N-1)\alpha_H+\alpha_L+(1-\theta_H)\mu_H+\theta_H\mu_H]$$
$$\pi(N,1,0,4)$$
$$=\lambda_H\pi(N-1,1,0,4)+\lambda_L\pi(N,0,0,4)+\lambda_{H2}\pi(N-2,1,0,4)$$
$$+\lambda_{H2}\pi(N-1,1,0,4)+\lambda_{L2}\pi(N,0,0,4).$$

Case 162: For $i=N+1,\ j=0,\ x=0$, and $y=4$,

$$[\beta+N\alpha_H+(1-\theta_H)\mu_H+\theta_H\mu_H]\pi(N+1,0,0,4)$$
$$=\lambda_H\pi(N,0,0,4)+\lambda_{H2}\pi(N-1,0,0,4)+\lambda_{H2}\pi(N,0,0,4).$$

Case 163: For $i=1,\ j=0,\ 1\le x\le K-1$, and $y=4$,

$$(\lambda_H+\lambda_L+\lambda_{H2}+\lambda_{L2}+\beta+\mu_H)\pi(1,0,x,4)$$
$$=\beta\pi(1,0,x-1,4)+\alpha_H\pi(2,0,x,4)+\alpha_L\pi(1,1,x,4).$$

Case 164: For $i=1,\ j=1,\ 1\le x\le K-1$, and $y=4$,

$$(\lambda_H+\lambda_L+\lambda_{H2}+\lambda_{L2}+\beta+j\alpha_L+\mu_H)\pi(1,j,x,4)$$
$$=\lambda_L\pi(1,0,x,4)+\beta\pi(1,1,x-1,4)+\alpha_H\pi(2,1,x,4)+$$
$$2\alpha_L\pi(1,2,x,4).$$

Case 165: For $i = 1,\ 2 \le j \le N-1,\ 1 \le x \le K-1$, and $y = 4$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + j\alpha_L + \mu_H)\pi(1,j,x,4)$$
$$= \lambda_L\pi(1,j-1,x,4) + \lambda_{L2}\pi(1,j-2,x,4) + \beta\pi(1,j,x-1,4) + \alpha_H\pi(2,j,x,4) + (j+1)\alpha_L\pi(1,j+1,x,4).$$

Case 166: For $i = 1,\ j = N,\ 1 \le x \le K-1$, and $y = 4$,

$$(\beta + N\alpha_L + \mu_H)\pi(1,N,x,4)$$
$$= \lambda_L\pi(1,N-1,x,4) + \lambda_{L2}\pi(1,N-2,x,4) + \lambda_{L2}\pi(1,N-1,x,4) + \beta\pi(1,N,x-1,4).$$

Case 167: For $i = 2,\ j = 0,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \mu_H]\pi(2,0,x,4)$$
$$= \lambda_H\pi(1,0,x,4) + \beta\pi(2,0,x-1,4) + 2\alpha_H\pi(3,0,x,4) + \alpha_L\pi(2,1,x,4).$$

Case 168: For $i = 2,\ j = 1,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + \alpha_L + \mu_H]\pi(2,1,x,4)$$
$$= \lambda_H\pi(1,1,x,4) + \lambda_L\pi(2,0,x,4) + \beta\pi(2,1,x-1,4) + 2\alpha_H\pi(3,1,x,4) + 2\alpha_L\pi(2,2,x,4).$$

Case 169: For $i = 2,\ 2 \le j \le N-2,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + \alpha_H + j\alpha_L + \mu_H]\pi(2,j,x,4)$$
$$= \lambda_H\pi(1,j,x,4) + \lambda_L\pi(2,j-1,x,4) + \lambda_{L2}\pi(2,j-2,x,4) + \beta\pi(2,j,x-1,4) + 2\alpha_H\pi(3,j,x,4) + (j+1)\alpha_L\pi(2,j+1,x,4).$$

Case 170: For $i = 2,\ j = N-1,\ 1 \le x \le K-1$, and $y = 4$,

$$[\beta + \alpha_H + (N-1)\alpha_L + \mu_H]\pi(i,N-1,x,4)$$
$$= \lambda_H\pi(1,N-1,x,4) + \lambda_L\pi(2,N-2,x,4) + \lambda_{H2}\pi(1,N-1,x,4) + \lambda_{L2}\pi(2,N-3,x,4) + \lambda_{L2}\pi(2,N-2,x,4) + \beta\pi(2,N-1,x-1,4).$$

Case 171: For $3 \le i \le N,\ j = 0,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \mu_H]\pi(i,0,x,4)$$

$$= \lambda_H \pi(i-1,0,x,4) + \lambda_{H2} \pi(i-2,0,x,4) + \beta \pi(i,0,x-1,4) + i\alpha_H \pi(i + 1,0,x,4) + \alpha_L \pi(i,1,x,4).$$

Case 172: For $3 \le i \le N-1,\ j = 1,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + \alpha_L + \mu_H]\pi(i,1,x,4)$$
$$= \lambda_H \pi(i-1,1,x,4) + \lambda_L \pi(i,0,x,4) + \lambda_{H2} \pi(i-2,1,x,4) + \beta \pi(i,1,x-1,4) + i\alpha_H \pi(i+1,1,x,4) + 2\alpha_L \pi(i,2,x,4).$$

Case 173: For $3 \le i \le N-2,\ 2 \le j \le N-i,\ 1 \le x \le K-1$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + j\alpha_L + \mu_H]\pi(i,j,x,4)$$
$$= \lambda_H \pi(i-1,j,x,4) + \lambda_L \pi(i,j-1,x,4) + \lambda_{H2} \pi(i-2,j,x,4) + \lambda_{L2} \pi(i,j-2,x,4) + \beta \pi(i,j,x-1,4) + i\alpha_H \pi(i+1,j,x,4) + (j+1)\alpha_L \pi(i,j+1,x,4).$$

Case 174: For $3 \le i \le N-1,\ j = N-i+1,\ 1 \le x \le K-1$, and $y = 4$,

$$[\beta + (i-1)\alpha_H + (N-i+1)\alpha_L + \mu_H]\pi(i,N-i+1,x,4)$$
$$= \lambda_H \pi(i-1,N-i+1,x,4) + \lambda_L \pi(i,N-i,x,4) + \lambda_{H2} \pi(i-2,N-i+1,x,4) + \lambda_{H2} \pi(i-1,N-i+1,x,4) + \lambda_{L2} \pi(i,N-i-1,x,4) + \lambda_{L2} \pi(i,N-i,x,4) + \beta \pi(i,N-i+1,x-1,4).$$

Case 175: For $i = N,\ j = 1,\ 1 \le x \le K-1$, and $y = 4$,

$$[\beta + (N-1)\alpha_H + \alpha_L + \mu_H]\pi(N,1,x,4)$$
$$= \lambda_H \pi(N-1,1,x,4) + \lambda_L \pi(i,0,x,4) + \lambda_{H2} \pi(N-2,1,x,4) + \lambda_{H2} \pi(N-1,1,x,4) + \beta \pi(i,1,x-1,4).$$

Case 176: For $i = N+1,\ j = 0,\ 1 \le x \le K-1$, and $y = 4$,

$$(\beta + N\alpha_H + \mu_H)\pi(N+1,0,x,4)$$
$$= \lambda_H \pi(N,0,x,4) + \lambda_{H2} \pi(N-2,0,x,4) + \lambda_{H2} \pi(N-1,0,x,4) + \beta \pi(N+1,0,x-1,4).$$

Case 177: For $i = 1,\ j = 0,\ x = K$, and $y = 4$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \mu_H)\pi(1,0,K,4)$$

$$= \beta\pi(1,0,K-1,4) + \alpha_H\pi(2,0,K,4) + \alpha_L\pi(1,1,K,4).$$

Case 178: For $i = 1,\ j = 1,\ x = K$, and $y = 4$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_L + \mu_H)\pi(1,1,K,4)$$
$$= \lambda_L\pi(1,0,K,4) + \beta\pi(1,1,K-1,4) + \alpha_H\pi(2,1,K,4) + 2\alpha_L\pi(1,2,K,4).$$

Case 179: For $i = 1,\ 2 \le j \le N-1,\ x = K$, and $y = 4$,

$$(\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + j\alpha_L + \mu_H)\pi(1,j,K,4)$$
$$= \lambda_L\pi(1,j-1,K,4) + \lambda_{L2}\pi(1,j-2,K,4) + \beta\pi(1,j,K-1,4)$$
$$+ \alpha_H\pi(2,j,K,4) + (j+1)\alpha_L\pi(1,j+1,K,4).$$

Case 180: For $i = 1,\ j = N,\ x = K$, and $y = 4$,

$$(N\alpha_L + \mu_H)\pi(1,N,K,4)$$
$$= \lambda_L\pi(1,N-1,K,4) + \lambda_{L2}\pi(1,N-2,K,4) + \lambda_{L2}\pi(1,N-1,K,4)$$
$$+ \beta\pi(1,N,K-1,4).$$

Case 181: For $i = 2,\ j = 0,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \mu_H]\pi(2,0,K,4)$$
$$= \lambda_H\pi(1,0,K,4) + \beta\pi(2,0,K-1,4) + 2\alpha_H\pi(3,0,K,4) +$$
$$\alpha_L\pi(2,1,K,4).$$

Case 182: For $i = 2,\ j = 1,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \alpha_H + \alpha_L + \mu_H]\pi(2,1,K,4)$$
$$= \lambda_H\pi(1,1,K,4) + \lambda_L\pi(2,0,K,4) + \beta\pi(2,1,K-1,4) +$$
$$2\alpha_H\pi(3,1,K,4) + 2\alpha_L\pi(2,2,K,4).$$

Case 183: For $i = 2,\ 2 \le j \le N-2,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \alpha_H + j\alpha_L + \mu_H]\pi(2,j,K,4)$$
$$= \lambda_H\pi(1,j,K,4) + \lambda_L\pi(2,j-1,K,4) + \lambda_{L2}\pi(2,j-2,K,4)$$
$$+ \beta\pi(2,j,K-1,4) + 2\alpha_H\pi(3,j,K,4) + (j+1)\alpha_L\pi(2,j+1,K,4).$$

Case 184: For $i = 2,\ j = N-1,\ x = K$, and $y = 4$,

$$[\alpha_H + (N-1)\alpha_L + \mu_H]\pi(2, N-1, K, 4)$$
$$= \lambda_H \pi(1, N-1, K, 4) + \lambda_L \pi(2, N-2, K, 4) + \lambda_{H2} \pi(1, N-2, K, 4)$$
$$+ \lambda_{L2} \pi(2, N-3, K, 4) + \lambda_{L2} \pi(2, N-2, K, 4) + \beta \pi(2, N-1, K-1, 4).$$

Case 185: For $3 \le i \le N,\ j = 0,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + \mu_H]\pi(i, 0, K, 4)$$
$$= \lambda_H \pi(i-1, 0, K, 4) + \lambda_{H2} \pi(i-2, 0, K, 4) + \beta \pi(i, 0, K-1, 4)$$
$$+ i\alpha_H \pi(i+1, 0, K, 4) + \alpha_L \pi(i, 1, K, 4).$$

Case 186: For $3 \le i \le N-1,\ j = 1,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + \alpha_L + \mu_H]\pi(i, 1, K, 4)$$
$$= \lambda_H \pi(i-1, 1, K, 4) + \lambda_L \pi(i, 0, K, 4) + \lambda_{H2} \pi(i-2, 1, K, 4)$$
$$+ \beta \pi(i, 1, K-1, 4) + i\alpha_H \pi(i+1, 1, K, 4) + 2\alpha_L \pi(i, 2, K, 4).$$

Case 187: For $3 \le i \le N-2,\ 2 \le j \le N-i,\ x = K$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + (i-1)\alpha_H + j\alpha_L + \mu_H]\pi(i, j, K, 4)$$
$$= \lambda_H \pi(i-1, j, K, 4) + \lambda_L \pi(i, j-1, K, 4) + \lambda_{H2} \pi(i-2, j, K, 4)$$
$$+ \lambda_{L2} \pi(i, j-2, K, 4) + \beta \pi(i, j, K-1, 4) + i\alpha_H \pi(i+1, j, K, 4)$$
$$+ (j+1)\alpha_L \pi(i, j+1, K, 4).$$

Case 188: For $3 \le i \le N-1,\ j = N-i+1,\ x = K$, and $y = 4$,

$$[(i-1)\alpha_H + (N-i+1)\alpha_L + \mu_H]\pi(i, N-i+1, K, 4)$$
$$= \lambda_H \pi(i-1, N-i+1, K, 4) + \lambda_L \pi(i, N-i, K, 4)$$
$$+ \lambda_{H2} \pi(i-2, N-i+1, K, 4) + \lambda_{H2} \pi(i-1, N-i+1, K, 4)$$
$$+ \lambda_{L2} \pi(i, N-i-1, K, 4) + \lambda_{L2} \pi(i, N-i, K, 4)$$
$$+ \beta \pi(i, N-i+1, K-1, 4).$$

Case 189: For $i = N,\ j = 1,\ x = K$, and $y = 4$,

$$[(N-1)\alpha_H + \alpha_L + \mu_H]\pi(N, 1, K, 4)$$
$$= \lambda_H \pi(N-1, 1, K, 4) + \lambda_L \pi(N, 0, K, 4) + \lambda_{H2} \pi(N-2, 1, K, 4)$$
$$+ \lambda_{H2} \pi(N-1, 1, K, 4) + \lambda_{L2} \pi(N, 0, K, 4) + \beta \pi(N, 1, K-1, 4).$$

Case 190: For $i = N+1,\ j = 0,\ x = K$, and $y = 4$,

$$(N\alpha_H + \mu_H)\pi(N+1,0,K,4)$$
$$= \lambda_H\pi(N,0,K,4) + \lambda_{H2}\pi(N-1,0,K,4) + \lambda_{H2}\pi(N,0,K,4) + \beta\pi(N+1,0,K-1,4).$$

Since there are many equations presented above, discussing each one separately would be challenging. Therefore, we focus on a relatively complicated case, specifically case 159, to provide an illustrative example. This state occurs when there are more than or equal to three but less than or equal to N-3 HP packets and N-i LP packets in the system, and there is only one seat left in the packet queue, while the energy queue is empty. The HP packet being served in the server is using the regular battery. The corresponding detailed state transition diagram can be found in Fig. 3 - 2.

Case 159: For $3 \le i \le N-3$, $j = N-i$, $x = 0$, and $y = 4$,

$$[\lambda_H + \lambda_L + \lambda_{H2} + \lambda_{L2} + \beta + (i-1)\alpha_H + (N-i)\alpha_L + (1-\theta_H)\mu_H + \theta_H\mu_H]$$
$$\pi(i,N-i,0,4)$$
$$= \theta_H\lambda_H\pi(i-1,N-i,0,0) + \lambda_H\pi(i-1,N-i,0,4) +$$
$$\theta_H\lambda_L\pi(i,N-i-1,0,0) + \lambda_L\pi(i,N-i-1,0,4)$$
$$+ \theta_H\lambda_{H2}\pi(i-2,N-i,0,0) + \lambda_{H2}\pi(i-2,N-i,0,4)$$
$$+ \theta_H\lambda_{H2}\pi(i-1,N-i,0,0) + \theta_H\lambda_{L2}\pi(i,N-i-2,0,0)$$
$$+ \lambda_{L2}\pi(i,N-i-2,0,4) + \theta_H\lambda_{L2}\pi(i,N-i-1,0,0) +$$
$$i\alpha_H\pi(i+1,N-i,0,4) + (N-i+1)\alpha_L\pi(i,N-i+1,0,4) +$$
$$\theta_H\mu_H\pi(i+1,N-i,0,2) + \theta_H\mu_L\pi(i,N-i+1,0,1) +$$
$$\theta_H\mu_H\pi(i+1,N-i,0,4) + \theta_H\mu_L\pi(i,N-i+1,0,3).$$

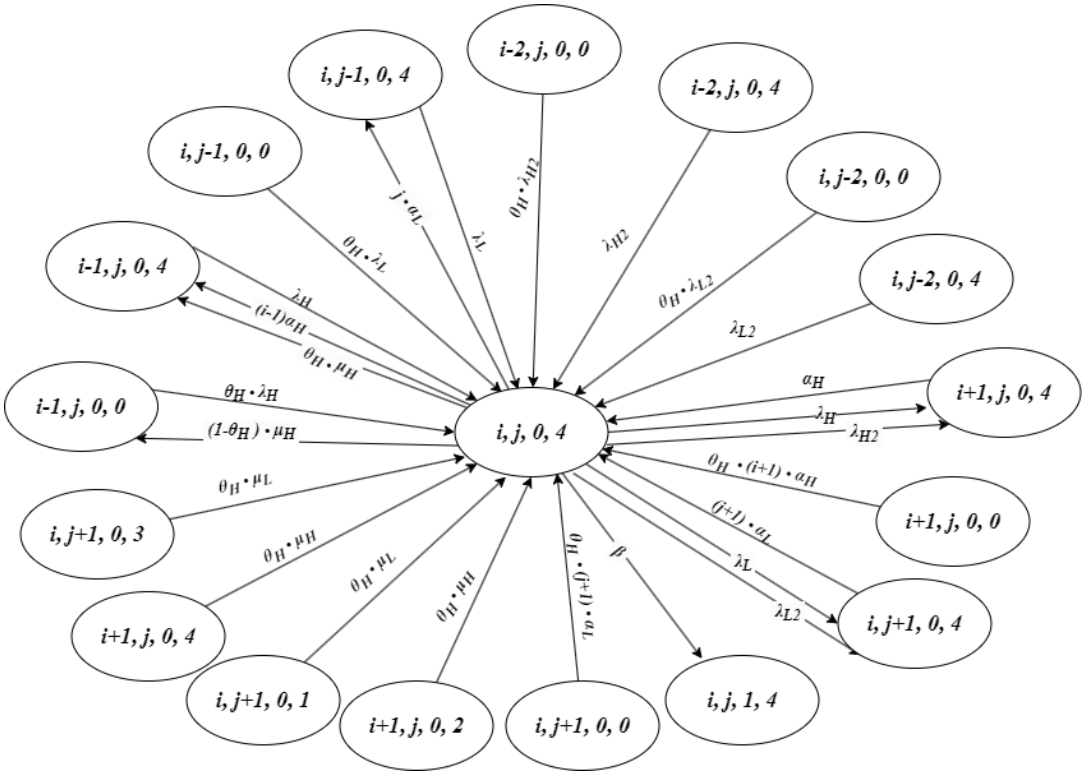

Fig. 3 - 2: The state transition diagram for $3 \leq i \leq N-3$, $j = N - i$, $x = 0$, and $y = 4$.

3.1.3 Iterative algorithm

Using the iterative algorithm provided below, we perform calculations on the state balance equations until they converge, allowing us to determine the steady-state distribution of the system.

Iterative algorithm:

Step 1: Select a group of initial values for $\pi(i,j,x,y)^{old} = \frac{1}{|S|}$, $\forall i,j,x,y$, where $|S|$ is the total number of feasible states.

Step 2: Substitute $\pi(i,j,x,y)^{old}$ into *Case 1* to *Case 190* to find $\pi(i,j,x,y)^{new}$, $\forall i,j,x,y$.

Step 3: Normalize $\pi(i,j,x,y)^{new}$, $\forall i,j,x,y$.

Step 4: If $\sqrt{\sum\sum\sum\sum_{(i,j,x,y)\in S}|\pi(i,j,x,y)^{old} - \pi(i,j,x,y)^{new}|^2} < \varepsilon$, stop the iterative algorithm, where ε is the stopping criterion. Otherwise, set $\pi(i,j,x,y)^{old} = \pi(i,j,x,y)^{new}$, and return to Step 2.

In the analytical experiments, we set $\varepsilon = 10^{-8}$. It takes about 200 to 7000 iterations for the algorithm to converge.

3.1.4 Performance measures

We obtain different performance measures of interest from the steady-state probability $\pi(i,j,x,y)$ in order to evaluate the system's measures of effectiveness. These measures are presented below.

$E[N]$ $(E[N]_H, E[N]_L)$, the expected number of all $(\mathrm{HP}, \mathrm{LP})$ packets in the system, is provided below.

$$E[N] = \sum_{i=0}^{N}\sum_{j=0}^{N-i}(i+j)\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i}(i+j)[\pi(i,j,x,1)+\pi(i,j,x,3)]$$
$$+\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i}(i+j)[\pi(i,j,x,2)+\pi(i,j,x,4)] \quad (3\text{-}2)$$

$$E[N]_H = \sum_{i=0}^{N}\sum_{j=0}^{N-i} i\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} i[\pi(i,j,x,1)+\pi(i,j,x,3)]$$
$$+\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} i[\pi(i,j,x,2)+\pi(i,j,x,4)] \quad (3\text{-}3)$$

$$E[N]_L = \sum_{i=0}^{N}\sum_{j=0}^{N-i} j\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} j[\pi(i,j,x,1)+\pi(i,j,x,3)]$$
$$+\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} j[\pi(i,j,x,2)+\pi(i,j,x,4)] \quad (3\text{-}4)$$

$E[N_q]$ $\left(E[N_q]_H,\ E[N_q]_L\right)$, the expected number of all (HP, LP) packets in the queue, is provided below.

$$E[N_q] = \sum_{i=0}^{N}\sum_{j=0}^{N-i}(i+j)\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i}(i+j-1)[\pi(i,j,x,1)+\pi(i,j,x,3)]$$
$$+\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i}(i+j-1)[\pi(i,j,x,2)+\pi(i,j,x,4)]$$

(3-5)

$$E[N_q]_H = \sum_{i=0}^{N}\sum_{j=0}^{N-i} i\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} i[\pi(i,j,x,1) + \pi(i,j,x,3)]$$

$$+ \sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} (i-1)[\pi(i,j,x,2) + \pi(i,j,x,4)]$$

(3-6)

$$E[N_q]_L = \sum_{i=0}^{N}\sum_{j=0}^{N-i} j\pi(i,j,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} (j-1)[\pi(i,j,x,1) + \pi(i,j,x,3)]$$

$$+ \sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} j[\pi(i,j,x,2) + \pi(i,j,x,4)]$$

(3-7)

TH (TH_H, TH_L), the throughput of all (HP, LP) packets, is provided below.

$$TH = \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} \mu_L[\pi(i,j,x,1) + \pi(i,j,x,3)]$$

$$+ \sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} \mu_H[\pi(i,j,x,2) + \pi(i,j,x,4)]$$

(3-8)

$$TH_H = \sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i} \mu_H[\pi(i,j,x,2) + \pi(i,j,x,4)]$$

(3-9)

$$TH_L = \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i} \mu_L[\pi(i,j,x,1) + \pi(i,j,x,3)]$$

(3-10)

P_{bl}, the blocking probability of each arrived packet, without considering its priority, is provided below.

$$P_{bl} = \sum_{i=0}^{N} \pi(i,N-i,0,0) + \sum_{x=0}^{K}\sum_{i=0}^{N} [\pi(i,N+1-i,x,1) + \pi(i,N+1-i,x,3)]$$

$$+ \sum_{x=0}^{K}\sum_{i=1}^{N+1} [\pi(i,N+1-i,x,2) + \pi(i,N+1-i,x,4)]$$

(3-11)

P_{el}, the energy loss probability, is provided below.

$$P_{el} = \pi(0,0,K,0) + \sum_{i=0}^{N} \sum_{j=1}^{N+1-i} [\pi(i,j,K,1) + \pi(i,j,K,3)]$$

$$+ \sum_{i=1}^{N+1} \sum_{j=0}^{N+1-i} [\pi(i,j,K,2) + \pi(i,j,K,4)]$$

(3-12)

$E[W]$ $(E[W]_H, E[W]_L)$, the mean waiting time of all (HP, LP) packets in the system, , which refers to all packets that have exited the system, either after receiving service or due to impatience, is provided below.

$$E[W] = \frac{E[N]}{(\lambda_H + \lambda_L)(1 - P_{bl})}$$

(3-13)

$$E[W]_H = \frac{E[N]_H}{\lambda_H(1 - P_{bl})}$$

(3-14)

$$E[W]_L = \frac{E[N]_L}{\lambda_L(1 - P_{bl})}$$

(3-15)

$P_{imp(arr)}$ $(P_{imp(arr)\_H},\ P_{imp(arr)\_L})$, the impatient loss probability of arrived (HP, LP) packets, is provided below.

$$P_{imp(arr)} = \frac{\alpha_H E[N_q]_H + \alpha_L E[N_q]_L}{\lambda_H + \lambda_L}$$

(3-16)

$$P_{imp(arr)\_H} = \frac{\alpha_H E[N_q]_H}{\lambda_H}$$

(3-17)

$$P_{imp(arr)\_L} = \frac{\alpha_L E[N_q]_L}{\lambda_L}$$

(3-18)

$P_{imp(adm)}$ $(P_{imp(adm)\_H},\ P_{imp(adm)\_L})$, the impatient loss probability of admitted (HP, LP) packets, is provided below.

$$P_{imp(adm)} = \frac{\alpha_H E[N_q]_H + \alpha_L E[N_q]_L}{(\lambda_H + \lambda_L)(1 - P_{bl})}$$

(3-19)

$$P_{imp(adm)\_H} = \frac{\alpha_H E[N_q]_H}{\lambda_H(1 - P_{bl})} \tag{3-20}$$

$$P_{imp(adm)\_L} = \frac{\alpha_L E[N_q]_L}{\lambda_L(1 - P_{bl})} \tag{3-21}$$

P_{tl} $\left(P_{tl\_H}, P_{tl\_L}\right)$, the total loss probability of arrived (HP, LP) packets, is provided below.

$$P_{tl} = 1 - \left(\frac{TH}{\lambda_H + \lambda_L}\right) \tag{3-22}$$

$$P_{tl\_H} = 1 - \left(\frac{TH_H}{\lambda_H}\right) \tag{3-23}$$

$$P_{tl\_L} = 1 - \left(\frac{TH_L}{\lambda_L}\right) \tag{3-24}$$

$RECR$ $(RECR_H, RECR_L)$, the regular energy consumption ratio of all (HP, LP) packets, is provided below.

$$RECR = \frac{\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i}\mu_H\pi(i,j,x,4) + \sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i}\mu_L\pi(i,j,x,3)}{TH_H + TH_L} \tag{3-25}$$

$$RECR_H = \frac{\sum_{x=0}^{K}\sum_{i=1}^{N+1}\sum_{j=0}^{N+1-i}\mu_H\pi(i,j,x,4)}{TH_H + TH_L} \tag{3-26}$$

$$RECR_L = \frac{\sum_{x=0}^{K}\sum_{i=0}^{N}\sum_{j=1}^{N+1-i}\mu_L\pi(i,j,x,3)}{TH_H + TH_L} \tag{3-27}$$

3.2 Scenario 2

In this section we discuss a network with three connected nodes, where packets come in batches and each batch consists of one packet or two packets. All batches can be divided into two priorities: high priority (HP) and low priority (LP). Both types of packets require the same amount of energy. The first node is an "entry node", the second node is an "exit node", and the third node is a "control node". After passing through the

control node, packets are directed to the correct path to the exit node and leave the network. Impatient packets may leave the queue at any time. Whenever a node is ready to serve a packet, it first checks the energy queue for sufficient energy units. If there are not enough energy units, the packet may use a regular battery with a given probability. The model diagram, state balance equations, iterative algorithm, and performance measures are provided below.

3.2.1 Model diagram

In scenario 2, as illustrated in Fig. 3 - 3, we have an open network model with three nodes. Each node in the network consists of a finite packet queue, a finite energy queue, a regular battery, and a single server. The packet queue size is denoted by N, $(0 < N < \infty)$, the energy queue size is denoted by K, $(0 < K < \infty)$, and the regular battery has an unlimited supply of energy. HP and LP packets have the same energy requirement, which is one energy unit. The arrivals of HP and LP packets from outside the system follow a Poisson process with specific arrival rates λ_H, λ_{H2} and λ_L, λ_{L2} respectively. The arrivals of energy units of node n follow a Poisson process with arrival rate β_n. The waiting time for each HP (LP) packet in node n's queue is determined by an exponential distribution with corresponding rates α_{Hn} (α_{Ln}). Node n's service time for each HP (LP) packet is assumed to follow an exponential distribution with rates μ_{Hn} (μ_{Ln}), respectively. We assume that when the energy queue lacks the necessary harvested energy to support an HP or LP packet, the regular battery will be used based on probabilities θ_{Hn} and θ_{Ln}, respectively. Additionally, when a packet finishes service at node i, it is either removed from the network or forwarded to the next node j with an assigned routing probability r_{ij}, where $i, j = 1, 2, 3$ and $i \neq j$. However, there are some restrictions on the routing policy. First, packets cannot be routed from the exit node to the entry node. Second, packets from the entry and exit nodes are only permitted to pass through the control node once before being forwarded to the previous node.

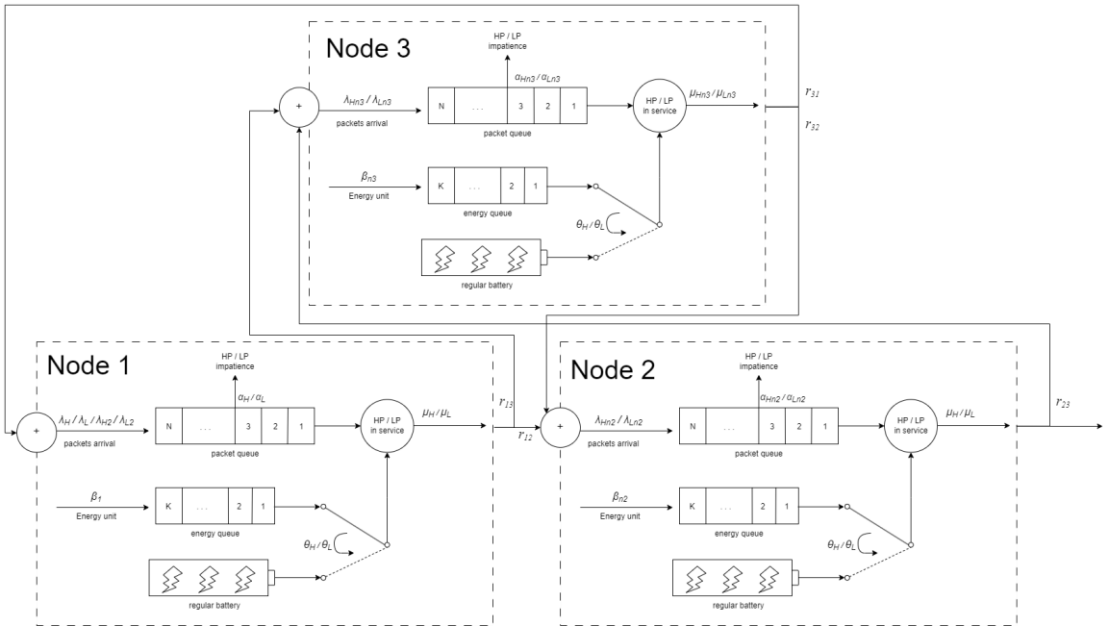

Fig. 3 - 3: The model diagram for scenario 2

3.2.2 State balance equations

To achieve a simplification in the derivation, we assume that each node in the network is independent of the other nodes. We can represent node n as a four-dimensional Markov chain with the state (i_n, j_n, x_n, y_n), where i_n represents the number of HP packets in node n, j_n represents the number of LP packets in node n, x_n represents the number of harvested energy units in the energy queue in node n, and y_n represents the server status and the energy resource being used in node n. Note that y_n can take five values: (1) "0" indicates that the server is idle; (2) "1" indicates that an LP packet has entered the server and consumed one energy unit from the energy queue; (3) "2" indicates that an HP packet has entered the server and consumed one energy unit from the energy queue; (4) "3" indicates that an LP packet has entered the server and consumed one energy unit from the regular battery; (5) "4" indicates that an HP packet has entered the server and consumed one energy unit from the regular battery. The steady state probability of the system is represented by $\pi(i_n, j_n, x_n, y_n)$, and the state space is defined as follows:

$$\begin{aligned} S_n = \{(i_n, j_n, x_n, y_n) |\ & 0 \le i_n + j_n \le N, x_n = 0, y_n = 0; \\ & i_n + j_n = 0, 1 \le x_n \le K, y_n = 0; \\ & 1 \le i_n + j_n \le N+1, j_n \ge 1, 0 \le x_n \le K, y_n = 1 \text{、} 3; \\ & 1 \le i_n + j_n \le N+1, i_n \ge 1, 0 \le x_n \le K, y_n = 2 \text{、} 4\} \end{aligned} \tag{3-28}$$

As a result, we can calculate the total count of feasible states

$$|S_n| = \left(2K + \frac{5}{2}\right) N^2 + \left(6K + \frac{15}{2}\right) N + 5K + 5.$$

In addition, the analysis of the arrival rate of each node should take into account five internal arrival rates.

To begin with, we calculate the arrival rates of HP and LP packets from node 1 to node 3.

$$\begin{cases} \lambda_{H13} = \frac{(\lambda_H+2\lambda_{H2})(1-P_{tl\_H-1})r_{13}}{(\lambda_H+2\lambda_{H2}+\lambda_{H31})(1-P_{tl\_H-1})} TH_{H-1} \\ \lambda_{L13} = \frac{(\lambda_L+2\lambda_{L2})(1-P_{tl\_L-1})r_{13}}{(\lambda_L+2\lambda_{L2}+\lambda_{L31})(1-P_{tl\_L-1})} TH_{L-1} \end{cases} \tag{3-29}$$

The equation's right-hand side has two parts. The first part shows the percentage of HP or LP packets completing service in node 1 and then directed to node 3. The denominator shows the total HP or LP packet arrival rate for node 1, including the unblocked arrival rates from outside and node 3. The numerator represents the unblocked arrival rate from outside, which is directed to node 3 according to r_{13}. It should be noted that in this scenario, only external packets can be routed to node 3, as each packet can visit node 3 only once.

Next, we calculate the arrival rates of HP and LP packets from node 3 to node 1.

$$\begin{cases} \lambda_{H31} = \frac{\lambda_{H13}(1-P_{tl\_H-3})}{(\lambda_{H13}+\lambda_{H23})(1-P_{tl\_H-3})} TH_{H-3} \\ \lambda_{L31} = \frac{\lambda_{L13}(1-P_{tl\_L-3})}{(\lambda_{L13}+\lambda_{L23})(1-P_{tl\_L-3})} TH_{L-3} \end{cases} \tag{3-30}$$

The equation's right-hand side has two parts. The first part shows the percentage of HP or LP packets that have completed their service at node 3 and will be sent back to node 1. The denominator shows the total HP or LP packet arrival rate of node 3, which includes the unblocked arrival rates from node 1 and node 2. The numerator indicates the unblocked arrival rate from node 1. It should be noted that all packets that pass through the control node will be forwarded to the previous node, so the packets that have finished service at node 3 will be routed back to node 1.

Next, we calculate the arrival rates of HP and LP packets from node 1 to node 2.

$$\begin{cases} \lambda_{H12} = \frac{(\lambda_H+2\lambda_{H2})(1-P_{tl\_H-1})r_{12}+\lambda_{H31}(1-P_{tl\_H-1})}{(\lambda_H+2\lambda_{H2}+\lambda_{H31})(1-P_{tl\_H-1})} TH_{H-1} \\ \lambda_{L12} = \frac{(\lambda_L+2\lambda_{L2})(1-P_{tl\_L-1})r_{12}+\lambda_{L31}(1-P_{tl\_L-1})}{(\lambda_L+2\lambda_{L2}+\lambda_{L31})(1-P_{tl\_L-1})} TH_{L-1} \end{cases} \tag{3-31}$$

The equation's right-hand side calculates the HP or LP packet arrival rates from node 1 to node 2. The first term indicates the proportion of HP or LP packets finishing service in node 1 and moving on to node 2. The denominator part represents the total HP or LP packet arrival rate of node 1, which includes those unblocked arrival rates from outside and node 3. The numerator part represents the unblocked arrival rates from outside that is routed to node 2 based on r_{12} and the unblocked arrival rates from node 3. It is

important to note that each packet passing through the control node will be forwarded to the previous node, so the packets that have completed service at node 1 will be routed back to node 2.

Next, we calculate the HP and LP packet arrival rates from node 2 to node 3.

$$\begin{cases} \lambda_{H23} = \frac{\lambda_{H12}(1-P_{tl\_H-2})}{(\lambda_{H12}+\lambda_{H32})(1-P_{tl\_H-2})} \frac{(\lambda_H+2\lambda_{H2})r_{12}r_{23}}{\lambda_{H31}+\lambda_H r_{12}} TH_{H-2} \\ \lambda_{L23} = \frac{\lambda_{L12}(1-P_{tl\_L-2})}{(\lambda_{L12}+\lambda_{L32})(1-P_{tl\_L-2})} \frac{(\lambda_L+2\lambda_{L2})r_{12}r_{23}}{\lambda_{L31}+\lambda_L r_{12}} TH_{L-2} \end{cases} \tag{3-32}$$

To calculate the HP and LP packet arrival rates from node 2 to node 3, we use the equation. The first term on the right-hand side shows the proportion of HP or LP packets that complete service at node 2 and are routed to node 3. The denominator of this term indicates the total HP or LP packet arrival rate of node 2, including the unblocked arrival rates from node 1 and node 3. The numerator represents the unblocked arrival rate from node 1. The second term on the right-hand side shows the portion of HP or LP packets that complete service at node 1 and are routed to node 3. The denominator of this term shows the HP or LP packet arrival rates being routed from node 1 to node 2, which includes the arrival rates from node 3 to node 1 and the outside arrival rates that are routed to node 2 based on r_{12}. The numerator of the second term shows the outside packet arrival rates from node 1 routed to node 2 based on r_{12} and then routed to node 3 based on r_{23}.

Last, we calculate the HP and LP packet arrival rates from node 3 to node 2.

$$\begin{cases} \lambda_{H32} = \frac{\lambda_{H23}(1-P_{tl\_H-3})}{(\lambda_{H13}+\lambda_{H23})(1-P_{tl\_H-3})} TH_{H-3} \\ \lambda_{L32} = \frac{\lambda_{L23}(1-P_{tl\_L-3})}{(\lambda_{L13}+\lambda_{L23})(1-P_{tl\_L-3})} TH_{L-3} \end{cases} \tag{3-33}$$

In this equation, the first term on the right-hand side indicates the fraction of HP or LP packets that will be directed from node 3 to node 2 after finishing their service in node 3. The denominator represents the total arrival rate of HP or LP packets at node 3, which includes the unblocked arrival rates from node 1 and node 2. The numerator represents the unblocked arrival rate from node 2. It is important to note that any packet that passes through the control node will be forwarded to the previous node. Therefore, packets that have finished their service at node 3 will be sent back to node 2.

Based on the explanation of the model provided earlier, the HP and LP packet arrival rates, denoted as λ_{H-n} and λ_{L-n}, respectively, are determined for each node n.

$$\lambda_{H-n} = \begin{cases} \lambda_H + \lambda_{H31}, n = 1 \\ \lambda_{H12} + \lambda_{H32}, n = 2 \\ \lambda_{H13} + \lambda_{H23}, n = 3 \end{cases}, \quad \lambda_{L-n} = \begin{cases} \lambda_L + \lambda_{L31}, n = 1 \\ \lambda_{L12} + \lambda_{L32}, n = 2 \\ \lambda_{L13} + \lambda_{L23}, n = 3 \end{cases} \tag{3-34}$$

$$\lambda_{H2-n}=\begin{cases}\lambda_{H2}, n=1\\ 0, n=2\\ 0, n=3\end{cases},\quad \lambda_{L2-n}=\begin{cases}\lambda_{L2}, n=1\\ 0, n=2\\ 0, n=3\end{cases} \tag{3-35}$$

Furthermore, the system states of node 1 can be classified into 190 cases and numbered as case A1, A2, A3, and so on. The system states of node 2 and 3 can be classified into 101 cases and numbered as case B1, B2, B3, and so on. The corresponding equations for balancing the states are demonstrated below:

Case 191: For $i_1=0,\ j_1=0,\ x_1=0$, and $y_1=0$,

$$\begin{aligned}&[(1-\theta_{H1})\lambda_{H-1}+\theta_{H1}\lambda_{H-1}+(1-\theta_{L1})\lambda_{L-1}+\theta_{L1}\lambda_{L-1}+(1-\theta_{H1})\lambda_{H2}\\&\qquad+\theta_{H1}\lambda_{H2}+(1-\theta_{L1})\lambda_{L2}+\theta_{L1}\lambda_{L2}+\beta_1]\pi(0,0,0,0)\\&=\alpha_{H1}\pi(1,0,0,0)+\alpha_{L1}\pi(0,1,0,0)+\mu_{H1}\pi(1,0,0,2)+\mu_{L1}\pi(0,1,0,1)+\\&\quad\mu_{H1}\pi(1,0,0,4)+\mu_{L1}\pi(0,1,0,3).\end{aligned}$$

Case 192: For $i_1=0,\ j_1=1,\ x_1=0$, and $y_1=0$,

$$\begin{aligned}&[(1-\theta_{H1})\lambda_{H-1}+\theta_{H1}\lambda_{H-1}+(1-\theta_{L1})\lambda_{L-1}+\theta_{L1}\lambda_{L-1}+(1-\theta_{H1})\lambda_{H2}\\&\qquad+\theta_{H1}\lambda_{H2}+(1-\theta_{L1})\lambda_{L2}+\theta_{L1}\lambda_{L2}+\beta_1+\alpha_{L1}]\pi(0,1,0,0)\\&=(1-\theta_{L1})\lambda_{L-1}\pi(0,0,0,0)+(1-\theta_{L1})\alpha_{H1}\pi(1,1,0,0)+\\&\quad(1-\theta_{L1})2\alpha_{L1}\pi(0,2,0,0)+(1-\theta_{L1})\mu_{H1}\pi(1,1,0,2)+\\&\quad(1-\theta_{L1})\mu_{L1}\pi(0,2,0,1)+(1-\theta_{L1})\mu_{H1}\pi(1,1,0,4)+\\&\quad(1-\theta_{L1})\mu_{L1}\pi(0,2,0,3).\end{aligned}$$

Case 193: For $i_1=0,\ 2\le j_1\le N-1,\ x_1=0$, and $y_1=0$,

$$\begin{aligned}&[(1-\theta_{H1})\lambda_{H-1}+\theta_{H1}\lambda_{H-1}+(1-\theta_{L1})\lambda_{L-1}+\theta_{L1}\lambda_{L-1}+(1-\theta_{H1})\lambda_{H2}\\&\qquad+\theta_{H1}\lambda_{H2}+(1-\theta_{L1})\lambda_{L2}+\theta_{L1}\lambda_{L2}+\beta_1+(1-\theta_{L1})j_1\alpha_{L1}\\&\qquad+\theta_{L1}j_1\alpha_{L1}]\pi(0,j_1,0,0)\\&=(1-\theta_{L1})\lambda_{L-1}\pi(0,j_1-1,0,0)+(1-\theta_{L1})\lambda_{L2}\pi(0,j_1-2,0,0)\\&\qquad+(1-\theta_{L1})\alpha_{H1}\pi(1,j_1,0,0)+\\&\quad(1-\theta_{L1})(j_1+1)\alpha_{L1}\pi(0,j_1+1,0,0)+(1-\theta_{L1})\mu_{H1}\pi(1,j_1,0,2)+\\&\quad(1-\theta_{L1})\mu_{L1}\pi(0,j_1+1,0,1)+(1-\theta_{L1})\mu_{H1}\pi(1,j_1,0,4)+\\&\quad(1-\theta_{L1})\mu_{L1}\pi(0,j_1+1,0,3).\end{aligned}$$

Case 194: For $i_1=0,\ j_1=N,\ x_1=0$, and $y_1=0$,

$$[\beta_1+(1-\theta_{L1})N\alpha_{L1}+\theta_{L1}N\alpha_{L1}]\pi(0,N,0,0)$$

$$= (1-\theta_{L1})\lambda_{L-1}\pi(0,N-1,0,0) + (1-\theta_{L1})\lambda_{L2}\pi(0,N-2,0,0)$$
$$+ (1-\theta_{L1})\lambda_{L2}\pi(0,N-1,0,0) + (1-\theta_{L1})\mu_{H1}\pi(1,N,0,2) +$$
$$(1-\theta_{L1})\mu_{L1}\pi(0,N+1,0,1) + (1-\theta_{L1})\mu_{H1}\pi(1,N,0,4) +$$
$$(1-\theta_{L1})\mu_{L1}\pi(0,N+1,0,3).$$

Case 195: For $i_1 = 1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1} + \theta_{H1}\lambda_{H-1} + (1-\theta_{H1})\lambda_{L-1} + \theta_{H1}\lambda_{L-1} + (1-\theta_{H1})\lambda_{H2}$$
$$+ \theta_{H1}\lambda_{H2} + (1-\theta_{H1})\lambda_{L2} + \theta_{H1}\lambda_{L2} + \beta_1 + \alpha_{H1}]\pi(1,0,0,0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(0,0,0,0) + (1-\theta_{H1})2\alpha_{H1}\pi(2,0,0,0) +$$
$$(1-\theta_{H1})\alpha_{L1}\pi(1,1,0,0) + (1-\theta_{H1})\mu_{H1}\pi(2,0,0,2) +$$
$$(1-\theta_{H1})\mu_{L1}\pi(1,1,0,1) + (1-\theta_{H1})\mu_{H1}\pi(2,0,0,4) +$$
$$(1-\theta_{H1})\mu_{L1}\pi(1,1,0,3).$$

Case 196: For $i_1 = 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1} + \theta_{H1}\lambda_{H-1} + (1-\theta_{H1})\lambda_{L-1} + \theta_{H1}\lambda_{L-1} + (1-\theta_{H1})\lambda_{H2}$$
$$+ \theta_{H1}\lambda_{H2} + (1-\theta_{H1})\lambda_{L2} + \theta_{H1}\lambda_{L2} + \beta_1 + (1-\theta_{L1})\alpha_{H1}$$
$$+ \theta_{L1}\alpha_{H1} + (1-\theta_{H1})\alpha_{L1} + \theta_{H1}\alpha_{L1}]\pi(1,1,0,0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(0,1,0,0) + (1-\theta_{H1})\lambda_{L-1}\pi(1,0,0,0) +$$
$$(1-\theta_{H1})2\alpha_{H1}\pi(2,1,0,0) + (1-\theta_{H1})2\alpha_{L1}\pi(1,2,0,0) +$$
$$(1-\theta_{H1})\mu_{H1}\pi(2,1,0,2) + (1-\theta_{H1})\mu_{L1}\pi(1,2,0,1)$$
$$(1-\theta_{H1})\mu_{H1}\pi(2,1,0,4) + (1-\theta_{H1})\mu_{L1}\pi(1,2,0,3).$$

Case 197: For $i_1 = 1,\ 2 \le j_1 \le N-2,\ x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1} + \theta_{H1}\lambda_{H-1} + (1-\theta_{H1})\lambda_{L-1} + \theta_{H1}\lambda_{L-1} + (1-\theta_{H1})\lambda_{H2}$$
$$+ \theta_{H1}\lambda_{H2} + (1-\theta_{H1})\lambda_{L2} + \theta_{H1}\lambda_{L2} + \beta_1 + (1-\theta_{L1})\alpha_{H1}$$
$$+ \theta_{L1}\alpha_{H1} + (1-\theta_{H1})j_1\alpha_{L1} + \theta_{H1}j\alpha_{L1}]\pi(1,j_1,0,0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(0,j_1,0,0) + (1-\theta_{H1})\lambda_{L-1}\pi(1,j_1-1,0,0)$$
$$+ (1-\theta_{H1})\lambda_{L2}\pi(1,j_1-2,0,0) +$$
$$(1-\theta_{H1})2\alpha_{H1}\pi(2,j_1,0,0) + (1-\theta_{H1})(j_1+1)\alpha_{L1}\pi(1,j_1+1,0,0) +$$
$$(1-\theta_{H1})\mu_{H1}\pi(2,j_1,0,2) + (1-\theta_{H1})\mu_{L1}\pi(1,j_1+1,0,1)$$
$$(1-\theta_{H1})\mu_{H1}\pi(2,j_1,0,4) + (1-\theta_{H1})\mu_{L1}\pi(1,j_1+1,0,3).$$

Case 198: For $i_1 = 1,\ j_1 = N-1,\ x_1 = 0$, and $y_1 = 0$,

$$[\beta_1 + (1-\theta_{L1})\alpha_{H1} + \theta_{L1}\alpha_{H1} + (1-\theta_{H1})(N-1)\alpha_{L1} + \theta_{H1}(N-1)\alpha_{L1}]$$
$$\pi(1, N-1, 0, 0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(0, N-1, 0, 0) + (1-\theta_{H1})\lambda_{L-1}\pi(1, N-2, 0, 0)$$
$$+ (1-\theta_{H1})\lambda_{H2}\pi(0, N-1, 0, 0)$$
$$+ (1-\theta_{H1})\lambda_{L2}\pi(1, N-3, 0, 0) + (1-\theta_{H1})\lambda_{L2}\pi(1, N-2, 0, 0)$$
$$+ (1-\theta_{H1})\mu_{H1}\pi(2, N-1, 0, 2) + (1-\theta_{H1})\mu_{L1}\pi(1, N, 0, 1)$$
$$+ (1-\theta_{H1})\mu_{H1}\pi(2, N-1, 0, 4) + (1-\theta_{H1})\mu_{L1}\pi(1, N, 0, 3).$$

Case 199: For $2 \le i_1 \le N-1$, $j_1 = 0$, $x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1} + \theta_{H1}\lambda_{H-1} + (1-\theta_{H1})\lambda_{L-1} + \theta_{H1}\lambda_{L-1} + (1-\theta_{H1})\lambda_{H2}$$
$$+ \theta_{H1}\lambda_{H2} + (1-\theta_{H1})\lambda_{L2} + \theta_{H1}\lambda_{L2} + \beta_1 + (1-\theta_{H1})i_1\alpha_{H1}$$
$$+ \theta_{H1}i_1\alpha_{H1}]\pi(i_1, 0, 0, 0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(i_1 - 1, 0, 0, 0) + (1-\theta_{H1})\lambda_{H2}\pi(i_1 - 2, 0, 0, 0)$$
$$+ (1-\theta_{H1})(i_1+1)\alpha_{H1}\pi(i_1 + 1, 0, 0, 0)$$
$$+ (1-\theta_{H1})\alpha_{L1}\pi(i_1, 1, 0, 0) + (1-\theta_{H1})\mu_{H1}\pi(i_1 + 1, 0, 0, 2)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(i_1, 1, 0, 1) + (1-\theta_{H1})\mu_{H1}\pi(i_1 + 1, 0, 0, 4)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(i_1, 1, 0, 3).$$

Case 200: For $2 \le i_1 \le N-2$, $j_1 = 1$, $x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1} + \theta_{H1}\lambda_{H-1} + (1-\theta_{H1})\lambda_{L-1} + \theta_{H1}\lambda_{L-1} + (1-\theta_{H1})\lambda_{H2}$$
$$+ \theta_{H1}\lambda_{H2} + (1-\theta_{H1})\lambda_{L2} + \theta_{H1}\lambda_{L2} + \beta_1 + (1-\theta_{H1})i_1\alpha_{H1}$$
$$+ \theta_{H1}i_1\alpha_{H1} + (1-\theta_{H1})\alpha_{L1} + \theta_{H1}\alpha_{L1}]\pi(i_1, 1, 0, 0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(i_1 - 1, 1, 0, 0) + (1-\theta_{H1})\lambda_{L-1}\pi(i_1, 0, 0, 0)$$
$$+ (1-\theta_{H1})\lambda_{H2}\pi(i_1 - 2, 1, 0, 0)$$
$$+ (1-\theta_{H1})(i_1+1)\alpha_{H1}\pi(i_1 + 1, 1, 0, 0)$$
$$+ (1-\theta_{H1})2\alpha_{L1}\pi(i_1, 1, 0, 0) + (1-\theta_{H1})\mu_{H1}\pi(i_1 + 1, 1, 0, 2)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(i_1, 2, 0, 1) + (1-\theta_{H1})\mu_{H1}\pi(i_1 + 1, 1, 0, 4)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(i_1, 2, 0, 3).$$

Case 201: For $2 \le i_1 \le N-3$, $2 \le j_1 \le N - i_1 - 1$, $x_1 = 0$, and $y_1 = 0$,

$$[(1-\theta_{H1})\lambda_{H-1}+\theta_{H1}\lambda_{H-1}+(1-\theta_{H1})\lambda_{L-1}+\theta_{H1}\lambda_{L-1}+(1-\theta_{H1})\lambda_{H2}$$
$$+\theta_{H1}\lambda_{H2}+(1-\theta_{H1})\lambda_{L2}+\theta_{H1}\lambda_{L2}+\beta_1+(1-\theta_{H1})i_1\alpha_{H1}$$
$$+\theta_{H1}i_1\alpha_{H1}+(1-\theta_{H1})j_1\alpha_{L1}+\theta_{H1}j_1\alpha_{L1}]\pi(i_1,j_1,0,0)$$
$$=(1-\theta_{H1})\lambda_{H-1}\pi(i_1-1,j_1,0,0)+(1-\theta_{H1})\lambda_{L-1}\pi(i_1,j_1-1,0,0)$$
$$+(1-\theta_{H1})\lambda_{H2}\pi(i_1-2,j_1,0,0)$$
$$+(1-\theta_{H1})\lambda_{L2}\pi(i_1,j_1-2,0,0)$$
$$+(1-\theta_{H1})(i_1+1)\alpha_{H1}\pi(i_1+1,j_1,0,0)$$
$$+(1-\theta_{H1})(j_1+1)\alpha_{L1}\pi(i_1,j_1+1,0,0)$$
$$+(1-\theta_{H1})\mu_{H1}\pi(i_1+1,j_1,0,2)$$
$$+(1-\theta_{H1})\mu_{L1}\pi(i_1,j_1+1,0,1)$$
$$+(1-\theta_{H1})\mu_{H1}\pi(i_1+1,j_1,0,4)$$
$$+(1-\theta_{H1})\mu_{L1}\pi(i_1,j_1+1,0,3).$$

Case 202: For $2\le i_1\le N-2,\ j_1=N-i_1,\ x_1=0$, and $y_1=0$,

$$[\beta_1+(1-\theta_{H1})i_1\alpha_{H1}+\theta_{H1}i_1\alpha_{H1}+(1-\theta_{H1})(N-i_1)\alpha_{L1}+\theta_{H1}(N-i_1)\alpha_{L1}]$$
$$\pi(i_1,N-i_1,0,0)$$
$$=(1-\theta_{H1})\lambda_{H-1}\pi(i_1-1,N-i_1,0,0)+(1-\theta_{H1})\lambda_{L-1}\pi(i,N-i_1-1,0,0)$$
$$+(1-\theta_{H1})\lambda_{H2}\pi(i_1-2,N-i_1,0,0)$$
$$+(1-\theta_{H})\lambda_{H2}\pi(i_1-1,N-i_1,0,0)$$
$$+(1-\theta_{H})\lambda_{L2}\pi(i_1,N-i_1-2,0,0)$$
$$+(1-\theta_{H})\lambda_{L2}\pi(i_1,N-i_1-1,0,0)$$
$$+(1-\theta_{H1})\mu_{H1}\pi(i_1+1,N-i_1,0,2)$$
$$+(1-\theta_{H1})\mu_{L1}\pi(i_1,N-i_1+1,0,1)$$
$$+(1-\theta_{H1})\mu_{H1}\pi(i_1+1,N-i_1,0,4)+(1-\theta_{H1})\mu_{L1}\pi(i_1,N$$
$$-i_1+1,0,3).$$

Case 203: For $i_1=N-1,\ j_1=1,\ x_1=0$, and $y_1=0$,

$$[\beta_1+(1-\theta_{H1})(N-1)\alpha_{H1}+\theta_{H1}(N-1)\alpha_{H1}+(1-\theta_{H1})\alpha_{L1}+\theta_{H1}\alpha_{L1}]$$
$$\pi(N-1,1,0,0)$$

$$= (1-\theta_{H1})\lambda_{H-1}\pi(N-2,1,0,0) + (1-\theta_{H1})\lambda_{L-1}\pi(N-1,0,0,0)$$
$$+ (1-\theta_{H1})\lambda_{H2}\pi(N-3,1,0,0) + (1-\theta_{H1})\lambda_{H2}\pi(N-2,1,0,0)$$
$$+ (1-\theta_{H1})\lambda_{L2}\pi(N-1,0,0,0) + (1-\theta_{H1})\mu_{H1}\pi(N,1,0,2)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(N-1,2,0,1) + (1-\theta_{H1})\mu_{H1}\pi(N,1,0,4)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(N-1,2,0,3).$$

Case 204: For $i_1 = N,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 0$,

$$[\beta_1 + (1-\theta_{H1})N\alpha_{H1} + \theta_{H1}N\alpha_{H1}]\pi(N,0,0,0)$$
$$= (1-\theta_{H1})\lambda_{H-1}\pi(N-1,0,0,0) + (1-\theta_{H1})\lambda_{H2}\pi(N-2,0,0,0)$$
$$+ (1-\theta_{H1})\lambda_{H2}\pi(N-1,0,0,0) + (1-\theta_{H1})\mu_{H1}\pi(N+1,0,0,2)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(N,1,0,1) + (1-\theta_{H1})\mu_{H1}\pi(N+1,0,0,4)$$
$$+ (1-\theta_{H1})\mu_{L1}\pi(N,1,0,3).$$

Case 205: For $i_1 = 0,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 0$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1)\pi(0,0,x_1,0)$$
$$= \beta_1\pi(0,0,x_1-1,0) + \mu_{H1}\pi(1,0,x_1,2) + \mu_{L1}\pi(0,1,x_1,1) + \mu_{H1}\pi(1,0,x_1,4)$$
$$+ \mu_{L1}\pi(0,1,x_1,3).$$

Case 206: For $i_1 = 0,\ j_1 = 0,\ x_1 = K$, and $y_1 = 0$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2})\pi(0,0,K,0)$$
$$= \beta_1\pi(0,0,K-1,0) + \mu_{H1}\pi(1,0,K,2) + \mu_{L1}\pi(0,1,K,1) + \mu_{H1}\pi(1,0,K,4) +$$
$$\mu_{L1}\pi(0,1,K,3).$$

Case 207: For $i_1 = 0,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 1$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{L1})\pi(0,1,0,1)$$
$$= \lambda_{L-1}\pi(0,0,1,0) + \beta_1\pi(0,1,0,0) + \alpha_{H1}\pi(1,1,0,1) + \alpha_{L1}\pi(0,2,0,1) +$$
$$\mu_{H1}\pi(1,1,1,2) + \mu_{L1}\pi(0,2,1,1) + \mu_{H1}\pi(1,1,1,4) + \mu_{L1}\pi(0,2,1,3).$$

Case 208: For $i_1 = 0,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + (1-\theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0,2,0,1)$$
$$= \lambda_{L-1}\pi(0,1,0,1) + \lambda_{L2}\pi(0,0,1,0) + \beta_1\pi(0,2,0,0) + \alpha_{H1}\pi(1,2,0,1)$$
$$+ 2\alpha_{L1}\pi(0,3,0,1) + \mu_{H1}\pi(1,2,1,2) + \mu_{L1}\pi(0,3,1,1)$$
$$+ \mu_{H1}\pi(1,2,1,4) + \mu_{L1}\pi(0,3,1,3).$$

Case 209: For $i_1 = 0, 3 \leq j_1 \leq N,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (j_1 - 1)\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} \\
&\qquad + \theta_{L1}\mu_{L1}]\pi(0, j_1, 0, 1) \\
&= \lambda_{L-1}\pi(0, j_1 - 1, 0, 1) + \lambda_{L2}\pi(0, j_1 - 2, 0, 1) + \beta_1\pi(0, j_1, 0, 0) \\
&\qquad + \alpha_{H1}\pi(1, j_1, 0, 1) + j_1\alpha_{L1}\pi(0, j_1 + 1, 0, 1) + \mu_{H1}\pi(1, j_1, 1, 2) \\
&\qquad + \mu_{L1}\pi(0, j_1 + 1, 1, 1) + \mu_{H1}\pi(1, j_1, 1, 4) + \mu_{L1}\pi(0, j_1 + 1, 1, 3).
\end{aligned}$$

Case 210: For $i_1 = 0,\ j_1 = N + 1,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\beta_1 + N\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0, N + 1, 0, 1) \\
&= \lambda_{L-1}\pi(0, N, 0, 1) + \lambda_{L2}\pi(0, N - 1, 0, 1) + \lambda_{L2}\pi(0, N, 0, 1).
\end{aligned}$$

Case 211: For $i_1 = 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(1, 1, 0, 1) \\
&= \lambda_{H-1}\pi(0, 1, 0, 1) + 2\alpha_{H1}\pi(2, 1, 0, 1) + \alpha_{L1}\pi(1, 2, 0, 1).
\end{aligned}$$

Case 212: For $i_1 = 1,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{L1} \\
&\qquad + \theta_{H1}\mu_{L1}]\pi(1, 2, 0, 1) \\
&= \lambda_{H-1}\pi(0, 2, 0, 1) + \lambda_{L-1}\pi(1, 1, 0, 1) + 2\alpha_{H1}\pi(2, 2, 0, 1) + 2\alpha_{L1}\pi(1, 3, 0, 1).
\end{aligned}$$

Case 213: For $i_1 = 1,\ 3 \leq j_1 \leq N - 1,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (j_1 - 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{L1} \\
&\qquad + \theta_{H1}\mu_{L1}]\pi(1, j_1, 0, 1) \\
&= \lambda_{H-1}\pi(0, j_1, 0, 1) + \lambda_{L-1}\pi(1, j_1 - 1, 0, 1) + \lambda_{L2}\pi(1, j_1 - 2, 0, 1) \\
&\qquad + 2\alpha_{H1}\pi(2, j_1, 0, 1) + j_1\alpha_{L1}\pi(1, j_1 + 1, 0, 1).
\end{aligned}$$

Case 214: For $i_1 = 1,\ j_1 = N,\ x_1 = 0$, and $y_1 = 1$,

$$\begin{aligned}
&[\beta_1 + \alpha_{H1} + (N - 1)\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(1, N, 0, 1) \\
&= \lambda_{H-1}\pi(0, N, 0, 1) + \lambda_{L-1}\pi(1, N - 1, 0, 1) + \lambda_{H2}\pi(0, N, 0, 1) \\
&\qquad + \lambda_{L2}\pi(1, N - 2, 0, 1) + \lambda_{L2}\pi(1, N - 1, 0, 1) + 2\alpha_{H1}\pi(2, N, 0, 1) \\
&\qquad + N\alpha_{L1}\pi(1, N + 1, 0, 1).
\end{aligned}$$

Case 215: For $2 \leq i_1 \leq N-1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, 1, 0, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, 0, 1) + \lambda_{H2}\pi(i_1 - 2, 1, 0, 1) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 1, 0, 1)$$
$$+ \alpha_{L1}\pi(i_1, 2, 0, 1)$$

Case 216: For $2 \leq i_1 \leq N-2,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \alpha_{L1} + (1-\theta_{H1})\mu_{L1}$$
$$+ \theta_{H1}\mu_{L1}]\pi(i_1, 2, 0, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 2, 0, 1) + \lambda_{H2}\pi(i_1 - 2, 2, 0, 1) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 2, 0, 1)$$
$$+ 2\alpha_{L1}\pi(i_1, 3, 0, 1)$$

Case 217: For $2 \leq i_1 \leq N-3,\ 3 \leq j_1 \leq N - i_1,\ x_1 = 0$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (j_1 - 1)\alpha_{L1} + (1-\theta_{H1})\mu_{L1}$$
$$+ \theta_{H1}\mu_{L1}]\pi(i_1, j_1, 0, 1)$$
$$= \lambda_{H1}\pi(i_1 - 1, j_1, 0, 1) + \lambda_{H2}\pi(i_1 - 2, j_1, 0, 1) + \lambda_{L2}\pi(i_1, j_1 - 2, 0, 1)$$
$$+ (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, j_1, 0, 1) + \alpha_{L1}\pi(i_1, j_1 + 1, 0, 1)$$

Case 218: For $2 \leq i_1 \leq N-2,\ j_1 = N - i_1 + 1,\ x_1 = 0$, and $y_1 = 1$,

$$[\beta_1 + i_1\alpha_{H1} + (N - i_1)\alpha_{L1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, N - i_1 + 1, 0, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, 0, 1) + \lambda_{L-1}\pi(i_1, N - i_1, 0, 1)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, 0, 1) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, 0, 1)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1 - 1, 0, 1) + \lambda_{L2}\pi(i_1, N - i_1, 0, 1).$$

Case 219: For $i_1 = N-1,\ j_1 = 2\ x_1 = 0$, and $y_1 = 1$,

$$[\beta_1 + (N-1)\alpha_{H1} + \alpha_{L1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N-1, 2, 0, 1)$$
$$= \lambda_{H-1}\pi(N-2, 2, 0, 1) + \lambda_{L-1}\pi(N-1, 1, 0, 1) + \lambda_{H2}\pi(N-3, 2, 0, 1)$$
$$+ \lambda_{H2}\pi(N-2, 2, 0, 1) + \lambda_{L2}\pi(N-1, 1, 0, 1).$$

Case 220: For $i_1 = N,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 1$,

$$[\beta_1 + N\alpha_{H1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N, 1, 0, 1)$$
$$= \lambda_{H-1}\pi(N-1, 1, 0, 1) + \lambda_{H2}\pi(N-2, 1, 0, 1) + \lambda_{H2}\pi(N-1, 1, 0, 1).$$

Case 221: For $i_1 = 0,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{L1})\pi(0,1,x_1,1) \\
&= \lambda_{L-1}\pi(0,0,x_1+1,0) + \beta_1\pi(0,1,x_1-1,1) + \alpha_{H1}\pi(1,1,x_1,1) \\
&\qquad + \alpha_{L1}\pi(0,2,x_1,1) + \mu_{H1}\pi(1,1,x_1+1,2) + \mu_{L1}\pi(0,2,x_1+1,1) \\
&\qquad + \mu_{H1}\pi(1,1,x_1+1,4) + \mu_{L1}\pi(0,2,x_1+1,3).
\end{aligned}
$$

Case 222: For $i_1 = 0,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + \mu_{L1}]\pi(0,2,x_1,1) \\
&= \lambda_{L-1}\pi(0,1,x_1,1) + \lambda_{L2}\pi(0,0,x_1+1,0) + \beta_1\pi(0,2,x_1-1,1) \\
&\qquad + \alpha_{H1}\pi(1,2,x_1,1) + 2\alpha_{L1}\pi(0,3,x_1,1) + \mu_{H1}\pi(1,2,x_1+1,2) \\
&\qquad + \mu_{L1}\pi(0,3,x_1+1,1) + \mu_{H1}\pi(1,2,x_1+1,4) \\
&\qquad + \mu_{L1}\pi(0,3,x_1+1,3).
\end{aligned}
$$

Case 223: For $i_1 = 0,\ 3 \le j_1 \le N,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(0,j_1,x_1,1) \\
&= \lambda_{L-1}\pi(0,j_1-1,x_1,1) + \lambda_{L2}\pi(0,j_1-2,x_1,1) + \beta_1\pi(0,j_1,x_1-1,1) \\
&\qquad + \alpha_{H1}\pi(1,j_1,x_1,1) + j_1\alpha_{L1}\pi(0,j_1+1,x_1,1) \\
&\qquad + \mu_{H1}\pi(1,j_1,x_1+1,2) + \mu_{L1}\pi(0,j_1+1,x_1+1,1) \\
&\qquad + \mu_{H1}\pi(1,j_1,x_1+1,4) + \mu_{L1}\pi(0,j_1+1,x_1+1,3).
\end{aligned}
$$

Case 224: For $i_1 = 0,\ j_1 = N+1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
\begin{aligned}
&(\beta_1 + N\alpha_{L1} + \mu_{L1})\pi(0,N+1,x_1,1) \\
&= \lambda_{L-1}\pi(0,N,x_1,1) + \lambda_{L2}\pi(0,N-1,x_1,1) + \lambda_{L2}\pi(0,N,x_1,1) \\
&\qquad + \beta_1\pi(0,N+1,x_1-1,1).
\end{aligned}
$$

Case 225: For $i_1 = 1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \mu_{L1})\pi(1,1,x_1,1) \\
&= \lambda_{H-1}\pi(0,1,x_1,1) + \beta_1\pi(1,1,x_1-1,1) + 2\alpha_{H1}\pi(2,1,x_1,1) + \\
&\quad \alpha_{L1}\pi(1,2,x_1,1).
\end{aligned}
$$

Case 226: For $i_1 = 1,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$
(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + \mu_{L1})\pi(1,2,x_1,1)
$$

$$= \lambda_{H-1}\pi(0,2,x_1,1) + \lambda_{L-1}\pi(1,1,x_1,1) + \beta_1\pi(1,2,x_1-1,1)$$
$$+ 2\alpha_{H1}\pi(2,2,x_1,1) + 2\alpha_{L1}\pi(1,3,x_1,1).$$

Case 227: For $i_1 = 1,\ 3 \le j_1 \le N-1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (j_1-1)\alpha_{L1} + \mu_{L1})\pi(1,j_1,x_1,1)$$
$$= \lambda_{H-1}\pi(0,j_1,x_1,1) + \lambda_{L-1}\pi(1,j_1-1,x_1,1) + \lambda_{L2}\pi(1,j_1-2,x_1,1)$$
$$+ \beta_1\pi(i,1,x_1-1,1) + 2\alpha_{H1}\pi(2,j_1,x_1,1)$$
$$+ j_1\alpha_{L1}\pi(1,j_1+1,x_1,1).$$

Case 228: For $i_1 = 1,\ j_1 = N,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$(\beta_1 + \alpha_{H1} + (N-1)\alpha_{L1} + \mu_{L1})\pi(1,N,x_1,1)$$
$$= \lambda_{H-1}\pi(0,N,x_1,1) + \lambda_{L-1}\pi(1,N-1,x_1,1) + \lambda_{H2}\pi(0,N,x_1,1)$$
$$+ \lambda_{L2}\pi(1,N-2,x_1,1) + \lambda_{L2}\pi(1,N-1,x_1,1) + \beta_1\pi(1,N,x_1-1,1)$$
$$+ 2\alpha_{H1}\pi(2,N,x_1,1) + N\alpha_{L1}\pi(1,N+1,x_1,1).$$

Case 229: For $2 \le i_1 \le N-1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \mu_{L1}]\pi(i_1,1,x_1,1)$$
$$= \lambda_{H-1}\pi(i_1-1,1,x_1,1) + \lambda_{H2}\pi(i_1-2,1,x_1,1) + \beta_1\pi(i_1,1,x_1-1,1) +$$
$$(i_1+1)\alpha_{H1}\pi(i_1+1,1,x_1,1) + \alpha_{L1}\pi(i_1,2,x_1,1).$$

Case 230: For $2 \le i_1 \le N-1,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(i_1,2,x_1,1)$$
$$= \lambda_{H-1}\pi(i_1-1,2,x_1,1) + \lambda_{H2}\pi(i_1-2,2,x_1,1) + \lambda_{L-1}\pi(i_1,1,x_1,1)$$
$$+ \beta_1\pi(i_1,2,x_1-1,1) + (i_1+1)\alpha_{H1}\pi(i_1+1,2,x_1,1)$$
$$+ 2\alpha_{L1}\pi(i_1,3,x_1,1).$$

Case 231: For $2 \le i_1 \le N-3,\ 3 \le j_1 \le N-i_1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(i_1,j_1,x_1,1)$$
$$= \lambda_{H-1}\pi(i_1-1,j_1,x_1,1) + \lambda_{L-1}\pi(i_1,j_1-1,x_1,1) + \lambda_{H2}\pi(i_1-2,j_1,x_1,1)$$
$$+ \lambda_{L2}\pi(i_1,j_1-2,x_1,1) + \beta_1\pi(i_1,j_1,x_1-1,1)$$
$$+ (i_1+1)\alpha_{H1}\pi(i_1+1,j_1,x_1,1) + j_1\alpha_{L1}\pi(i_1,j_1+1,x_1,1).$$

Case 232: For $2 \le i_1 \le N-2,\ j_1 = N-i_1+1,\ 1 \le x_1 \le K-1$, and $y_1 = 1$,

$$[\beta_1 + i_1\alpha_{H1} + (N - i_1)\alpha_{L1} + \mu_{L1}]\pi(i_1, N - i_1 + 1, x_1, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, x_1, 1) + \lambda_{L-1}\pi(i_1, N - i_1, x_1, 1)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, x_1, 1)$$
$$+ \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, x_1, 1) + \lambda_{L2}\pi(i_1, N - i_1 - 1, x_1, 1)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1, x_1, 1) + \beta_1\pi(i_1, N - i_1 + 1, x_1 - 1, 1).$$

Case 233: For $i_1 = N - 1,\ j_1 = 2,\ 1 \le x_1 \le K - 1$, and $y_1 = 1$,

$$[\beta_1 + (N - 1)\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(N - 1, 2, x_1, 1)$$
$$= \lambda_{H-1}\pi(N - 2, 2, x_1, 1) + \lambda_{L-1}\pi(N - 1, 1, x_1, 1) + \lambda_{H2}\pi(N - 3, 2, x_1, 1)$$
$$+ \lambda_{H2}\pi(N - 2, 2, x_1, 1) + \lambda_{L2}\pi(N - 1, 1, x_1, 1)$$
$$+ \beta_1\pi(N - 1, 2, x_1 - 1, 1).$$

Case 234: For $i_1 = N,\ j_1 = 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 1$,

$$(\beta_1 + N\alpha_{H1} + \mu_{L1})\pi(N, 1, x_1, 1)$$
$$= \lambda_{H-1}\pi(N - 1, 1, x_1, 1) + \lambda_{H2}\pi(N - 2, 1, x_1, 1) + \lambda_{H2}\pi(N - 1, 1, x_1, 1)$$
$$+ \beta_1\pi(N, 1, x_1 - 1, 1).$$

Case 235: For $i_1 = 0,\ j_1 = 1,\ x_1 = K$, and $y_1 = 1$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \mu_{L1})\pi(0, 1, K, 1)$$
$$= \lambda_{L-1}\pi(0, 0, K, 0) + \beta_1\pi(0, 1, K - 1, 1) + \alpha_{H1}\pi(1, 1, K, 1) + \alpha_{L1}\pi(0, 2, K, 1).$$

Case 236: For $i_1 = 0,\ j_1 = 2,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{L1} + \mu_{L1}]\pi(0, 2, K, 1)$$
$$= \lambda_{L-1}\pi(0, 1, K, 1) + \lambda_{L2}\pi(0, 0, K, 0) + \beta_1\pi(0, 2, K - 1, 1) + \alpha_{H1}\pi(1, 2, K, 1)$$
$$+ 2\alpha_{L1}\pi(0, 3, K, 1).$$

Case 237: For $i_1 = 0,\ 3 \le j_1 \le N,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (j_1 - 1)\alpha_{L1} + \mu_{L1}]\pi(0, j_1, K, 1)$$
$$= \lambda_{L-1}\pi(0, j_1 - 1, K, 1) + \lambda_{L2}\pi(0, j_1 - 2, K, 1) + \beta_1\pi(0, j_1, K - 1, 1)$$
$$+ \alpha_{H1}\pi(1, j_1, K, 1) + j_1\alpha_L\pi(0, j_1 + 1, K, 1).$$

Case 238: For $i_1 = 0,\ j_1 = N + 1,\ x_1 = K$, and $y_1 = 1$,

$$(N\alpha_{L1} + \mu_{L1})\pi(0, N+1, K, 1)$$
$$= \lambda_{L-1}\pi(0, N, K, 1) + \lambda_{L2}\pi(0, N-1, K, 1) + \lambda_{L2}\pi(0, N, K, 1)$$
$$+ \beta_1\pi(0, N+1, K-1, 1).$$

Case 239: For $i_1 = 1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 1$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \mu_{L1})\pi(1, 1, K, 1)$$
$$= \lambda_{H-1}\pi(0, 1, K, 1) + \beta_1\pi(1, 1, K-1, 1) + 2\alpha_{H1}\pi(2, 1, K, 1) +$$
$$\alpha_{L1}\pi(1, 2, K, 1).$$

Case 240: For $i_1 = 1,\ j_1 = 2,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(1, 2, K, 1)$$
$$= \lambda_{H-1}\pi(0, 2, K, 1) + \lambda_{L-1}\pi(1, 1, K, 1) + \beta_1\pi(1, 2, K-1, 1) +$$
$$2\alpha_{H1}\pi(2, 2, K, 1) + 2\alpha_{L1}\pi(1, 3, K, 1).$$

Case 241: For $i_1 = 1,\ 3 \le j_1 \le N-1,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + (j_1 - 1)\alpha_{L1} + \mu_{L1}]\pi(1, j_1, K, 1)$$
$$= \lambda_{H-1}(0, j_1, K, 1) + \lambda_{L-1}\pi(1, j_1 - 1, K, 1) + \lambda_{L2}\pi(1, j_1 - 2, K, 1)$$
$$+ \beta_1\pi(1, j_1, K-1, 1) + 2\alpha_{H1}\pi(2, j_1, K, 1) + j_1\alpha_{L1}\pi(1, j_1 + 1, K, 1).$$

Case 242: For $i_1 = 1,\ j_1 = N,\ x_1 = K$, and $y_1 = 1$,

$$[\alpha_{H1} + (N-1)\alpha_{L1} + \mu_{L1}]\pi(1, N, K, 1)$$
$$= \lambda_{H-1}\pi(0, N, K, 1) + \lambda_{L-1}\pi(1, N-1, K, 1) + \lambda_{H2}\pi(0, N, K, 1)$$
$$+ \lambda_{L2}\pi(1, N-2, K, 1) + \lambda_{L2}\pi(1, N-1, K, 1) + \beta_1\pi(i, N-i+1, K-1, 1).$$

Case 243: For $2 \le i_1 \le N-1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + \mu_{L1}]\pi(i_1, 1, K, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, K, 1) + \lambda_{H2}\pi(i_1 - 2, 1, K, 1) + \beta_1\pi(i_1, 1, K-1, 1) +$$
$$(i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 1, K, 1) + \alpha_{L1}\pi(i_1, 2, K, 1).$$

Case 244: For $2 \le i_1 \le N-1,\ j_1 = 2,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(i_1, 2, K, 1)$$

$$= \lambda_{H-1}\pi(i_1 - 1, 2, K, 1) + \lambda_{H2}\pi(i_1 - 2, 2, K, 1) + \lambda_{L-1}\pi(i_1, 1, K, 1)$$
$$+ \beta_1\pi(i_1, 2, K-1, 1) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 2, K, 1) + 2\alpha_{L1}\pi(i_1, 3, K, 1).$$

Case 245: For $2 \le i_1 \le N-3,\ 3 \le j_1 \le N - i_1,\ x_1 = K$, and $y_1 = 1$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + (j_1 - 1)\alpha_{L1} + \mu_{L1}]\pi(i_1, j_1, K, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, K, 1) + \lambda_{L-1}\pi(i_1, j_1 - 1, K, 1) + \lambda_{H2}\pi(i_1 - 2, j_1, K, 1)$$
$$+ \lambda_{L2}\pi(i_1, j_1 - 2, K, 1) + \beta_1\pi(i_1, j_1, K-1, 1) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, j_1, K, 1)$$
$$+ j_1\alpha_{L1}\pi(i_1, j_1 + 1, K, 1).$$

Case 246: For $2 \le i_1 \le N-2,\ j_1 = N - i_1 + 1,\ x_1 = K$, and $y_1 = 1$,

$$[i_1\alpha_{H1} + (N - i_1)\alpha_{L1} + \mu_{L1}]\pi(i_1, N - i_1 + 1, K, 1)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, K, 1) + \lambda_{L-1}\pi(i_1, N - i_1, K, 1)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, K, 1) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, K, 1)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1 - 1, K, 1) + \lambda_{L2}\pi(i_1, N - i_1, K, 1)$$
$$+ \beta_1\pi(i_1, N - i_1 + 1, K-1, 1).$$

Case 247: For $i_1 = N-1,\ j_1 = 2,\ x_1 = K$, and $y_1 = 1$,

$$[(N-1)\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(N-1, 2, K, 1)$$
$$= \lambda_{H-1}\pi(N-2, 2, K, 1) + \lambda_{L-1}\pi(N-1, 1, K, 1) + \lambda_{H2}\pi(N-3, 2, K, 1)$$
$$+ \lambda_{H2}\pi(N-2, 2, K, 1) + \lambda_{L2}\pi(N-1, 1, K, 1)$$
$$+ \beta_1\pi(N-1, 2, K-1, 1).$$

Case 248: For $i_1 = N,\ j_1 = 1,\ x_1 = K$, and $y_1 = 1$,

$$(N\alpha_{H1} + \mu_{L1})\pi(N, 1, K, 1)$$
$$= \lambda_{H-1}\pi(N-1, 1, K, 1) + \lambda_{H2}\pi(N-2, 1, x, 1) + \lambda_{H2}\pi(N-1, 1, x, 1)$$
$$+ \beta_1\pi(N, 1, K-1, 1).$$

Case 249: For $i_1 = 1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 2$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{H1})\pi(1, 0, 0, 2)$$
$$= \lambda_{H-1}\pi(0, 0, 1, 0) + \beta_1\pi(1, 0, 0, 0) + \alpha_{H1}\pi(2, 0, 0, 2) + \alpha_{L1}\pi(1, 1, 0, 2) +$$
$$\mu_{H1}\pi(2, 0, 1, 2) + \mu_{L1}\pi(1, 1, 1, 1) + \mu_{H1}\pi(2, 0, 1, 4) + \mu_{L1}\pi(1, 1, 1, 3).$$

Case 250: For $i_1 = 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 2$,

$$[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+\alpha_{L1}+(1-\theta_{L1})\mu_{H1}+\theta_{L1}\mu_{H1}]\pi(1,1,0,2)$$
$$=\lambda_{L-1}\pi(1,0,0,2)+\beta_1\pi(1,1,0,0)+\alpha_{H-1}\pi(2,1,0,2)+2\alpha_{L1}\pi(1,2,0,2)$$
$$+\mu_{H1}\pi(2,1,1,2)+\mu_{L1}\pi(1,2,1,1)+\mu_{H1}\pi(2,1,1,4)$$
$$+\mu_{L1}\pi(1,2,1,3).$$

Case 251: For $i_1=1,\ 2\le j_1\le N-1,\ x_1=0$, and $y_1=2$,

$$[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+j_1\alpha_{L1}+(1-\theta_{L1})\mu_{H1}+\theta_{L1}\mu_{H1}]\pi(1,j_1,0,2)$$
$$=\lambda_{L-1}\pi(1,j_1-1,0,2)+\lambda_{L2}\pi(1,j_1-2,0,2)+\beta_1\pi(1,j_1,0,0)$$
$$+\alpha_{H1}\pi(2,j_1,0,2)+(j_1+1)\alpha_{L1}\pi(1,j_1+1,0,2)$$
$$+\mu_{H1}\pi(2,j_1,1,2)+\mu_{L1}\pi(1,j_1+1,1,1)+\mu_{H1}\pi(2,j_1,1,4)$$
$$+\mu_{L1}\pi(1,j_1+1,1,3).$$

Case 252: For $i_1=1,\ j_1=N,\ x_1=0$, and $y_1=2$,

$$[\beta_1+N\alpha_{L1}+(1-\theta_{L1})\mu_{H1}+\theta_{L1}\mu_{H1}]\pi(1,N,0,2)$$
$$=\lambda_{L-1}\pi(1,N-1,0,2)+\lambda_{L2}\pi(1,N-2,0,2)+\lambda_{L2}\pi(1,N-1,0,2).$$

Case 253: For $i_1=2,\ j_1=0,\ x_1=0$, and $y_1=2$,

$$[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+\alpha_{H1}+(1-\theta_{H1})\mu_{H1}+\theta_{H1}\mu_{H1}]\pi(2,0,0,2)$$
$$=\lambda_{H-1}\pi(1,0,0,2)+\lambda_{H2}\pi(0,0,1,0)+\beta_1\pi(2,0,0,0)+2\alpha_{H1}\pi(3,0,0,2)+$$
$$\alpha_{L1}\pi(2,1,0,2)+\mu_{H1}\pi(3,0,1,2)+\mu_{L1}\pi(2,1,1,1)+\mu_{H1}\pi(3,0,1,4)+$$
$$\mu_{L1}\pi(2,1,1,3).$$

Case 254: For $i_1=2,\ j_1=1,\ x_1=0$, and $y_1=2$,

$$[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+\alpha_{H1}+\alpha_{L1}+(1-\theta_{H1})\mu_{H1}$$
$$+\theta_{H1}\mu_{H1}]\pi(2,1,0,2)$$
$$=\lambda_{H-1}\pi(1,1,0,2)+\lambda_{L-1}\pi(2,0,0,2)+\beta_1\pi(2,1,0,0)+2\alpha_{H1}\pi(3,1,0,2)+$$
$$\alpha_{L1}\pi(2,2,0,2)+\mu_{H1}\pi(3,1,1,2)+\mu_{L1}\pi(2,2,1,1)+\mu_{H1}\pi(3,1,1,4)+$$
$$\mu_{L1}\pi(2,2,1,3).$$

Case 255: For $i_1=2,\ 2\le j_1\le N-2,\ x_1=0$, and $y_1=2$,

$$[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+\alpha_{H1}+j_1\alpha_{L1}+(1-\theta_{H1})\mu_{H1}$$
$$+\theta_{H1}\mu_{H1}]\pi(2,j_1,0,2)$$

$$= \lambda_{H-1}\pi(1, j_1, 0, 2) + \lambda_{L-1}\pi(2, j_1 - 1, 0, 2) + \lambda_{L2}\pi(2, j_1 - 2, 0, 2)$$
$$+ \beta_1\pi(2, j_1, 0, 0) + 2\alpha_{H1}\pi(3, j_1, 0, 2)$$
$$+ (j_1 + 1)\alpha_{L1}\pi(2, j_1 + 1, 0, 2) + \mu_{H1}\pi(3, j_1, 1, 2)$$
$$+ \mu_{L1}\pi(2, j_1 + 1, 1, 1) + \mu_{H1}\pi(3, j_1, 1, 4) + \mu_{L1}\pi(2, j_1 + 1, 1, 3).$$

Case 256: For $i_1 = 2,\ j_1 = N - 1,\ x_1 = 0$, and $y_1 = 2$,

$$[\beta_1 + \alpha_{H1} + (N-1)\alpha_{L1} + (1 - \theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(2, N-1, 0, 2)$$
$$= \lambda_{H-1}\pi(1, N-1, 0, 2) + \lambda_{L-1}\pi(2, N-2, 0, 2) + \lambda_{H2}\pi(1, N-1, 0, 2)$$
$$+ \lambda_{L2}\pi(2, N-3, 0, 2) + \lambda_{L2}\pi(2, N-2, 0, 2).$$

Case 257: For $3 \le i_1 \le N,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + (1 - \theta_{H1})\mu_{H1}$$
$$+ \theta_{H1}\mu_{H1}]\pi(i_1, 0, 0, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 0, 0, 2) + \lambda_{H2}\pi(i_1 - 2, 0, 0, 2) + i_1\alpha_{H1}\pi(i_1 + 1, 0, 0, 2)$$
$$+ \alpha_{L1}\pi(i_1, 1, 0, 2) + \mu_{H1}\pi(i_1 + 1, 0, 1, 2) + \mu_{L1}\pi(i_1, 1, 1, 1)$$
$$+ \mu_{H1}\pi(i_1 + 1, 0, 1, 4) + \mu_{L1}\pi(i_1, 1, 1, 3).$$

Case 258: For $3 \le i_1 \le N - 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{H1}$$
$$+ \theta_{H1}\mu_{H1}]\pi(i_1, 1, 0, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, 0, 2) + \lambda_{L-1}\pi(i_1, 0, 0, 2) + \lambda_{H2}\pi(i_1 - 2, 1, 0, 2)$$
$$+ i_1\alpha_{H1}\pi(i_1 + 1, 1, 0, 2) + 2\alpha_{L1}\pi(i_1, 2, 0, 2)$$
$$+ \mu_{H1}\pi(i_1 + 1, 1, 1, 2) + \mu_{L1}\pi(i_1, 2, 1, 1) + \mu_{H1}\pi(i_1 + 1, 1, 1, 4)$$
$$+ \mu_{L1}\pi(i_1, 2, 1, 3).$$

Case 259: For $3 \le i_1 \le N - 2,\ 2 \le j_1 \le N - i_1,\ x_1 = 0$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + j_1\alpha_{L1} + (1 - \theta_{H1})\mu_{H1}$$
$$+ \theta_{H1}\mu_{H1}]\pi(i_1, j_1, 0, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, 0, 2) + \lambda_{L-1}\pi(i_1, j_1 - 1, 0, 2) + \lambda_{H2}\pi(i_1 - 2, j_1, 0, 2)$$
$$+ \lambda_{L2}\pi(i_1, j_1 - 2, 0, 2) + i_1\alpha_{H1}\pi(i_1 + 1, j_1, 0, 2) + (j_1$$
$$+ 1)\alpha_{L1}\pi(i_1, j_1 + 1, 0, 2) + \mu_{H1}\pi(i_1 + 1, j_1, 1, 2)$$
$$+ \mu_{L1}\pi(i_1, j_1 + 1, 1, 1) + \mu_{H1}\pi(i_1 + 1, j_1, 1, 4)$$
$$+ \mu_{L1}\pi(i_1, j_1 + 1, 1, 3).$$

Case 260: For $3 \le i_1 \le N-1,\ j_1 = N - i_1 + 1,\ x_1 = 0$, and $y_1 = 2$,

$$
\begin{aligned}
&[\beta_1 + (i_1 - 1)\alpha_{H1} + (N - i_1 + 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, N - i_1 \\
&\qquad + 1, 0, 2) \\
&= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, 0, 2) + \lambda_{L-1}\pi(i_1, N - i_1, 0, 2) \\
&\qquad + \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, 0, 2) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, 0, 2) \\
&\qquad + \lambda_{L2}\pi(i_1, N - i_1 - 1, 0, 2) + \lambda_{L2}\pi(i_1, N - i_1, 0, 2).
\end{aligned}
$$

Case 261: For $i_1 = N,\ j_1 = 1\ x_1 = 0$, and $y_1 = 2$,

$$
\begin{aligned}
&[\beta_1 + (N - 1)\alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N, 1, 0, 2) \\
&= \lambda_{H-1}\pi(N - 1, 1, 0, 2) + \lambda_{L-1}\pi(N, 0, 0, 2) + \lambda_{H2}\pi(N - 2, 1, 0, 2) \\
&\qquad + \lambda_{H2}\pi(N - 1, 1, 0, 2) + \lambda_{L2}\pi(N, 0, 0, 2)
\end{aligned}
$$

Case 262: For $i_1 = N + 1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 2$,

$$
\begin{aligned}
&[\beta_1 + N\alpha_{H1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N + 1, 0, 0, 2) \\
&= \lambda_{H-1}\pi(N, 0, 0, 2) + \lambda_{H2}\pi(N - 1, 0, 0, 2).
\end{aligned}
$$

Case 263: For $i_1 = 1,\ j_1 = 0,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{H1})\pi(1, 0, x_1, 2) \\
&= \lambda_{H-1}\pi(0, 0, x_1 + 1, 0) + \beta_1\pi(1, 0, x_1 - 1, 2) + \alpha_{H1}\pi(2, 0, x_1, 2) \\
&\qquad + \alpha_{L1}\pi(1, 1, x_1, 2) + \mu_{H1}\pi(2, 0, x_1 + 1, 2) + \mu_{L1}\pi(1, 1, x_1 + 1, 1) \\
&\qquad + \mu_{H1}\pi(2, 0, x_1 + 1, 4) + \mu_{L1}\pi(1, 1, x_1 + 1, 3).
\end{aligned}
$$

Case 264: For $i_1 = 1,\ j_1 = 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + \mu_{H1})\pi(1, 1, x_1, 2) \\
&= \lambda_{L-1}\pi(1, 0, x_1, 2) + \beta_1\pi(1, 1, x_1 - 1, 2) + \alpha_{H1}\pi(2, 1, x_1, 2) + \\
&\quad 2\alpha_{L1}\pi(1, 2, x_1, 2) + \mu_{H1}\pi(2, 1, x_1 + 1, 2) + \mu_{L1}\pi(1, 2, x_1 + 1, 1) + \\
&\quad \mu_{H1}\pi(2, 1, x_1 + 1, 4) + \mu_{L1}\pi(1, 2, x_1 + 1, 3).
\end{aligned}
$$

Case 265: For $i_1 = 1,\ 2 \le j_1 \le N - 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$
(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + j_1\alpha_{L1} + \mu_{H1})\pi(1, j_1, x_1, 2)
$$

$$= \lambda_{L-1}\pi(1, j_1 - 1, x_1, 2) + \lambda_{L2}\pi(1, j_1 - 2, x_1, 2) + \beta_1\pi(1, j_1, x_1 - 1, 2)$$
$$+ \alpha_{H1}\pi(2, j_1, x_1, 2) + (j_1 + 1)\alpha_{L1}\pi(1, j_1 + 1, x_1, 2)$$
$$+ \mu_{H1}\pi(2, j_1, x_1 + 1, 2) + \mu_{L1}\pi(1, j_1 + 1, x_1 + 1, 1)$$
$$+ \mu_{H1}\pi(2, j_1, x_1 + 1, 4) + \mu_{L1}\pi(1, j_1 + 1, x_1 + 1, 3).$$

Case 266: For $i_1 = 1,\ j_1 = N,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$(\beta_1 + N\alpha_{L1} + \mu_{H1})\pi(1, N, x_1, 2)$$
$$= \lambda_{L-1}\pi(1, N - 1, x_1, 2) + \lambda_{L2}\pi(1, N - 2, x_1, 2) + \lambda_{L2}\pi(1, N - 1, x_1, 2)$$
$$+ \beta_1\pi(1, N, x_1 - 1, 2).$$

Case 267: For $i_1 = 2,\ j_1 = 0,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \mu_{H1}]\pi(2, 0, x_1, 2)$$
$$= \lambda_{H-1}\pi(1, 0, x_1, 2) + \beta_1\pi(2, 0, x_1 - 1, 2) + 2\alpha_{H1}\pi(3, 0, x_1, 2) +$$
$$\alpha_{L1}\pi(2, 1, x_1, 2) + \mu_{H1}\pi(3, 0, x_1 + 1, 2) + \mu_{L1}\pi(2, 1, x_1 + 1, 1) +$$
$$\mu_{H1}\pi(2 + 1, 0, x_1 + 1, 4) + \mu_{L1}\pi(2, 1, x_1 + 1, 3).$$

Case 268: For $i_1 = 2,\ j_1 = 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(2, 1, x_1, 2)$$
$$= \lambda_{H-1}\pi(1, 1, x_1, 2) + \lambda_{L-1}\pi(2, 0, x_1, 2) + \beta_1\pi(2, 1, x_1 - 1, 2) +$$
$$2\alpha_{H1}\pi(3, 1, x_1, 2) + 2\alpha_{L1}\pi(2, 2, x_1, 2) + \mu_{H1}\pi(3, 1, x_1 + 1, 2) +$$
$$\mu_{L1}\pi(2, 2, x_1 + 1, 1) + \mu_{H1}\pi(3, 1, x_1 + 1, 4) + \mu_{L1}\pi(2, 2, x_1 + 1, 3).$$

Case 269: For $i_1 = 2,\ 2 \le j_1 \le N - 2,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(2, j_1, x_1, 2)$$
$$= \lambda_{H-1}\pi(1, j_1, x_1, 2) + \lambda_{L-1}\pi(2, j_1 - 1, x_1, 2) + \lambda_{L2}\pi(2, j_1 - 2, x_1, 2)$$
$$+ \beta_1\pi(2, j_1, x_1 - 1, 2) + 2\alpha_{H1}\pi(3, j_1, x_1, 2)$$
$$+ (j_1 + 1)\alpha_{L1}\pi(2, j_1 + 1, x_1, 2) + \mu_{H1}\pi(3, j_1, x_1 + 1, 2)$$
$$+ \mu_{L1}\pi(2, j_1 + 1, x_1 + 1, 1) + \mu_{H1}\pi(3, j_1, x_1 + 1, 4)$$
$$+ \mu_{L1}\pi(2, j_1 + 1, x_1 + 1, 3).$$

Case 270: For $i_1 = 2,\ j_1 = N - 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 2$,

$$[\beta_1 + \alpha_{H1} + (N - 1)\alpha_{L1} + \mu_{H1}]\pi(2, N - 1, x_1, 2)$$

$$= \lambda_{H-1}\pi(1, N-1, x_1, 2) + \lambda_{L-1}\pi(2, N-2, x_1, 2) + \lambda_{L2}\pi(2, N-3, x_1, 2)$$
$$+ \lambda_{L2}\pi(2, N-2, x_1, 2) + \beta_1\pi(2, N-1, x_1 - 1, 2).$$

Case 271: For $3 \le i_1 \le N,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + \mu_{H1}]\pi(i_1, 0, x_1, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 0, x_1, 2) + \lambda_{H2}\pi(i_1 - 2, 0, x_1, 2) + \beta_1\pi(i_1, 0, x_1 - 1, 2) +$$
$$i_1\alpha_{H1}\pi(i_1 + 1, 0, x_1, 2) + \alpha_{L1}\pi(i_1, 1, x_1, 2) + \mu_{H1}\pi(i_1 + 1, 0, x_1 + 1, 2)$$
$$+ \mu_{L1}\pi(i_1, 1, x_1 + 1, 1) + \mu_{H1}\pi(i_1 + 1, 0, x_1 + 1, 4) + \mu_{L1}\pi(i_1, 1, x_1 + 1, 3).$$

Case 272: For $3 \le i_1 \le N-1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(i_1, 1, x_1, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, x_1, 2) + \lambda_{H2}\pi(i_1 - 2, 1, x_1, 2) + \lambda_{L-1}\pi(i_1, 0, x_1, 2)$$
$$+ \beta_1\pi(i_1, 1, x_1 - 1, 2) + i_1\alpha_{H1}\pi(i_1 + 1, 1, x_1, 2)$$
$$+ 2\alpha_{L1}\pi(i_1, 2, x_1, 2) + \mu_{H1}\pi(i_1 + 1, 1, x_1 + 1, 2)$$
$$+ \mu_{L1}\pi(i_1, 2, x_1 + 1, 1) + \mu_{H1}\pi(i_1 + 1, 1, x_1 + 1, 4)$$
$$+ \mu_{L1}\pi(i_1, 2, x_1 + 1, 3).$$

Case 273: For $3 \le i_1 \le N-2,\ 2 \le j_1 \le N - i_1,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(i_1, j_1, x_1, 2)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, x_1, 2) + \lambda_{L-1}\pi(i_1, j_1 - 1, x_1, 2) + \lambda_{H2}\pi(i_1 - 2, j_1, x_1, 2)$$
$$+ \lambda_{L2}\pi(i_1, j_1 - 2, x_1, 2) + \beta_1\pi(i_1, j_1, x_1 - 1, 2)$$
$$+ i_1\alpha_{H1}\pi(i_1 + 1, j_1, x_1, 2) + (j_1 + 1)\alpha_{L1}\pi(i_1, j_1 + 1, x_1, 2)$$
$$+ \mu_{H1}\pi(i_1 + 1, j_1, x_1 + 1, 2) + \mu_{L1}\pi(i_1, j_1 + 1, x_1 + 1, 1)$$
$$+ \mu_{H1}\pi(i_1 + 1, j_1, x_1 + 1, 4) + \mu_{L1}\pi(i_1, j_1 + 1, x_1 + 1, 3).$$

Case 274: For $3 \le i_1 \le N-1,\ j_1 = N - i_1 + 1,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$[\beta_1 + (i_1 - 1)\alpha_{H1} + (N - i_1 + 1)\alpha_{L1} + \mu_{H1}]\pi(i_1, N - i_1 + 1, x_1, 2)$$
$$= \lambda_{H1}\pi(i_1 - 1, N - i_1 + 1, x_1, 2) + \lambda_{L1}\pi(i_1, N - i_1, x_1, 2)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, x_1, 2)$$
$$+ \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, x_1, 2) + \lambda_{L2}\pi(i_1, N - i_1 - 1, x_1, 2)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1, x_1, 2) + \beta_1\pi(i_1, N - i_1 + 1, x_1 - 1, 2).$$

Case 275: For $i_1 = N,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$[\beta_1 + (N-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(N, 1, x_1, 2)$$
$$= \lambda_{H-1}\pi(N-1, 1, x_1, 2) + \lambda_{L-1}\pi(N, 0, x_1, 2) + \lambda_{H2}\pi(N-2, 1, x_1, 2)$$
$$+ \lambda_{H2}\pi(N-1, 1, x_1, 2) + \lambda_{L2}\pi(N, 0, x_1, 2)$$
$$+ \beta_1\pi(N, 1, x_1 - 1, 2).$$

Case 276: For $i_1 = N+1,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 2$,

$$(\beta_1 + N\alpha_{H1} + \mu_{H1})\pi(N+1, 0, x_1, 2)$$
$$= \lambda_{H-1}\pi(N, 0, x_1, 2) + \lambda_{H2}\pi(N-1, 0, x_1, 2) + \lambda_{H2}\pi(N, 0, x_1, 2) + \beta_1\pi(N$$
$$+ 1, 0, x_1 - 1, 2).$$

Case 277: For $i_1 = 1,\ j_1 = 0,\ x_1 = K$, and $y_1 = 2$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \mu_{H1})\pi(1, 0, K, 2)$$
$$= \beta_1\pi(1, 0, K-1, 2) + \alpha_{H1}\pi(2, 0, K, 2) + \alpha_{L1}\pi(1, 1, K, 2).$$

Case 278: For $i_1 = 1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 2$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{L1} + \mu_{H1})\pi(1, 1, K, 2)$$
$$= \lambda_{L1}\pi(1, 0, K, 2) + \beta_1\pi(1, 1, K-1, 2) + \alpha_{H1}\pi(2, 1, K, 2) +$$
$$2\alpha_{L1}\pi(1, 2, K, 2).$$

Case 279: For $i_1 = 1,\ 2 \le j_1 \le N-1,\ x_1 = K$, and $y_1 = 2$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + j_1\alpha_{L1} + \mu_{H1})\pi(1, j_1, K, 2)$$
$$= \lambda_{L-1}\pi(1, j_1 - 1, K, 2) + \lambda_{L2}\pi(1, j_1 - 2, K, 2) + \beta_1\pi(1, j_1, K-1, 2)$$
$$+ \alpha_{H1}\pi(2, j_1, K, 2) + (j_1 + 1)\alpha_{L1}\pi(1, j_1 + 1, K, 2).$$

Case 280: For $i_1 = 1,\ j_1 = N,\ x_1 = K$, and $y_1 = 2$,

$$(N\alpha_{L1} + \mu_{H1})\pi(1, N, K, 2)$$
$$= \lambda_{L-1}\pi(1, N-1, K, 2) + \lambda_{L2}\pi(1, N-2, K, 2) + \lambda_{L2}\pi(1, N-1, K, 2)$$
$$+ \beta_1\pi(1, N, K-1, 2).$$

Case 281: For $i_1 = 2,\ j_1 = 0,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \mu_{H1}]\pi(2, 0, K, 2)$$
$$= \lambda_{H-1}\pi(1, 0, K, 2) + \beta_1\pi(2, 0, K-1, 2) + 2\alpha_{H1}\pi(3, 0, K, 2) +$$

$\alpha_{L1}\pi(2,1,K,2)$.

Case 282: For $i_1 = 2,\ j_1 = 1,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(2,1,K,2)$$
$$= \lambda_{H1}\pi(1,1,K,2) + \lambda_{L1}\pi(2,0,K,2) + \beta_1\pi(2,1,K-1,2) +$$
$$2\alpha_{H1}\pi(3,1,K,2) + 2\alpha_{L1}\pi(2,2,K,2).$$

Case 283: For $i_1 = 2,\ 2 \le j_1 \le N-2,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(2,j_1,K,2)$$
$$= \lambda_{H-1}\pi(1,j_1,K,2) + \lambda_{L-1}\pi(2,j_1-1,K,2) + \lambda_{L2}\pi(2,j_1-2,K,2)$$
$$+ \beta_1\pi(2,j_1,K-1,2) + 2\alpha_{H1}\pi(3,j_1,K,2) + (j_1+1)\alpha_{L1}\pi(2,j_1+1,K,2).$$

Case 284: For $i_1 = 2,\ j_1 = N-1,\ x_1 = K$, and $y_1 = 2$,

$$[\alpha_{H1} + (N-1)\alpha_{L1} + \mu_{H1}]\pi(2,N-1,K,2)$$
$$= \lambda_{H-1}\pi(1,N-1,K,2) + \lambda_{L-1}\pi(2,N,K,2) + \lambda_{H2}\pi(1,N-1,K,2)$$
$$+ \lambda_{L2}\pi(2,N-1,K,2) + \lambda_{L2}\pi(2,N,K,2) + \beta_1\pi(2,N-1,K-1,2).$$

Case 285: For $3 \le i_1 \le N,\ j_1 = 0,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + \mu_{H1}]\pi(i_1,0,K,2)$$
$$= \lambda_{H-1}\pi(i_1-1,0,K,2) + \lambda_{H2}\pi(i_1-2,0,K,2) + \beta_1\pi(i_1,0,K-1,2) +$$
$$i\alpha_{H1}\pi(i_1+1,0,K,2) + \alpha_{L1}\pi(i_1,1,K,2).$$

Case 286: For $3 \le i_1 \le N-1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(i_1,1,K,2)$$
$$= \lambda_{H-1}\pi(i_1-1,1,K,2) + \lambda_{H2}\pi(i_1-2,1,K,2) + \lambda_{L-1}\pi(i_1,0,K,2)$$
$$+ \beta_1\pi(i_1,1,K-1,2) + i_1\alpha_{H1}\pi(i_1+1,1,K,2) + 2\alpha_{L1}\pi(i_1,2,K,2).$$

Case 287: For $3 \le i_1 \le N-2,\ 2 \le j_1 \le N-i_1,\ x_1 = K$, and $y_1 = 2$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(i_1,j_1,K,2)$$
$$= \lambda_{H-1}\pi(i_1-1,j_1,K,2) + \lambda_{L-11}\pi(i_1,j_1-1,K,2) + \lambda_{H2}\pi(i_1-2,j_1,K,2)$$
$$+ \lambda_{L2}\pi(i_1,j_1-2,K,2) + \beta_1\pi(i_1,j_1,K-1,2) + i_1\alpha_{H1}\pi(i_1+1,j_1,K,2) + (j_1$$
$$+ 1)\alpha_{L1}\pi(i_1,j_1+1,K,2).$$

Case 288: For $3 \le i_1 \le N-1,\ j_1 = N - i_1 + 1,\ x_1 = K$, and $y_1 = 2$,

$$
\begin{aligned}
&[(i_1 - 1)\alpha_{H1} + (N - i_1 + 1)\alpha_{L1} + \mu_{H1}]\pi(i_1, N - i_1 + 1, K, 2) \\
&= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, K, 2) + \lambda_{L-1}\pi(i_1, N - i_1, K, 2) \\
&\qquad + \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, K, 2) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, K, 2) \\
&\qquad + \lambda_{L2}\pi(i_1, N - i_1 - 1, K, 2) + \lambda_{L2}\pi(i_1, N - i_1, K, 2) \\
&\qquad + \beta_1\pi(i_1, N - i_1 + 1, K - 1, 2).
\end{aligned}
$$

Case 289: For $i_1 = N,\ j_1 = 1,\ x_1 = K$, and $y_1 = 2$,

$$
\begin{aligned}
&[(N-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(N, 1, K, 2) \\
&= \lambda_{H-1}\pi(N - 1, 1, K, 2) + \lambda_{L-1}\pi(N, 0, K, 2) + \lambda_{H2}\pi(N - 2, 1, K, 2) \\
&\qquad + \lambda_{H2}\pi(N - 1, 1, K, 2) + \lambda_{L2}\pi(N, 0, K, 2) + \beta_1\pi(N, 1, K - 1, 2).
\end{aligned}
$$

Case 290: For $i_1 = N + 1,\ j_1 = 0,\ x_1 = K$, and $y_1 = 2$,

$$
\begin{aligned}
&(N\alpha_{H1} + \mu_{H1})\pi(N + 1, 0, K, 2) \\
&= \lambda_{H-1}\pi(N, 0, K, 2) + \lambda_{H2}\pi(N - 1, 0, K, 2) + \lambda_{H2}\pi(N, 0, K, 2) + \beta_1\pi(N \\
&\qquad + 1, 0, K - 1, 2).
\end{aligned}
$$

Case 291: For $i_1 = 0,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 3$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{L1})\pi(0, 1, 0, 3) \\
&= \theta_{L1}\lambda_{L-1}\pi(0, 0, 0, 0) + \alpha_{H1}\pi(1, 1, 0, 3) + \theta_{L1}\alpha_{H1}\pi(1, 1, 0, 0) \\
&\qquad + \alpha_{L1}\pi(0, 2, 0, 3) + \theta_{L1}2\alpha_{L1}\pi(0, 2, 0, 0) + \theta_{L1}\mu_{H1}\pi(1, 1, 0, 2) \\
&\qquad + \theta_{L1}\mu_{L1}\pi(0, 2, 0, 1) + \theta_{L1}\mu_{H1}\pi(1, 1, 0, 4) + \theta_{L1}\mu_{L1}\pi(0, 2, 0, 3).
\end{aligned}
$$

Case 292: For $i_1 = 0,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 3$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0, 2, 0, 3) \\
&= \theta_{L1}\lambda_{L-1}\pi(0, 1, 0, 0) + \lambda_{L-1}\pi(0, 1, 0, 3) + \theta_{L1}\lambda_{L2}\pi(0, 0, 0, 0) \\
&\qquad + \alpha_{H1}\pi(1, 2, 0, 3) + \theta_{L1}\alpha_{H1}\pi(1, 2, 0, 0) + 2\alpha_{L1}\pi(0, 3, 0, 3) \\
&\qquad + \theta_{L1}3\alpha_{L1}\pi(0, 3, 0, 0) + \theta_{L1}\mu_{H1}\pi(1, 2, 0, 2) + \theta_{L1}\mu_{L1}\pi(0, 3, 0, 1) \\
&\qquad + \theta_{L1}\mu_{H1}\pi(1, 2, 0, 4) + \theta_{L1}\mu_{L1}\pi(0, 3, 0, 3).
\end{aligned}
$$

Case 293: For $i_1 = 0,\ 3 \le j_1 \le N - 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (j_1 - 1)\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0, j_1, 0, 3)$$
$$= \theta_{L1}\lambda_{L-1}\pi(0, j_1 - 1, 0, 0) + \lambda_{L-1}\pi(0, j_1 - 1, 0, 3) + \theta_{L1}\lambda_{L2}\pi(0, j_1 - 2, 0, 0) + \lambda_{L2}\pi(0, j_1 - 2, 0, 3) + \alpha_{H1}\pi(1, j_1, 0, 3) + \theta_{L1}\alpha_{H1}\pi(1, j_1, 0, 0) + j_1\alpha_{L1}\pi(0, j_1 + 1, 0, 3) + \theta_{L1}(j_1 + 1)\alpha_{L1}\pi(0, j_1 + 1, 0, 0) + \theta_{L1}\mu_{H1}\pi(1, j_1, 0, 2) + \theta_{L1}\mu_{L1}\pi(0, j_1 + 1, 0, 1) + \theta_{L1}\mu_{H1}\pi(1, j_1, 0, 4) + \theta_{L1}\mu_{L1}\pi(0, j_1 + 1, 0, 3).$$

Case 294: For $i_1 = 0,\ j_1 = N,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (N - 1)\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0, N, 0, 3)$$
$$= \theta_{L1}\lambda_{L-1}\pi(0, N - 1, 0, 0) + \lambda_{L-1}\pi(0, N - 1, 0, 3) + \theta_{L1}\lambda_{L2}\pi(0, N - 2, 0, 0) + \lambda_{L2}\pi(0, N - 2, 0, 3) + \lambda_{L2}\pi(0, N - 1, 0, 3) + \alpha_{H1}\pi(1, N, 0, 3) + N\alpha_{L1}\pi(0, N + 1, 0, 3) + \theta_{L1}\mu_{H1}\pi(1, N, 0, 2) + \theta_{L1}\mu_{L1}\pi(0, N + 1, 0, 1) + \theta_{L1}\mu_{H1}\pi(1, N, 0, 4) + \theta_{L1}\mu_{L1}\pi(0, N + 1, 0, 3).$$

Case 295: For $i_1 = 0,\ j_1 = N + 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\beta_1 + N\alpha_{L1} + (1 - \theta_{L1})\mu_{L1} + \theta_{L1}\mu_{L1}]\pi(0, N + 1, 0, 3)$$
$$= \lambda_{L-1}\pi(0, N, 0, 3) + \lambda_{L2}\pi(0, N - 2, 0, 3) + \lambda_{L2}\pi(0, N - 1, 0, 3).$$

Case 296: For $i_1 = 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(1, 1, 0, 3)$$
$$= \lambda_{H-1}\pi(0, 1, 0, 3) + 2\alpha_{H1}\pi(2, 1, 0, 3) + \alpha_{L1}\pi(1, 2, 0, 3).$$

Case 297: For $i_1 = 1,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(1, 2, 0, 3)$$
$$= \lambda_{H-1}\pi(0, 2, 0, 3) + \lambda_{L-1}\pi(1, 1, 0, 3) + 2\alpha_{H1}\pi(2, 2, 0, 3) + 2\alpha_{L1}\pi(1, 3, 0, 3).$$

Case 298: For $i_1 = 1,\ 3 \le j_1 \le N - 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (j_1 - 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(1, j_1, 0, 3)$$
$$= \lambda_{H-1}\pi(0, j_1, 0, 3) + \lambda_{L-1}\pi(1, j_1 - 1, 0, 3) + \lambda_{L2}\pi(1, j_1 - 2, 0, 3) + 2\alpha_{H1}\pi(2, j_1, 0, 3) + j_1\alpha_{L1}\pi(1, j_1 + 1, 0, 3).$$

Case 299: For $i_1 = 1,\ j_1 = N,\ x_1 = 0$, and $y_1 = 3$,

$$[\beta_1 + \alpha_{H1} + (N - 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(1, N, 0, 3)$$
$$= \lambda_{H-1}\pi(0, N, 0, 3) + \lambda_{L-1}\pi(1, N - 1, 0, 3) + \lambda_{L2}\pi(1, N - 2, 0, 3).$$

Case 300: For $2 \le i_1 \le N - 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, 1, 0, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, 0, 3) + \lambda_{H2}\pi(i_1 - 2, 1, 0, 3) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 1, 0, 3) + \alpha_{L1}\pi(i_1, 2, 0, 3).$$

Case 301: For $2 \le i_1 \le N - 2,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, 2, 0, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 2, 0, 3) + \lambda_{H2}\pi(i_1 - 2, 2, 0, 3) + \lambda_{L-1}\pi(i_1, 1, 0, 3) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 2, 0, 3) + 2\alpha_{L1}\pi(i_1, 3, 0, 3).$$

Case 302: For $2 \le i_1 \le N - 3,\ 3 \le j_1 \le N - i_1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (j_1 - 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, j_1, 0, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, 0, 3) + \lambda_{L-1}\pi(i_1, j_1 - 1, 0, 3) + \lambda_{H2}\pi(i_1 - 2, j_1, 0, 3) + \lambda_{L2}\pi(i_1, j_1 - 2, 0, 3) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, j_1, 0, 3) + j_1\alpha_{L1}\pi(i_1, j_1 + 1, 0, 3).$$

Case 303: For $2 \le i_1 \le N - 2,\ j_1 = N - i + 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\beta_1 + i_1\alpha_{H1} + (N - i_1)\alpha_{L1} + (1 - \theta_H)\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(i_1, N - i_1 + 1, 0, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, 0, 3) + \lambda_{L-1}\pi(i_1, N - i_1, 0, 3) + \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, 0, 3) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, 0, 3) + \lambda_{L2}\pi(i_1, N - i_1 - 1, 0, 3) + \lambda_{L2}\pi(i_1, N - i_1, 0, 3).$$

Case 304: For $i_1 = N-1,\ j_1 = 2,\ x_1 = 0$, and $y_1 = 3$,

$$[\beta_1 + (N-1)\alpha_{H1} + \alpha_{L1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N-1,2,0,3)$$
$$= \lambda_{H-1}\pi(N-2,2,0,3) + \lambda_{L-1}\pi(N-1,1,0,3) + \lambda_{H2}\pi(N-3,2,0,3)$$
$$+ \lambda_{H2}\pi(N-2,2,0,3) + \lambda_{L2}\pi(N-1,1,0,3).$$

Case 305: For $i_1 = N,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\beta_1 + N\alpha_{H1} + (1-\theta_{H1})\mu_{L1} + \theta_{H1}\mu_{L1}]\pi(N,1,0,3)$$
$$= \lambda_{H1}\pi(N-1,1,0,3) + \lambda_{H2}\pi(N-2,2,0,3) + \lambda_{H2}\pi(N-1,2,0,3).$$

Case 306: For $i_1 = 0,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{L1})\pi(0,1,x_1,3)$$
$$= \beta_1\pi(0,1,x_1-1,3) + \alpha_{H1}\pi(1,1,x_1,3) + \alpha_{L1}\pi(0,2,x_1,3).$$

Case 307: For $i_1 = 0,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + \mu_{L1}]\pi(0,2,x_1,3)$$
$$= \lambda_{L1}\pi(0,1,x_1,3) + \beta_1\pi(0,2,x_1-1,3) + \alpha_{H1}\pi(1,2,x_1,3)$$
$$+ 2\alpha_{L1}\pi(0,3,x_1,3).$$

Case 308: For $i_1 = 0,\ 3 \le j_1 \le N,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(0,j_1,x_1,3)$$
$$= \lambda_{L-1}\pi(0,j_1-1,x_1,3) + \lambda_{L2}\pi(0,j_1-2,x_1,3) + \beta_1\pi(0,j_1,x_1-1,3)$$
$$+ \alpha_{H1}\pi(1,j_1,x_1,3) + j_1\alpha_{L1}\pi(0,j_1+1,x_1,3).$$

Case 309: For $i_1 = 0,\ j_1 = N+1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$(\beta_1 + N\alpha_{L1} + \mu_{L1})\pi(0,N+1,x_1,3)$$
$$= \lambda_{L-1}\pi(0,N,x_1,3) + \lambda_{L2}\pi(0,N-1,x_1,3) + \lambda_{L2}\pi(0,N,x_1,3)$$
$$+ \beta_1\pi(0,N+1,x_1-1,3).$$

Case 310: For $i_1 = 1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \mu_{L1})\pi(1,1,x_1,3)$$
$$= \lambda_{H-1}\pi(0,1,x_1,3) + \beta_1\pi(1,1,x_1-1,3) + 2\alpha_{H1}\pi(2,1,x_1,3) +$$
$$\alpha_{L1}\pi(1,2,x_1,3).$$

Case 311: For $i_1 = 1,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(1,2,x_1,3) \\
&= \lambda_{H-1}\pi(0,2,x_1,3) + \lambda_{L-1}\pi(1,1,x_1,3) + \beta_1\pi(1,2,x_1-1,3) + \\
&\quad 2\alpha_{H1}\pi(2,2,x_1,3) + 2\alpha_{L1}\pi(1,3,x_1,3).
\end{aligned}
$$

Case 312: For $i_1 = 1,\ 3 \le j_1 \le N-1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(1,j_1,x_1,3) \\
&= \lambda_{H-1}\pi(0,j_1,x_1,3) + \lambda_{L-1}\pi(1,j_1-1,x_1,3) + \lambda_{L2}\pi(1,j_1-2,x_1,3) \\
&\qquad + \beta_1\pi(1,j_1,x_1-1,3) + 2\alpha_{H1}\pi(2,j_1,x_1,3) \\
&\qquad + j_1\alpha_{L1}\pi(1,j_1+1,x_1,3).
\end{aligned}
$$

Case 313: For $i_1 = 1,\ j_1 = N,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&[\beta_1 + \alpha_{H1} + (N-1)\alpha_{L1} + \mu_{L1}]\pi(1,N,x_1,3) \\
&= \lambda_{H-1}\pi(0,N,x_1,3) + \lambda_{L-1}\pi(1,N-1,x_1,3) + \lambda_{L2}\pi(1,N-2,x_1,3) \\
&\qquad + \lambda_{H2}\pi(0,N,x_1,3) + \lambda_{L2}\pi(1,N-1,x_1,3) \\
&\qquad + \beta_1\pi(1,N,x_1-1,3).
\end{aligned}
$$

Case 314: For $2 \le i_1 \le N-1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \mu_{L1})\pi(i_1,1,x_1,3) \\
&= \lambda_{H-1}\pi(i_1-1,1,x_1,3) + \lambda_{H2}\pi(i_1-2,1,x_1,3) + \beta_1\pi(i_1,1,x_1-1,3) \\
&\qquad + (i_1+1)\alpha_{H1}\pi(i_1+1,1,x_1,3) + \alpha_{L1}\pi(i_1,2,x_1,3).
\end{aligned}
$$

Case 315: For $2 \le i_1 \le N-2,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(i_1,2,x_1,3) \\
&= \lambda_{H-1}\pi(i_1-1,2,x_1,3) + \lambda_{L-1}\pi(i_1,1,x_1,3) + \lambda_{H2}\pi(i_1-2,2,x_1,3) \\
&\qquad + \beta_1\pi(i_1,2,x_1-1,3) + (i_1+1)\alpha_{H1}\pi(i_1+1,2,x_1,3) \\
&\qquad + 2\alpha_{L1}\pi(i_1,3,x_1,3).
\end{aligned}
$$

Case 316: For $2 \le i_1 \le N-3,\ 3 \le j_1 \le N-i_1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$
\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + i_1\alpha_{H1} + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(i_1,j_1,x_1,3) \\
&= \lambda_{H-1}\pi(i_1-1,j_1,x_1,3) + \lambda_{L-1}\pi(i_1,j_1-1,x_1,3) + \lambda_{H2}\pi(i_1-2,j_1,x_1,3) \\
&\qquad + \lambda_{L2}\pi(i_1,j_1-2,x_1,3) + \beta_1\pi(i_1,j_1,x_1-1,3) +
\end{aligned}
$$

$$(i_1+1)\alpha_{H1}\pi(i_1+1,j_1,x_1,3)+j_1\alpha_{L1}\pi(i_1,j_1+1,x_1,3).$$

Case 317: For $2 \le i_1 \le N-2,\ j_1 = N-i_1+1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$\begin{aligned}
&[\beta_1 + i_1\alpha_{H1} + (N-i_1)\alpha_{L1} + \mu_{L1}]\pi(i_1, N-i_1+1, x_1, 3)\\
&= \lambda_{H-1}\pi(i_1-1, N-i_1+1, x_1, 3) + \lambda_{L-1}\pi(i_1, N-i_1, x_1, 3)\\
&\qquad + \lambda_{H2}\pi(i_1-2, N-i_1+1, x_1, 3) + \lambda_{L2}\pi(i_1, N-i_1-1, x_1, 3)\\
&\qquad + \lambda_{H2}\pi(i_1-1, N-i_1+1, x_1, 3) + \lambda_{L2}\pi(i_1, N-i_1, x_1, 3)\\
&\qquad + \beta_1\pi(i_1, N-i_1+1, x_1-1, 3).
\end{aligned}$$

Case 318: For $i_1 = N-1,\ j_1 = 2,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$\begin{aligned}
&[\beta_1 + (N-1)\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(N-1, 2, x_1, 3)\\
&= \lambda_{H-1}\pi(N-2, 2, x_1, 3) + \lambda_{L-1}\pi(N-1, 1, x_1, 3) + \lambda_{H2}\pi(N-3, 2, x_1, 3)\\
&\qquad + \lambda_{H2}\pi(N-2, 2, x_1, 3) + \lambda_{L2}\pi(N-1, 1, x_1, 3)\\
&\qquad + \beta_1\pi(N-1, 2, x_1-1, 3).
\end{aligned}$$

Case 319: For $i_1 = N,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 3$,

$$\begin{aligned}
&(\beta_1 + N\alpha_{H1} + \mu_{L1})\pi(N, 1, x_1, 3)\\
&= \lambda_{H-1}\pi(N-1, 1, x_1, 3) + \lambda_{H2}\pi(N-2, 1, x_1, 3) + \lambda_{H2}\pi(N-1, 1, x_1, 3)\\
&\qquad + \beta_1\pi(N, 1, x_1-1, 3).
\end{aligned}$$

Case 320: For $i_1 = 0,\ j_1 = 1,\ x_1 = K$, and $y_1 = 3$,

$$\begin{aligned}
&(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \mu_{L1})\pi(0, 1, K, 3)\\
&= \beta_1\pi(0, 1, K-1, 3) + \alpha_{H1}\pi(1, 1, K, 3) + \alpha_{L1}\pi(0, 2, K, 3).
\end{aligned}$$

Case 321: For $i_1 = 0,\ j_1 = 2,\ x_1 = K$, and $y_1 = 3$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{L1} + \mu_{L1}]\pi(0, 2, K, 3)\\
&= \lambda_{L-1}\pi(0, 1, K, 3) + \beta_1\pi(0, 2, K-1, 3) + \alpha_{H1}\pi(1, 2, K, 3) +\\
&\quad 2\alpha_{L1}\pi(0, 3, K, 3).
\end{aligned}$$

Case 322: For $i_1 = 0,\ 3 \le j_1 \le N,\ x_1 = K$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (j_1-1)\alpha_{L1} + \mu_{L1}]\pi(0, j_1, K, 3)$$

$$= \lambda_{L-1}\pi(0, j_1 - 1, K, 3) + \lambda_{L2}\pi(0, j_1 - 2, K, 3) + \beta_1\pi(0, j_1, K - 1, 3) + \alpha_{H1}\pi(1, j_1, K, 3) + j_1\alpha_{L1}\pi(0, j_1 + 1, K, 3).$$

Case 323: For $i_1 = 0,\ j_1 = N + 1,\ x_1 = K$, and $y_1 = 3$,

$$(N\alpha_{L1} + \mu_{L1})\pi(0, N + 1, K, 3)$$
$$= \lambda_{L-1}\pi(0, N, K, 3) + \lambda_{L2}\pi(0, N - 1, K, 3) + \lambda_{L2}\pi(0, N, K, 3) + \beta_1\pi(0, N + 1, K - 1, 3).$$

Case 324: For $i_1 = 1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 3$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \mu_{L1})\pi(1, 1, K, 3)$$
$$= \lambda_{H-1}\pi(0, 1, K, 3) + \beta_1\pi(1, 1, K - 1, 3) + 2\alpha_{H1}\pi(2, 1, K, 3) + \alpha_{L1}\pi(1, 2, K, 3).$$

Case 325: For $i_1 = 1,\ j_1 = 2,\ x_1 = K$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(1, 2, K, 3)$$
$$= \lambda_{H-1}\pi(0, 2, K, 3) + \lambda_{L-1}\pi(1, 1, K, 3) + \beta_1\pi(1, 2, K - 1, 3) + 2\alpha_{H1}\pi(2, 2, K, 3) + 2\alpha_{L1}\pi(1, 3, K, 3).$$

Case 326: For $i_1 = 1,\ 3 \le j_1 \le N - 1,\ x_1 = K$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + (j_1 - 1)\alpha_{L1} + \mu_{L1}]\pi(1, j_1, K, 3)$$
$$= \lambda_{H-1}\pi(0, j_1, K, 3) + \lambda_{L-1}\pi(1, j_1 - 1, K, 3) + \lambda_{L2}\pi(1, j_1 - 2, K, 3) + \beta_1\pi(1, j_1, K - 1, 3) + 2\alpha_{H1}\pi(2, j_1, K, 3) + j_1\alpha_{L1}\pi(1, j_1 + 1, K, 3).$$

Case 327: For $i_1 = 1,\ j_1 = N,\ x_1 = K$, and $y_1 = 3$,

$$[\alpha_{H1} + (N - 1)\alpha_{L1} + \mu_{L1}]\pi(1, N, K, 3)$$
$$= \lambda_{H-1}\pi(0, N, K, 3) + \lambda_{L-1}\pi(1, N - 1, K, 3) + \lambda_{L2}\pi(1, N - 2, K, 3) + \lambda_{L2}\pi(1, N - 1, K, 3) + \beta_1\pi(1, N, K - 1, 3).$$

Case 328: For $2 \le i_1 \le N - 1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 3$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + \mu_{L1})\pi(i_1, 1, K, 3)$$

$$= \lambda_{H-1}\pi(i_1 - 1, 1, K, 3) + \lambda_{H2}\pi(i_1 - 2, 1, K, 3) + \beta_1\pi(i_1, 1, K - 1, 3)$$
$$+ (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 1, K, 3) + \alpha_{L1}\pi(i_1, 2, K, 3).$$

Case 329: For $2 \le i_1 \le N - 2,\ j_1 = 2,\ x_1 = K$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(i_1, 2, K, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 2, K, 3) + \lambda_{L-1}\pi(i_1, 1, K, 3) + \lambda_{H2}\pi(i_1 - 2, 2, K, 3)$$
$$+ \beta_1\pi(i_1, 2, K - 1, 3) + (i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 2, K, 3) + 2\alpha_{L1}\pi(i_1, 3, K, 3).$$

Case 330: For $2 \le i_1 \le N - 3,\ 3 \le j_1 \le N - i_1,\ x_1 = K$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + i_1\alpha_{H1} + (j_1 - 1)\alpha_{L1} + \mu_{L1}]\pi(i_1, j_1, K, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, K, 3) + \lambda_{L-1}\pi(i_1, j_1 - 1, K, 3) + \lambda_{H2}\pi(i_1 - 2, j_1, K, 3)$$
$$+ \lambda_{L2}\pi(i_1, j_1 - 2, K, 3) + \beta_1\pi(i_1, j_1, K - 1, 3) +$$
$$(i_1 + 1)\alpha_{H1}\pi(i_1 + 1, j_1, K, 3) + j_1\alpha_{L1}\pi(i_1, j_1 + 1, K, 3).$$

Case 331: For $2 \le i_1 \le N - 2,\ j_1 = N - i_1 + 1,\ x_1 = K$, and $y_1 = 3$,

$$[i_1\alpha_{H1} + (N - i_1)\alpha_{L1} + \mu_{L1}]\pi(i_1, N - i_1 + 1, K, 3)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, K, 3) + \lambda_{L-1}\pi(i_1, N - i_1, K, 3)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, K, 3) + \lambda_{L2}\pi(i_1, N - i_1 - 1, K, 3)$$
$$+ \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, K, 3) + \lambda_{L2}\pi(i_1, N - i_1, K, 3)$$
$$+ \beta_1\pi(i, N - i_1 + 1, K - 1, 3).$$

Case 332: For $i_1 = N - 1,\ j_1 = 2,\ x_1 = K$, and $y_1 = 3$,

$$[(N - 1)\alpha_{H1} + \alpha_{L1} + \mu_{L1}]\pi(N - 1, 2, K, 3)$$
$$= \lambda_{H-1}\pi(N - 2, 2, K, 3) + \lambda_{L-1}\pi(N - 1, 1, K, 3) + \lambda_{H2}\pi(N - 3, 2, K, 3)$$
$$+ \lambda_{H2}\pi(N - 2, 2, K, 3) + \lambda_{L2}\pi(N - 1, 1, K, 3) + \beta_1\pi(N - 1, 2, K$$
$$- 1, 3)$$

Case 333: For $i_1 = N,\ j_1 = 1,\ x_1 = K$, and $y_1 = 3$,

$$(N\alpha_{H1} + \mu_{L1})\pi(N, 1, K, 3)$$
$$= \lambda_{H-1}\pi(N - 1, 1, K, 3) + \lambda_{H2}\pi(N - 2, 1, K, 3) + \lambda_{H2}\pi(N - 1, 1, K, 3)$$
$$+ \beta_1\pi(N, 1, K - 1, 3)$$

Case 334: For $i_1 = 1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 4$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{H1})\pi(1,0,0,4)$$
$$= \theta_{H1}\lambda_{H1}\pi(0,0,0,0) + \alpha_{H1}\pi(2,0,0,4) + \theta_{H1}2\alpha_{H1}\pi(2,0,0,0) +$$
$$\alpha_{L1}\pi(1,1,0,4) + \theta_{H1}\alpha_{L1}\pi(1,1,0,0) + \theta_{H1}\mu_{H1}\pi(2,0,0,2) +$$
$$\theta_{H1}\mu_{L1}\pi(1,1,0,1) + \theta_{H1}\mu_{H1}\pi(2,0,0,4) + \theta_{H1}\mu_{L1}\pi(1,1,0,3).$$

Case 335: For $i_1 = 1,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{L1} + (1-\theta_{L1})\mu_{H1} + \theta_{L1}\mu_{H1}]\pi(1,1,0,4)$$
$$= \theta_{H1}\lambda_{H1}\pi(0,1,0,0) + \theta_{H1}\lambda_{L1}\pi(1,0,0,0) + \lambda_{L1}\pi(1,0,0,4) +$$
$$\alpha_{H1}\pi(2,1,0,4) + \theta_{H1}2\alpha_{H1}\pi(2,1,0,0) + 2\alpha_{L1}\pi(1,2,0,4) +$$
$$\theta_{H1}2\alpha_{L1}\pi(1,2,0,0) + \theta_{H1}\mu_{H1}\pi(2,1,0,2) + \theta_{H1}\mu_{L1}\pi(1,2,0,1) +$$
$$\theta_{H1}\mu_{H1}\pi(2,1,0,4) + \theta_{H1}\mu_{L1}\pi(1,2,0,3).$$

Case 336: For $i_1 = 1,\ 2 \le j_1 \le N-2,\ x_1 = 0$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + j_1\alpha_{L1} + (1-\theta_{L1})\mu_{H1} + \theta_{L1}\mu_{H1}]\pi(1,j_1,0,4)$$
$$= \theta_{H1}\lambda_{H-1}\pi(0,j_1,0,0) + \theta_{H1}\lambda_{L-1}\pi(1,j_1-1,0,0) + \lambda_{L-1}\pi(1,j_1-1,0,4)$$
$$+ \theta_{H1}\lambda_{L2}\pi(1,j_1-2,0,0) + \lambda_{L2}\pi(1,j_1-2,0,4)$$
$$+ \alpha_{H1}\pi(2,j_1,0,4) + \theta_{H1}2\alpha_{H1}\pi(2,j_1,0,0)$$
$$+ (j_1+1)\alpha_{L1}\pi(1,j_1+1,0,4) + \theta_{H1}(j_1+1)\alpha_{L1}\pi(1,j_1+1,0,0)$$
$$+ \theta_{H1}\mu_{H1}\pi(2,j_1,0,2) + \theta_{H1}\mu_{L1}\pi(1,j_1+1,0,1)$$
$$+ \theta_{H1}\mu_{H1}\pi(2,j_1,0,4) + \theta_{H1}\mu_{L1}\pi(1,j_1+1,0,3).$$

Case 337: For $i_1 = 1,\ j_1 = N-1,\ x_1 = 0$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (N-1)\alpha_{L1} + (1-\theta_{L1})\mu_{H1} + \theta_{L1}\mu_{H1}]\pi(1,N-1,0,4)$$
$$= \theta_{H1}\lambda_{H-1}\pi(0,N-1,0,0) + \theta_{H1}\lambda_{L-1}\pi(1,N-2,0,0) + \lambda_{L-1}\pi(1,N-2,0,4)$$
$$+ \theta_{H1}\lambda_{L2}\pi(1,N-3,0,0) + \lambda_{L2}\pi(1,N-3,0,4)$$
$$+ \alpha_{H1}\pi(2,N-1,0,4) + N\alpha_{L1}\pi(1,N,0,4)$$
$$+ \theta_{H1}\mu_{H1}\pi(2,N-1,0,2) + \theta_{H1}\mu_{L1}\pi(1,N,0,1)$$
$$+ \theta_{H1}\mu_{H1}\pi(2,N-1,0,4) + \theta_{H1}\mu_{L1}\pi(1,N,0,3).$$

Case 338: For $i_1 = 1,\ j_1 = N,\ x_1 = 0$, and $y_1 = 4$,

$$[\beta_1 + N\alpha_{L1} + (1-\theta_{L1})\mu_{H1} + \theta_{L1}\mu_{H1}]\pi(1,N,0,4)$$

$$= \lambda_{L-1}\pi(1, N-1, 0, 4) + \lambda_{L2}\pi(1, N-2, 0, 4) + \lambda_{L2}\pi(1, N-1, 0, 4).$$

Case 339: For $i_1 = 2,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 4$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (1-\theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(2, 0, 0, 4) \\
&= \theta_{H1}\lambda_{H-1}\pi(1, 0, 0, 0) + \lambda_{H-1}\pi(1, 0, 0, 4) + \theta_{H1}\lambda_{H2}\pi(0, 0, 0, 0) \\
&\qquad + 2\alpha_{H1}\pi(3, 0, 0, 4) + \theta_{H1}3\alpha_{H1}\pi(3, 0, 0, 0) + \alpha_{L1}\pi(2, 1, 0, 4) \\
&\qquad + \theta_{H1}\alpha_{L1}\pi(2, 1, 0, 0) + \theta_{H1}\mu_{H1}\pi(3, 0, 0, 2) + \theta_{H1}\mu_{L1}\pi(2, 1, 0, 1) \\
&\qquad + \theta_{H1}\mu_{H1}\pi(3, 0, 0, 4) + \theta_{H1}\mu_{L1}\pi(2, 1, 0, 3).
\end{aligned}$$

Case 340: For $i_1 = 2,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 4$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + (1-\theta_{H1})\mu_{H1} \\
&\qquad + \theta_{H1}\mu_{H1}]\pi(2, 1, 0, 4) \\
&= \theta_{H1}\lambda_{H-1}\pi(1, 1, 0, 0) + \lambda_{H-1}\pi(1, 1, 0, 4) + \theta_{H1}\lambda_{L-1}\pi(2, 0, 0, 0) + \\
&\quad \lambda_{L-1}\pi(2, 0, 0, 4) + \theta_{H1}\lambda_{H2}\pi(0, 1, 0, 0) + 2\alpha_{H1}\pi(3, 1, 0, 4) \\
&\qquad + \theta_{H1}3\alpha_{H1}\pi(3, 1, 0, 0) + 2\alpha_{L1}\pi(2, 2, 0, 4) + \theta_{H1}2\alpha_{L1}\pi(2, 2, 0, 0) \\
&\qquad + \theta_{H1}\mu_{H1}\pi(3, 1, 0, 2) + \theta_{H1}\mu_{L1}\pi(2, 2, 0, 1) + \theta_{H1}\mu_{H1}\pi(3, 1, 0, 4) \\
&\qquad + \theta_{H1}\mu_{L1}\pi(2, 2, 0, 3).
\end{aligned}$$

Case 341: For $i_1 = 2,\ 2 \le j_1 \le N-3,\ x_1 = 0$, and $y_1 = 4$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + j_1\alpha_{L1} + (1-\theta_{H1})\mu_{H1} \\
&\qquad + \theta_{H1}\mu_{H1}]\pi(2, j_1, 0, 4) \\
&= \theta_{H1}\lambda_{H-1}\pi(1, j_1, 0, 0) + \lambda_{H-1}\pi(1, j_1, 0, 4) + \theta_{H1}\lambda_{L-1}\pi(2, j_1 - 1, 0, 0) + \\
&\quad \lambda_{L-1}\pi(2, j_1 - 1, 0, 4) + \theta_{H1}\lambda_{H2}\pi(0, j_1, 0, 0) + \theta_{H1}\lambda_{L2}\pi(2, j_1 - 2, 0, 0) \\
&\qquad + \lambda_{H2}\pi(2, j_1 - 2, 0, 4) + 2\alpha_{H1}\pi(3, j_1, 0, 4) \\
&\qquad + \theta_{H1}3\alpha_{H1}\pi(3, j_1, 0, 0) + (j_1 + 1)\alpha_{L1}\pi(2, j_1 + 1, 0, 4) \\
&\qquad + \theta_{H1}(j_1 + 1)\alpha_{L1}\pi(2, j_1 + 1, 0, 0) + \theta_{H1}\mu_{H1}\pi(3, j_1, 0, 2) \\
&\qquad + \theta_{H1}\mu_{L1}\pi(2, j_1 + 1, 0, 1) + \theta_{H1}\mu_{H1}\pi(3, j_1, 0, 4) \\
&\qquad + \theta_{H1}\mu_{L1}\pi(2, j_1 + 1, 0, 3).
\end{aligned}$$

Case 342: For $i_1 = 2,\ j_1 = N-2,\ x_1 = 0$, and $y_1 = 4$,

$$\begin{aligned}
&[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + (N-2)\alpha_{L1} + (1-\theta_{H1})\mu_{H1} \\
&\qquad + \theta_{H1}\mu_{H1}]\pi(2, N-2, 0, 4)
\end{aligned}$$

$$= \theta_{H1}\lambda_{H-1}\pi(1, N-2, 0, 0) + \lambda_{H-1}\pi(1, N-2, 0, 4) + \theta_{H1}\lambda_{L-1}\pi(2, N-3, 0, 0)$$
$$+ \lambda_{L-1}\pi(2, N-3, 0, 4) + \theta_{H1}\lambda_{H2}\pi(0, N-2, 0, 0)$$
$$+ \theta_{H1}\lambda_{L2}\pi(2, N-4, 0, 0) + \theta_{H1}\lambda_{L2}\pi(2, N-4, 0, 4)$$
$$+ \theta_{H1}\lambda_{L2}\pi(2, N-3, 0, 0) + 2\alpha_{H1}\pi(3, N-2, 0, 4)$$
$$+ (N-1)\alpha_{L1}\pi(2, N-1, 0, 4) + \theta_{H1}\mu_{H1}\pi(3, N-2, 0, 2)$$
$$+ \theta_{H1}\mu_{L1}\pi(2, N-1, 0, 1) + \theta_{H1}\mu_{H1}\pi(3, N-2, 0, 4)$$
$$+ \theta_{H1}\mu_{L1}\pi(2, N-1, 0, 3).$$

Case 343: For $i_1 = 2,\ j_1 = N-1,\ x_1 = 0$, and $y_1 = 4$,

$$[\beta_1 + \alpha_{H1} + (N-1)\alpha_{L1} + (1-\theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(2, N-1, 0, 4)$$
$$= \lambda_{H-1}\pi(1, N-1, 0, 4) + \lambda_{L-1}\pi(2, N-2, 0, 4) + \lambda_{H2}\pi(1, N-1, 0, 4)$$
$$+ \lambda_{L2}\pi(2, N-3, 0, 4) + \lambda_{L2}\pi(2, N-2, 0, 4).$$

Case 344: For $3 \le i_1 \le N-1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + (1-\theta_{H1})\mu_{H1}$$
$$+ \theta_{H1}\mu_{H1}]\pi(i_1, 0, 0, 4)$$
$$= \theta_{H1}\lambda_{H-1}\pi(i_1 - 1, 0, 0, 0) + \lambda_{H-1}\pi(i_1 - 1, 0, 0, 4) + \theta_{H1}\lambda_{H2}\pi(i_1 - 2, 0, 0, 0)$$
$$+ \lambda_{H2}\pi(i_1 - 2, 0, 0, 4) + i_1\alpha_{H1}\pi(i_1 + 1, 0, 0, 4)$$
$$+ \theta_{H1}(i_1 + 1)\alpha_{H1}\pi(i_1 + 1, 0, 0, 0) + \alpha_{L1}\pi(i_1, 1, 0, 4)$$
$$+ \theta_{H1}\alpha_{L1}\pi(i_1, 1, 0, 0) + \theta_{H1}\mu_{H1}\pi(i_1 + 1, 0, 0, 2)$$
$$+ \theta_{H1}\mu_{L1}\pi(i_1, 1, 0, 1) + \theta_{H1}\mu_{H1}\pi(i_1 + 1, 0, 0, 4)$$
$$+ \theta_{H1}\mu_{L1}\pi(i_1, 1, 0, 3).$$

Case 345: For $i_1 = N,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (N-1)\alpha_{H1} + (1-\theta_{H1})\mu_{H1}$$
$$+ \theta_{H1}\mu_{H1}]\pi(N, 0, 0, 4)$$
$$= \theta_{H1}\lambda_{H-1}\pi(N-1, 0, 0, 0) + \lambda_{H-1}\pi(N-1, 0, 0, 4) + \theta_{H1}\lambda_{H2}\pi(N-2, 0, 0, 0)$$
$$+ \lambda_{H2}\pi(N-2, 0, 0, 4) + \theta_{H1}\lambda_{H2}\pi(N-1, 0, 0, 0) + N\alpha_{H1}\pi(N+1, 0, 0, 4) +$$
$$\alpha_{L1}\pi(N, 1, 0, 4) + \theta_{H1}\mu_{H1}\pi(N+1, 0, 0, 2) + \theta_{H1}\mu_{L1}\pi(N, 1, 0, 1) +$$
$$\theta_{H1}\mu_{H1}\pi(N+1, 0, 0, 4) + \theta_{H1}\mu_{L1}\pi(N, 1, 0, 3).$$

Case 346: For $3 \le i_1 \le N-2,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 4$,

$$
\begin{aligned}
&[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+(i_1-1)\alpha_{H1}+\alpha_{L1}+(1-\theta_{H1})\mu_{H1}\\
&\qquad+\theta_{H1}\mu_{H1}]\pi(i_1,1,0,4)\\
&=\theta_{H1}\lambda_{H-1}\pi(i_1-1,1,0,0)+\lambda_{H-1}\pi(i_1-1,1,0,4)+\theta_{H1}\lambda_{L-1}\pi(i_1,0,0,0)\\
&\qquad+\lambda_{L-1}\pi(i_1,0,0,4)+\theta_{H1}\lambda_{H2}\pi(i_1-2,1,0,0)\\
&\qquad+\lambda_{H2}\pi(i_1-2,1,0,4)+i_1\alpha_{H1}\pi(i_1+1,1,0,4)\\
&\qquad+\theta_{H1}(i_1+1)\alpha_{H1}\pi(i_1+1,1,0,0)+2\alpha_{L1}\pi(i_1,2,0,4)\\
&\qquad+\theta_{H1}2\alpha_{L1}\pi(i_1,2,0,0)+\theta_{H1}\mu_{H1}\pi(i_1+1,1,0,2)\\
&\qquad+\theta_{H1}\mu_{L1}\pi(i_1,2,0,1)+\theta_{H1}\mu_{H1}\pi(i_1+1,1,0,4)\\
&\qquad+\theta_{H1}\mu_{L1}\pi(i_1,2,0,3).
\end{aligned}
$$

Case 347: For $i_1=N-1,\ j_1=1,\ x_1=0,$ and $y_1=4,$

$$
\begin{aligned}
&[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+(N-2)\alpha_{H1}+\alpha_{L1}+(1-\theta_{H1})\mu_{H1}\\
&\qquad+\theta_{H1}\mu_{H1}]\pi(N-1,1,0,4)\\
&=\theta_{H1}\lambda_{H-1}\pi(N-2,1,0,0)+\lambda_{H-1}\pi(N-2,1,0,4)+\theta_{H1}\lambda_{L-1}\pi(N-1,0,0,0)\\
&+\lambda_{L-1}\pi(N-1,0,0,4)+\theta_{H1}\lambda_{H2}\pi(N-3,1,0,0)+\lambda_{H2}\pi(N-3,1,0,4)\\
&+\theta_{H1}\lambda_{H2}\pi(N-2,1,0,0)+\theta_{H1}\lambda_{L2}\pi(N-1,0,0,0)+(N-1)\alpha_{H1}\pi(N,1,0,4)\\
&+2\alpha_{L1}\pi(N-1,2,0,4)+\theta_{H1}\mu_{H1}\pi(N,1,0,2)+\theta_{H1}\mu_{L1}\pi(N-1,2,0,1)+\\
&\quad\theta_{H1}\mu_{H1}\pi(N,1,0,4)+\theta_{H1}\mu_{L1}\pi(N-1,2,0,3).
\end{aligned}
$$

Case 348: For $3\le i_1\le N-3,\ 2\le j_1\le N-i_1-1,\ x_1=0,$ and $y_1=4,$

$$
\begin{aligned}
&[\lambda_{H-1}+\lambda_{L-1}+\lambda_{H2}+\lambda_{L2}+\beta_1+(i_1-1)\alpha_{H1}+j_1\alpha_{L1}+(1-\theta_{H1})\mu_{H1}\\
&\qquad+\theta_{H1}\mu_{H1}]\pi(i_1,j_1,0,4)\\
&=\theta_{H1}\lambda_{H-1}\pi(i_1-1,j_1,0,0)+\lambda_{H-1}\pi(i_1-1,j_1,0,4)\\
&\qquad+\theta_{H1}\lambda_{L-1}\pi(i_1,j_1-1,0,0)+\lambda_{L-1}\pi(i_1,j_1-1,0,4)\\
&\qquad+\theta_{H1}\lambda_{H2}\pi(i_1-2,j_1,0,0)+\lambda_{H2}\pi(i_1-2,j_1,0,4)\\
&\qquad+\theta_{H1}\lambda_{L2}\pi(i_1,j_1-2,0,0)+\lambda_{L2}\pi(i_1,j_1-2,0,4)\\
&\qquad+i_1\alpha_{H1}\pi(i_1+1,j_1,0,4)+\theta_{H1}(i_1+1)\alpha_{H1}\pi(i_1+1,j_1,0,0)\\
&\qquad+(j_1+1)\alpha_{L1}\pi(i_1,j_1+1,0,4)\\
&\qquad+\theta_{H1}(j_1+1)\alpha_{L1}\pi(i_1,j_1+1,0,0)+\theta_{H1}\mu_{H1}\pi(i_1+1,j_1,0,2)\\
&\qquad+\theta_{H1}\mu_{L1}\pi(i_1,j_1+1,0,1)+\theta_{H1}\mu_{H1}\pi(i_1+1,j_1,0,4)\\
&\qquad+\theta_{H1}\mu_{L1}\pi(i_1,j_1+1,0,3).
\end{aligned}
$$

Case 349: For $3\le i_1\le N-3,\ j_1=N-i_1,\ x_1=0,$ and $y_1=4,$

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1 - 1)\alpha_{H1} + (N - i_1)\alpha_{L1} + (1 - \theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(i_1, N - i, 0, 4)$$
$$= \theta_{H1}\lambda_{H-1}\pi(i_1 - 1, N - i_1, 0, 0) + \lambda_{H-1}\pi(i_1 - 1, N - i_1, 0, 4) +$$
$$\theta_{H1}\lambda_{L-1}\pi(i_1, N - i_1 - 1, 0, 0) + \lambda_{L-1}\pi(i_1, N - i_1 - 1, 0, 4)$$
$$+ \theta_{H1}\lambda_{H2}\pi(i_1 - 2, N - i_1, 0, 0) + \lambda_{H2}\pi(i_1 - 2, N - i_1, 0, 4)$$
$$+ \theta_{H1}\lambda_{H2}\pi(i_1 - 1, N - i_1, 0, 0) + \theta_{H1}\lambda_{L2}\pi(i_1, N - i_1 - 2, 0, 0)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1 - 2, 0, 4) + \theta_{H1}\lambda_{L2}\pi(i_1, N - i_1 - 1, 0, 0)$$
$$+ i_1\alpha_{H1}\pi(i_1 + 1, N - i_1, 0, 4)$$
$$+ (N - i_1 + 1)\alpha_{L1}\pi(i_1, N - i_1 + 1, 0, 4)$$
$$+ \theta_{H1}\mu_{H1}\pi(i_1 + 1, N - i_1, 0, 2) + \theta_{H1}\mu_{L1}\pi(i_1, N - i_1 + 1, 0, 1)$$
$$+ \theta_{H1}\mu_{H1}\pi(i_1 + 1, N - i_1, 0, 4) + \theta_{H1}\mu_{L1}\pi(i_1, N - i_1 + 1, 0, 3).$$

Case 350: For $3 \le i_1 \le N - 1,\ j_1 = N - i_1 + 1,\ x_1 = 0$, and $y_1 = 4$,

$$[\beta_1 + (i_1 - 1)\alpha_{H1} + (N - i_1 + 1)\alpha_{L1} + (1 - \theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]$$
$$\pi(i, N - i_1 + 1, 0, 4)$$
$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, 0, 4) + \lambda_{L-1}\pi(i_1, N - i_1, 0, 4)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, 0, 4) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, 0, 4)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1 - 1, 0, 4) + \lambda_{L2}\pi(i_1, N - i_1, 0, 4).$$

Case 351: For $i_1 = N,\ j_1 = 1,\ x_1 = 0$, and $y_1 = 4$,

$$[\beta_1 + (N - 1)\alpha_{H1} + \alpha_{L1} + (1 - \theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(N, 1, 0, 4)$$
$$= \lambda_{H-1}\pi(N - 1, 1, 0, 4) + \lambda_{L-1}\pi(N, 0, 0, 4) + \lambda_{H2}\pi(N - 2, 1, 0, 4)$$
$$+ \lambda_{H2}\pi(N - 1, 1, 0, 4) + \lambda_{L2}\pi(N, 0, 0, 4).$$

Case 352: For $i_1 = N + 1,\ j_1 = 0,\ x_1 = 0$, and $y_1 = 4$,

$$[\beta_1 + N\alpha_{H1} + (1 - \theta_{H1})\mu_{H1} + \theta_{H1}\mu_{H1}]\pi(N + 1, 0, 0, 4)$$
$$= \lambda_{H-1}\pi(N, 0, 0, 4) + \lambda_{H2}\pi(N - 1, 0, 0, 4) + \lambda_{H2}\pi(N, 0, 0, 4).$$

Case 353: For $i_1 = 1,\ j_1 = 0,\ 1 \le x_1 \le K - 1$, and $y_1 = 4$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \mu_{H1})\pi(1, 0, x_1, 4)$$
$$= \beta_1\pi(1, 0, x_1 - 1, 4) + \alpha_{H1}\pi(2, 0, x_1, 4) + \alpha_{L1}\pi(1, 1, x_1, 4).$$

Case 354: For $i_1 = 1,\ j_1 = 1,\ 1 \le x_1 \le K - 1$, and $y_1 = 4$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + j\alpha_{L1} + \mu_{H1})\pi(1, j, x_1, 4)$$
$$= \lambda_{L-1}\pi(1, 0, x_1, 4) + \beta_1\pi(1, 1, x_1 - 1, 4) + \alpha_{H1}\pi(2, 1, x_1, 4) +$$
$$2\alpha_{L1}\pi(1, 2, x_1, 4).$$

Case 355: For $i_1 = 1,\ 2 \le j_1 \le N-1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$(\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + j_1\alpha_{L1} + \mu_{H1})\pi(1, j_1, x_1, 4)$$
$$= \lambda_{L-1}\pi(1, j_1 - 1, x_1, 4) + \lambda_{L2}\pi(1, j_1 - 2, x_1, 4) + \beta_1\pi(1, j_1, x_1 - 1, 4)$$
$$+ \alpha_{H1}\pi(2, j_1, x_1, 4) + (j_1 + 1)\alpha_{L1}\pi(1, j_1 + 1, x_1, 4).$$

Case 356: For $i_1 = 1,\ j_1 = N,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$(\beta_1 + N\alpha_{L1} + \mu_{H1})\pi(1, N, x_1, 4)$$
$$= \lambda_{L-1}\pi(1, N-1, x_1, 4) + \lambda_{L2}\pi(1, N-2, x_1, 4) + \lambda_{L2}\pi(1, N-1, x_1, 4)$$
$$+ \beta_1\pi(1, N, x_1 - 1, 4).$$

Case 357: For $i_1 = 2,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \mu_{H1}]\pi(2, 0, x_1, 4)$$
$$= \lambda_{H-1}\pi(1, 0, x_1, 4) + \beta_1\pi(2, 0, x_1 - 1, 4) + 2\alpha_{H1}\pi(3, 0, x_1, 4)$$
$$+ \alpha_{L1}\pi(2, 1, x_1, 4).$$

Case 358: For $i_1 = 2,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(2, 1, x_1, 4)$$
$$= \lambda_{H-1}\pi(1, 1, x_1, 4) + \lambda_{L-1}\pi(2, 0, x_1, 4) + \beta_1\pi(2, 1, x_1 - 1, 4) +$$
$$2\alpha_{H1}\pi(3, 1, x_1, 4) + 2\alpha_{L1}\pi(2, 2, x_1, 4).$$

Case 359: For $i_1 = 2,\ 2 \le j_1 \le N-2,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + \alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(2, j_1, x_1, 4)$$
$$= \lambda_{H-1}\pi(1, j_1, x_1, 4) + \lambda_{L-1}\pi(2, j_1 - 1, x_1, 4) + \lambda_{L2}\pi(2, j_1 - 2, x_1, 4)$$
$$+ \beta_1\pi(2, j_1, x_1 - 1, 4) + 2\alpha_{H1}\pi(3, j_1, x_1, 4) + (j_1 + 1)\alpha_{L1}\pi(2, j_1$$
$$+ 1, x_1, 4).$$

Case 360: For $i_1 = 2,\ j_1 = N-1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\beta_1 + \alpha_{H1} + (N-1)\alpha_{L1} + \mu_{H1}]\pi(i, N-1, x_1, 4)$$

$$= \lambda_{H-1}\pi(1, N-1, x_1, 4) + \lambda_{L-1}\pi(2, N-2, x_1, 4) + \lambda_{H2}\pi(1, N-1, x_1, 4) + \lambda_{L2}\pi(2, N-3, x_1, 4) + \lambda_{L2}\pi(2, N-2, x_1, 4) + \beta_1\pi(2, N-1, x_1-1, 4).$$

Case 361: For $3 \le i_1 \le N,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1-1)\alpha_{H1} + \mu_{H1}]\pi(i_1, 0, x_1, 4)$$
$$= \lambda_{H-1}\pi(i_1-1, 0, x_1, 4) + \lambda_{H2}\pi(i_1-2, 0, x_1, 4) + \beta_1\pi(i_1, 0, x_1-1, 4) + i_1\alpha_{H1}\pi(i_1+1, 0, x_1, 4) + \alpha_{L1}\pi(i_1, 1, x_1, 4).$$

Case 362: For $3 \le i_1 \le N-1,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(i_1, 1, x_1, 4)$$
$$= \lambda_{H-1}\pi(i_1-1, 1, x_1, 4) + \lambda_{L-1}\pi(i_1, 0, x_1, 4) + \lambda_{H2}\pi(i_1-2, 1, x_1, 4) + \beta_1\pi(i_1, 1, x_1-1, 4) + i_1\alpha_{H1}\pi(i_1+1, 1, x_1, 4) + 2\alpha_{L1}\pi(i_1, 2, x_1, 4).$$

Case 363: For $3 \le i_1 \le N-2,\ 2 \le j_1 \le N-i_1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (i_1-1)\alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(i_1, j_1, x_1, 4)$$
$$= \lambda_{H-1}\pi(i_1-1, j_1, x_1, 4) + \lambda_{L-1}\pi(i_1, j_1-1, x_1, 4) + \lambda_{H2}\pi(i_1-2, j_1, x_1, 4) + \lambda_{L2}\pi(i_1, j_1-2, x_1, 4) + \beta_1\pi(i_1, j_1, x_1-1, 4) + i_1\alpha_{H1}\pi(i_1+1, j_1, x_1, 4) + (j_1+1)\alpha_{L1}\pi(i_1, j_1+1, x_1, 4).$$

Case 364: For $3 \le i_1 \le N-1,\ j_1 = N-i_1+1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\beta_1 + (i_1-1)\alpha_{H1} + (N-i_1+1)\alpha_{L1} + \mu_{H1}]\pi(i_1, N-i_1+1, x_1, 4)$$
$$= \lambda_{H-1}\pi(i_1-1, N-i_1+1, x_1, 4) + \lambda_{L-1}\pi(i_1, N-i_1, x_1, 4) + \lambda_{H2}\pi(i_1-2, N-i_1+1, x_1, 4) + \lambda_{H2}\pi(i_1-1, N-i_1+1, x_1, 4) + \lambda_{L2}\pi(i_1, N-i_1-1, x_1, 4) + \lambda_{L2}\pi(i_1, N-i_1, x_1, 4) + \beta_1\pi(i_1, N-i_1+1, x_1-1, 4).$$

Case 365: For $i_1 = N,\ j_1 = 1,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\beta_1 + (N-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(N, 1, x_1, 4)$$
$$= \lambda_{H-1}\pi(N-1, 1, x_1, 4) + \lambda_{L-1}\pi(i, 0, x_1, 4) + \lambda_{H2}\pi(N-2, 1, x_1, 4) + \lambda_{H2}\pi(N-1, 1, x_1, 4) + \beta_1\pi(i, 1, x_1-1, 4).$$

Case 366: For $i_1 = N+1,\ j_1 = 0,\ 1 \le x_1 \le K-1$, and $y_1 = 4$,

$$[\beta_1 + N\alpha_{H1} + \mu_{H1}]\pi(N+1, 0, x_1, 4)$$
$$= \lambda_{H-1}\pi(N, 0, x_1, 4) + \lambda_{H2}\pi(N-2, 0, x_1, 4) + \lambda_{H2}\pi(N-1, 0, x_1, 4) + \beta_1\pi(N + 1, 0, x_1 - 1, 4).$$

Case 367: For $i_1 = 1,\ j_1 = 0,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \mu_{H1}]\pi(1, 0, K, 4)$$
$$= \beta_1\pi(1, 0, K-1, 4) + \alpha_{H1}\pi(2, 0, K, 4) + \alpha_{L1}\pi(1, 1, K, 4).$$

Case 368: For $i_1 = 1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{L1} + \mu_{H1}]\pi(1, 1, K, 4)$$
$$= \lambda_{L-1}\pi(1, 0, K, 4) + \beta_1\pi(1, 1, K-1, 4) + \alpha_{H1}\pi(2, 1, K, 4) + 2\alpha_{L1}\pi(1, 2, K, 4).$$

Case 369: For $i_1 = 1,\ 2 \le j_1 \le N-1,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + j_1\alpha_{L1} + \mu_{H1}]\pi(1, j_1, K, 4)$$
$$= \lambda_{L-1}\pi(1, j_1 - 1, K, 4) + \lambda_{L2}\pi(1, j_1 - 2, K, 4) + \beta_1\pi(1, j_1, K-1, 4) + \alpha_{H1}\pi(2, j_1, K, 4) + (j_1 + 1)\alpha_{L1}\pi(1, j_1 + 1, K, 4).$$

Case 370: For $i_1 = 1,\ j_1 = N,\ x_1 = K$, and $y_1 = 4$,

$$[N\alpha_{L1} + \mu_{H1}]\pi(1, N, K, 4)$$
$$= \lambda_{L-1}\pi(1, N-1, K, 4) + \lambda_{L2}\pi(1, N-2, K, 4) + \lambda_{L2}\pi(1, N-1, K, 4) + \beta_1\pi(1, N, K-1, 4).$$

Case 371: For $i_1 = 2,\ j_1 = 0,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \mu_{H1}]\pi(2, 0, K, 4)$$
$$= \lambda_{H-1}\pi(1, 0, K, 4) + \beta_1\pi(2, 0, K-1, 4) + 2\alpha_{H1}\pi(3, 0, K, 4) + \alpha_{L1}\pi(2, 1, K, 4).$$

Case 372: For $i_1 = 2,\ j_1 = 1,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(2, 1, K, 4)$$
$$= \lambda_{H-1}\pi(1, 1, K, 4) + \lambda_{L-1}\pi(2, 0, K, 4) + \beta_1\pi(2, 1, K-1, 4) + 2\alpha_{H1}\pi(3, 1, K, 4) + 2\alpha_{L1}\pi(2, 2, K, 4).$$

Case 373: For $i_1 = 2,\ 2 \le j_1 \le N-2,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(2, j_1, K, 4)$$
$$= \lambda_{H-1}\pi(1, j_1, K, 4) + \lambda_{L-1}\pi(2, j_1 - 1, K, 4) + \lambda_{L2}\pi(2, j_1 - 2, K, 4)$$
$$+ \beta_1\pi(2, j_1, K-1, 4) + 2\alpha_{H1}\pi(3, j_1, K, 4) + (j_1+1)\alpha_{L1}\pi(2, j_1+1, K, 4).$$

Case 374: For $i_1 = 2,\ j_1 = N-1,\ x_1 = K$, and $y_1 = 4$,

$$[\alpha_{H1} + (N-1)\alpha_{L1} + \mu_{H1}]\pi(2, N-1, K, 4)$$
$$= \lambda_{H-1}\pi(1, N-1, K, 4) + \lambda_{L-1}\pi(2, N-2, K, 4) + \lambda_{H2}\pi(1, N-2, K, 4)$$
$$+ \lambda_{L2}\pi(2, N-3, K, 4) + \lambda_{L2}\pi(2, N-2, K, 4) + \beta_1\pi(2, N-1, K-1, 4).$$

Case 375: For $3 \le i_1 \le N,\ j_1 = 0,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + \mu_{H1}]\pi(i_1, 0, K, 4)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 0, K, 4) + \lambda_{H2}\pi(i_1 - 2, 0, K, 4) + \beta_1\pi(i_1, 0, K-1, 4)$$
$$+ i_1\alpha_{H1}\pi(i_1+1, 0, K, 4) + \alpha_{L1}\pi(i_1, 1, K, 4).$$

Case 376: For $3 \le i_1 \le N-1,\ j_1 = 1,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(i_1, 1, K, 4)$$
$$= \lambda_{H-1}\pi(i_1 - 1, 1, K, 4) + \lambda_{L-1}\pi(i_1, 0, K, 4) + \lambda_{H2}\pi(i_1 - 2, 1, K, 4)$$
$$+ \beta_1\pi(i_1, 1, K-1, 4) + i_1\alpha_{H1}\pi(i_1+1, 1, K, 4)$$
$$+ 2\alpha_{L1}\pi(i_1, 2, K, 4).$$

Case 377: For $3 \le i_1 \le N-2,\ 2 \le j_1 \le N - i_1,\ x_1 = K$, and $y_1 = 4$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + (i_1-1)\alpha_{H1} + j_1\alpha_{L1} + \mu_{H1}]\pi(i_1, j_1, K, 4)$$
$$= \lambda_{H-1}\pi(i_1 - 1, j_1, K, 4) + \lambda_{L-1}\pi(i_1, j_1 - 1, K, 4) + \lambda_{H2}\pi(i_1 - 2, j_1, K, 4)$$
$$+ \lambda_{L2}\pi(i_1, j_1 - 2, K, 4) + \beta_1\pi(i_1, j_1, K-1, 4) + i_1\alpha_{H1}\pi(i_1+1, j_1, K, 4)$$
$$+ (j_1+1)\alpha_{L1}\pi(i_1, j_1+1, K, 4).$$

Case 378: For $3 \le i_1 \le N-1,\ j_1 = N - i_1 + 1,\ x_1 = K$, and $y_1 = 4$,

$$[(i_1-1)\alpha_{H1} + (N - i_1 + 1)\alpha_{L1} + \mu_{H1}]\pi(i_1, N - i_1 + 1, K, 4)$$

$$= \lambda_{H-1}\pi(i_1 - 1, N - i_1 + 1, K, 4) + \lambda_{L-1}\pi(i_1, N - i_1, K, 4)$$
$$+ \lambda_{H2}\pi(i_1 - 2, N - i_1 + 1, K, 4) + \lambda_{H2}\pi(i_1 - 1, N - i_1 + 1, K, 4)$$
$$+ \lambda_{L2}\pi(i_1, N - i_1 - 1, K, 4) + \lambda_{L2}\pi(i_1, N - i_1, K, 4)$$
$$+ \beta_1\pi(i_1, N - i_1 + 1, K - 1, 4).$$

Case 379: For $i_1 = N,\ j_1 = 1,\ x_1 = K$, and $y_1 = 4$,

$$[(N-1)\alpha_{H1} + \alpha_{L1} + \mu_{H1}]\pi(N, 1, K, 4)$$
$$= \lambda_{H-1}\pi(N - 1, 1, K, 4) + \lambda_{L-1}\pi(N, 0, K, 4) + \lambda_{H2}\pi(N - 2, 1, K, 4)$$
$$+ \lambda_{H2}\pi(N - 1, 1, K, 4) + \lambda_{L2}\pi(N, 0, K, 4) + \beta_1\pi(N, 1, K - 1, 4).$$

Case 380: For $i_1 = N + 1,\ j_1 = 0,\ x_1 = K$, and $y_1 = 4$,

$$(N\alpha_{H1} + \mu_{H1})\pi(N + 1, 0, K, 4)$$
$$= \lambda_{H-1}\pi(N, 0, K, 4) + \lambda_{H2}\pi(N - 1, 0, K, 4) + \lambda_{H2}\pi(N, 0, K, 4) + \beta_1\pi(N$$
$$+ 1, 0, K - 1, 4).$$

Since there are many equations presented above, discussing each one separately would be challenging. Therefore, we focus on a relatively complicated case, specifically case *A103*, to provide an illustrative example. This state occurs when there are more than or equal to three but less than or equal to N-1 LP packets and 0 HP packet in the system, and there are at least 2 seats left in the packet queue, while the energy queue is empty. The LP packet being served in the server is using the regular battery. The corresponding detailed state transition diagram can be found in Fig. 3 - 2.

Case A103: For $i_1 = 0,\ 3 \le j_1 \le N - 1,\ x_1 = 0$, and $y_1 = 3$,

$$[\lambda_{H-1} + \lambda_{L-1} + \lambda_{H2} + \lambda_{L2} + \beta_1 + (j_1 - 1)\alpha_{L1} + (1 - \theta_{L1})\mu_{L1}$$
$$+ \theta_{L1}\mu_{L1}]\pi(0, j_1, 0, 3)$$
$$= \theta_{L1}\lambda_{L-1}\pi(0, j_1 - 1, 0, 0) + \lambda_{L-1}\pi(0, j_1 - 1, 0, 3) + \theta_{L1}\lambda_{L2}\pi(0, j_1 - 2, 0, 0)$$
$$+ \lambda_{L2}\pi(0, j_1 - 2, 0, 3) + \alpha_{H1}\pi(1, j_1, 0, 3) + \theta_{L1}\alpha_{H1}\pi(1, j_1, 0, 0)$$
$$+ j_1\alpha_{L1}\pi(0, j_1 + 1, 0, 3) + \theta_{L1}(j_1 + 1)\alpha_{L1}\pi(0, j_1 + 1, 0, 0)$$
$$+ \theta_{L1}\mu_{H1}\pi(1, j_1, 0, 2) + \theta_{L1}\mu_{L1}\pi(0, j_1 + 1, 0, 1)$$
$$+ \theta_{L1}\mu_{H1}\pi(1, j_1, 0, 4) + \theta_{L1}\mu_{L1}\pi(0, j_1 + 1, 0, 3).$$

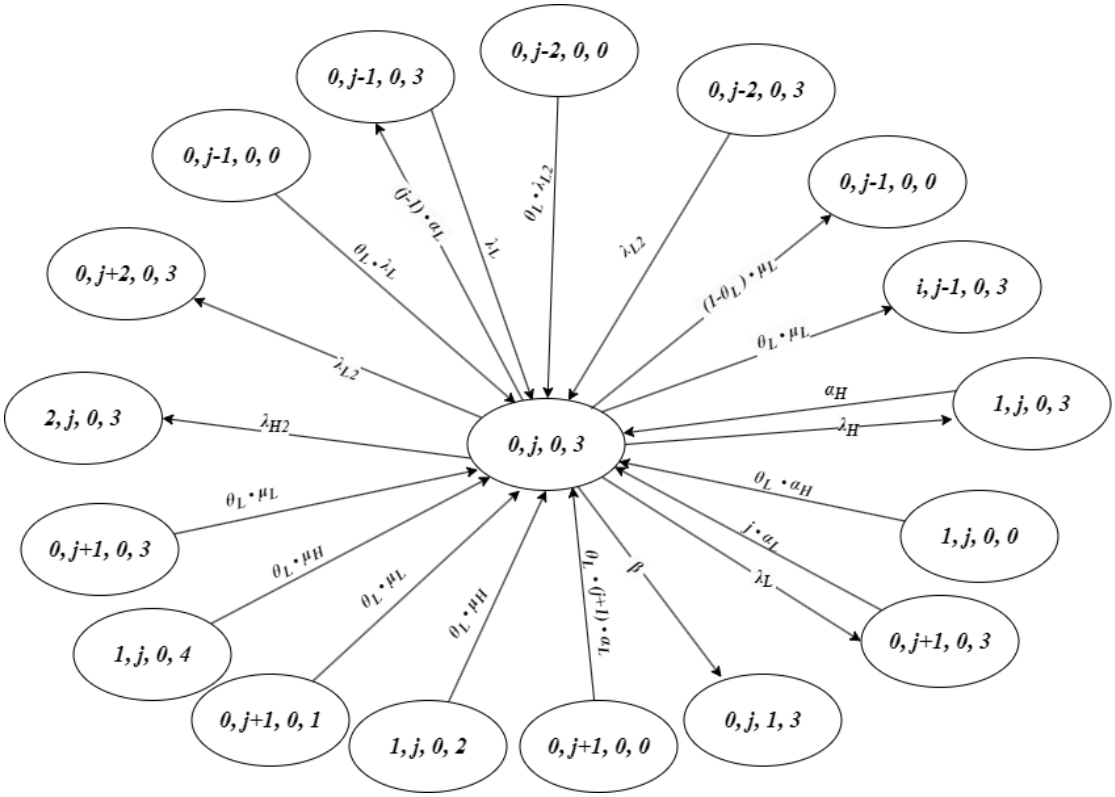

Fig. 3 - 4: The state transition diagram for $i_1 = 0,\ 3 \le j_1 \le N-1,\ x_1 = 0$, and $y_1 = 3$.

Case 1: For $i_n = 0,\ j_n = 0,\ x_n = 0$, and $y_n = 0$,

$$[(1-\theta_{Hn})\lambda_{H-n} + \theta_{Hn}\lambda_{H-n} + (1-\theta_{Ln})\lambda_{L-n} + \theta_{Ln}\lambda_{L-n} + \beta_n]\pi(0,0,0,0)$$
$$= \alpha_{Hn}\pi(1,0,0,0) + \alpha_{Ln}\pi(0,1,0,0) + \mu_{Hn}\pi(1,0,0,2) + \mu_{Ln}\pi(0,1,0,1) +$$
$$\mu_{Hn}\pi(1,0,0,4) + \mu_{Ln}\pi(0,1,0,3).$$

Case 2: For $i_n = 0,\ j_n = 1,\ x_n = 0$, and $y_n = 0$,

$$[(1-\theta_{Hn})\lambda_{H-n} + \theta_{Hn}\lambda_{H-n} + (1-\theta_{Ln})\lambda_{L-n} + \theta_{Ln}\lambda_{L-n} + \beta_n$$
$$+ \alpha_{Ln}]\pi(0,1,0,0)$$
$$= (1-\theta_{Ln})\lambda_{L-n}\pi(0,0,0,0) + (1-\theta_{Ln})\alpha_{Hn}\pi(1,1,0,0) +$$
$$(1-\theta_{Ln})2\alpha_{Ln}\pi(0,2,0,0) + (1-\theta_{Ln})\mu_{Hn}\pi(1,1,0,2) +$$
$$(1-\theta_{Ln})\mu_{Ln}\pi(0,2,0,1) + (1-\theta_{Ln})\mu_{Hn}\pi(1,1,0,4) +$$
$$(1-\theta_{Ln})\mu_{Ln}\pi(0,2,0,3).$$

Case 3: For $i_n = 0,\ 2 \le j_n \le N-1,\ x_n = 0$, and $y_n = 0$,

$$
\begin{aligned}
&[(1-\theta_{Hn})\lambda_{H-n}+\theta_{Hn}\lambda_{H-n}+(1-\theta_{Ln})\lambda_{L-n}+\theta_{Ln}\lambda_{L-n}+\beta_n \\
&\qquad +(1-\theta_{Ln})j_n\alpha_{Ln}+\theta_{Ln}j_n\alpha_{Ln}]\pi(0,j_n,0,0) \\
&=(1-\theta_{Ln})\lambda_{L-n}\pi(0,j_n-1,0,0)+(1-\theta_{Ln})\alpha_{Hn}\pi(1,j_n,0,0)+ \\
&\quad (1-\theta_{Ln})(j_n+1)\alpha_{Ln}\pi(0,j_n+1,0,0)+(1-\theta_{Ln})\mu_{Hn}\pi(1,j_n,0,2)+ \\
&\quad (1-\theta_{Ln})\mu_{Ln}\pi(0,j_n+1,0,1)+(1-\theta_{Ln})\mu_{Hn}\pi(1,j_n,0,4)+ \\
&\quad (1-\theta_{Ln})\mu_{Ln}\pi(0,j_n+1,0,3).
\end{aligned}
$$

Case 4: For $i_n=0,\ j_n=N,\ x_n=0$, and $y_n=0$,

$$
\begin{aligned}
&[\beta_n+(1-\theta_{Ln})N\alpha_{Ln}+\theta_{Ln}N\alpha_{Ln}]\pi(0,N,0,0) \\
&=(1-\theta_{Ln})\lambda_{L-n}\pi(0,N-1,0,0)+(1-\theta_{Ln})\mu_{Hn}\pi(1,N,0,2)+ \\
&\quad (1-\theta_{Ln})\mu_{Ln}\pi(0,N+1,0,1)+(1-\theta_{Ln})\mu_{Hn}\pi(1,N,0,4)+ \\
&\quad (1-\theta_{Ln})\mu_{Ln}\pi(0,N+1,0,3).
\end{aligned}
$$

Case 5: For $i_n=1,\ j_n=0,\ x_n=0$, and $y_n=0$,

$$
\begin{aligned}
&[(1-\theta_{Hn})\lambda_{H-n}+\theta_{Hn}\lambda_{H-n}+(1-\theta_{Hn})\lambda_{L-n}+\theta_{Hn}\lambda_{L-n}+\beta_n \\
&\qquad +\alpha_{Hn}]\pi(1,0,0,0) \\
&=(1-\theta_{Hn})\lambda_{H-n}\pi(0,0,0,0)+(1-\theta_{Hn})2\alpha_{Hn}\pi(2,0,0,0)+ \\
&\quad (1-\theta_{Hn})\alpha_{Ln}\pi(1,1,0,0)+(1-\theta_{Hn})\mu_{Hn}\pi(2,0,0,2)+ \\
&\quad (1-\theta_{Hn})\mu_{Ln}\pi(1,1,0,1)+(1-\theta_{Hn})\mu_{Hn}\pi(2,0,0,4)+ \\
&\quad (1-\theta_{Hn})\mu_{Ln}\pi(1,1,0,3).
\end{aligned}
$$

Case 6: For $i_n=1,\ 1\le j_n\le N-2,\ x_n=0$, and $y_n=0$,

$$
\begin{aligned}
&[(1-\theta_{Hn})\lambda_{H-n}+\theta_{Hn}\lambda_{H-n}+(1-\theta_{Hn})\lambda_{L-n}+\theta_{Hn}\lambda_{L-n}+\beta_n+(1-\theta_{Ln})\alpha_{Hn} \\
&\qquad +\theta_{Ln}\alpha_{Hn}+(1-\theta_{Hn})j_n\alpha_{Ln}+\theta_{Hn}j_n\alpha_{Ln}]\pi(1,j_n,0,0) \\
&=(1-\theta_{Hn})\lambda_{H-n}\pi(0,j_n,0,0)+(1-\theta_{Hn})\lambda_{L-n}\pi(1,j_n-1,0,0)+ \\
&\quad (1-\theta_{Hn})2\alpha_{Hn}\pi(2,j_n,0,0)+(1-\theta_{Hn})(j_n+1)\alpha_{Ln}\pi(1,j_n+1,0,0)+ \\
&\quad (1-\theta_{Hn})\mu_{Hn}\pi(2,j_n,0,2)+(1-\theta_{Hn})\mu_{Ln}\pi(1,j_n+1,0,1) \\
&\quad (1-\theta_{Hn})\mu_{Hn}\pi(2,j_n,0,4)+(1-\theta_{Hn})\mu_{Ln}\pi(1,j_n+1,0,3).
\end{aligned}
$$

Case 7: For $i_n=1,\ j_n=N-1,\ x_n=0$, and $y_n=0$,

$$
\begin{aligned}
&[\beta_n+(1-\theta_{Ln})\alpha_{Hn}+\theta_{Ln}\alpha_{Hn}+(1-\theta_{Hn})(N-1)\alpha_{Ln}+\theta_{Hn}(N-1)\alpha_{Ln}] \\
&\pi(1,N-1,0,0)
\end{aligned}
$$

$$= (1-\theta_{Hn})\lambda_{H-n}\pi(0,N-1,0,0) + (1-\theta_{Hn})\lambda_{L-n}\pi(1,N-2,0,0) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(2,N-1,0,2) + (1-\theta_{Hn})\mu_{Ln}\pi(1,N,0,1) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(2,N-1,0,4) + (1-\theta_{Hn})\mu_{Ln}\pi(1,N,0,3).$$

Case 8: For $2 \le i_n \le N-1,\ j_n = 0,\ x_n = 0$, and $y_n = 0$,

$$[(1-\theta_{Hn})\lambda_{H-n} + \theta_{Hn}\lambda_{H-n} + (1-\theta_{Hn})\lambda_{L-n} + \theta_{Hn}\lambda_{L-n} + \beta_n$$
$$+ (1-\theta_{Hn})i_n\alpha_{Hn} + \theta_{Hn}i_n\alpha_{Hn}]\pi(i_n,0,0,0)$$
$$= (1-\theta_{Hn})\lambda_{H-n}\pi(i_n-1,0,0,0) + (1-\theta_{Hn})(i_n+1)\alpha_{Hn}\pi(i_n+1,0,0,0) +$$
$$(1-\theta_{Hn})\alpha_{Ln}\pi(i_n,1,0,0) + (1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,0,0,2) +$$
$$(1-\theta_{Hn})\mu_{Ln}\pi(i_n,1,0,1) + (1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,0,0,4) +$$
$$(1-\theta_{Hn})\mu_{Ln}\pi(i_n,1,0,3).$$

Case 9: For $2 \le i_n \le N-2,\ 1 \le j_n \le N-i_n-1,\ x_n = 0$, and $y_n = 0$,

$$[(1-\theta_{Hn})\lambda_{H-n} + \theta_{Hn}\lambda_{H-n} + (1-\theta_{Hn})\lambda_{L-n} + \theta_{Hn}\lambda_{L-n} + \beta_n$$
$$+ (1-\theta_{Hn})i_n\alpha_{Hn} + \theta_{Hn}i_n\alpha_{Hn} + (1-\theta_{Hn})j_n\alpha_{Ln}$$
$$+ \theta_{Hn}j_n\alpha_{Ln}]\pi(i_n,j_n,0,0)$$
$$= (1-\theta_{Hn})\lambda_{H-n}\pi(i_n-1,j_n,0,0) + (1-\theta_{Hn})\lambda_{L-n}\pi(i_n,j_n-1,0,0) +$$
$$(1-\theta_{Hn})(i_n+1)\alpha_{Hn}\pi(i_n+1,j_n,0,0) +$$
$$(1-\theta_{Hn})(j_n+1)\alpha_{Ln}\pi(i_n,j_n+1,0,0) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,j_n,0,2) + (1-\theta_{Hn})\mu_{Ln}\pi(i_n,j_n+1,0,1) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,j_n,0,4) + (1-\theta_{Hn})\mu_{Ln}\pi(i_n,j_n+1,0,3).$$

Case 10: For $2 \le i_n \le N-1,\ j_n = N-i_n,\ x_n = 0$, and $y_n = 0$,

$$[\beta_n + (1-\theta_{Hn})i_n\alpha_{Hn} + \theta_{Hn}i_n\alpha_{Hn} + (1-\theta_{Hn})(N-i_n)\alpha_{Ln} +$$
$$\theta_{Hn}(N-i_n)\alpha_{Ln}]\pi(i_n,N-i_n,0,0)$$
$$= (1-\theta_{Hn})\lambda_{H-n}\pi(i_n-1,N-i_n,0,0)$$
$$+ (1-\theta_{Hn})\lambda_{L-n}\pi(i_n,N-i_n-1,0,0) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,N-i_n,0,2) + (1-\theta_{Hn})\mu_{Ln}\pi(i_n,N-i_n+1,0,1) +$$
$$(1-\theta_{Hn})\mu_{Hn}\pi(i_n+1,N-i_n,0,4) + (1-\theta_{Hn})\mu_{Ln}\pi(i_n,N-i_n+1,0,3).$$

Case 11: For $i_n = N,\ j_n = 0,\ x_n = 0$, and $y_n = 0$,

$$[\beta_n + (1-\theta_{Hn})N\alpha_{Hn} + \theta_{Hn}N\alpha_{Hn}]\pi(N,0,0,0)$$
$$= (1-\theta_{Hn})\lambda_{H-n}\pi(N-1,0,0,0) + (1-\theta_{Hn})\mu_{Hn}\pi(N+1,0,0,2) +$$

$$(1-\theta_{Hn})\mu_{Ln}\pi(N,1,0,1)+(1-\theta_{Hn})\mu_{Hn}\pi(N+1,0,0,4)+$$
$$(1-\theta_{Hn})\mu_{Ln}\pi(N,1,0,3).$$

Case 12: For $i_n=0,\ j_n=0,\ 1\le x_n\le K-1$, and $y_n=0$,

$$(\lambda_{H-n}+\lambda_{L-n}+\beta_n)\pi(0,0,x_n,0)$$
$$=\beta_n\pi(0,0,x_n-1,0)+\mu_{Hn}\pi(1,0,x_n,2)+\mu_{Ln}\pi(0,1,x_n,1)+$$
$$\mu_{Hn}\pi(1,0,x_n,4)+\mu_{Ln}\pi(0,1,x_n,3).$$

Case 13: For $i_n=0,\ j_n=0,\ x_n=K$, and $y_n=0$,

$$(\lambda_{H-n}+\lambda_{L-n})\pi(0,0,K,0)$$
$$=\beta_n\pi(0,0,K-1,0)+\mu_{Hn}\pi(1,0,K,2)+\mu_{Ln}\pi(0,1,K,1)+$$
$$\mu_{Hn}\pi(1,0,K,4)+\mu_{Ln}\pi(0,1,K,3).$$

Case 14: For $i_n=0,\ j_n=1,\ x_n=0$, and $y_n=1$,

$$(\lambda_{H-n}+\lambda_{L-n}+\beta_n+\mu_{Ln})\pi(0,1,0,1)$$
$$=\lambda_{L-n}\pi(0,0,1,0)+\beta_n\pi(0,1,0,0)+\alpha_{Hn}\pi(1,1,0,1)+\alpha_{Ln}\pi(0,2,0,1)+$$
$$\mu_{Hn}\pi(1,1,1,2)+\mu_{Ln}\pi(0,2,1,1)+\mu_{Hn}\pi(1,1,1,4)+\mu_{Ln}\pi(0,2,1,3).$$

Case 15: For $i_n=0,\ 2\le j_n\le N,\ x_n=0$, and $y_n=1$,

$$[\lambda_{H-n}+\lambda_{L-n}+\beta_n+(j_n-1)\alpha_{Ln}+(1-\theta_{Ln})\mu_{Ln}+\theta_{Ln}\mu_{Ln}]\pi(0,j_n,0,1)$$
$$=\lambda_{L-n}\pi(0,j_n-1,0,1)+\beta_n\pi(0,j_n,0,0)+\alpha_{Hn}\pi(1,j_n,0,1)+$$
$$j_n\alpha_{Ln}\pi(0,j_n+1,0,1)+\mu_{Hn}\pi(1,j_n,1,2)+\mu_{Ln}\pi(0,j_n+1,1,1)+$$
$$\mu_{Hn}\pi(1,j_n,1,4)+\mu_{Ln}\pi(0,j_n+1,1,3).$$

Case 16: For $i_n=0,\ j_n=N+1,\ x_n=0$, and $y_n=1$,

$$[\beta_n+N\alpha_{Ln}+(1-\theta_{Ln})\mu_{Ln}+\theta_{Ln}\mu_{Ln}]\pi(0,N+1,0,1)$$
$$=\lambda_{L-n}\pi(0,N,0,1).$$

Case 17: For $1\le i_n\le N-1,\ j_n=1,\ x_n=0$, and $y_n=1$,

$$[\lambda_{H-n}+\lambda_{L-n}+\beta_n+i_n\alpha_{Hn}+(1-\theta_{Hn})\mu_{Ln}+\theta_{Hn}\mu_{Ln}]\pi(i_n,1,0,1)$$
$$=\lambda_{H-n}\pi(i_n-1,1,0,1)+(i_n+1)\alpha_{Hn}\pi(i_n+1,1,0,1)+\alpha_{Ln}\pi(i_n,2,0,1).$$

Case 18: For $1\le i_n\le N-2,\ 2\le j_n\le N-i_n,\ x_n=0$, and $y_n=1$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + (j_n - 1)\alpha_{Ln} + (1 - \theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]$$
$$\pi(i_n, j_n, 0, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, 0, 1) + \lambda_{L-n}\pi(i_n, j_n - 1, 0, 1) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, 0, 1) + j_n\alpha_{Ln}\pi(i_n, j_n + 1, 0, 1).$$

Case 19: For $1 \le i_n \le N - 1$, $j_n = N - i_n + 1$, $x_n = 0$, and $y_n = 1$,

$$[\beta_n + i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + (1 - \theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]\pi(i_n, N - i_n + 1, 0, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, 0, 1) + \lambda_{L-n}\pi(i_n, N - i_n, 0, 1).$$

Case 20: For $i_n = N$, $j_n = 1$, $x_n = 0$, and $y_n = 1$,

$$[\beta_n + N\alpha_{Hn} + (1 - \theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]\pi(N, 1, 0, 1)$$
$$= \lambda_{H-n}\pi(N - 1, 1, 0, 1).$$

Case 21: For $i_n = 0$, $j_n = 1$, $1 \le x_n \le K - 1$, and $y_n = 1$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Ln})\pi(0, 1, x_n, 1)$$
$$= \lambda_{L-n}\pi(0, 0, x_n + 1, 0) + \beta_n\pi(0, 1, x_n - 1, 1) +$$
$$\alpha_{Hn}\pi(1, 1, x_n, 1) + \alpha_{Ln}\pi(0, 2, x_n, 1) +$$
$$\mu_{Hn}\pi(1, 1, x_n + 1, 2) + \mu_{Ln}\pi(0, 2, x_n + 1, 1) +$$
$$\mu_{Hn}\pi(1, 1, x_n + 1, 4) + \mu_{Ln}\pi(0, 2, x_n + 1, 3).$$

Case 22: For $i_n = 0$, $2 \le j_n \le N$, $1 \le x_n \le K - 1$, and $y_n = 1$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(0, j_n, x_n, 1)$$
$$= \lambda_{L-n}\pi(0, j_n - 1, x_n, 1) + \beta_n\pi(0, j_n, x_n - 1, 1) +$$
$$\alpha_{Hn}\pi(1, j_n, x_n, 1) + j_n\alpha_{Ln}\pi(0, j_n + 1, x_n, 1) +$$
$$\mu_{Hn}\pi(1, j_n, x_n + 1, 2) + \mu_{Ln}\pi(0, j_n + 1, x_n + 1, 1) +$$
$$\mu_{Hn}\pi(1, j_n, x_n + 1, 4) + \mu_{Ln}\pi(0, j_n + 1, x_n + 1, 3).$$

Case 23: For $i_n = 0$, $j_n = N + 1$, $1 \le x_n \le K - 1$, and $y_n = 1$,

$$(\beta_n + N\alpha_{Ln} + \mu_{Ln})\pi(0, N + 1, x_n, 1)$$
$$= \lambda_{L-n}\pi(0, N, x_n, 1) + \beta_n\pi(0, N + 1, x_n - 1, 1).$$

Case 24: For $1 \le i_n \le N - 1$, $j_n = 1$, $1 \le x_n \le K - 1$, and $y_n = 1$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + \mu_{Ln})\pi(i_n, 1, x_n, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1,1, x_n, 1) + \beta_n\pi(i_n, 1, x_n - 1, 1) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1,1, x_n, 1) + \alpha_{Ln}\pi(i_n, 2, x_n, 1).$$

Case 25: For $1 \le i_n \le N-2,\ 2 \le j_n \le N - i_n,\ 1 \le x_n \le K-1$, and $y_n = 1$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, j_n, x_n, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, x_n, 1) + \lambda_{L-n}\pi(i_n, j_n - 1, x_n, 1) + \beta_n\pi(i_n, j_n, x_n - 1, 1) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, x_n, 1) + j_n\alpha_{Ln}\pi(i_n, j_n + 1, x_n, 1).$$

Case 26: For $1 \le i_n \le N-1,\ j_n = N - i_n + 1,\ 1 \le x_n \le K-1$, and $y_n = 1$,

$$[\beta_n + i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, N - i_n + 1, x_n, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, x_n, 1) + \lambda_{L-n}\pi(i_n, N - i_n, x_n, 1) +$$
$$\beta_n\pi(i_n, N - i_n + 1, x_n - 1, 1).$$

Case 27: For $i_n = N,\ j_n = 1,\ 1 \le x_n \le K-1$, and $y_n = 1$,

$$(\beta_n + N\alpha_{Hn} + \mu_{Ln})\pi(N, 1, x_n, 1)$$
$$= \lambda_{H-n}\pi(N - 1, 1, x_n, 1) + \beta_n\pi(N, 1, x_n - 1, 1).$$

Case 28: For $i_n = 0,\ j_n = 1,\ x_n = K$, and $y_n = 1$,

$$(\lambda_{H-n} + \lambda_{L-n} + \mu_{Ln})\pi(0, 1, K, 1)$$
$$= \beta_n\pi(0, 1, K - 1, 1) + \alpha_{Hn}\pi(1, 1, K, 1) + \alpha_{Ln}\pi(0, 2, K, 1).$$

Case 29: For $i_n = 0,\ 2 \le j_n \le N,\ x_n = K$, and $y_n = 1$,

$$[\lambda_{H-n} + \lambda_{L-n} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(0, j_n, K, 1)$$
$$= \lambda_{L-n}\pi(0, j_n - 1, K, 1) + \beta_n\pi(0, j_n, K - 1, 1) + \alpha_{Hn}\pi(1, j_n, K, 1) +$$
$$j_n\alpha_{Ln}\pi(0, j_n + 1, K, 1).$$

Case 30: For $i_n = 0,\ j_n = N + 1,\ x_n = K$, and $y_n = 1$,

$$(N\alpha_{Ln} + \mu_{Ln})\pi(0, N + 1, K, 1)$$
$$= \lambda_{L-n}\pi(0, N, K, 1) + \beta_n\pi(0, N + 1, K - 1, 1).$$

Case 31: For $1 \le i_n \le N-1,\ j_n = 1,\ x_n = K$, and $y_n = 1$,

$$(\lambda_{H-n} + \lambda_{L-n} + i_n\alpha_{Hn} + \mu_{Ln})\pi(i_n, 1, K, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, 1, K, 1) + \beta_n\pi(i_n, 1, K - 1, 1) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, 1, K, 1) + \alpha_{Ln}\pi(i_n, 2, K, 1).$$

Case 32: For $1 \le i_n \le N - 2$, $2 \le j_n \le N - i_n$, $x_n = K$, and $y_n = 1$,

$$[\lambda_{H-n} + \lambda_{L-n} + i_n\alpha_{Hn} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, j_n, K, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, K, 1) + \lambda_{L-n}\pi(i_n, j_n - 1, K, 1) + \beta_n\pi(i_n, j_n, K - 1, 1) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, K, 1) + j_n\alpha_{Ln}\pi(i_n, j_n + 1, K, 1).$$

Case 33: For $1 \le i_n \le N - 1$, $j_n = N - i_n + 1$, $x_n = K$, and $y_n = 1$,

$$[i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, N - i_n + 1, K, 1)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, K, 1) + \lambda_{L-n}\pi(i_n, N - i_n, K, 1) +$$
$$\beta_n\pi(i_n, N - i_n + 1, K - 1, 1).$$

Case 34: For $i_n = N$, $j_n = 1$, $x_n = K$, and $y_n = 1$,

$$(N\alpha_{Hn} + \mu_{Ln})\pi(N, 1, K, 1)$$
$$= \lambda_{H-n}\pi(N - 1, 1, K, 1) + \beta_n\pi(N, 1, K - 1, 1).$$

Case 35: For $i_n = 1$, $j_n = 0$, $x_n = 0$, and $y_n = 2$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Hn})\pi(1, 0, 0, 2)$$
$$= \lambda_{H-n}\pi(0, 0, 1, 0) + \beta_n\pi(1, 0, 0, 0) + \alpha_{Hn}\pi(2, 0, 0, 2) + \alpha_{Ln}\pi(1, 1, 0, 2) +$$
$$\mu_{Hn}\pi(2, 0, 1, 2) + \mu_{Ln}\pi(1, 1, 1, 1) + \mu_{Hn}\pi(2, 0, 1, 4) + \mu_{Ln}\pi(1, 1, 1, 3).$$

Case 36: For $i_n = 1$, $1 \le j_n \le N - 1$, $x_n = 0$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + j_n\alpha_{Ln} + (1 - \theta_{Ln})\mu_{Hn} + \theta_{Ln}\mu_{Hn}]\pi(1, j_n, 0, 2)$$
$$= \lambda_{L-n}\pi(1, j_n - 1, 0, 2) + \beta_n\pi(1, j_n, 0, 0) + \alpha_{Hn}\pi(2, j_n, 0, 2) +$$
$$(j_n + 1)\alpha_{Ln}\pi(1, j_n + 1, 0, 2) + \mu_{Hn}\pi(2, j_n, 1, 2) + \mu_{Ln}\pi(1, j_n + 1, 1, 1) +$$
$$\mu_{Hn}\pi(2, j_n, 1, 4) + \mu_{Ln}\pi(1, j_n + 1, 1, 3).$$

Case 37: For $i_n = 1$, $j_n = N$, $x_n = 0$, and $y_n = 2$,

$$[\beta_n + N\alpha_{Ln} + (1 - \theta_{Ln})\mu_{Hn} + \theta_{Ln}\mu_{Hn}]\pi(1, N, 0, 2)$$
$$= \lambda_{L-n}\pi(1, N - 1, 0, 2).$$

Case 38: For $2 \le i_n \le N,\ j_n = 0,\ x_n = 0$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + (1 - \theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(i_n, 0, 0, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, 0, 0, 2) + \beta_n\pi(i_n, 0, 0, 0) + i_n\alpha_{Hn}\pi(i_n + 1, 0, 0, 2) +$$
$$\alpha_{Ln}\pi(i_n, 1, 0, 2) + \mu_{Hn}\pi(i_n + 1, 0, 1, 2) + \mu_{Ln}\pi(i_n, 1, 1, 1) +$$
$$\mu_{Hn}\pi(i_n + 1, 0, 1, 4) + \mu_{Ln}\pi(i_n, 1, 1, 3).$$

Case 39: For $2 \le i_n \le N - 1,\ 1 \le j_n \le N - i_n,\ x_n = 0$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + j_n\alpha_{Ln} + (1 - \theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]$$
$$\pi(i_n, j_n, 0, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, 0, 2) + \lambda_{L-n}\pi(i_n, j_n - 1, 0, 2) + \beta_n\pi(i_n, j_n, 0, 0) +$$
$$i_n\alpha_{Hn}\pi(i_n + 1, j_n, 0, 2) + (j_n + 1)\alpha_{Ln}\pi(i_n, j_n + 1, 0, 2) +$$
$$\mu_{Hn}\pi(i_n + 1, j_n, 1, 2) + \mu_{Ln}\pi(i_n, j_n + 1, 1, 1) +$$
$$\mu_{Hn}\pi(i_n + 1, j_n, 1, 4) + \mu_{Ln}\pi(i_n, j_n + 1, 1, 3).$$

Case 40: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ x_n = 0$, and $y_n = 2$,

$$[\beta_n + (i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + (1 - \theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]$$
$$\pi(i_n, N - i_n + 1, 0, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, 0, 2) + \lambda_{L-n}\pi(i_n, N - i_n, 0, 2).$$

Case 41: For $i_n = N + 1,\ j_n = 0,\ x_n = 0$, and $y_n = 2$,

$$[\beta_n + N\alpha_{Hn} + (1 - \theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(N + 1, 0, 0, 2)$$
$$= \lambda_{H-n}\pi(N, 0, 0, 2).$$

Case 42: For $i_n = 1,\ j_n = 0,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Hn})\pi(1, 0, x_n, 2)$$
$$= \lambda_{H-n}\pi(0, 0, x_n + 1, 0) + \beta_n\pi(1, 0, x_n - 1, 2) +$$
$$\alpha_{Hn}\pi(2, 0, x_n, 2) + \alpha_{Ln}\pi(1, 1, x_n, 2) + \mu_{Hn}\pi(2, 0, x_n + 1, 2) +$$
$$\mu_{Ln}\pi(1, 1, x_n + 1, 1) + \mu_{Hn}\pi(2, 0, x_n + 1, 4) + \mu_{Ln}\pi(1, 1, x_n + 1, 3).$$

Case 43: For $i_n = 1,\ 1 \le j_n \le N - 1,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + j_n\alpha_{Ln} + \mu_{Hn})\pi(1, j_n, x_n, 2)$$
$$= \lambda_{L-n}\pi(1, j_n - 1, x_n, 2) + \beta_n\pi(1, j_n, x_n - 1, 2) +$$

$$\alpha_{Hn}\pi(2, j_n, x_n, 2) + (j_n + 1)\alpha_{Ln}\pi(1, j_n + 1, x_n, 2) +$$
$$\mu_{Hn}\pi(2, j_n, x_n + 1, 2) + \mu_{Ln}\pi(1, j_n + 1, x_n + 1, 1) +$$
$$\mu_{Hn}\pi(2, j_n, x_n + 1, 4) + \mu_{Ln}\pi(1, j_n + 1, x_n + 1, 3).$$

Case 44: For $i_n = 1,\ j_n = N,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$(\beta_n + N\alpha_{Ln} + \mu_{Hn})\pi(1, N, x_n, 2)$$
$$= \lambda_{L-n}\pi(1, N - 1, x_n, 2) + \beta_n\pi(1, N, x_n - 1, 2).$$

Case 45: For $2 \le i_n \le N,\ j_n = 0,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + \mu_{Hn}]\pi(i_n, 0, x_n, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, 0, x_n, 2) + \beta_n\pi(i_n, 0, x_n - 1, 2) + i_n\alpha_{Hn}\pi(i_n + 1, 0, x_n, 2) +$$
$$\alpha_{Ln}\pi(i_n, 1, x_n, 2) + \mu_{Hn}\pi(i_n + 1, 0, x_n + 1, 2) + \mu_{Ln}\pi(i_n, 1, x_n + 1, 1) +$$
$$\mu_{Hn}\pi(i_n + 1, 0, x_n + 1, 4) + \mu_{Ln}\pi(i_n, 1, x_n + 1, 3).$$

Case 46: For $2 \le i_n \le N - 1,\ 1 \le j_n \le N - i_n,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + j_n\alpha_{Ln} + \mu_{Hn}]\pi(i_n, j_n, x_n, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, x_n, 2) + \lambda_{L-n}\pi(i_n, j_n - 1, x_n, 2) + \beta_n\pi(i_n, j_n, x_n - 1, 2) +$$
$$i_n\alpha_{Hn}\pi(i_n + 1, j_n, x_n, 2) + (j_n + 1)\alpha_{Ln}\pi(i_n, j_n + 1, x_n, 2) +$$
$$\mu_{Hn}\pi(i_n + 1, j_n, x_n + 1, 2) + \mu_{Ln}\pi(i_n, j_n + 1, x_n + 1, 1) +$$
$$\mu_{Hn}\pi(i_n + 1, j_n, x_n + 1, 4) + \mu_{Ln}\pi(i_n, j_n + 1, x_n + 1, 3).$$

Case 47: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$[\beta_n + (i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + \mu_{Hn}]\pi(i_n, N - i_n + 1, x_n, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, x_n, 2) + \lambda_{L-n}\pi(i_n, N - i_n, x_n, 2) +$$
$$\beta_n\pi(i_n, N - i_n + 1, x_n - 1, 2).$$

Case 48: For $i_n = N + 1,\ j_n = 0,\ 1 \le x_n \le K - 1$, and $y_n = 2$,

$$(\beta_n + N\alpha_{Hn} + \mu_{Hn})\pi(N + 1, 0, x_n, 2)$$
$$= \lambda_{H-n}\pi(N, 0, x_n, 2) + \beta_n\pi(N + 1, 0, x_n - 1, 2).$$

Case 49: For $i_n = 1,\ j_n = 0,\ x_n = K$, and $y_n = 2$,

$$(\lambda_{H-n} + \lambda_{L-n} + \mu_{Hn})\pi(1, 0, K, 2)$$

$$= \beta_n \pi(1, 0, K-1, 2) + \alpha_{Hn} \pi(2, 0, K, 2) + \alpha_{Ln} \pi(1, 1, K, 2).$$

Case 50: For $i_n = 1,\ 1 \le j_n \le N-1,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&(\lambda_{H-n} + \lambda_{L-n} + j_n \alpha_{Ln} + \mu_{Hn}) \pi(1, j_n, K, 2) \\ &= \lambda_{L-n} \pi(1, j_n - 1, K, 2) + \beta_n \pi(1, j_n, K-1, 2) + \\ &\quad \alpha_{Hn} \pi(2, j_n, K, 2) + (j_n + 1) \alpha_{Ln} \pi(1, j_n + 1, K, 2).\end{aligned}$$

Case 51: For $i_n = 1,\ j_n = N,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&(N\alpha_{Ln} + \mu_{Hn}) \pi(1, N, K, 2) \\ &= \lambda_{L-n} \pi(1, N-1, K, 2) + \beta_n \pi(1, N, K-1, 2).\end{aligned}$$

Case 52: For $2 \le i_n \le N,\ j_n = 0,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&[\lambda_{H-n} + \lambda_{L-n} + (i_n - 1)\alpha_{Hn} + \mu_{Hn}] \pi(i_n, 0, K, 2) \\ &= \lambda_{H-n} \pi(i_n - 1, 0, K, 2) + \beta_n \pi(i_n, 0, K-1, 2) + \\ &\quad i_n \alpha_{Hn} \pi(i_n + 1, 0, K, 2) + \alpha_{Ln} \pi(i_n, 1, K, 2).\end{aligned}$$

Case 53: For $2 \le i_n \le N-1,\ 1 \le j_n \le N - i_n,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&[\lambda_{H-n} + \lambda_{L-n} + (i_n - 1)\alpha_{Hn} + j_n \alpha_{Ln} + \mu_{Hn}] \pi(i_n, j_n, K, 2) \\ &= \lambda_{H-n} \pi(i_n - 1, j_n, K, 2) + \lambda_{L-n} \pi(i_n, j_n - 1, K, 2) + \beta_n \pi(i_n, j_n, K-1, 2) + \\ &\quad i_n \alpha_{Hn} \pi(i_n + 1, j_n, K, 2) + (j_n + 1) \alpha_{Ln} \pi(i_n, j_n + 1, K, 2).\end{aligned}$$

Case 54: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&[(i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + \mu_{Hn}] \pi(i_n, N - i_n + 1, K, 2) \\ &= \lambda_{H-n} \pi(i_n - 1, N - i_n + 1, K, 2) + \lambda_{L-n} \pi(i_n, N - i_n, K, 2) + \\ &\quad \beta_n \pi(i_n, N - i_n + 1, K-1, 2).\end{aligned}$$

Case 55: For $i_n = N+1,\ j_n = 0,\ x_n = K$, and $y_n = 2$,

$$\begin{aligned}&(N\alpha_{Hn} + \mu_{Hn}) \pi(N+1, 0, K, 2) \\ &= \lambda_{H-n} \pi(N, 0, K, 2) + \beta_n \pi(N+1, 0, K-1, 2).\end{aligned}$$

Case 56: For $i_n = 0,\ j_n = 1,\ x_n = 0$, and $y_n = 3$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Ln}) \pi(0, 1, 0, 3)$$

$$= \theta_{Ln}\lambda_{L-n}\pi(0,0,0,0) + \alpha_{Hn}\pi(1,1,0,3) + \theta_{Ln}\alpha_{Hn}\pi(1,1,0,0) +$$
$$\alpha_{Ln}\pi(0,2,0,3) + \theta_{Ln}2\alpha_{Ln}\pi(0,2,0,0) +$$
$$\theta_{Ln}\mu_{Hn}\pi(1,1,0,2) + \theta_{Ln}\mu_{Ln}\pi(0,2,0,1) +$$
$$\theta_{Ln}\mu_{Hn}\pi(1,1,0,4) + \theta_{Ln}\mu_{Ln}\pi(0,2,0,3).$$

Case 57: For $i_n = 0,\ 2 \le j_n \le N-1,\ x_n = 0$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (j_n - 1)\alpha_{Ln} + (1-\theta_{Ln})\mu_{Ln} + \theta_{Ln}\mu_{Ln}]\pi(0,j_n,0,3)$$
$$= \theta_{Ln}\lambda_{L-n}\pi(0,j_n-1,0,0) + \lambda_{L-n}\pi(0,j_n-1,0,3) +$$
$$\alpha_{Hn}\pi(1,j_n,0,3) + \theta_{Ln}\alpha_{Hn}\pi(1,j_n,0,0) +$$
$$j_n\alpha_{Ln}\pi(0,j_n+1,0,3) + \theta_{Ln}(j_n+1)\alpha_{Ln}\pi(0,j_n+1,0,0) +$$
$$\theta_{Ln}\mu_{Hn}\pi(1,j_n,0,2) + \theta_{Ln}\mu_{Ln}\pi(0,j_n+1,0,1) +$$
$$\theta_{Ln}\mu_{Hn}\pi(1,j_n,0,4) + \theta_{Ln}\mu_{Ln}\pi(0,j_n+1,0,3).$$

Case 58: For $i_n = 0,\ j_n = N,\ x_n = 0$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (N-1)\alpha_{Ln} + (1-\theta_{Ln})\mu_{Ln} + \theta_{Ln}\mu_{Ln}]\pi(0,N,0,3)$$
$$= \theta_{Ln}\lambda_{L-n}\pi(0,N-1,0,0) + \lambda_{L-n}\pi(0,N-1,0,3) + \alpha_{Hn}\pi(1,N,0,3) +$$
$$N\alpha_{Ln}\pi(0,N+1,0,3) + \theta_{Ln}\mu_{Hn}\pi(1,N,0,2) + \theta_{Ln}\mu_{Ln}\pi(0,N+1,0,1) +$$
$$\theta_{Ln}\mu_{Hn}\pi(1,N,0,4) + \theta_{Ln}\mu_{Ln}\pi(0,N+1,0,3).$$

Case 59: For $i_n = 0,\ j_n = N+1,\ x_n = 0$, and $y_n = 3$,

$$[\beta_n + N\alpha_{Ln} + (1-\theta_{Ln})\mu_{Ln} + \theta_{Ln}\mu_{Ln}]\pi(0,N+1,0,3)$$
$$= \lambda_{L-n}\pi(0,N,0,3).$$

Case 60: For $1 \le i_n \le N-1,\ j_n = 1,\ x_n = 0$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + (1-\theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]\pi(i_n,1,0,3)$$
$$= \lambda_{H-n}\pi(i_n-1,1,0,3) + (i_n+1)\alpha_{Hn}\pi(i_n+1,1,0,3) + \alpha_{Ln}\pi(i_n,2,0,3).$$

Case 61: For $1 \le i_n \le N-2,\ 2 \le j_n \le N-i_n,\ x_n = 0$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + (j_n-1)\alpha_{Ln} + (1-\theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]$$
$$\pi(i_n,j_n,0,3)$$
$$= \lambda_{H-n}\pi(i_n-1,j_n,0,3) + \lambda_{L-n}\pi(i_n,j_n-1,0,3) +$$
$$(i_n+1)\alpha_{Hn}\pi(i_n+1,j_n,0,3) + j_n\alpha_{Ln}\pi(i_n,j_n+1,0,3).$$

Case 62: For $1 \le i_n \le N-1,\ j_n = N - i_n + 1,\ x_n = 0$, and $y_n = 3$,

$$[\beta_n + i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + (1 - \theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]\pi(i_n, N - i_n + 1, 0, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, 0, 3) + \lambda_{L-n}\pi(i_n, N - i_n, 0, 3).$$

Case 63: For $i_n = N,\ j_n = 1,\ x_n = 0$, and $y_n = 3$,

$$[\beta_n + N\alpha_{Hn} + (1 - \theta_{Hn})\mu_{Ln} + \theta_{Hn}\mu_{Ln}]\pi(N, 1, 0, 3)$$
$$= \lambda_{H-n}\pi(N - 1, 1, 0, 3).$$

Case 64: For $i_n = 0,\ j_n = 1,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Ln})\pi(0, 1, x_n, 3)$$
$$= \beta_n\pi(0, 1, x_n - 1, 3) + \alpha_{Hn}\pi(1, 1, x_n, 3) + \alpha_{Ln}\pi(0, 2, x_n, 3).$$

Case 65: For $i_n = 0,\ 2 \le j_n \le N,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(0, j_n, x_n, 3)$$
$$= \lambda_{L-n}\pi(0, j_n - 1, x_n, 3) + \beta_n\pi(0, j_n, x_n - 1, 3) +$$
$$\alpha_{Hn}\pi(1, j_n, x_n, 3) + j_n\alpha_{Ln}\pi(0, j_n + 1, x_n, 3).$$

Case 66: For $i_n = 0,\ j_n = N + 1,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$(\beta_n + N\alpha_{Ln} + \mu_{Ln})\pi(0, N + 1, x_n, 3)$$
$$= \lambda_{L-n}\pi(0, N, x_n, 3) + \beta_n\pi(0, N + 1, x_n - 1, 3).$$

Case 67: For $1 \le i_n \le N-1,\ j_n = 1,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + \mu_{Ln})\pi(i_n, 1, x_n, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, 1, x_n, 3) + \beta_n\pi(i_n, 1, x_n - 1, 3) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, 1, x_n, 3) + \alpha_{Ln}\pi(i_n, 2, x_n, 3).$$

Case 68: For $1 \le i_n \le N-2,\ 2 \le j_n \le N - i_n,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + i_n\alpha_{Hn} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, j_n, x_n, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, x_n, 3) + \lambda_{L-n}\pi(i_n, j_n - 1, x_n, 3) + \beta_n\pi(i_n, j_n, x_n - 1, 3) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, x_n, 3) + j_n\alpha_{Ln}\pi(i_n, j_n + 1, x_n, 3).$$

Case 69: For $1 \le i_n \le N-1,\ j_n = N - i_n + 1,\ 1 \le x_n \le K-1$, and $y_n = 3$,

$$[\beta_n + i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, N - i_n + 1, x_n, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, x_n, 3) + \lambda_{L-n}\pi(i_n, N - i_n, x_n, 3) +$$
$$\beta_n\pi(i_n, N - i_n + 1, x_n - 1, 3).$$

Case 70: For $i_n = N,\ j_n = 1,\ 1 \le x_n \le K - 1$, and $y_n = 3$,

$$(\beta_n + N\alpha_{Hn} + \mu_{Ln})\pi(N, 1, x_n, 3)$$
$$= \lambda_{H-n}\pi(N - 1, 1, x_n, 3) + \beta_n\pi(N, 1, x_n - 1, 3).$$

Case 71: For $i_n = 0,\ j_n = 1,\ x_n = K$, and $y_n = 3$,

$$(\lambda_{H-n} + \lambda_{L-n} + \mu_{Ln})\pi(0, 1, K, 3)$$
$$= \beta_n\pi(0, 1, K - 1, 3) + \alpha_{Hn}\pi(1, 1, K, 3) + \alpha_{Ln}\pi(0, 2, K, 3).$$

Case 72: For $i_n = 0,\ 2 \le j_n \le N,\ x_n = K$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(0, j_n, K, 3)$$
$$= \lambda_{L-n}\pi(0, j_n - 1, K, 3) + \beta_n\pi(0, j_n, K - 1, 3) + \alpha_{Hn}\pi(1, j_n, K, 3) +$$
$$j_n\alpha_{Ln}\pi(0, j_n + 1, K, 3).$$

Case 73: For $i_n = 0,\ j_n = N + 1,\ x_n = K$, and $y_n = 3$,

$$(N\alpha_{Ln} + \mu_{Ln})\pi(0, N + 1, K, 3)$$
$$= \lambda_{L-n}\pi(0, N, K, 3) + \beta_n\pi(0, N + 1, K - 1, 3).$$

Case 74: For $1 \le i_n \le N - 1,\ j_n = 1,\ x_n = K$, and $y_n = 3$,

$$(\lambda_{H-n} + \lambda_{L-n} + i_n\alpha_{Hn} + \mu_{Ln})\pi(i_n, 1, K, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, 1, K, 3) + \beta_n\pi(i_n, 1, K - 1, 3) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, 1, K, 3) + \alpha_{Ln}\pi(i_n, 2, K, 3).$$

Case 75: For $1 \le i_n \le N - 2,\ 2 \le j_n \le N - i_n,\ x_n = K$, and $y_n = 3$,

$$[\lambda_{H-n} + \lambda_{L-n} + i_n\alpha_{Hn} + (j_n - 1)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, j_n, K, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, K, 3) + \lambda_{L-n}\pi(i_n, j_n - 1, K, 3) + \beta_n\pi(i_n, j_n, K - 1, 3) +$$
$$(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, K, 3) + j_n\alpha_{Ln}\pi(i_n, j_n + 1, K, 3).$$

Case 76: For $1 \le i_n \le N - 1,\ j_n = N - i_n + 1,\ x_n = K$, and $y_n = 3$,

$$[i_n\alpha_{Hn} + (N - i_n)\alpha_{Ln} + \mu_{Ln}]\pi(i_n, N - i_n + 1, K, 3)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, K, 3) + \lambda_{L-n}\pi(i_n, N - i_n, K, 3) +$$
$$\beta_n\pi(i_n, N - i_n + 1, K - 1, 3).$$

Case 77: For $i_n = N$, $j_n = 1$, $x_n = K$, and $y_n = 3$,

$$(N\alpha_{Hn} + \mu_{Ln})\pi(N, 1, K, 3)$$
$$= \lambda_{H-n}\pi(N - 1, 1, K, 3) + \beta_n\pi(N, 1, K - 1, 3)$$

Case 78: For $i_n = 1$, $j_n = 0$, $x_n = 0$, and $y_n = 4$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Hn})\pi(1, 0, 0, 4)$$
$$= \theta_{Hn}\lambda_{H-n}\pi(0, 0, 0, 0) + \alpha_{Hn}\pi(2, 0, 0, 4) + \theta_{Hn}2\alpha_{Hn}\pi(2, 0, 0, 0) +$$
$$\alpha_{Ln}\pi(1, 1, 0, 4) + \theta_{Hn}\alpha_{Ln}\pi(1, 1, 0, 0) + \theta_{Hn}\mu_{Hn}\pi(2, 0, 0, 2) +$$
$$\theta_{Hn}\mu_{Ln}\pi(1, 1, 0, 1) + \theta_{Hn}\mu_{Hn}\pi(2, 0, 0, 4) + \theta_{Hn}\mu_{Ln}\pi(1, 1, 0, 3).$$

Case 79: For $i_n = 1$, $1 \le j_n \le N - 2$, $x_n = 0$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + j_n\alpha_{Ln} + (1 - \theta_{Ln})\mu_{Hn} + \theta_{Ln}\mu_{Hn}]\pi(1, j_n, 0, 4)$$
$$= \theta_{Hn}\lambda_{H-n}\pi(0, j_n, 0, 0) + \theta_{Hn}\lambda_{L-n}\pi(1, j_n - 1, 0, 0) + \lambda_{L-n}\pi(1, j_n - 1, 0, 4) +$$
$$\alpha_{Hn}\pi(2, j_n, 0, 4) + \theta_{Hn}2\alpha_{Hn}\pi(2, j_n, 0, 0) + (j_n + 1)\alpha_{Ln}\pi(1, j_n + 1, 0, 4) +$$
$$\theta_{Hn}(j_n + 1)\alpha_{Ln}\pi(1, j_n + 1, 0, 0) + \theta_{Hn}\mu_{Hn}\pi(2, j_n, 0, 2) +$$
$$\theta_{Hn}\mu_{Ln}\pi(1, j_n + 1, 0, 1) + \theta_{Hn}\mu_{Hn}\pi(2, j_n, 0, 4) + \theta_{Hn}\mu_{Ln}\pi(1, j_n + 1, 0, 3).$$

Case 80: For $i_n = 1$, $j_n = N - 1$, $x_n = 0$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (N - 1)\alpha_{Ln} + (1 - \theta_{Ln})\mu_{Hn} + \theta_{Ln}\mu_{Hn}]\pi(1, N - 1, 0, 4)$$
$$= \theta_{Hn}\lambda_{H-n}\pi(0, N - 1, 0, 0) + \theta_{Hn}\lambda_{L-n}\pi(1, N - 2, 0, 0)$$
$$+ \lambda_{L-n}\pi(1, N - 2, 0, 4) +$$
$$\alpha_{Hn}\pi(2, N - 1, 0, 4) + N\alpha_{Ln}\pi(1, N, 0, 4) + \theta_{Hn}\mu_{Hn}\pi(2, N - 1, 0, 2) +$$
$$\theta_{Hn}\mu_{Ln}\pi(1, N, 0, 1) + \theta_{Hn}\mu_{Hn}\pi(2, N - 1, 0, 4) + \theta_{Hn}\mu_{Ln}\pi(1, N, 0, 3).$$

Case 81: For $i_n = 1$, $j_n = N$, $x_n = 0$, and $y_n = 4$,

$$[\beta_n + N\alpha_{Ln} + (1 - \theta_{Ln})\mu_{Hn} + \theta_{Ln}\mu_{Hn}]\pi(1, N, 0, 4)$$
$$= \lambda_{L-n}\pi(1, N - 1, 0, 4).$$

Case 82: For $2 \le i_n \le N-1,\ j_n = 0,\ x_n = 0$, and $y_n = 4$,

$$
\begin{aligned}
&[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(i_n, 0, 0, 4) \\
&= \theta_{Hn}\lambda_{H-n}\pi(i_n - 1, 0, 0, 0) + \lambda_{H-n}\pi(i_n - 1, 0, 0, 4) + i_n\alpha_{Hn}\pi(i_n + 1, 0, 0, 4) \\
&\qquad\qquad + \\
&\quad \theta_{Hn}(i_n + 1)\alpha_{Hn}\pi(i_n + 1, 0, 0, 0) + \alpha_{Ln}\pi(i_n, 1, 0, 4) + \theta_{Hn}\alpha_{Ln}\pi(i_n, 1, 0, 0) + \\
&\quad \theta_{Hn}\mu_{Hn}\pi(i_n + 1, 0, 0, 2) + \theta_{Hn}\mu_{Ln}\pi(i_n, 1, 0, 1) + \theta_{Hn}\mu_{Hn}\pi(i_n + 1, 0, 0, 4) + \\
&\quad \theta_{Hn}\mu_{Ln}\pi(i_n, 1, 0, 3).
\end{aligned}
$$

Case 83: For $2 \le i_n \le N-2,\ 1 \le j_n \le N - i_n - 1,\ x_n = 0$, and $y_n = 4$,

$$
\begin{aligned}
&[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + j_n\alpha_{Ln} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}] \\
&\pi(i_n, j_n, 0, 4) \\
&= \theta_{Hn}\lambda_{H-n}\pi(i_n - 1, j_n, 0, 0) + \lambda_{H-n}\pi(i_n - 1, j_n, 0, 4) + \\
&\quad \theta_{Hn}\lambda_{L-n}\pi(i_n, j_n - 1, 0, 0) + \lambda_{L-n}\pi(i_n, j_n - 1, 0, 4) + \\
&\quad i_n\alpha_{Hn}\pi(i_n + 1, j_n, 0, 4) + \theta_{Hn}(i_n + 1)\alpha_{Hn}\pi(i_n + 1, j_n, 0, 0) + \\
&\quad (j_n + 1)\alpha_{Ln}\pi(i_n, j_n + 1, 0, 4) + \theta_{Hn}(j_n + 1)\alpha_{Ln}\pi(i_n, j_n + 1, 0, 0) + \\
&\quad \theta_{Hn}\mu_{Hn}\pi(i_n + 1, j_n, 0, 2) + \theta_{Hn}\mu_{Ln}\pi(i_n, j_n + 1, 0, 1) + \\
&\quad \theta_{Hn}\mu_{Hn}\pi(i_n + 1, j_n, 0, 4) + \theta_{Hn}\mu_{Ln}\pi(i_n, j_n + 1, 0, 3).
\end{aligned}
$$

Case 84: For $2 \le i_n \le N-1,\ j_n = N - i_n,\ x_n = 0$, and $y_n = 4$,

$$
\begin{aligned}
&[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + (N - i_n)\alpha_{Ln} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}] \\
&\pi(i_n, N - i_n, 0, 4) \\
&= \theta_{Hn}\lambda_{H-n}\pi(i_n - 1, N - i_n, 0, 0) + \lambda_{H-n}\pi(i_n - 1, N - i_n, 0, 4) + \\
&\quad \theta_{Hn}\lambda_{L-n}\pi(i_n, N - i_n - 1, 0, 0) + \lambda_{L-n}\pi(i_n, N - i_n - 1, 0, 4) + \\
&\quad i_n\alpha_{Hn}\pi(i_n + 1, N - i_n, 0, 4) + (N - i_n + 1)\alpha_{Ln}\pi(i_n, N - i_n + 1, 0, 4) + \\
&\quad \theta_{Hn}\mu_{Hn}\pi(i_n + 1, N - i_n, 0, 2) + \theta_{Hn}\mu_{Ln}\pi(i_n, N - i_n + 1, 0, 1) + \\
&\quad \theta_{Hn}\mu_{Hn}\pi(i_n + 1, N - i_n, 0, 4) + \theta_{Hn}\mu_{Ln}\pi(i_n, N - i_n + 1, 0, 3).
\end{aligned}
$$

Case 85: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ x_n = 0$, and $y_n = 4$,

$$
\begin{aligned}
&[\beta_n + (i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}] \\
&\pi(i_n, N - i_n + 1, 0, 4) \\
&= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, 0, 4) + \lambda_{L-n}\pi(i_n, N - i_n, 0, 4).
\end{aligned}
$$

Case 86: For $i_n = N,\ j_n = 0,\ x_n = 0$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (N-1)\alpha_{Hn} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(N,0,0,4)$$
$$= \theta_{Hn}\lambda_{H-n}\pi(N-1,0,0,0) + \lambda_{H-n}\pi(N-1,0,0,4) + N\alpha_{Hn}\pi(N+1,0,0,4)$$
$$+$$
$$\alpha_{Ln}\pi(N,1,0,4) + \theta_{Hn}\mu_{Hn}\pi(N+1,0,0,2) + \theta_{Hn}\mu_{Ln}\pi(N,1,0,1) +$$
$$\theta_{Hn}\mu_{Hn}\pi(N+1,0,0,4) + \theta_{Hn}\mu_{Ln}\pi(N,1,0,3).$$

Case 87: For $i_n = N+1,\ j_n = 0,\ x_n = 0$, and $y_n = 4$,

$$[\beta_n + N\alpha_{Hn} + (1-\theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(N+1,0,0,4)$$
$$= \lambda_{H-n}\pi(N,0,0,4).$$

Case 88: For $i_n = 1,\ j_n = 0,\ 1 \le x_n \le K-1$, and $y_n = 4$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + \mu_{Hn})\pi(1,0,x_n,4)$$
$$= \beta_n\pi(1,0,x_n-1,4) + \alpha_{Hn}\pi(2,0,x_n,4) + \alpha_{Ln}\pi(1,1,x_n,4).$$

Case 89: For $i_n = 1,\ 1 \le j_n \le N-1,\ 1 \le x_n \le K-1$, and $y_n = 4$,

$$(\lambda_{H-n} + \lambda_{L-n} + \beta_n + j_n\alpha_{Ln} + \mu_{Hn})\pi(1,j_n,x_n,4)$$
$$= \lambda_{L-n}\pi(1,j_n-1,x_n,4) + \beta_n\pi(1,j_n,x_n-1,4) + \alpha_{Hn}\pi(2,j_n,x_n,4) +$$
$$(j_n+1)\alpha_{Ln}\pi(1,j_n+1,x_n,4).$$

Case 90: For $i_n = 1,\ j_n = N,\ 1 \le x_n \le K-1$, and $y_n = 4$,

$$(\beta_n + N\alpha_{Ln} + \mu_{Hn})\pi(1,N,x_n,4)$$
$$= \lambda_{L-n}\pi(1,N-1,x_n,4) + \beta_n\pi(1,N,x_n-1,4).$$

Case 91: For $2 \le i_n \le N,\ j_n = 0,\ 1 \le x_n \le K-1$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n-1)\alpha_{Hn} + \mu_{Hn}]\pi(i_n,0,x_n,4)$$
$$= \lambda_{H-n}\pi(i_n-1,0,x_n,4) + \beta_n\pi(i_n,0,x_n-1,4) +$$
$$i_n\alpha_{Hn}\pi(i_n+1,0,x_n,4) + \alpha_{Ln}\pi(i_n,1,x_n,4).$$

Case 92: For $2 \le i_n \le N-1,\ 1 \le j_n \le N-i_n,\ 1 \le x_n \le K-1$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n-1)\alpha_{Hn} + j_n\alpha_{Ln} + \mu_{Hn}]\pi(i_n,j_n,x_n,4)$$
$$= \lambda_{H-n}\pi(i_n-1,j_n,x_n,4) + \lambda_{L-n}\pi(i_n,j_n-1,x_n,4) + \beta_n\pi(i_n,j_n,x_n-1,4) +$$
$$i_n\alpha_{Hn}\pi(i_n+1,j_n,x_n,4) + (j_n+1)\alpha_{Ln}\pi(i_n,j_n+1,x_n,4).$$

Case 93: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ 1 \le x_n \le K - 1$, and $y_n = 4$,

$$[\beta_n + (i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + \mu_{Hn}]\pi(i_n, N - i_n + 1, x_n, 4)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, x_n, 4) + \lambda_{L-n}\pi(i_n, N - i_n, x_n, 4) +$$
$$\beta_n\pi(i_n, N - i_n + 1, x_n - 1, 4).$$

Case 94: For $i_n = N + 1,\ j_n = 0,\ 1 \le x_n \le K - 1$, and $y_n = 4$,

$$(\beta_n + N\alpha_{Hn} + \mu_{Hn})\pi(N + 1, 0, x_n, 4)$$
$$= \lambda_{H-n}\pi(N, 0, x_n, 4) + \beta_n\pi(N + 1, 0, x_n - 1, 4).$$

Case 95: For $i_n = 1,\ j_n = 0,\ x_n = K$, and $y_n = 4$,

$$(\lambda_{H-n} + \lambda_{L-n} + \mu_{Hn})\pi(1, 0, K, 4)$$
$$= \beta_n\pi(1, 0, K - 1, 4) + \alpha_{Hn}\pi(2, 0, K, 4) + \alpha_{Ln}\pi(1, 1, K, 4).$$

Case 96: For $i_n = 1,\ 1 \le j_n \le N - 1,\ x_n = K$, and $y_n = 4$,

$$(\lambda_{H-n} + \lambda_{L-n} + j_n\alpha_{Ln} + \mu_{Hn})\pi(1, j_n, K, 4)$$
$$= \lambda_{L-n}\pi(1, j_n - 1, K, 4) + \beta_n\pi(1, j_n, K - 1, 4) + \alpha_{Hn}\pi(2, j_n, K, 4) +$$
$$(j_n + 1)\alpha_{Ln}\pi(1, j_n + 1, K, 4).$$

Case 97: For $i_n = 1,\ j_n = N,\ x_n = K$, and $y_n = 4$,

$$(N\alpha_{Ln} + \mu_{Hn})\pi(1, N, K, 4)$$
$$= \lambda_{L-n}\pi(1, N - 1, K, 4) + \beta_n\pi(1, N, K - 1, 4).$$

Case 98: For $2 \le i_n \le N,\ j_n = 0,\ x_n = K$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + (i_n - 1)\alpha_{Hn} + \mu_{Hn}]\pi(i_n, 0, K, 4)$$
$$= \lambda_{H-n}\pi(i_n - 1, 0, K, 4) + \beta_n\pi(i_n, 0, K - 1, 4) + i_n\alpha_{Hn}\pi(i_n + 1, 0, K, 4) +$$
$$\alpha_{Ln}\pi(i_n, 1, K, 4).$$

Case 99: For $2 \le i_n \le N - 1,\ 1 \le j_n \le N - i_n,\ x_n = K$, and $y_n = 4$,

$$[\lambda_{H-n} + \lambda_{L-n} + (i_n - 1)\alpha_{Hn} + j_n\alpha_{Ln} + \mu_{Hn}]\pi(i_n, j_n, K, 4)$$
$$= \lambda_{H-n}\pi(i_n - 1, j_n, K, 4) + \lambda_{L-n}\pi(i_n, j_n - 1, K, 4) + \beta_n\pi(i_n, j_n, K - 1, 4) +$$
$$i_n\alpha_{Hn}\pi(i_n + 1, j_n, K, 4) + (j_n + 1)\alpha_{Ln}\pi(i_n, j_n + 1, K, 4).$$

Case 100: For $2 \le i_n \le N,\ j_n = N - i_n + 1,\ x_n = K$, and $y_n = 4$,

$$[(i_n - 1)\alpha_{Hn} + (N - i_n + 1)\alpha_{Ln} + \mu_{Hn}]\pi(i_n, N - i_n + 1, K, 4)$$
$$= \lambda_{H-n}\pi(i_n - 1, N - i_n + 1, K, 4) + \lambda_{L-n}\pi(i_n, N - i_n, K, 4) +$$
$$\beta_n\pi(i_n, N - i_n + 1, K - 1, 4).$$

Case 101: For $i_n = N + 1,\ j_n = 0,\ x_n = K$, and $y_n = 4$,

$$(N\alpha_{Hn} + \mu_{Hn})\pi(N + 1, 0, K, 4)$$
$$= \lambda_{H-n}\pi(N, 0, K, 4) + \beta_n\pi(N + 1, 0, K - 1, 4).$$

Since there are many equations presented above, discussing each one separately would be challenging. Therefore, we focus on a relatively complicated case, specifically case *B38*, to provide an illustrative example. This state occurs when there are more than or equal to two but less than or equal to N HP packets and 0 LP packet in the system, and there are more than 1 seat left in the packet queue, while the energy queue is empty. The HP packet being served in the server is using the harvested energy. The corresponding detailed state transition diagram can be found in Fig. 3 - 2.

Case B38: For $2 \le i_n \le N,\ j_n = 0,\ x_n = 0$, and $y_n = 2$,

$$[\lambda_{H-n} + \lambda_{L-n} + \beta_n + (i_n - 1)\alpha_{Hn} + (1 - \theta_{Hn})\mu_{Hn} + \theta_{Hn}\mu_{Hn}]\pi(i_n, 0, 0, 2)$$
$$= \lambda_{H-n}\pi(i_n - 1, 0, 0, 2) + \beta_n\pi(i_n, 0, 0, 0) + i_n\alpha_{Hn}\pi(i_n + 1, 0, 0, 2) +$$
$$\alpha_{Ln}\pi(i_n, 1, 0, 2) + \mu_{Hn}\pi(i_n + 1, 0, 1, 2) + \mu_{Ln}\pi(i_n, 1, 1, 1) +$$
$$\mu_{Hn}\pi(i_n + 1, 0, 1, 4) + \mu_{Ln}\pi(i_n, 1, 1, 3).$$

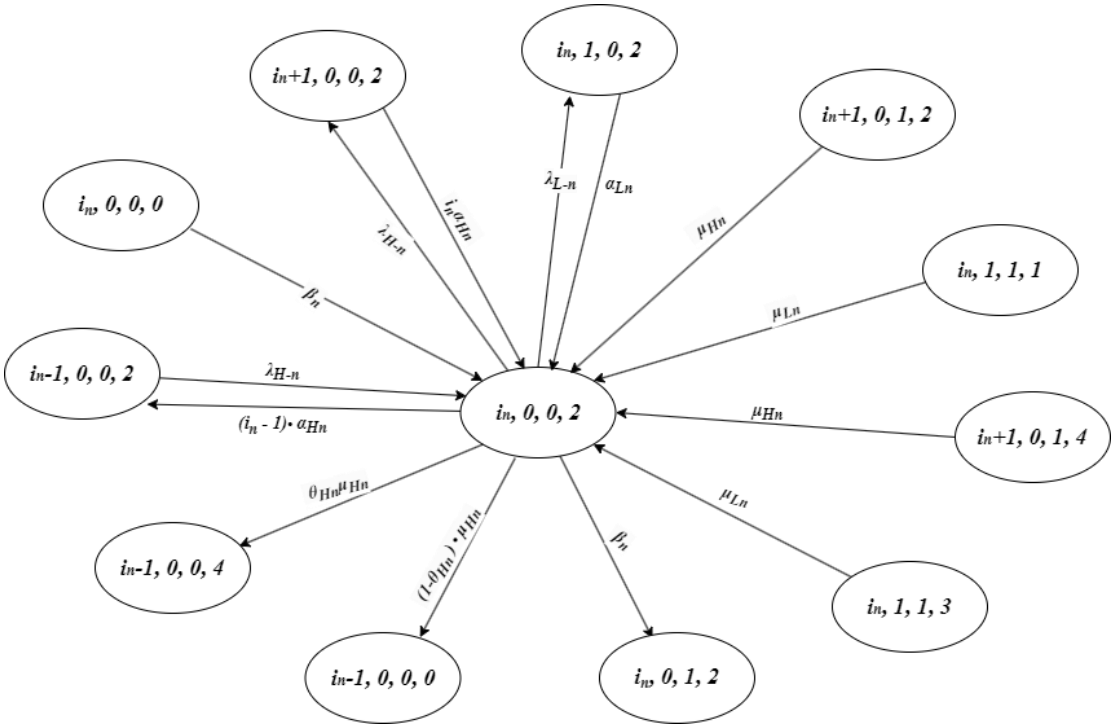

Fig. 3 - 5: The state transition diagram for $2 \leq i_n \leq N$, $j_n = 0$, $x_n = 0$, and $y_n = 2$.

3.3.3 Iterative algorithm

Using the iterative algorithm provided below, we perform calculations on the state balance equations until they converge, allowing us to determine the steady-state distribution of the system.

Iterative algorithm:

Step 1: Select a group of initial values for $\pi(i_n, j_n, x_n, y_n)^{old} = \frac{1}{|S_n|}$, $\forall i_n, j_n, x_n, y_n$, where $|S_n|$ is the total number of feasible states for node n, $n = 1 \sim 3$.

Step 2: Calculate TH_{H-n}, TH_{L-n}, P_{tl_H-n} and P_{tl_L-n} based on $\pi(i_n, j_n, x_n, y_n)^{old}$, $n = 1 \sim 3$.

Step 3: Substitute TH_{H-n}, TH_{L-n}, P_{tl_H-n} and P_{tl_L-n} into eqs. (3-29) - (3-35) to find λ_{Hn} and λ_{Ln}, $n = 1 \sim 3$.

Step 4: Substitute $\pi(i_n, j_n, x_n, y_n)^{old}$ into *Case A1* to *Case A190* and *Case B1* to *Case B101* to find $\pi(i_n, j_n, x_n, y_n)^{new}$, $\forall i_n, j_n, x_n, y_n$, $n = 1$ *and* $n = 2$ 、3, respectively.

Step 5: Normalize $\pi(i_n, j_n, x_n, y_n)^{new}$, $\forall i_n, j_n, x_n, y_n$, $n = 1 \sim 3$.

Step 6: If $\sum_{n=1}^{3}\sqrt{\sum\sum\sum\sum_{(i_n,j_n,x_n,y_n)\in S_n}|\pi(i_n,j_n,x_n,y_n)^{old}-\pi(i_n,j_n,x_n,y_n)^{new}|^2}<$ ε, stop the iterative algorithm, where ε is the stopping criterion. Otherwise, set $\pi(i_n,j_n,x_n,y_n)^{old}=\pi(i_n,j_n,x_n,y_n)^{new}$, and return to Step 2.

In the analytical experiments, we set $\varepsilon=10^{-8}$. It takes about 200 to 7000 iterations for the algorithm to converge.

3.3.4 Performance measures

To evaluate the network's effectiveness, we obtain several performance measures of interest from the steady-state probability $\pi(i_n,j_n,x_n,y_n)$ of each node n, as presented below.

$E[N]_n$ $(E[N]_{H-n},E[N]_{L-n})$, the expected number of all (HP, LP) packets in node n, is provided below.

$$\begin{aligned}E[N]_n=&\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n}(i_n+j_n)\pi(i_n,j_n,0,0)\\&+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}(i_n+j_n)[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]\\&+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}(i_n+j_n)[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]\end{aligned}\tag{3-36}$$

$$\begin{aligned}E[N]_{H-n}=&\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n}i_n\pi(i_n,j_n,0,0)\\&+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}i_n[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]\\&+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}i_n[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]\end{aligned}\tag{3-37}$$

$$E[N]_{L-n}=\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n}j_n\pi(i_n,j_n,0,0)$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n} j_n[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n} j_n[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]$$

(3-38)

$E[N]$ $(E[N]_H, E[N]_L)$, the expected number of all (HP, LP) packets for the network, is provided below.

$$E[N]=\sum_{n=1}^{3}E[N]_n$$

(3-39)

$$E[N]_H=\sum_{n=1}^{3}E[N]_{H-n}$$

(3-40)

$$E[N]_L=\sum_{n=1}^{3}E[N]_{L-n}$$

(3-41)

$E[N_q]_n$ $(E[N_q]_{H-n}, E[N_q]_{L-n})$, the expected number of all (HP, LP) packets in the queue of node n, is provided below.

$$E[N_q]_n=\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n}(i_n+j_n)\pi(i_n,j_n,0,0)$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}(i_n+j_n-1)[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}(i_n+j_n-1)[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]$$

(3-42)

$$E[N_q]_{H-n}=\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n} i_n\pi(i_n,j_n,0,0)$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n} i_n[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}(i_n-1)[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]$$

(3-43)

$$E[N_q]_{L-n}=\sum_{i_n=0}^{N}\sum_{j_n=0}^{N-i_n}j_n\pi(i_n,j_n,0,0)$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}(j_n-1)[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}j_n[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]$$

(3-44)

$E[N_q]$ $(E[N_q]_H, E[N_q]_L)$, the expected number of all (HP, LP) packets in the queue, is provided below.

$$E[N_q]=\sum_{n=1}^{3}E[N_q]_n$$

(3-45)

$$E[N_q]_H=\sum_{n=1}^{3}E[N_q]_{H-n}$$

(3-46)

$$E[N_q]_L=\sum_{n=1}^{3}E[N_q]_{L-n}$$

(3-47)

TH_n (TH_{H-n}, TH_{L-n}), the throughput of all (HP, LP) packets for node n, is provided below.

$$TH_n=\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}\mu_{Ln}[\pi(i_n,j_n,x_n,1)+\pi(i_n,j_n,x_n,3)]$$

$$+\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}\mu_{Hn}[\pi(i_n,j_n,x_n,2)+\pi(i_n,j_n,x_n,4)]$$

(3-48)

$$TH_{H-n} = \sum_{x_n=0}^{K} \sum_{i_n=1}^{N+1} \sum_{j_n=0}^{N+1-i_n} \mu_{Hn}[\pi(i_n, j_n, x_n, 2) + \pi(i_n, j_n, x_n, 4)] \tag{3-49}$$

$$TH_{L-n} = \sum_{x_n=0}^{K} \sum_{i_n=0}^{N} \sum_{j_n=1}^{N+1-i_n} \mu_{Ln}[\pi(i_n, j_n, x_n, 1) + \pi(i_n, j_n, x_n, 3)] \tag{3-50}$$

TH (TH_H, TH_L), the throughput of all (HP, LP) packets for the network, is provided below.

$$TH = TH_2 - (\lambda_{H23} + \lambda_{L23}) \tag{3-51}$$

$$TH_H = TH_{H-2} - \lambda_{H23} \tag{3-52}$$

$$TH_L = TH_{L-2} - \lambda_{L23} \tag{3-53}$$

P_{bl-n}, the blocking probability of each arrived packet for node n, regardless of priority, is provided below.

$$\begin{aligned} P_{bl-n} = & \sum_{i_n=0}^{N} \pi(i_n, N - i_n, 0, 0) \\ & + \sum_{x_n=0}^{K} \sum_{i_n=0}^{N} [\pi(i_n, N + 1 - i_n, x_n, 1) + \pi(i_n, N + 1 - i_n, x_n, 3)] \\ & + \sum_{x_n=0}^{K} \sum_{i_n=1}^{N+1} [\pi(i_n, N + 1 - i_n, x_n, 2) + \pi(i_n, N + 1 - i_n, x_n, 4)] \end{aligned} \tag{3-54}$$

P_{bl}, the blocking probability of each arrived packet for the network, regardless of priority, is provided below.

$$P_{bl} = \frac{\sum_{n=1}^{3} [(\lambda_{Hn} + \lambda_{Ln}) P_{bl-n}]}{\lambda_H + \lambda_L} \tag{3-55}$$

P_{el-n}, the energy loss probability for node n, is provided below.

$$P_{el-n} = \pi(0,0,K,0) + \sum_{i_n=0}^{N} \sum_{j_n=1}^{N+1-i_n} [\pi(i_n, j_n, K, 1) + \pi(i_n, j_n, K, 3)]$$

$$+\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}[\pi(i_n,j_n,K,2)+\pi(i_n,j_n,K,4)]$$

(3-56)

P_{el}, the energy loss probability for the network, is provided below.

$$P_{el}=\frac{\sum_{n=1}^{3}(\beta_n P_{el-n})}{\sum_{n=1}^{3}\beta_n}$$

(3-57)

$E[W]_n$ $(E[W]_{H-n}, E[W]_{L-n})$, the mean waiting time of all (HP, LP) packets in node n, including those that have finished their service and those that left the network due to impatience, is provided below.

$$E[W]_n=\frac{E[N]_n}{(\lambda_{Hn}+\lambda_{Ln})(1-P_{bl-n})}$$

(3-58)

$$E[W]_{H-n}=\frac{E[N]_{H-n}}{\lambda_{Hn}(1-P_{bl-n})}$$

(3-59)

$$E[W]_{L-n}=\frac{E[N]_{L-n}}{\lambda_{Ln}(1-P_{bl-n})}$$

(3-60)

$E[W]$ $(E[W]_H, E[W]_L)$, the mean waiting time of all (HP, LP) packets in the network, including those that have finished their service and those that left the network due to impatience, is provided below.

$$E[W]=\frac{E[N]}{TH+\sum_{n=1}^{3}(\alpha_{Hn}E[N_q]_{H-n}+\alpha_{Ln}E[N_q]_{L-n})}$$

(3-61)

$$E[W]_H=\frac{E[N]_H}{TH_H+\sum_{n=1}^{3}(\alpha_{Hn}E[N_q]_{H-n})}$$

(3-62)

$$E[W]_L=\frac{E[N]_L}{TH_L+\sum_{n=1}^{3}(\alpha_{Ln}E[N_q]_{L-n})}$$

(3-63)

$P_{imp(arr)-n}$ $(P_{imp(arr)\_H-n},\ P_{imp(arr)\_L-n})$, the impatient loss probability of arrived (HP, LP) packets for node n, is provided below.

$$P_{imp(arr)-n}=\frac{\alpha_{Hn}E[N_q]_{H-n}+\alpha_{Ln}E[N_q]_{L-n}}{\lambda_{Hn}+\lambda_{Ln}}$$

(3-64)

$$P_{imp(arr)\_H-n}=\frac{\alpha_{Hn}E[N_q]_{H-n}}{\lambda_{Hn}}$$

(3-65)

$$P_{imp(arr)\_L-n} = \frac{\alpha_{Ln} E[N_q]_{L-n}}{\lambda_{Ln}}$$

(3-66)

$P_{imp(arr)}$ $(P_{imp(arr)\_H},\ P_{imp(arr)\_L})$, the impatient loss probability of arrived (HP, LP) packets for the network, is provided below.

$$P_{imp(arr)} = \frac{\sum_{n=1}^{3}(\alpha_{Hn} E[N_q]_{H-n} + \alpha_{Ln} E[N_q]_{L-n})}{\lambda_H + \lambda_L}$$

(3-67)

$$P_{imp(arr)\_H} = \frac{\sum_{n=1}^{3}(\alpha_{Hn} E[N_q]_{H-n})}{\lambda_H}$$

(3-68)

$$P_{imp(arr)\_L} = \frac{\sum_{n=1}^{3}(\alpha_{Ln} E[N_q]_{L-n})}{\lambda_L}$$

(3-69)

$P_{imp(adm)-n}$ $(P_{imp(adm)\_H-n},\ P_{imp(adm)\_L-n})$, the impatient loss probability of admitted (HP, LP) packets for node n, is provided below.

$$P_{imp(adm)-n} = \frac{\alpha_{Hn} E[N_q]_{H-n} + \alpha_{Ln} E[N_q]_{L-n}}{(\lambda_{Hn} + \lambda_{Ln})(1 - P_{bl-n})}$$

(3-70)

$$P_{imp(adm)\_H-n} = \frac{\alpha_{Hn} E[N_q]_{H-n}}{\lambda_{Hn}(1 - P_{bl-n})}$$

(3-71)

$$P_{imp(adm)\_L-n} = \frac{\alpha_{Ln} E[N_q]_{L-n}}{\lambda_{Ln}(1 - P_{bl-n})}$$

(3-72)

$P_{imp(adm)}$ $(P_{imp(adm)\_H},\ P_{imp(adm)\_L})$, the impatient loss probability of admitted (HP, LP) packets for the network, is provided below.

$$P_{imp(adm)} = \frac{\sum_{n=1}^{3}(\alpha_{Hn} E[N_q]_{H-n} + \alpha_{Ln} E[N_q]_{L-n})}{(\lambda_H + \lambda_L)(1 - P_{bl})}$$

(3-73)

$$P_{imp(adm)\_H} = \frac{\sum_{n=1}^{3}(\alpha_{Hn} E[N_q]_{H-n})}{\lambda_H(1 - P_{bl})}$$

(3-74)

$$P_{imp(adm)\_L} = \frac{\sum_{n=1}^{3}(\alpha_{Ln} E[N_q]_{L-n})}{\lambda_L(1 - P_{bl})}$$

(3-75)

P_{tl-n} (P_{tl_H-n}, P_{tl_L-n}), the total loss probability of arrived (HP, LP) packets for node n, is provided below.

$$P_{tl-n} = 1 - \left(\frac{TH_n}{\lambda_{Hn} + \lambda_{Ln}}\right) \tag{3-76}$$

$$P_{tl\_H-n} = 1 - \left(\frac{TH_{Hn}}{\lambda_{Hn}}\right) \tag{3-77}$$

$$P_{tl\_L-n} = 1 - \left(\frac{TH_{Ln}}{\lambda_{Ln}}\right) \tag{3-78}$$

P_{tl} (P_{tl_H}, P_{tl_L}), the total loss probability of arrived (HP, LP) packets for the network, is provided below.

$$P_{tl} = 1 - \left(\frac{TH}{\lambda_H + \lambda_L}\right) \tag{3-79}$$

$$P_{tl\_H} = 1 - \left(\frac{TH_H}{\lambda_H}\right) \tag{3-80}$$

$$P_{tl\_L} = 1 - \left(\frac{TH_L}{\lambda_L}\right) \tag{3-81}$$

$RECR_n$ $(RECR_{H-n},\ RECR_{L-n})$, the regular energy consumption ratio of all (HP, LP) packets for node n, is provided below.

$$RECR_n = \frac{\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}\mu_{Hn}\pi(i_n, j_n, x_n, 4) + \sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}\mu_{Ln}\pi(i_n, j_n, x_n, 3)}{TH_{H-n} + TH_{L-n}} \tag{3-82}$$

$$RECR_{H-n} = \frac{\sum_{x_n=0}^{K}\sum_{i_n=1}^{N+1}\sum_{j_n=0}^{N+1-i_n}\mu_{Hn}\pi(i_n, j_n, x_n, 4)}{TH_{H-n} + TH_{L-n}} \tag{3-83}$$

$$RECR_{L-n} = \frac{\sum_{x_n=0}^{K}\sum_{i_n=0}^{N}\sum_{j_n=1}^{N+1-i_n}\mu_{Ln}\pi(i_n, j_n, x_n, 3)}{TH_{H-n} + TH_{L-n}} \tag{3-84}$$

$RECR$ $(RECR_H,\ RECR_L)$, the regular energy consumption ratio of all (HP, LP) packets for the network, is provided below.

$$RECR = \frac{\sum_{n=1}^{3}[RECR_n(TH_{H-n} + TH_{L-n})]}{\sum_{n=1}^{3}(TH_{H-n} + TH_{L-n})}$$

(3-85)

$$RECR_H = \frac{\sum_{n=1}^{3}[RECR_{H-n}(TH_{H-n} + TH_{L-n})]}{\sum_{n=1}^{3}(TH_{H-n} + TH_{L-n})}$$

(3-86)

$$RECR_L = \frac{\sum_{n=1}^{3}[RECR_{L-n}(TH_{H-n} + TH_{L-n})]}{\sum_{n=1}^{3}(TH_{H-n} + TH_{L-n})}$$

(3-87)