運用優先權與無耐性之區塊鏈交易模型研究
A Study on Blockchain Transaction Models with Priority and Impatience

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西元 2025 年 7 月 2 日

Outline

- Motivation
- System Model
- Analytical Model
- Numerical Results
- Conclusions and Future Works

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Motivation

- Key technical challenge: **interoperability**
  - Difficulty in communication and asset exchange between independent blockchains
- Solution: **cross-chain bridges**
  - Protocol-based mechanism which allow heterogeneous blockchains to interoperate

Motivation

- In [2], apply M/M/n/L queues modeling transaction processing and block generation in Bitcoin taking account of block generation rate
- In [7], Blockchain queuing model with non-preemptive limited-priority is proposed, illustrating the performance tradeoffs between high- and low-priority transaction classes
- In [10], a blockchain transaction system with consensus that realizes multi-chain interoperability and provides a practical cross-chain architecture is presented.

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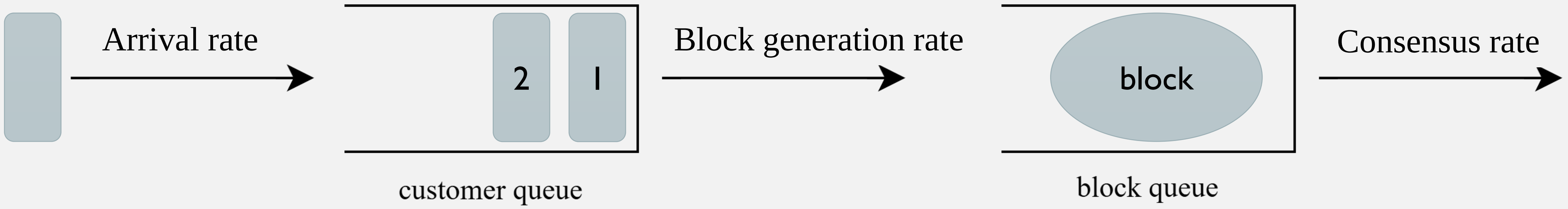
System Model

- Two sequential queues: **customer queue** and **block queue**
  - **Customer queue:** waiting to be selected for block formation with partial batch
  - **Block queue:** undergoing consensus
- **ON / OFF** mechanism
  - During ON period: arrival / block generation / consensus / (impatience)
  - During OFF period: arrival / (impatience)
- Four scenarios are considered
  - With / Without priority
  - With / Without impatience

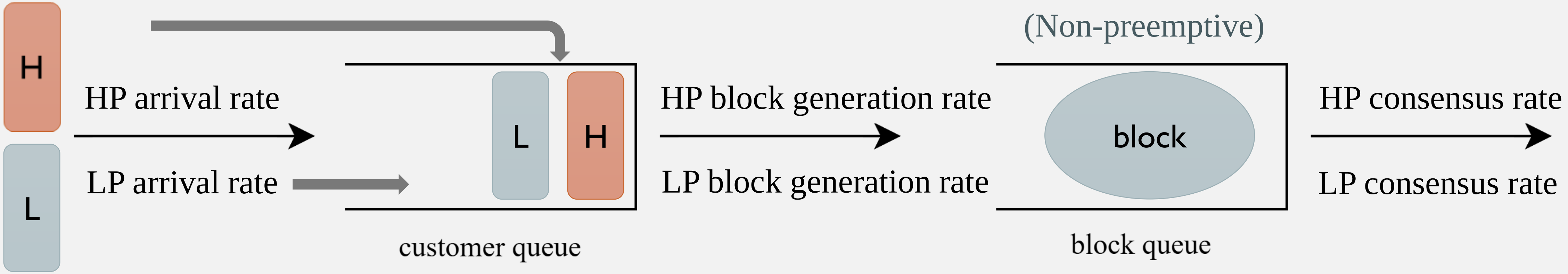
System Model – Scenario 1

single-class without impatience



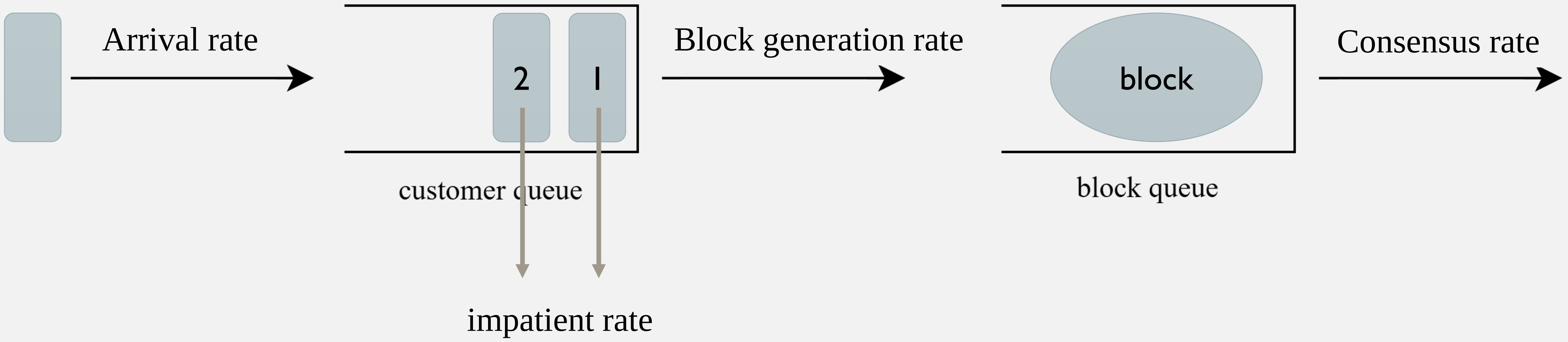
System Model – Scenario 2

two-class without impatience



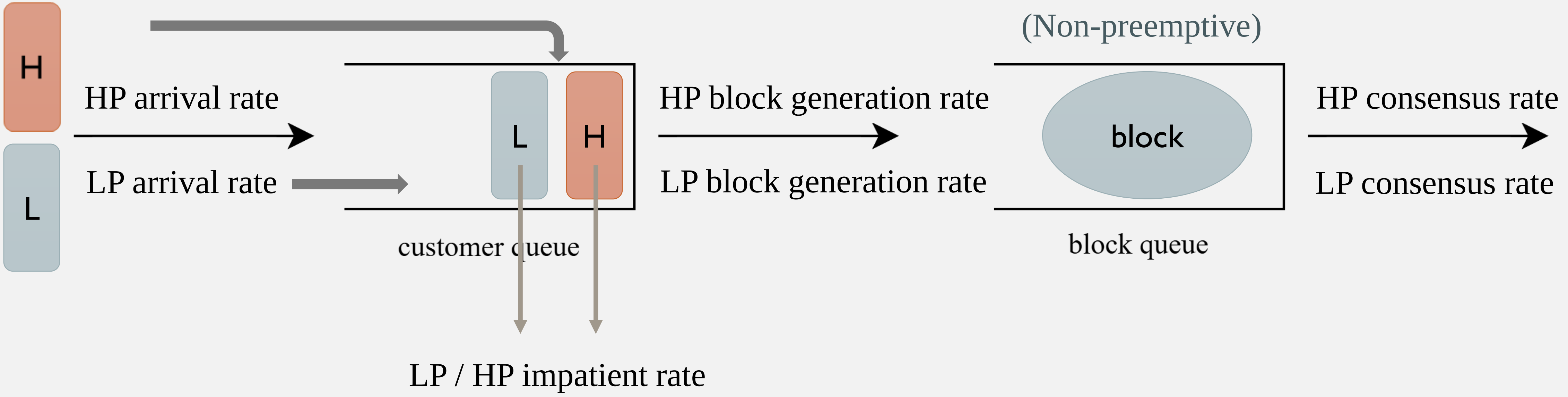
System Model – Scenario 3

single-class with impatience



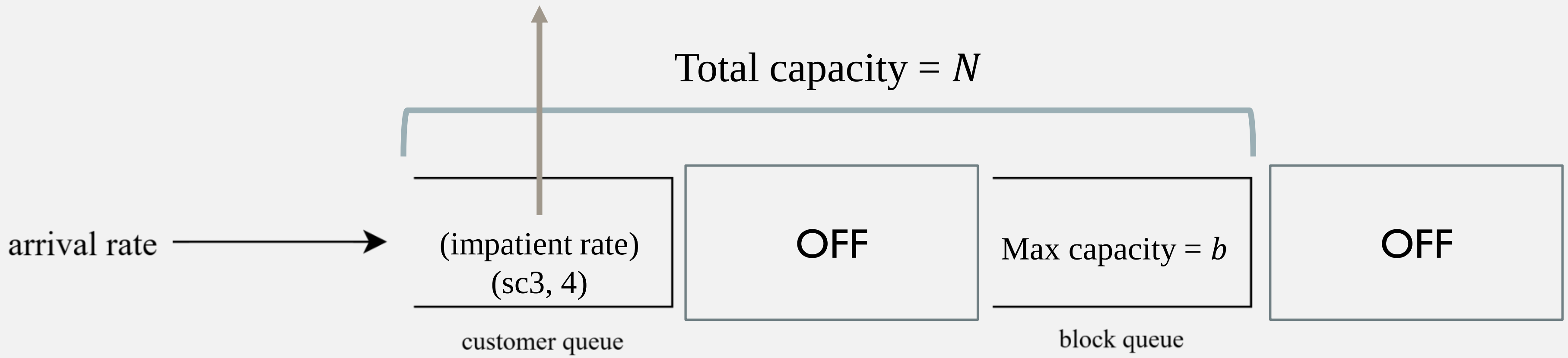
System Model – Scenario 4

two-class with impatience



System Model

ON OFF



num in block queue = 0	N	0
num in block queue > 0	$N-b$	$1 \sim b$
Capacity of LP (sc2, 4)	Always $N-b$	$0 \sim b$

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Analytical Model

- The arrivals follow a **Poisson process** with respective arrival rates λ / λ_H / λ_{L}
- Customers wait for block generation process in customer queue that determined by an **exponential distribution** with associate rates μ_1 / μ_{1_H} / μ_{1_L}
- Customers wait for consensus process in block queue that determined by an **exponential distribution** with associate rates μ_2 / μ_{2_H} / μ_{2_L}
- Impatience threshold is modeled as **exponential distribution** with associate rates γ / γ_H / γ_L
- The durations of both ON and OFF periods (the transition rate) are **exponentially distributed** (ON→OFF: α , OFF →ON: β)

Analytical Model

Description	Single-class	Two-class
Arrival rate	λ	λ_H
		λ_L
Block generation rate	μ_1	μ_{1_H}
		μ_{1_L}
Consensus rate	μ_2	μ_{2_H}
		μ_{2_L}
Impatience rate	γ	γ_H
		γ_L
Transition rate (ON $\rightarrow$ OFF)	α	α
Transition rate (OFF $\rightarrow$ ON)	β	β

Analytical Model

- The system models of **scenarios 1** and **3** are described based on a three-dimensional Markov chain with the state $\boldsymbol{(i, x, k)}$.

(1) i: the number of customers in the customer queue

(2) x: the number of customers in the block queue

(3) k: the channel state

  - 0: channel OFF
  - 1: channel ON

Analytical Model

- The state space for scenario 1 and scenario 3 can be expressed as :

$$S = \left\{ (i, x, k) \middle| \left\{ \begin{array}{c} 0 \le k \le 1 \\ if\ x = 0: 0 \le i \le N \\ if\ 1 < x \le b: 0 \le i \le N - b \end{array} \right. \right\}$$

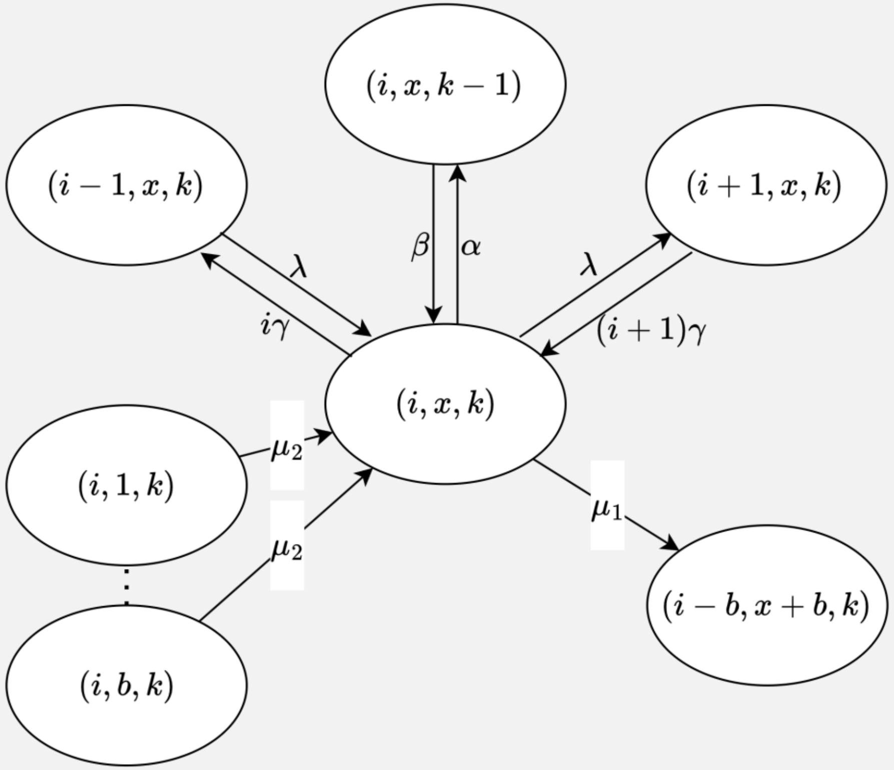
- We obtain the total number of feasible states

$$|S| = 2(N + 1) + 2b(N - b + 1)$$

- The steady state probability: $\pi_{i,x,k}$

- The feasible states can be categorized into 16 distinct cases

Analytical Model – State Balance Equations



- For scenario 3

  case12: $i = N - b, x = 0, k = 1$

$$[\alpha + \lambda + \mu_1 + (N-b)\gamma]\pi_{N-b,0,1}$$

$$= \beta\pi_{N-b,0,0} + \lambda\pi_{N-b-1,0,1} + \sum_{t=1}^{b} \mu_2\pi_{N-b,t,1}$$

$$+ (N-b+1)\gamma\pi_{N-b+1,0,1}$$

Analytical Model

- The system models of **scenarios 2** and **4** are described based on a five-dimensional Markov chain with the state $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{k})$.

(1) i: the number of HP customers in the customer queue

(2) j:the number of LP customers in the customer queue

(3) x: the number of HP customers in the block queue

(4) y: the number of LP customers in the block queue

(5) k: the channel state

- 0: channel OFF
- 1: channel ON

Analytical Model

- The state space for scenario 2 and scenario 4 can be expressed as :

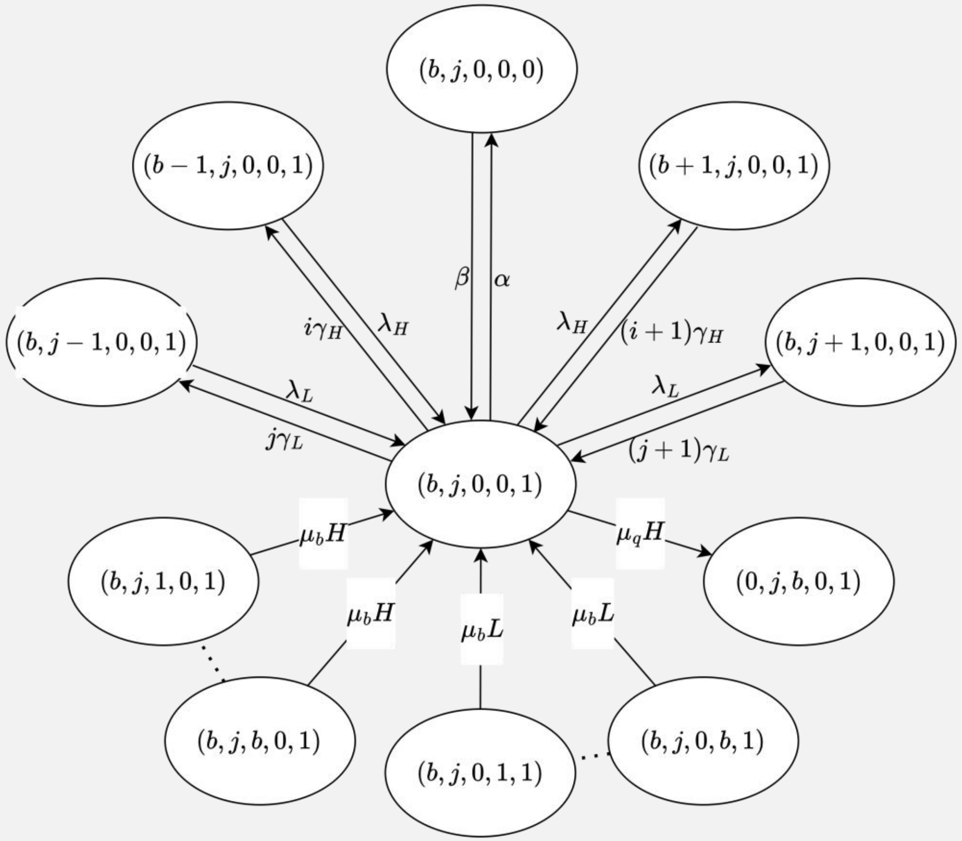
$$S = \left\{ (i,j,x,y,k) \middle| \left\{ \begin{matrix} 0 \leq i \leq N, 0 \leq j \leq N-b \\ if\ x = 0\ and\ y = 0: i+j \leq N \\ if\ 0 < x \leq b\ or\ 0 < y \leq b: i+j \leq N-b \end{matrix} \right\} \right\}$$

- We obtain the total number of feasible states

$$|S| = 2\left[(N+1)(N-b+1) - \frac{(N-b)(N-b+1)}{2}\right] + \frac{2b(N-b+1)(N-b+2)}{2}$$

- The steady state probability: $\pi_{i,j,x,y,k}$
- The feasible states can be categorized into 99 distinct cases

Analytical Model – State Balance Equations



- For scenario 4

  case 45:

$$i = b, 1 \le j \le N - 2b, x = 0, y = 0, k = 1$$

$$\left(\alpha + \lambda_H + \lambda_L + \mu_{1_H} + i\gamma_H + j\gamma_L\right)\pi_{2,j,0,0,1}$$
$$= \beta\pi_{2,j,0,0,0} + \lambda_H\pi_{2-1,j,0,0,1} + \lambda_L\pi_{2,j-1,0,0,1}$$
$$+ \sum_{t=1}^{b} \mu_{2_H}\pi_{2,j,t,0,1} + \sum_{t=1}^{b} \mu_{2_L}\pi_{2,j,0,t,1}$$
$$+ (b+1)\gamma_H\pi_{2+1,j,0,0,1} + (j+1)\gamma_L\pi_{2,j+1,0,0,1}$$

Analytical Model – Iterative Algorithm

1. Initialize $\pi(i, j, x, y, k)^{old} = \frac{1}{|s|}$ for all $(i, j, x, y, k) \in S$, where $|S|$ is the total number of feasible states.

2. Substitute $\pi(i, j, x, y, k)^{old}$ into the balance equations from Case 1 to Case 99 to find $\pi(i, j, x, y, k)^{new}$, $\forall i, j, x, y, k$..

3. Normalize $\pi(i, j, x, y, k)^{new}$, $\forall i, j, x, y, k$.

4. If $\sqrt{\sum\sum\sum\sum\sum_{(i,j,x,y,k)\in S} |\pi(i, j, x, y, k)^{old} - \pi(i, j, x, y, k)^{new}|^2} < \varepsilon$, then stop the iteration. Otherwise, set $\pi(i, j, x, y, k)^{old} = \pi(i, j, x, y, k)^{new}$, $\forall i, j, x, y, k$., and return to **Step 2**.

Analytical Model – Performance Measure

- The average number of HP/LP/overall customers in customer queue

$$L_{c_H} = \sum_{k=0}^{1}\sum_{j=0}^{N}\sum_{i=0}^{N-j} i\pi_{i,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} i\pi_{i,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} i\pi_{i,j,0,y,k}$$

$$L_{c_L} = \sum_{k=0}^{1}\sum_{j=0}^{N}\sum_{i=0}^{N-j} j\pi_{i,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} j\pi_{i,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} j\pi_{i,j,0,y,k}$$

$$L_c = L_{c_H} + L_{c_L}$$

Analytical Model – Performance Measure

- The average number of HP/LP/overall customers in block queue

$$L_{b_H} = \sum_{k=0}^{1} \sum_{x=1}^{b} \sum_{j=0}^{N-b} \sum_{i=0}^{N-b-j} x\pi_{i,j,x,0,k}$$

$$L_{b_L} = \sum_{k=0}^{1} \sum_{y=1}^{b} \sum_{j=0}^{N-b} \sum_{i=0}^{N-b-j} y\pi_{i,j,0,y,k}$$

$$L_b = L_{b_H} + L_{b_L}$$

Analytical Model – Performance Measure

- The average number of HP/LP/overall customers in system

$$L_H = \sum_{k=0}^{1}\sum_{j=0}^{N}\sum_{i=0}^{N-j} i\pi_{i,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} (i+x)\pi_{i,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} i\pi_{i,j,0,y,k}$$

$$L_L = \sum_{k=0}^{1}\sum_{j=0}^{N}\sum_{i=0}^{N-j} j\pi_{i,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} j\pi_{i,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\sum_{i=0}^{N-b-j} (j+y)\pi_{i,j,0,y,k}$$

$$L = L_H + L_L$$

Analytical Model – Performance Measure

- The blocking probability of HP/LP/overall

$$P_{b_H} = \sum_{k=0}^{1}\sum_{j=0}^{N-b}\pi_{N-j,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\pi_{N-b-j,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\pi_{N-b-j,j,0,y,k}$$

$$P_{b_L} = \sum_{k=0}^{1}\sum_{i=0}^{b}\pi_{i,N-b,0,0,k} + \sum_{k=0}^{1}\sum_{j=0}^{N-b-1}\pi_{N-j,j,0,0,k} + \sum_{k=0}^{1}\sum_{x=1}^{b}\sum_{j=0}^{N-b}\pi_{N-b-j,j,x,0,k} + \sum_{k=0}^{1}\sum_{y=1}^{b}\sum_{j=0}^{N-b}\pi_{N-b-j,j,0,y,k}$$

$$P_b = \frac{\lambda_H P_{b_H} + \lambda_L P_{b_L}}{\lambda_H + \lambda_L}$$

Analytical Model – Performance Measure

- The impatient probability of HP/LP/overall (scenario 4)

$$P_{im_H} = \frac{\gamma_H L_{c_H}}{\lambda_H(1 - P_{b_H})}$$

$$P_{im_L} = \frac{\gamma_L L_{c_L}}{\lambda_L(1 - P_{b_L})}$$

$$P_{im} = \frac{\gamma_H L_{c_H} + \gamma_L L_{c_L}}{\lambda_H\left(1 - P_{b_H}\right) + \lambda_L(1 - P_{b_L})}$$

Analytical Model – Performance Measure

- The throughput of HP/LP/overall

$$T_{h_H} = \sum_{x=1}^{b} \sum_{i=1}^{N-b} \sum_{j=0}^{N-b} \mu_{2_H} x \pi_{i,j,x,0,1}$$

$$T_{h_L} = \sum_{y=1}^{b} \sum_{i=1}^{N-b} \sum_{j=0}^{N-b} \mu_{2_L} y \pi_{i,j,x,0,1}$$

$$T_h = T_{h_H} + T_{h_L}$$

Analytical Model – Performance Measure

- The average waiting time of HP/LP/overall in the customer queue

$$W_{c_H} = \frac{L_{c_H}}{\lambda_H\left(1 - P_{b_H}\right)}$$

$$W_{c_L} = \frac{L_{c_L}}{\lambda_L\left(1 - P_{b_L}\right)}$$

$$W_c = \frac{L_c}{(\lambda_H + \lambda_L)(1 - P_b)}$$

Analytical Model – Performance Measure

- The average waiting time of HP/LP/overall in the block queue without impatience

$$W_{b_H} = \frac{L_{b_H}}{\lambda_H\left(1 - P_{b_H}\right)}$$

$$W_{b_L} = \frac{L_{b_L}}{\lambda_L\left(1 - P_{b_L}\right)}$$

$$W_b = \frac{L_b}{(\lambda_H + \lambda_L)(1 - P_b)}$$

Analytical Model – Performance Measure

- The average waiting time of HP/LP/overall in the block queue with impatience

$$W_{b_H} = \frac{L_{b_H}}{\lambda_H\left(1 - (P_{b_H} + \left(1 - P_{b_H}\right)P_{im_H}\right)}$$

$$W_{b_L} = \frac{L_{b_L}}{\lambda_L\left(1 - P_{b_L} + \left(1 - P_{b_L}\right)P_{im_L}\right)}$$

$$W_b = \frac{L_b}{\lambda_H\left(1 - (P_{b_H} + \left(1 - P_{b_H}\right)P_{im_H}\right) + \lambda_L\left(1 - P_{b_L} + \left(1 - P_{b_L}\right)P_{im_L}\right)}$$

Analytical Model – Performance Measure

- The average waiting time of HP/LP/overall in system

$$W_H = W_{c_H} + W_{b_H}$$

$$W_L = W_{c_L} + W_{b_L}$$

$$W = W_H + W_L$$

Analytical Model

- Without impatience (scenarios 1&2)
- $\lambda_{eff} = \lambda(1 - P_b)$
- $\mu_{eff} = b\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)^{-1} \frac{\beta}{\alpha+\beta}$

- With impatience (scenarios 3&4)
- $\lambda_{eff} = \lambda(1 - P_e)$
- $P_e = P_b + (1 - P_b)P_{im}$
- $\mu_{eff} = b\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)^{-1} \frac{\beta}{\alpha+\beta}$

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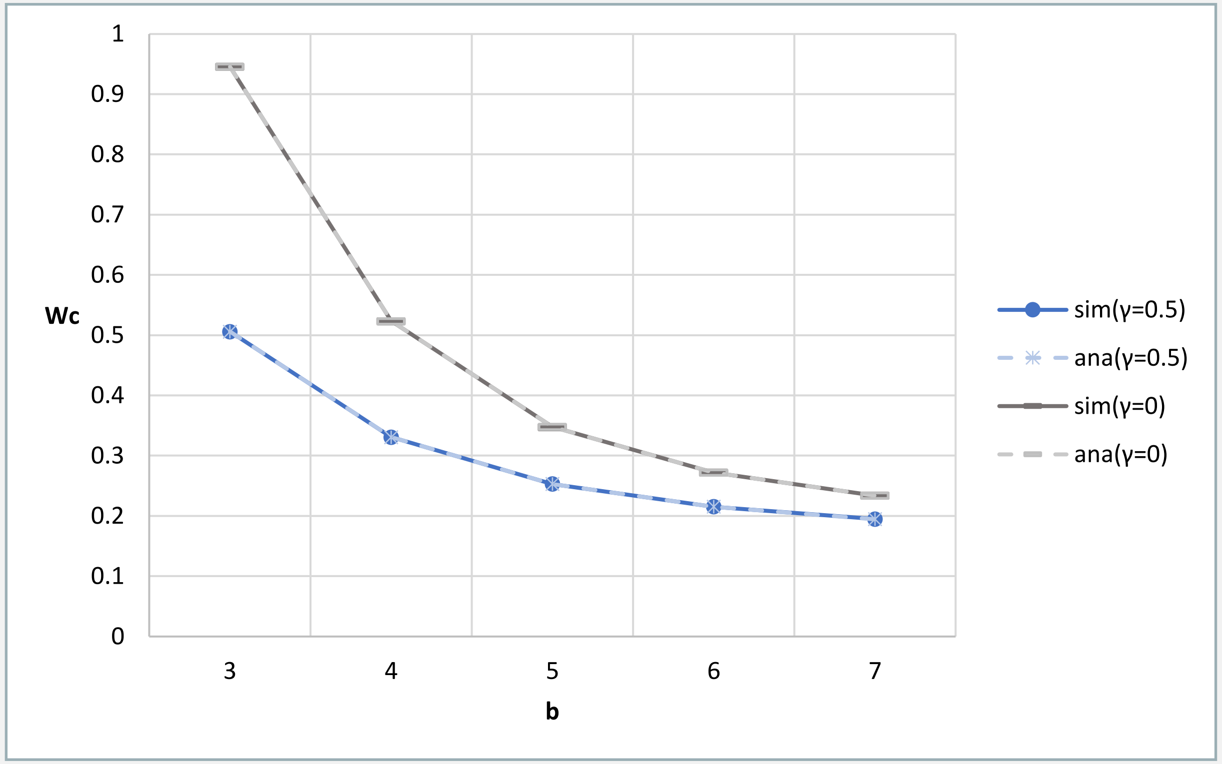
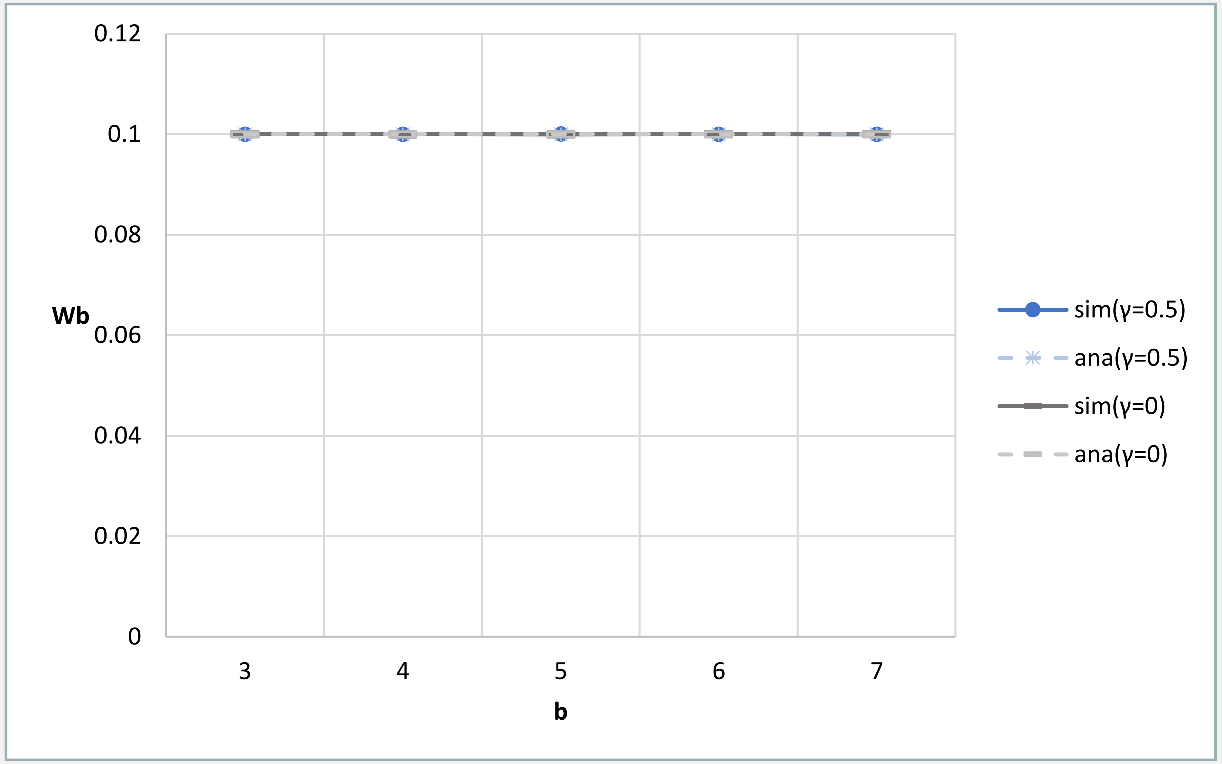
Numerical Result

Description	Single-class (scenario 1 & 3)	Two-class (scenario 2 & 4)
Arrival rate	$\lambda = 20$	$\lambda_H = 5$
		$\lambda_L = 15$
Block generation rate	$\mu_1 = 20$	$\mu_{1_H} = 20$
		$\mu_{1_L} = 20$
Consensus rate	$\mu_2 = 20$	$\mu_{2_H} = 25$
		$\mu_{2_L} = 20$
Impatience rate (scenarios 3 & 4)	$\gamma = 0.5$	$\gamma_H = 1$
		$\gamma_L = 0.5$
Transition rate (ON → OFF)	$\alpha = 15$	$\alpha = 15$
Transition rate (OFF → ON)	$\beta = 15$	$\beta = 15$

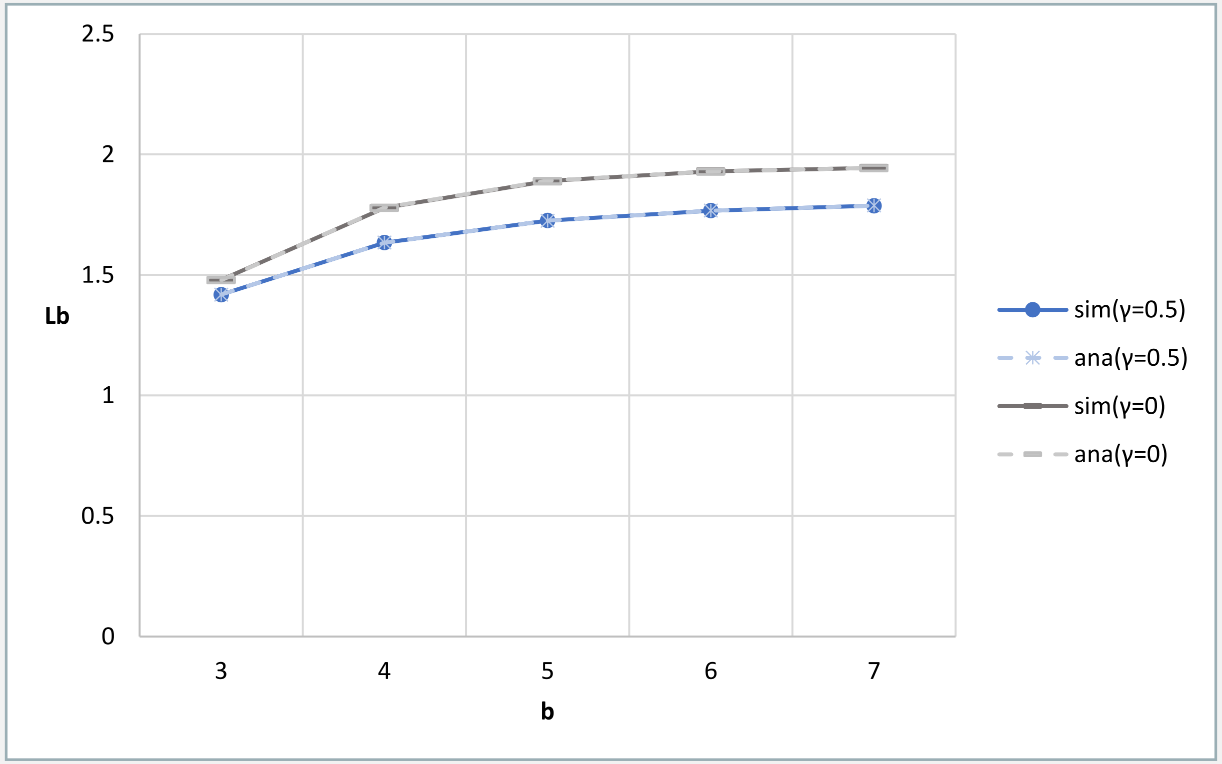
- $N = 20$, $b = 5$ for all scenarios

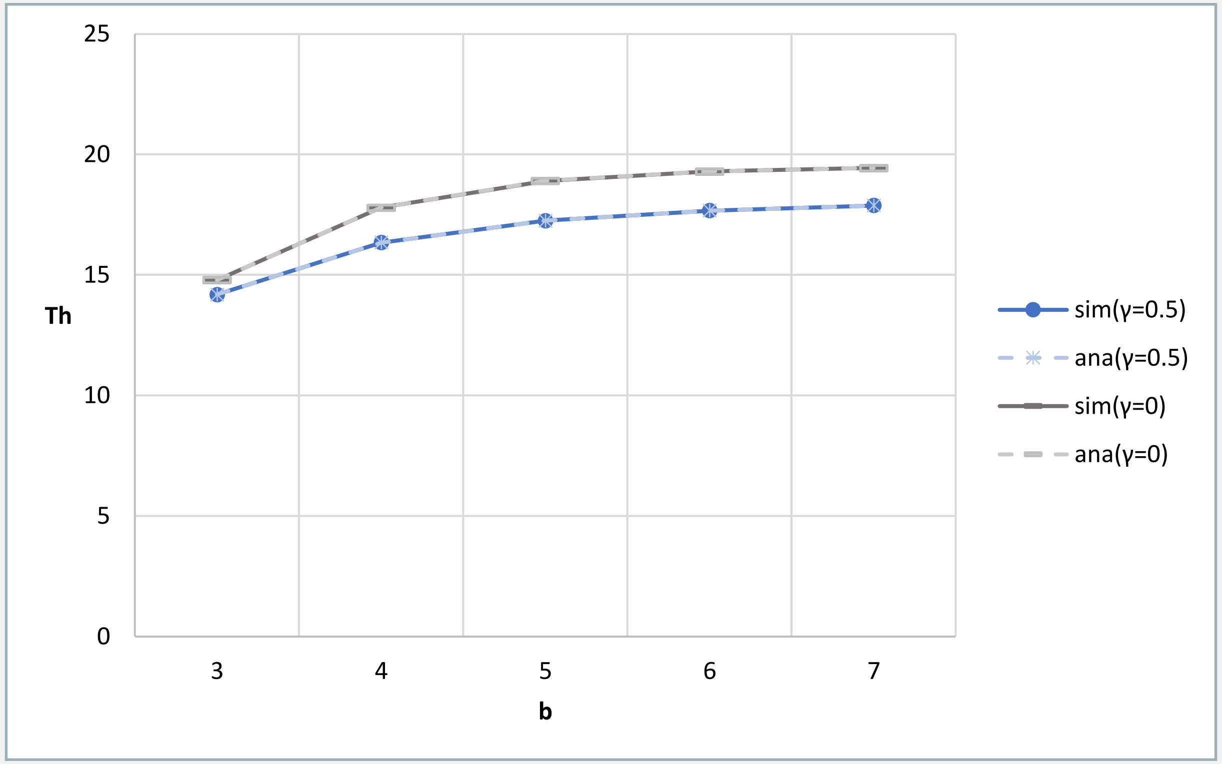
Numerical Result
– scenario 1 vs scenario 3



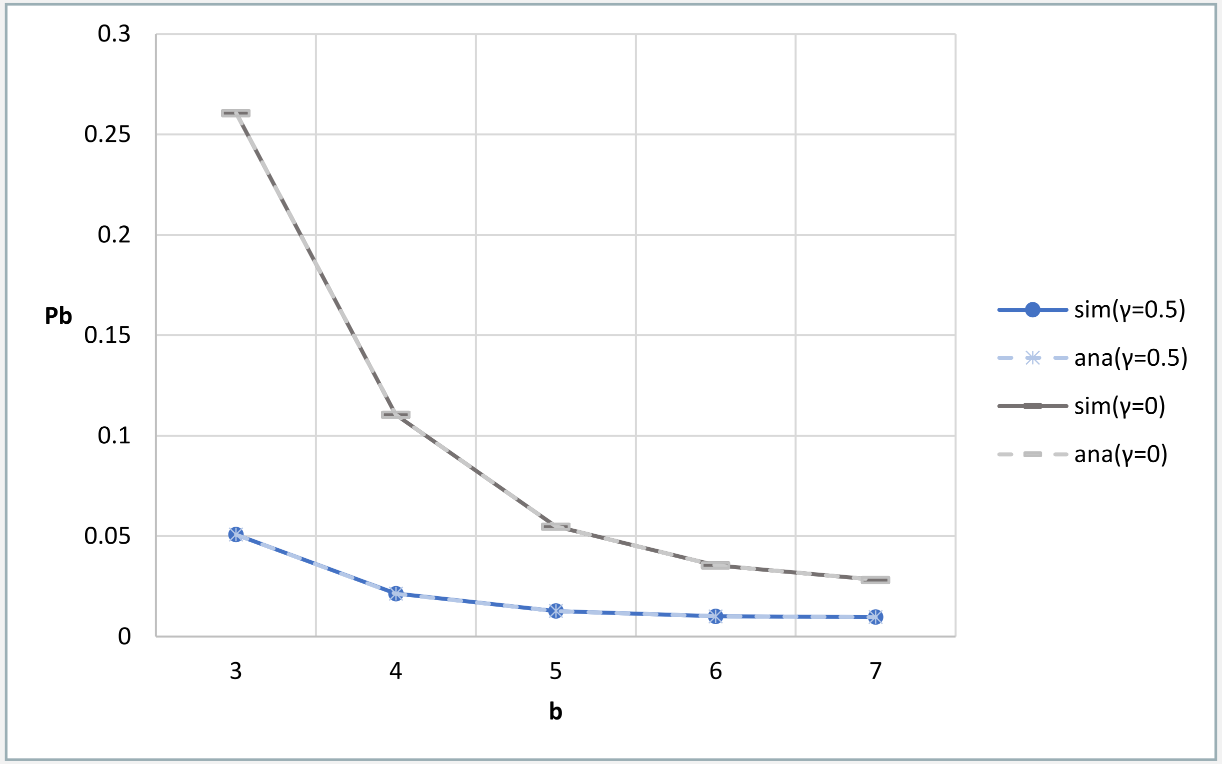


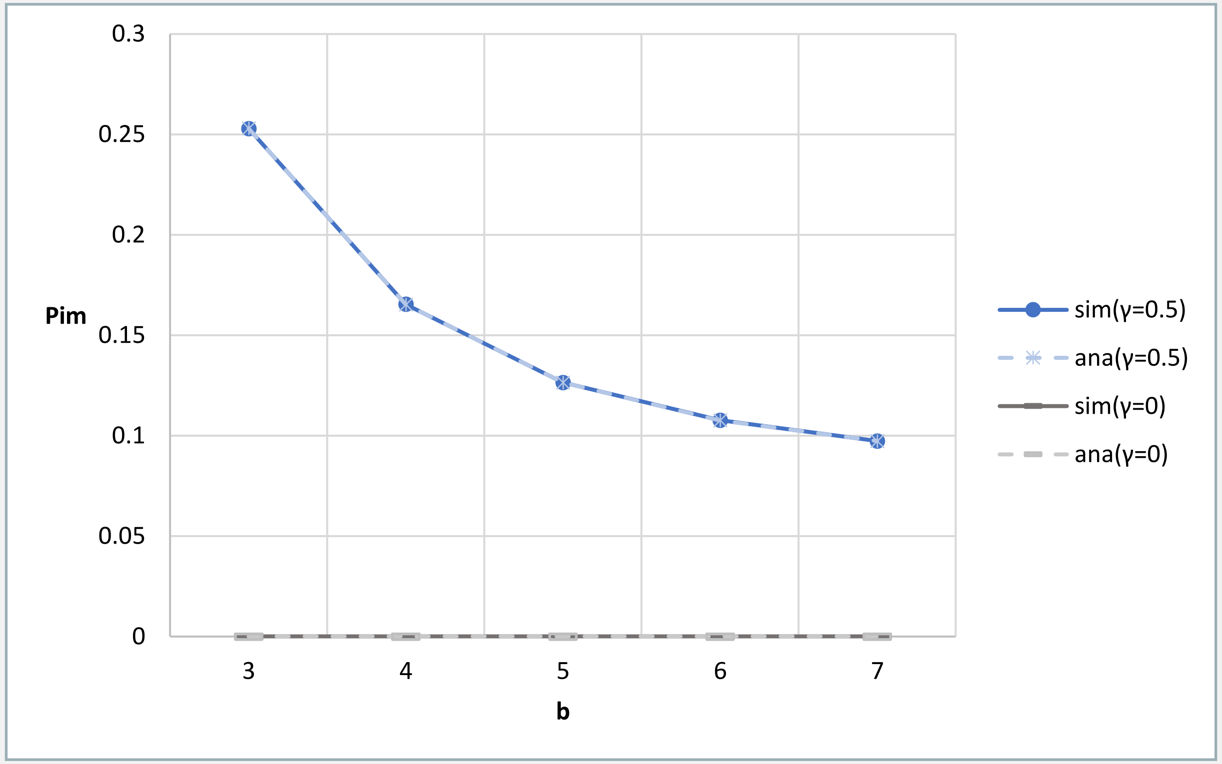
Numerical Result
– scenario 1 vs scenario 3



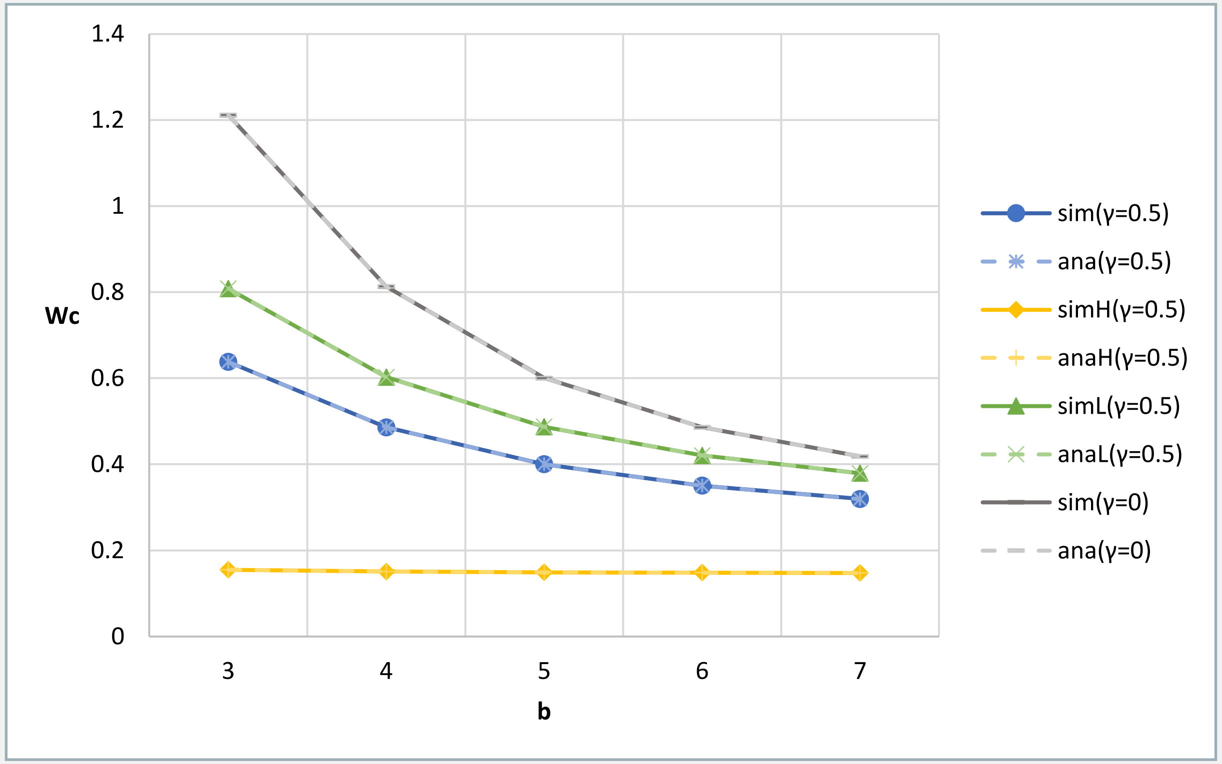


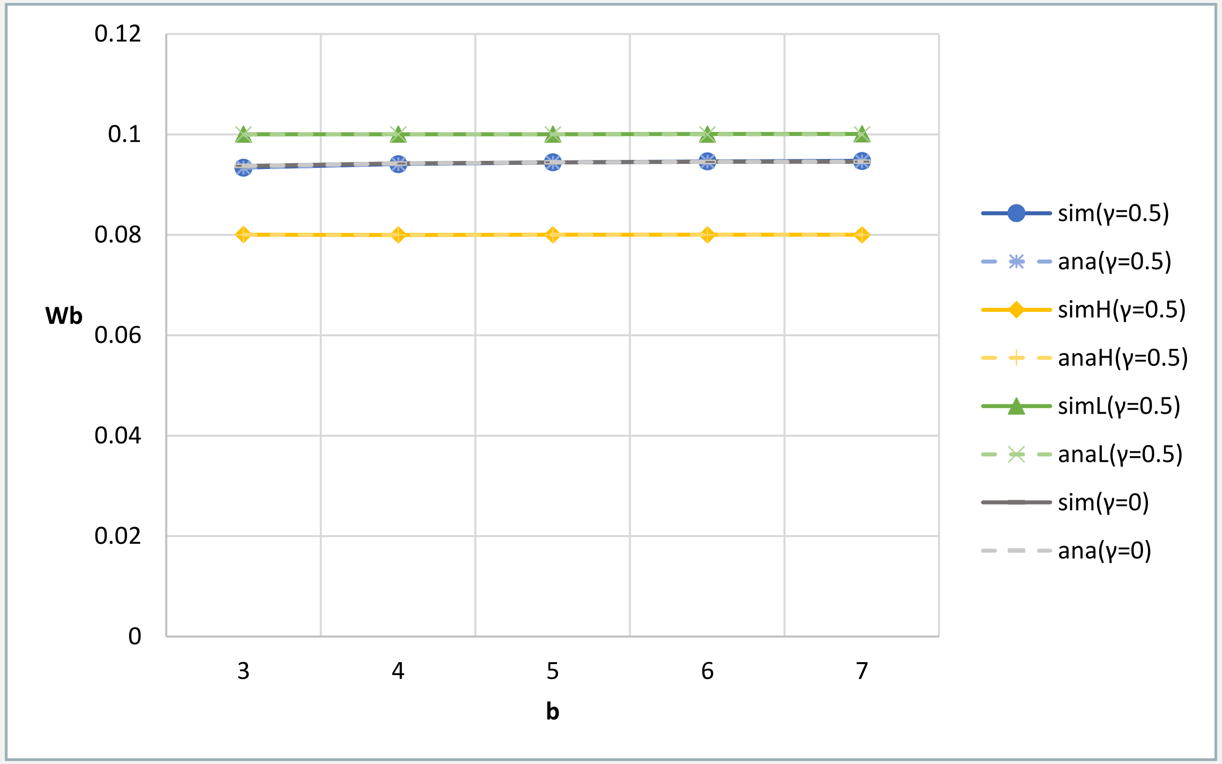
Numerical Result
– scenario 1 vs scenario 3



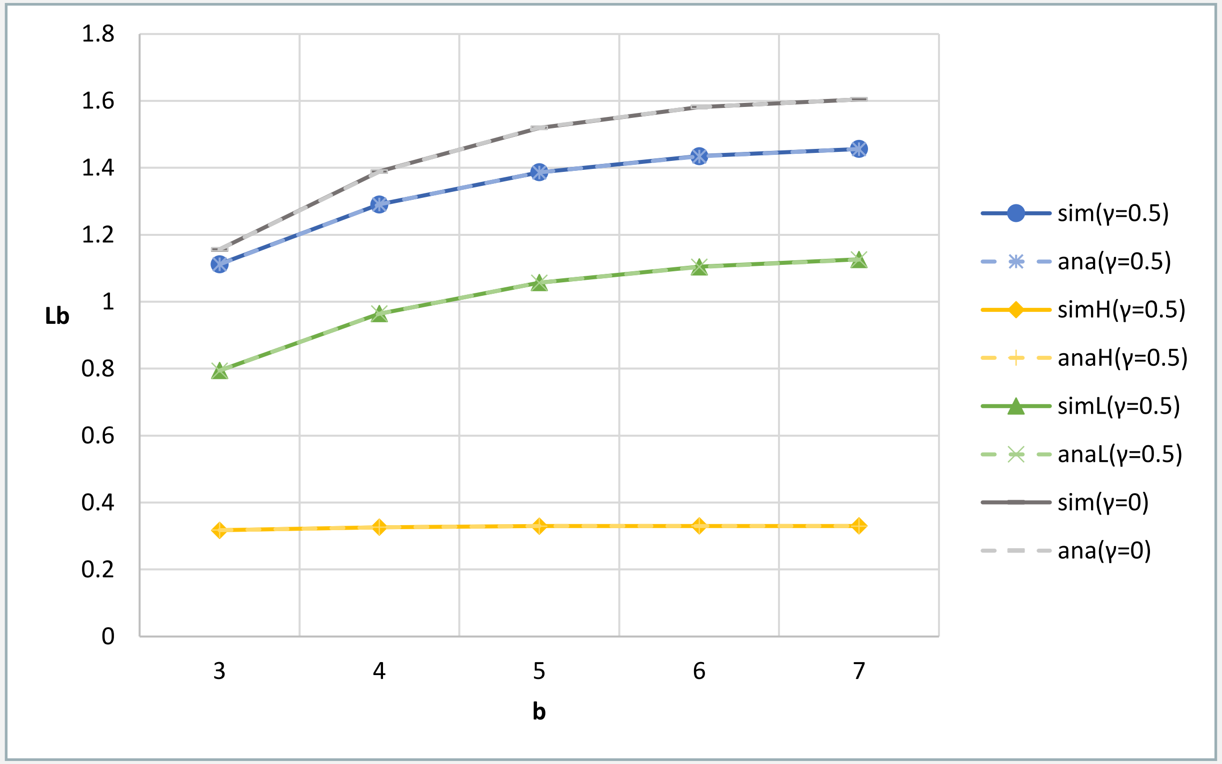


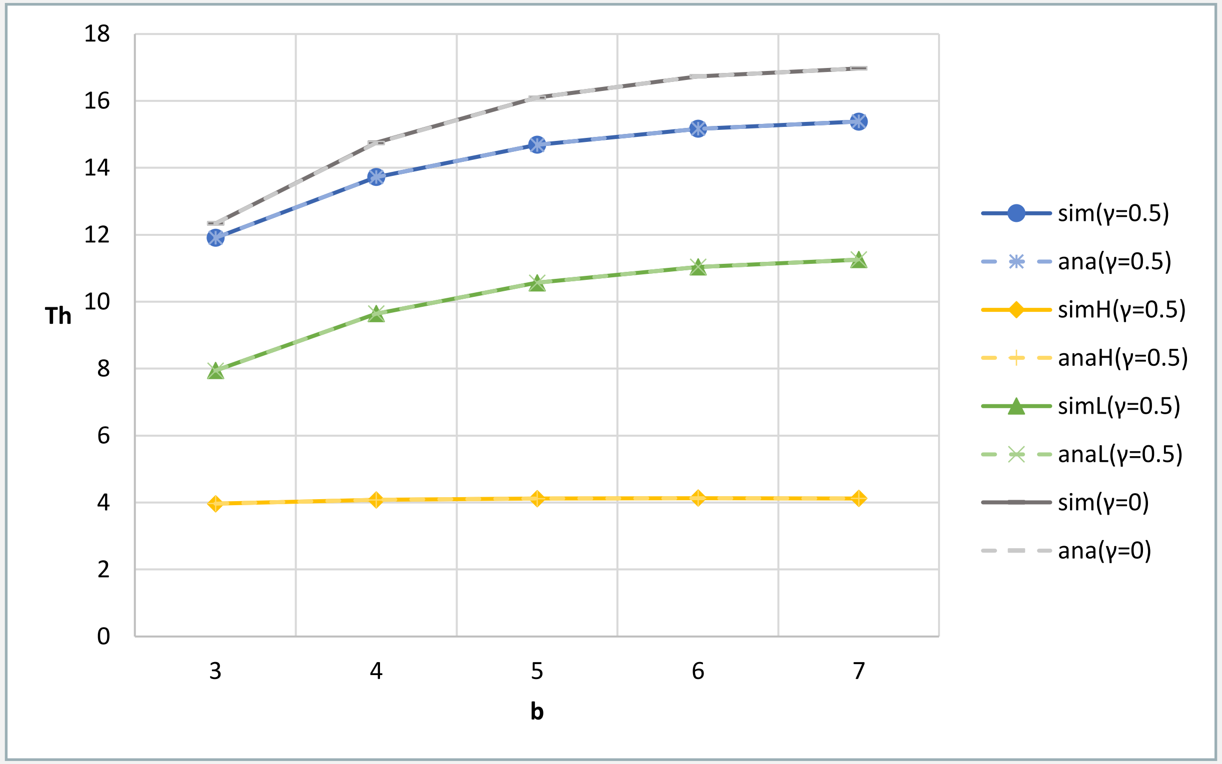
Numerical Result
– scenario 2 vs scenario 4



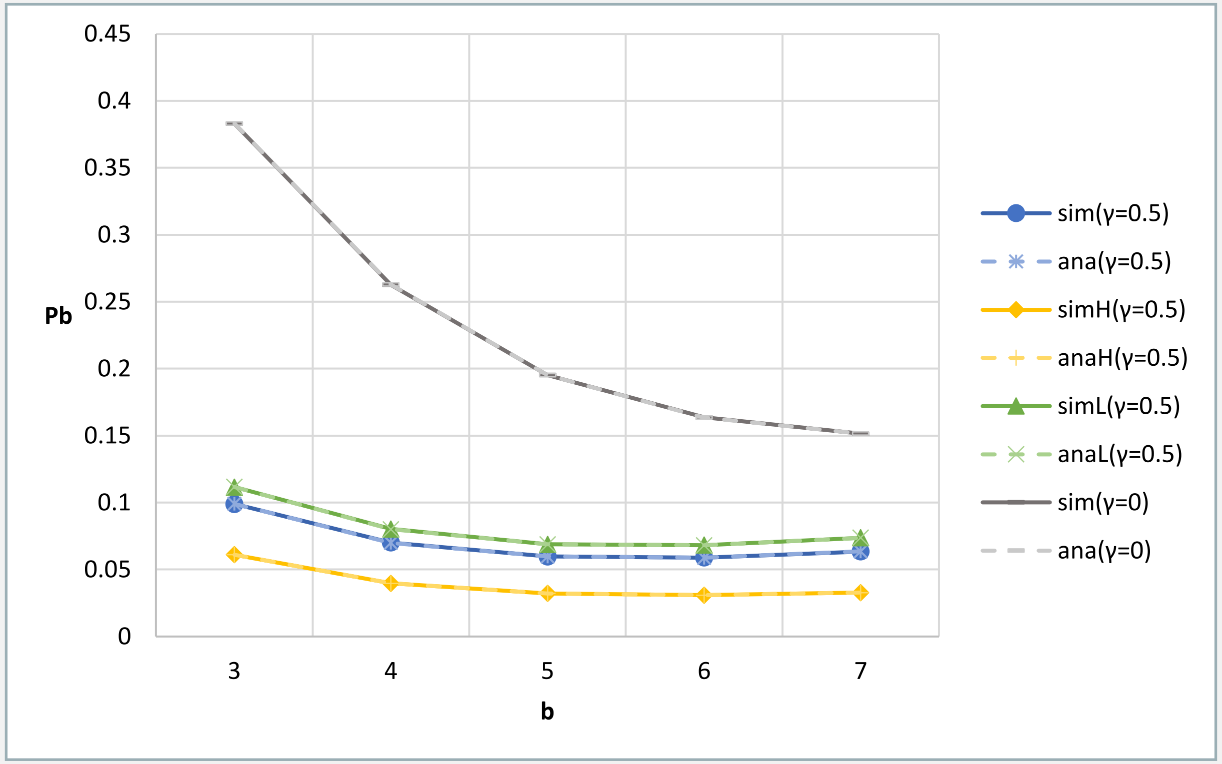


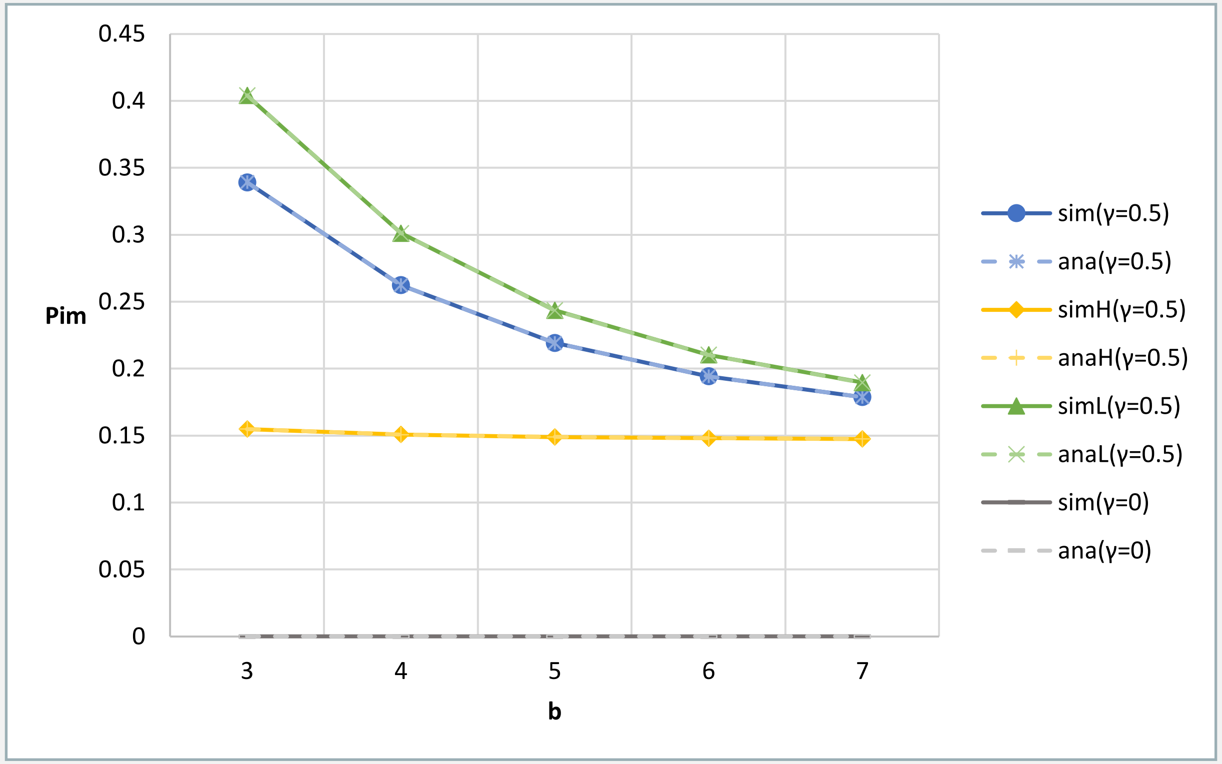
Numerical Result
– scenario 2 vs scenario 4



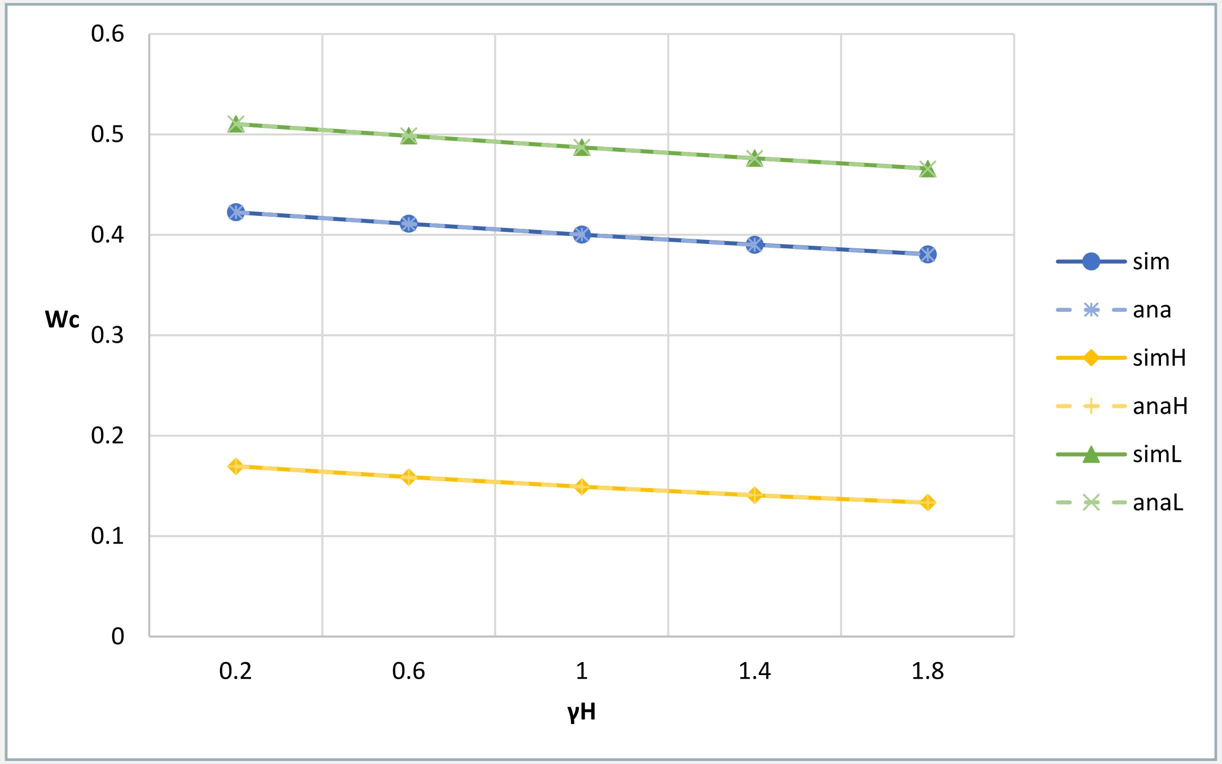


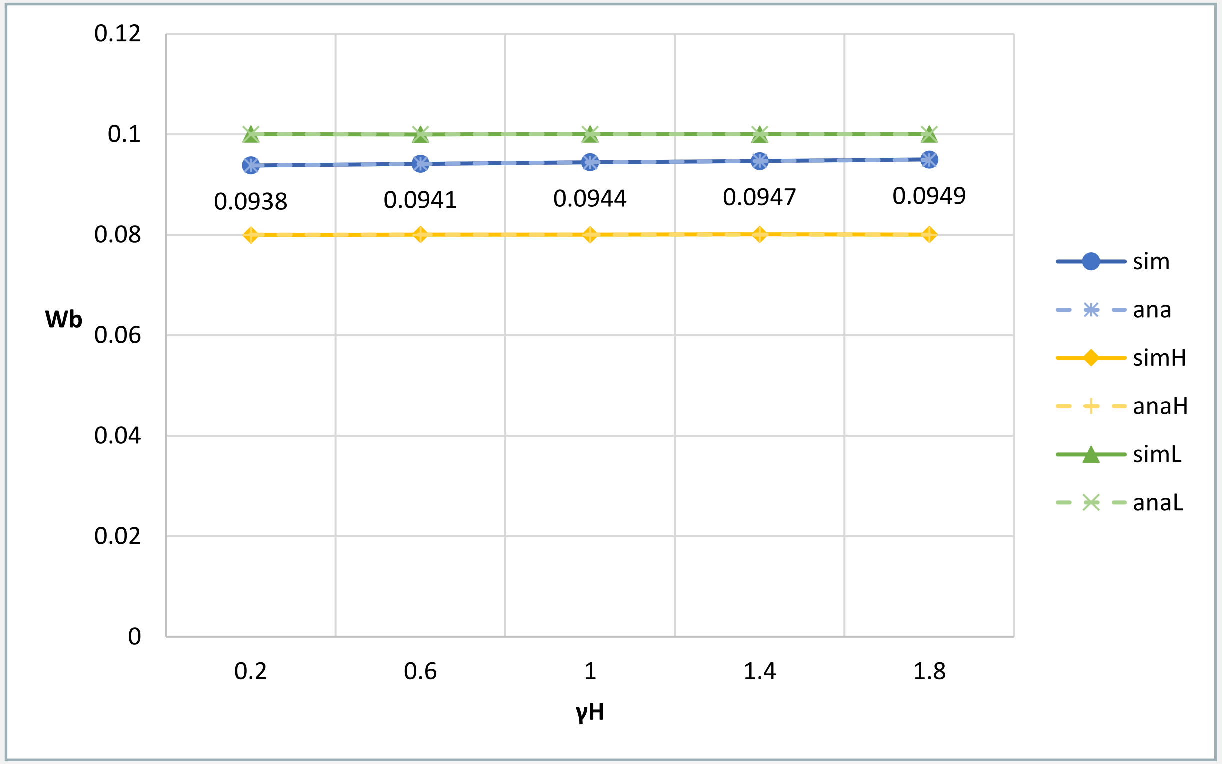
Numerical Result
– scenario 2 vs scenario 4



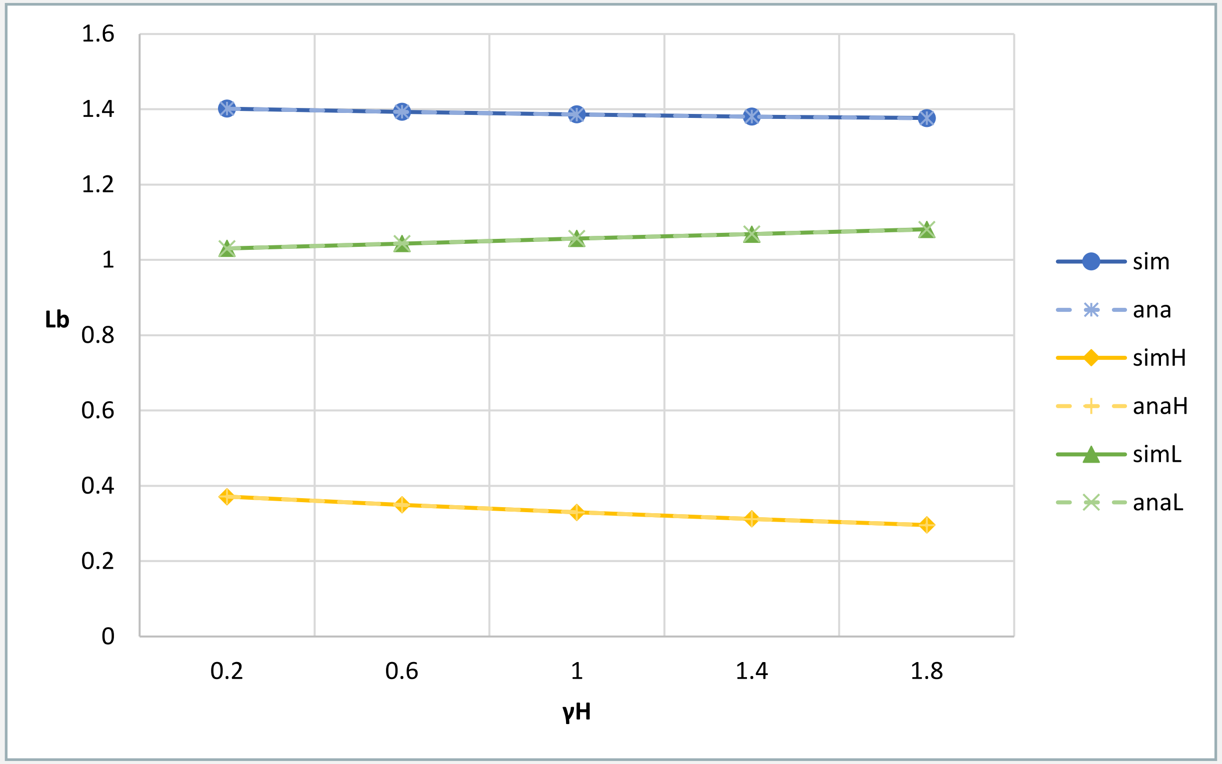


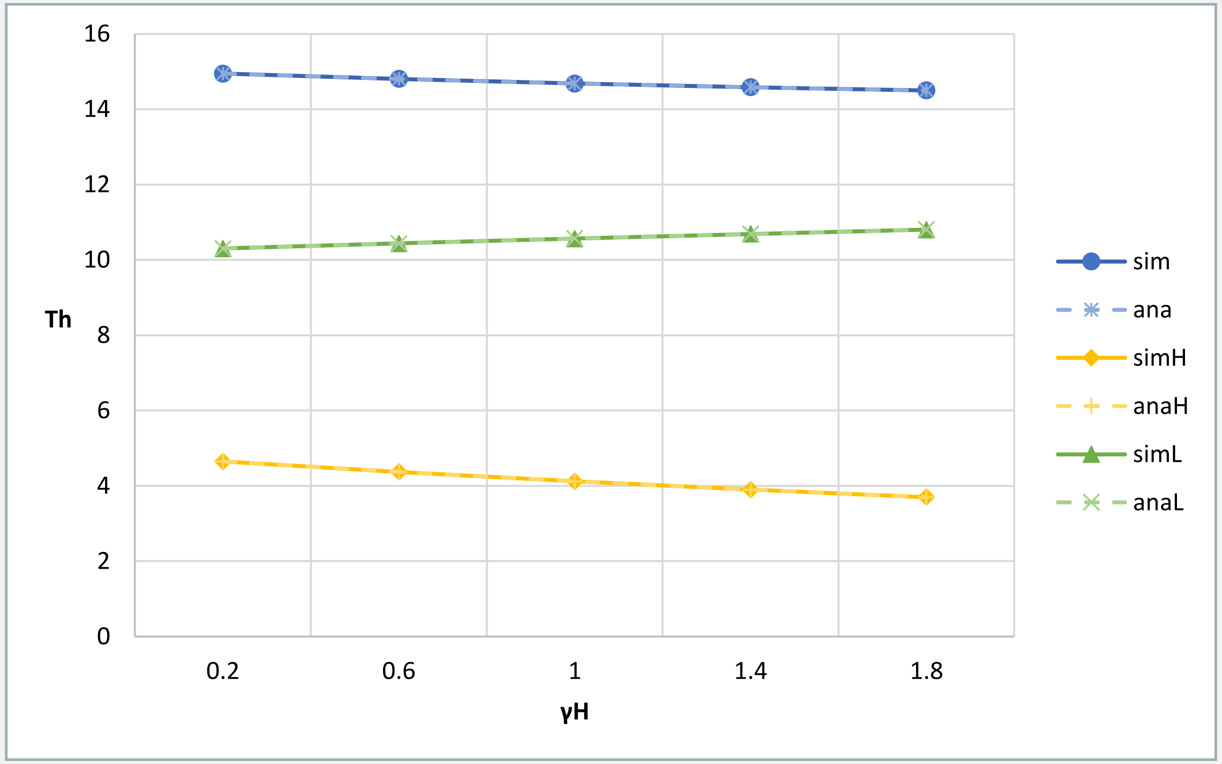
Numerical Result
–scenario 4



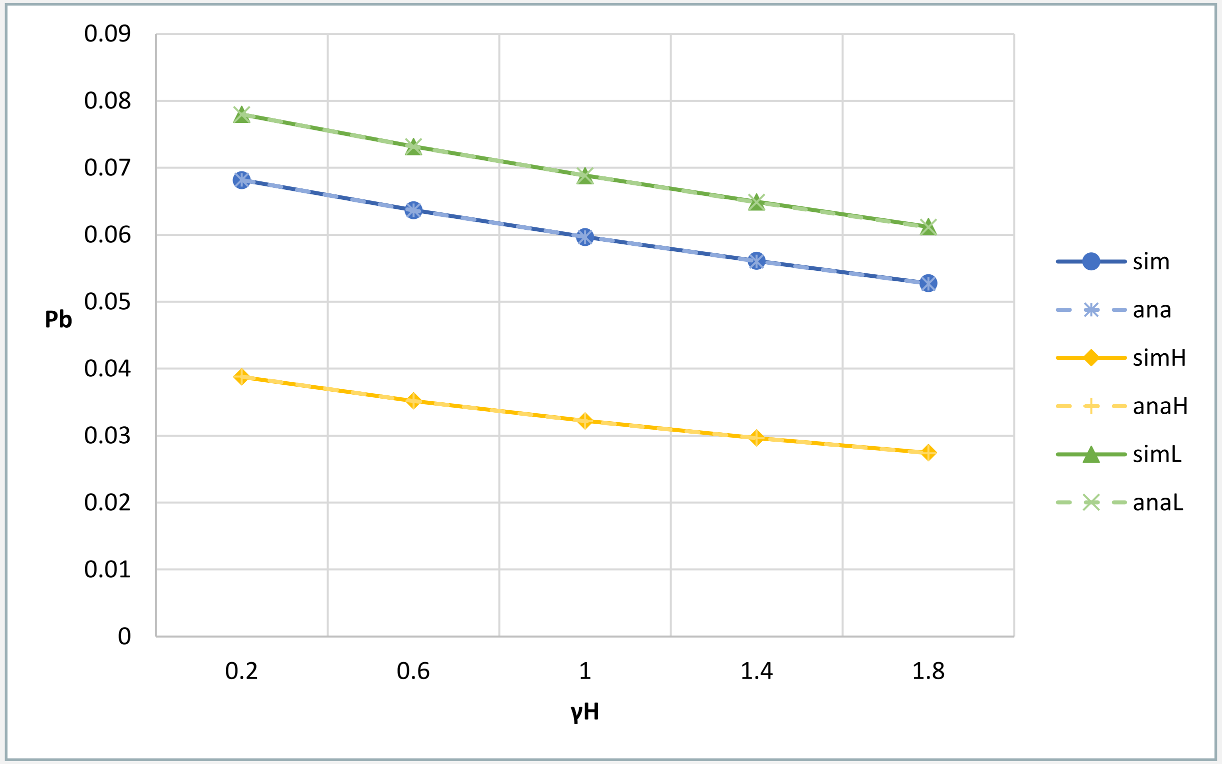


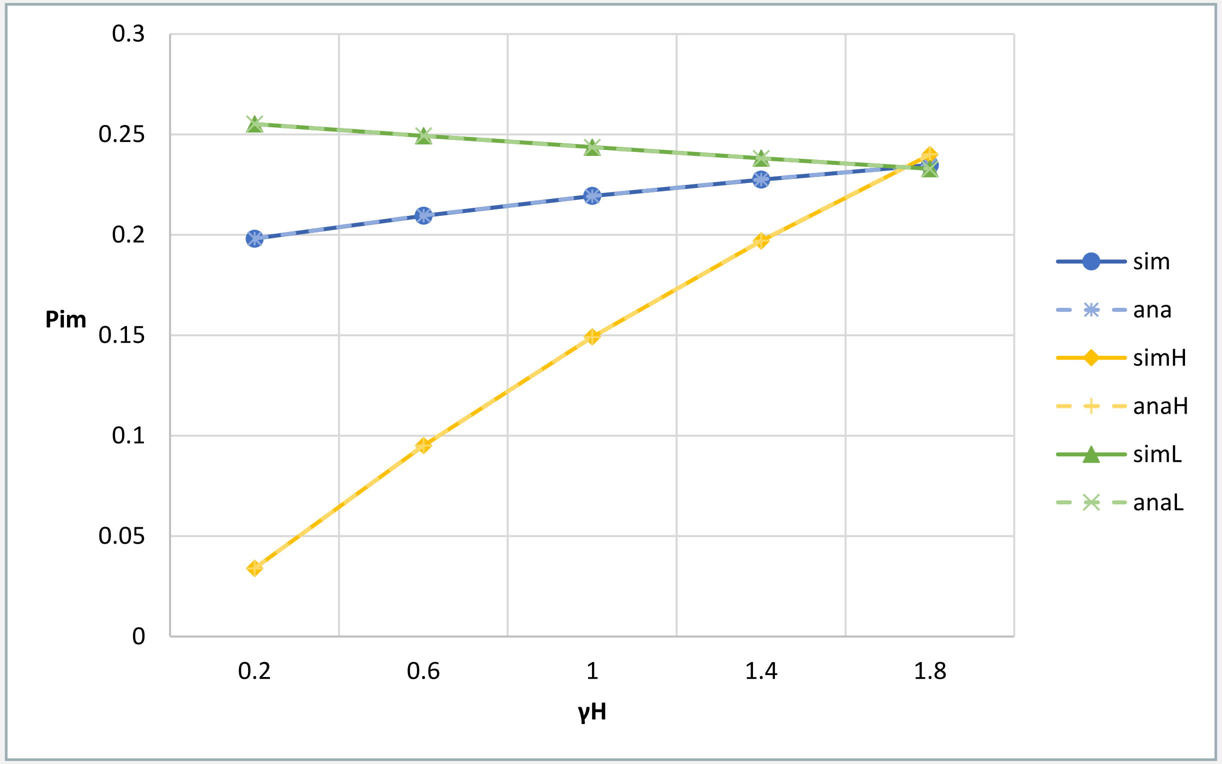
Numerical Result
–scenario 4



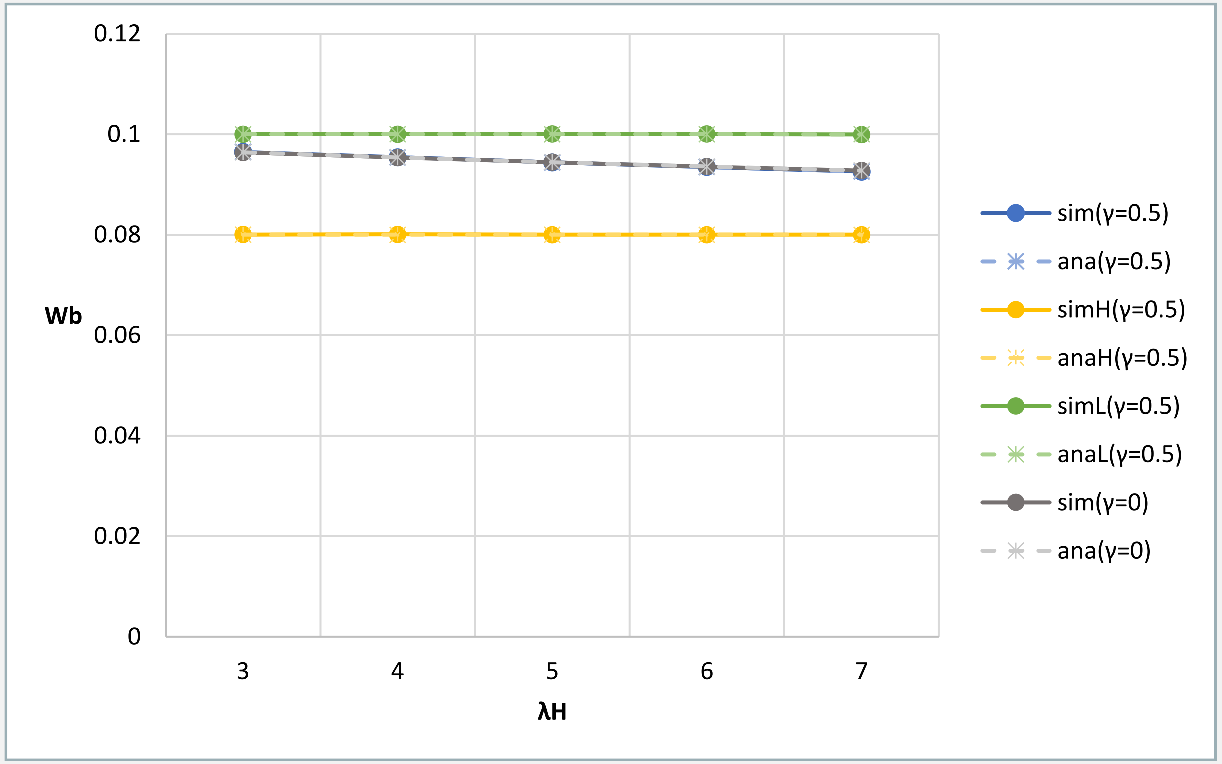


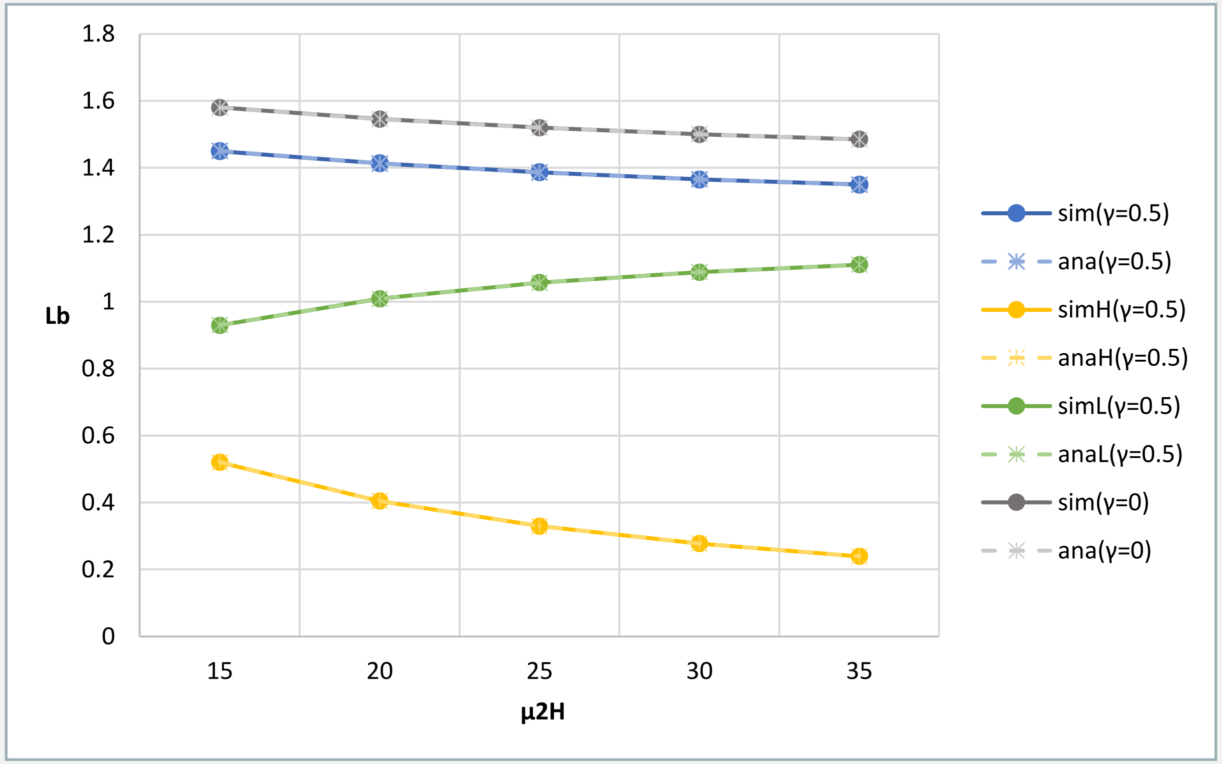
Numerical Result
–scenario 4





Numerical Result
- scenario 4





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Conclusion

1. **Effect of block size**
   - As b increases, L_b initially grows but eventually stabilizes around a constant value. (in all scenarios)
   - T_{h_H} remains nearly constant, but T_{h_L} increases with b but the increase gradually slows down (in scenario 2 & 4)
   - P_b decreases with increasing b at first, but shows a slight increase when b becomes large. (in all scenarios)

Conclusion

2. **Effect of HP arrival rate**
   - Increasing λ_H, W_{b_H} and W_{b_L} remains constant. However, W_b shows a downward trend. (in scenario 2 & 4)
3. **Effect of HP consensus rate**
   - Increasing μ_{2_H} leads to an upward trend in L_{b_L}. (in scenario 2 & 4)
4. **Effect of impatience mechanism**
   - Scenarios with impatience leads to improvements in most performance metrics except throughput, when compared to scenarios without impatience.

Future Work

- **Incorporate realistic voting mechanisms**

  Integrate more practical consensus voting procedures to reflect actual blockchain operations.

- **Dynamic batch policy optimization**

  Adapt the batch size based on the real-time state of the customer and block queues.

- **Enhanced impatience modeling**

  Extend impatience behavior to account for total waiting time until departure.

THANK YOU FOR LISTENING