less Go

Week 8 + Assignment

Matrices

$$\begin{pmatrix}2 & 3\\ 10 & 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}8\\ 13\end{pmatrix}$$

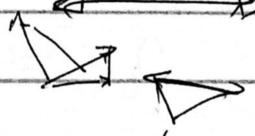
$$\begin{pmatrix}2a + 3b\\ 10a + b\end{pmatrix} = \begin{pmatrix}8\\ 13\end{pmatrix}$$

$$a\begin{pmatrix}2\\ 10\end{pmatrix} + b\begin{pmatrix}3\\ 1\end{pmatrix} = \begin{pmatrix}8\\ 13\end{pmatrix}$$

$$\begin{pmatrix}2a\\ 10a\end{pmatrix} + \begin{pmatrix}3b\\ b\end{pmatrix} = \begin{pmatrix}8\\ 13\end{pmatrix}$$

Remember

Changing basis vector would require you to take the normalized dot product with the new basis i.e to project r onto b.



Reminder

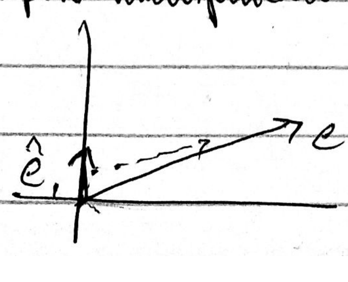
$$\text{① } \quad \overset{A}{\begin{bmatrix}2 & 3\\ 10 & 1\end{bmatrix}} \begin{bmatrix}0\\ 1\end{bmatrix} \rightarrow \begin{bmatrix}3\\ 1\end{bmatrix}$$

Lets rely on the column row method instead.

→ The multiplication transforms a vector using a matrix

this is the core concept.

Remember that for example ① above the basis vector $\begin{bmatrix}0\\1\end{bmatrix}$ gets transformed to $\begin{bmatrix}3\\1\end{bmatrix}$ upon multiplication with matrix A.



$\boxed{\text{Notation}}$ r' means r changed

$A \times r = r'$

$A \times nr = A\, nr'$

$A(r+s) = Ar' + As'$

$$\boxed{A(nr+ms) = nAr' + mAs'}$$

So first thing. every vector in the $\mathbb{R}^2$ space is made from a multiplo of the basis vector $\hat{e}_1$ and $\hat{e}_2$ which means that all tranformation of a vector in 2 dimensions can be written as sums of the matrix being multplied. This goes back to the column row concept.

$$A(nr+ms) = nA(r') + mA(s')$$

where r' and s' are the basis vectors

is the component of the basis vector

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\left(3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 6\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$= 3\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 6\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$3\begin{bmatrix} 2 \\ 10 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 1 \end{bmatrix} =$$

eg. $\begin{pmatrix} 7 & -6 \\ 12 & 8 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \end{pmatrix} = 8\begin{bmatrix} 7 \\ 2 \end{bmatrix} + 6\begin{bmatrix} -6 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} 35 \\ 10 \end{bmatrix} + \begin{bmatrix} -36 \\ 48 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 58 \end{bmatrix}$

$\overset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix}$

$x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Identify Matrix

Mirrored

$\hat{e}_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \hat{e}_1$

$I = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$ $\hat{e}_1$ $\hat{e}_2$

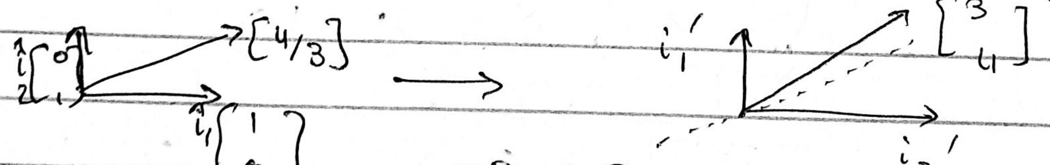
eg $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ 90° rotated anticlockwise

Inversion across the x axis

$I_{inv\,x} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

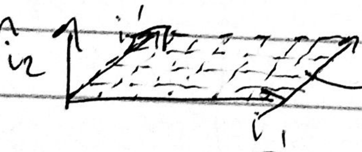
$I_{inv\,y} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $I_{mirr} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $I_{i} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

So this A matrix that we are defining basically changes the space / transforms the space itself. The whole unit stays equally spaced i.e. it is not warped but it does: scale or shear and This happens for every possible vector. for ex. in a normal unit vector basis (standard)



$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

applied



→ see how the grid also shears. That shows how the whole space has transformed due to the matrix.

$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ shears the unit square

Composition what does it do to the space of vectors itself

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$A_1 A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad n A(\hat{i}) + m($$

$$\begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 1 & 1 \\ & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} +}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

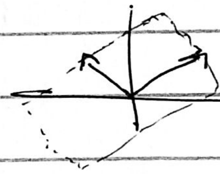
$$\boxed{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}$$

In composition.
the order of transforming matrices is not the same composition

Note
→ Not commutative. (Order)
→ They association though (Brackets)

Practice Quiz

$$A r = \begin{bmatrix} 1/2 & -1 \\ 0 & 3/4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$



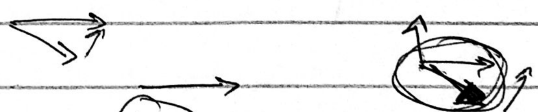
$$3 \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 3/4 \end{bmatrix}$$

$$\begin{bmatrix} 3/2 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 6/4 \end{bmatrix} = \frac{3/2 - 2}{3/2} \quad \begin{bmatrix} -1/2 \\ 3/2 \end{bmatrix}$$

$$-2 \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} -1 \\ 3/4 \end{bmatrix} = \begin{bmatrix} -1-4 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} \quad \{1 \; 1\}$$



Rescale

$$0.8 \qquad \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1/8 \end{bmatrix}} \longrightarrow \begin{bmatrix} 1 & 0 \\ +1/2 & 1 \end{bmatrix}$$

1/8

0.128

0

$$\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/8 \end{bmatrix} \Bigg| \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1/2 & 1/8 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 1 & 0 \\ -4 & 8 \end{bmatrix}$$

$\boxed{\text{Inverse}}$

$A^{-1}A = I$

$Ar = s$

$\boxed{A^{-1}Ar = A^{-1}s}$

for $Ar = s$ we dont need to calculate the inverse.

Gaussian Elimination

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Row echelon form

$R1 - 2R3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$R1 - 2R2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

3×3 3×1

Inverse using Gaussian Elimination

Note: We can use the row reduction method to make the inverse of A

$$\underbrace{\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}}_{A^{-1}} = I$$

this colum affects only the a's or applies

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \underbrace{\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}}_{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

first row of A^{-1}

what we can do instead is perform row elimination at the same time for all in I as the same operation will be performed for a's, b's and c's

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} A^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 - R_1$ $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$-1R_3$ $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$R_2 - R_1$ $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$R_1 - 3R_3$
$R_2 - R_3$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} -2 & 0 & 3 \\ -2 & 1 & +1 \\ 1 & 0 & -1 \end{bmatrix}$

$R_3 - R_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\boxed{\begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}}$ A^{-1}

QR decomposition

Important Note

Remember Sometime it makes sense Computationally to solve linear equations than finding an inverse.

$$Ar = S$$

$$\cancel{A^{-1}}A r = A^{-1} S$$

$$I r = A^{-1} S$$

Solve is ko karna

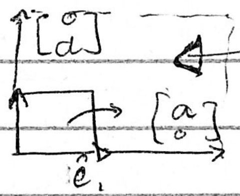
We could be trying to find the unit solutions and solve individual pixel consideration

$$A \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = S$$

Solve this.

So instead of finding this you could use row reduction and gaussian elimination to make a better impact on the computation

Determinants and Inverses



This space was scaled to this new space

$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

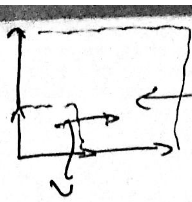
The determinant is basically the area of the scaled area of the transformation caused by matrix A.

Determinant. Area ? Inverse ? → why the $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} / ad - bc$ works ?

line area ?

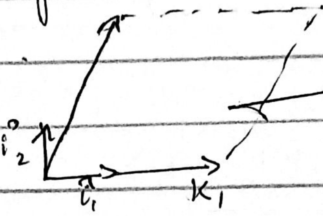
$$\frac{1}{ad - bc}$$

Area -

$\begin{bmatrix}2&0\\0&10\end{bmatrix} \longrightarrow$  $\leftarrow$ this area is basically the determinant

This transformation causes this up scale.

for a shear like the following

$\begin{bmatrix}2&1\\0&5\end{bmatrix} \longrightarrow$ 

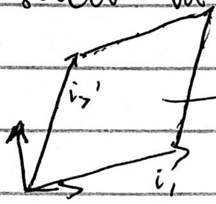
k_1 k_2

$\rightarrow$ this parallelogram has an area of

$b \times h = 2 \times 5 = 10$

$(a \times d) - bc$ $\quad ad - bc \overset{?}{=} 0$.

for a shear in both dir.

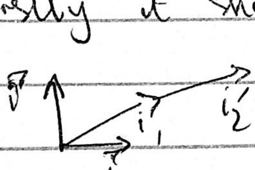


$\rightarrow$ this shape is proven to have area of $\boxed{ad - bc}$

This also true for 3D spaces where ~~you~~ the determinant will refer to the volume and not the plane.

Something that the course alludes to is linear independence and the process of Gaussian Elimination

Firstly it should be understood that



$\begin{bmatrix}1&2\\1&2\end{bmatrix}$ A_2 $\rightarrow$ This is because the area is 0 as its just a line the matrix transforms to. That second column is dependent on the first.

the determinant is $1(2) - 2(1) = 0$

Its also interesting to note the $A(x) =$ will get us a scaled vector x' on the line but getting x back from x' would be impossible as the answer is one dimensional. True for 3D scalar $\frac{r.s}{|r|}$

vector projection $\frac{r.s}{|r|} \cdot \frac{r}{|r|}$

Programming Assignment.

If an inverse exists.

Row echelon form where $c = 0$ ⟷ Singular
lin dependent columns ⟷ $\det = 0$

Week 4

Einstein Summation Convention. (Quick coding method).

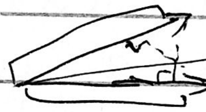
$AB = C$

$c_{ik} = a_{ij} b_{jk}$ Summation over all possible j's

$i \times k \times j$ computations — for row i in a and column k in b we multiply j element.

$u_i v_i$ for vector u → where u could be considered a $1 \times n$ matrix
and v → v could be n cross 1 matrix

This summation is basically the dot product.



line of symmetry

$\hat{u}$ he mentions is a unit vector which means that the projection is directly proportional to $(\hat{u} \cdot \hat{i})$ or even $(\hat{i} \cdot \hat{u})$

$|r| \cos\theta$

$|s| \cos\theta$

$\frac{r.s}{|s|} \neq \frac{s.r}{|r|}$

Practia

Q1

$a_2 b_1$

$4+0+1$

$= 5$

$2+12+10+6$

30

$4\times1 \quad 1\times4$

2 4 5 6

6 12

$c_{21} = a_{2j} b_{j1}$

$-1/53$

$c_{11} = a_i b_{ik} \quad a_1 b_1$

$(A(B)C)$

$3\times2 \quad 2\times4$

3×4

2 8 −4

$(5\times3)\, 3\times4$

5 7 7 4

5×4

$2\times2 \quad 2\times3$

2×3

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

$r_j \quad r_i$

r_i

ks

s

$1\times4 \quad 4\times1$

Practice Quiz

$r' = r + \lambda \hat{s}$

$\lambda = -\dfrac{r \cdot \hat{e}_3}{s_3}$ scalar projection

$$r' = r + -\left(\frac{r \cdot \hat{e}_3}{\hat{s} \cdot \hat{e}_3} \cdot \hat{s}\right)$$

$r' = r - \hat{s}\,(r \cdot \hat{e}_3)/s_3$

~~$r' = r\left(1 - \dfrac{\hat{e}_3 \cdot \hat{s}}{\hat{s} \cdot e_3}\right)$~~

$r' = r_i - s_i\,(r \cdot \hat{e}_3)/s_3$

$s_i = \hat{s} \cdot \hat{e}_3$

$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$r' = r\left(1 - \frac{s_3}{s_3}\right)$$

$s_3 = s_i e_{ik}$

s_3

$r_i - s_i\,(r_i \hat{e}_{3i})/s_3$

$1/3 \quad 4/13 \quad 12/13$

$r_i \left(1 - s_i \dfrac{e_{3i}}{s_3}\right)$

$\begin{Bmatrix} 4/13 \\ 3/13 \\ -12/13 \end{Bmatrix}$

$r_i v_i$

$1\times1 \quad 1\times5$

4×1

1×4

$r_i v_i$

r_i

$1\times4 \quad 4\times1$

$r\times c \quad r\times c$

$-12/13$

s_3

2×1

3×3

$u\, v$

$1\times4 \quad 4\times1$

2×1

2×3

$r \quad e_3$

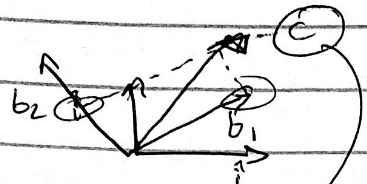
2^{+3}

$r'_i \quad A_{ij} \quad r'_i$

$$A = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1/4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1/2 \end{bmatrix}$$

Remember that for orthonormal new basis we can just find the projections of the vector c onto B_1 and B_2 to get r → the vector in bears axis.



assuming this is c_i

Also: for orthonormal basis of any dimension / Matrix the determinant = 1 which means that the inverse

$$\frac{1}{\cancel{adj}\ \det}\,adj = \boxed{\frac{1}{1}\left(\begin{bmatrix} b & -c \\ -d & a \end{bmatrix}\right)}$$

e.g.

$$\underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{\text{the transfor that allows us to go BACK to our world's coordinates}} \boxed{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Inverse

$$\boxed{\left(\frac{1}{2}\right)\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}}$$

$$\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ +2 \end{pmatrix}\frac{1}{2} \to \boxed{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

#Verified.

Matrix transformations work.

Matrices changing basis.

... You can imagine why taking all c's and converting them to Bear's world vector would be useful. Imagine a picture to be placed in standard basis to a slightly sheared basis.

Really Cool

Transformation in funny Basis

So imagine this. To perform a transformation in Bear's world for ex. rotation you can't just apply the standard Rotation vector

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad R_{45}$$

this can "only" be applied to normal vectors in my world. For vectors in Bear's world we first need to change their basis.

Remember that these are the basis vectors in Bear's space.

$$Br = c$$

$$R_{45}\, c = c_{rot}$$

$$B^{-1} c_{rot} = r_{45}$$

Back Sub

$$\boxed{B^{-1} R_{45}\, B}\,(r) = r_{45}$$

$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

Any tranformation matrix in standard basis needs to get converted as well.

Orthogonal Matrices

We can also say that a matrix of size m×n is a space with n basis vectors if they are linearly independent.

$a_i \cdot q_j \neq a_i$ ←

$\boxed{v_2 \cdot e_1}$

$\boxed{A^T A = I}$ A^T is a valid inverse of A

$A^T = A^{-1}$

$A^T = A^{-1}$

$(AA^T)^T = (I)^T \rightarrow \boxed{AA^T = I}$

here A would an Orthogonal matrix and its basis would be an Orthormal basis set (if they are perpendicular to each other and of unit length)

This also means that

A^T is an orthogonal matrix

and ITS columns are also an orthonormal basis set.

Gram Schmidt Process.

So its better to check linear independence by seeing if ~~two of them~~ all the basis vectors ~~are~~ have determinant $= 0$.

this will give us orthonormal basis vectors set.

$V = \{V_1, V_2 \cdots V_n\}$

our first basis vector

$V_1 \rightsquigarrow e_1 = \frac{V_1}{|V_1|}$

$v_2 = v$

$\frac{e_1 \cdot v_2}{|e_1|}$

Orthonormal basis vectors make our life much easier by allowing us dot product projections and easier tranformations $\boxed{B^{-1} C = r}$ just do $\boxed{B \cdot r = C}$

C onto B can be found insted of finding B^{-1}

To get orthonormal Basis from a nonorthogonal matrix like the following $\begin{bmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{bmatrix}$

$3 \times 1 + 1 \times 1 = 4$ which is not 1 so not orthogonal

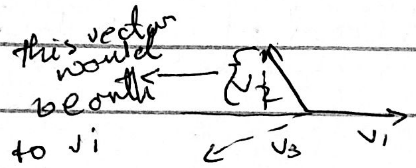
- Intro to Image
- Geometric modelling

We use the gram schmidt process to get an orthonormal set

$\{u\} = (\hat{u}_1, \hat{u}_2, \hat{u}_3 \ldots \hat{u}_n)$ which are orthogonal

from

$\{v\} = (v_1, v_2, v_3 \ldots v_n)$



$\hat{u}_1 = \frac{v_1}{|v_1|}$

$u_2 = v_2 -$ (projection of v_2 onto u_1) and then normalized

$$\boxed{u_2 = \underline{v_2} - \underbrace{(v_2 \cdot \hat{u}_1)\,\hat{u}_1}}$$

$\frac{u_2}{|u_2|} = \hat{u}_2$

(v_2's) component in direction of u_1 when subtracted from v_2 itself we get a magnitude vector exactly orthogonal to $\hat{u}_1$

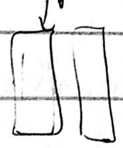
$u_3 = v_3 - (v_3 \cdot \hat{u}_1)\hat{u}_1 - (v_3 \cdot \hat{u}_2)\hat{u}_2$

$\frac{u_3}{|u_3|} = \hat{u}_3$

Another benefit of having orthonormal basis is the fact that the $A^T = A^{-1}$

$\boxed{A^T c = r}$ → which is also equivalent to c getting projected onto basis vectors in A.

fuckin' beautiful.



if anyone of my basis is not of unit length its not a basis of my transformed space..

$B(A) = C$ — real no. $E^{-1}T(E)$

$B^{-1}C(A)$ $A^T A$

$A^T = A^{-1}$?

$E^{-1}(C)$ $E(n)$

$\boxed{E^{-1}(n) = C}$ $\boxed{E(n) = C}$

Module/Week 8

Eigen (Characteristic) {Geometric approach}

$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}$ → vertical scaling

→ In a vertical scaling.
the two vectors ~~that~~ whose
direction remains unchanged.
are the eigen vector in this case
it would be i_1' and i_2' when $i_1' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $i_2' = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
the eigen value is the amount each eigen
vector is scaled by (?) / Stretched.
Vectors lying on the same (span)

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

/ Deception
/ Perception
↳ the
from
the
cent...

Eigen vector are / belong to a linear
transformations.

Uniform Scaling

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ would be a transform of reflection across the $y = -x$ axis?

See 2 days.
{Math
{CS231n

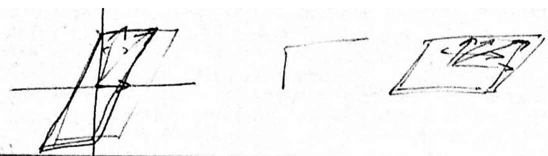
Open CV
practical

2 $\begin{bmatrix} \ \end{bmatrix}$

So for a 180° rotation
all vectors are eigen vectors
lets take the example of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ when
transformed goes to $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$
which could be written as $(-1)\boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$ → vector on the same span

scaling (~~span~~ eigenvalue).

Very similar to a
uniform scaling except $T = (-1)I$



3 the eigen vector

Beautiful Note: In 3d a rotation could ~~be~~ have ~~more than~~ a rotation unlike only - I for a 2D space. An eigenvector of a 3D ~~x~~ Linear transform would be its axis of rotation. The rest of uniform scaling could remain true. For a horizontal scaling in 3D space there will still exist 3 eigen vectors. Remember for a uniform scaling of -1 or 1 we will have ∞ eigen vectors.

is the scaling.

$\boxed{Ax = \lambda x}$ (Write this in english)

is the linear transformation

is to be the eigen vector

Why written like this?
When 'x', the eigen vector, is transformed by A, it should just be as if x was scaled by a scalar 'λ', the eigen value. How f***** beautiful.

Can we just do a simple solve on this equation. A has to be an $n \times n$ matrix as it has to multiply with x which is an $n \times 1$ ~~x~~ vector to generate an $n \times 1$ matrix. Could we do very simple row reduction. Well the subject x is unknown here so we cant solve with gaussian elimination.

$Ax = \lambda x$

$Ax = \lambda I x$

$Ax - \lambda I x = 0$

$\underbrace{(A - \lambda I)x = 0}$

for this to be zero. Either of the components of the multiplication have to be 0.

$(A - \lambda I)$ must $= 0$ and then we can just sub back into the prev eq to find x

$$\boxed{\det(A - \lambda I) = 0}$$

$$\det\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = 0$$

$$\cancel{\det\begin{pmatrix} a\lambda & b\lambda \\ c\lambda & d\lambda \end{pmatrix}}$$

$$\cancel{(a\lambda d\lambda) - (b\lambda c\lambda)}$$

$$\cancel{(ad\lambda^2) - (bd)\lambda^2}$$

$$\cancel{\lambda^2(ad - bd)}$$

$$\det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}$$

$$(a-\lambda)(d-\lambda) - (bc)$$

$$\begin{pmatrix} ad - a\lambda - d\lambda + \lambda^2 \\ -bc \end{pmatrix}$$

$$\lambda^2 + \overbrace{(ad - bc)}^{\text{determi}} - a\lambda - d\lambda$$

$$\lambda^2 + \det(A) - a\lambda - d\lambda \Big\} \text{polynomial.}$$

Ex: $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ vertical Scaling

$$\det(A - \lambda I) = 0$$

$$\lambda^2 + \det(A) - a\lambda - d\lambda$$

$$\lambda^2 + 2 - \lambda - 2\lambda$$

$$\lambda^2 + 2 - \lambda(1+2)$$

$$\boxed{\lambda^2 - 3\lambda + 2}$$

$$(1-\lambda)(2-\lambda)$$

$\lambda = 1$ and 2

$(A - \lambda I)x = 0$

$$\begin{pmatrix} 1-1 & 0 \\ 0 & 2-1 \end{pmatrix}\begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x$$

$$\begin{bmatrix} 0 \\ x_2 \end{bmatrix} = 0$$

second comp of x

$(A - \lambda I)x = 0$ | ex. $\begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$ $\lambda^2 + 15 - \lambda(8) = \lambda^2 - 8\lambda + 15 = 0$

this would be lambda^3 if we were 3 dims and so on

$\lambda^2 - 5\lambda - 3\lambda + 15$

$\lambda(\lambda - 5) - 3(\lambda - 5)$

for $\lambda = 2$ 1/30 hrs is up

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} (x) = \begin{bmatrix} -x_1 \\ 0 \end{bmatrix} = \boxed{0}$$

$\lambda = 1$ $\underbrace{\begin{bmatrix} \boxed{0} \\ x_2 \end{bmatrix} = 0}$ and $\lambda = 2$ $\begin{bmatrix} -x_1 \\ 0 \end{bmatrix} = 0$

~~this reads:~~ when $\lambda = 1$ the eigen vector must have x_2 comp $= 0$ but x_1 comp could be anything hence $\lambda = 1$

$e_1 = \begin{bmatrix} t \\ 0 \end{bmatrix}$

$e_2 = \begin{bmatrix} 0 \\ t \end{bmatrix}$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$A - \lambda I = \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix}$

$\lambda^2 + \det(A) - \lambda(a + d)$

$\lambda^2 + 1 - \lambda(0)$

$\boxed{(\lambda^2 + 1)} = 0$

$(\lambda^2 + 1) = 0$

$\lambda^2 = -1$ real values

$(\lambda = \sqrt{-1})$ does not exist.

why would we want to use eigen vectors as basis of a linear tranformation?

Orthogonality? Naah!

Maybe we're talking about

$\lambda^2 - \lambda(-2+1) + (-2 \mp (-3))$

$\lambda^2 + \lambda + 1$

→ why we change eigen basis

Changing Eigen Basis (Diagonilization)

apply same matrix transform several time

(A)

$$\boxed{v_n = \underbrace{T^n}_{\text{transformation}} v_0}$$

$$A^3 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

for a diagonal matrix (pow (diag$_{ii}$, n))

$$\boxed{T = CDC^{-1}}$$

$C \, D \, C^{-1} \, r$ → changing basis for r to be in the eigen basis

C → transform basis back

D → scaling

where $C = \begin{bmatrix} \begin{bmatrix} e_1 \\ | \end{bmatrix} & \begin{bmatrix} e_2 \\ \\ \end{bmatrix} & \begin{bmatrix} e_3 \\ \\ \end{bmatrix} \end{bmatrix}$ all eigen vectors arranged as column.

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$ → this is our simple transformation / Scaling

Remember

$C^{-1}(r) =$ r in basis of the eigen space.

$D(C^{-1}(r)) =$ scale r'

$C(D\,C^{-1}(r)) =$ r scaled in the real world.

10 — 60 pd 2 → 9:40

2:20

90 80%. 80%

6:00 4:40 40%

$T = CDC^{-1}$

$T^2 = CD\cancel{C^{-1}C}DC^{-1} = CDDC^{-1} = \boxed{CD^2C^{-1}}$ ← simpler calculation.

This is only possible because the eigen space has a transformation which is a diagonal matrix. In the eigen space remember that each vector is just a scaled version of the real thing

Quiz

$T = CDC^{-1}$ $\quad C^{-1}TC = D\cancel{C^{-1}C}$ $\quad C^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$\boxed{C^{-1}TC} = D.$

$$D = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 5 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$$

$$D = \frac{1}{3}\begin{bmatrix} 0 & \cancel{-1} \\ 3 & 7 \end{bmatrix}\begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -7 & 2 \\ 3 & 0 \end{bmatrix} \frac{1}{0-(-3)}\begin{bmatrix} 0 & -1 \\ 3 & 7 \end{bmatrix}$$

$$D = CTC^{-1} = \frac{1}{3}\begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 21 & 47 \\ -3 & -7 \end{bmatrix}$$

$\frac{1}{3}\begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix}$

$C^{-1}TC$

$$= \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 7 & 47/3 \\ -1 & -7/3 \end{bmatrix}$$

$-\frac{3}{3}$ -1

$\begin{bmatrix} 7 & -\frac{2}{3} + \frac{49}{3} \\ -1 & -7/3 \end{bmatrix}$

$\frac{6}{3} = 2$

$= \begin{bmatrix} 48 & 329/3 \end{bmatrix}$

$\begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 7 & \frac{47}{3} \\ -1 & -7/3 \end{bmatrix}$

$49 - 1$

$C^{-1}TC$ $\begin{bmatrix} \end{bmatrix}$

0 0 $\frac{1}{2}$ $\frac{1}{2}$ 0

$\boxed{T^3 = CD^8C^{-1}}$

$A = -\left(\frac{1}{\lambda}\right)$

$\boxed{f(A) - \frac{1}{\lambda}}$

T

Visualization **Page Rank**

$L_B = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$

$L_C = (0, 0, 0, 1)$

$L_D = (0, \frac{1}{2}, \frac{1}{2}, 0)$

$$A = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 1 & 0 \end{bmatrix}$$

$n \to$ all matrices

$$\boxed{R_A = \sum_{j=1}^{n} L_{A,j}\, r_j}$$

Rank

$r = \boxed{L r} \to$ rank of all web pages

$$r = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} \qquad r' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

C_1

$\boxed{r' = CDC^{-1} r}$ $\boxed{r' = Lr}$

$$\boxed{r^{i+1} = d(Lr^i) + \frac{1-d}{n}}$$

→ this cant be done because we dont know the eigen vectors, infact r'_{fin} is the (r^{∞}) eigen vector.

$\boxed{T^{\infty}(r)}$ = eigenvector ← power method.

↳ i.e. we need to find an r upon which this transformation has no effect. In $\mathbb{R}^2$ there exists only 2 such r's and we will reach one of them. → very interesting.

A principal eigen vector is the eigen vector that has the largest eigen value