Mathematics for Machine Learning (Linear Algebra)

Linear Algebra

1st Week:

Motivation for Linear Algebra:

→ Difference b/w Mathematical Objects (Vectors vs Matrices).

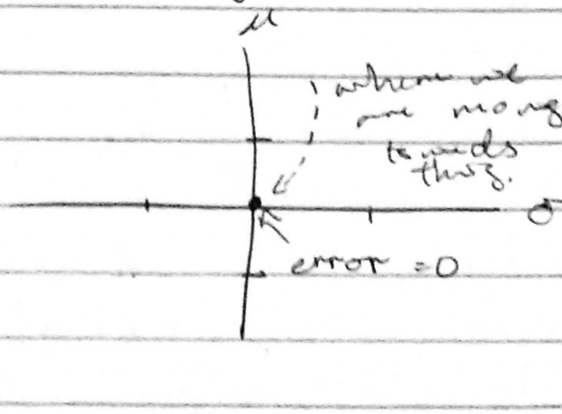
→ Types of problem we want to solve with LA
  → Solving Simultaneous equations.
  → Fitting an equation / line to some data

also partly connected to multivariate Calculus.

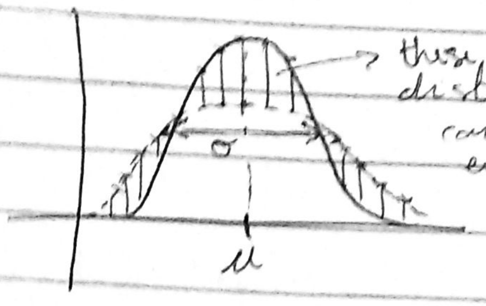
Module #1

Example of data

Parameter fitting → Maybe talking about something linear regression.



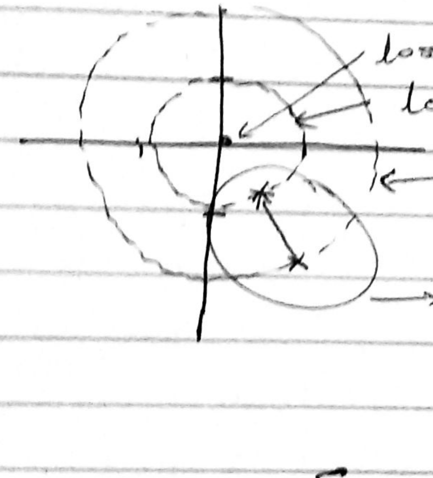
Parameter fitting



→ these errors are used to estimate better parameter.

In our current case μ and σ are the parameters : $\begin{bmatrix} \mu \\ \sigma \end{bmatrix}$ this vector needs to be found.

The equation is the gaussian equation.
The intuition is built on the following graph: of the parameter Space



where

$m > l > 0$

→ to take this step towards minima we use multivariate calculus.

[Practice Quiz]

Some extra information! in case you needed it

Sigma = $\boxed{S_x = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}}$

x_i → datapoint, $\bar{x}$ → mean

One sigma left and right should include 68% of all data in the probabilistic model.

It is important to note that in the case of σ and μ they depend ~~&~~ on each other which is also the case for other independent / dependent pairs of parameters. Maybe not in the physical space though.

The course defines σ as the characteristic width of the bell curve.

[Quiz related]

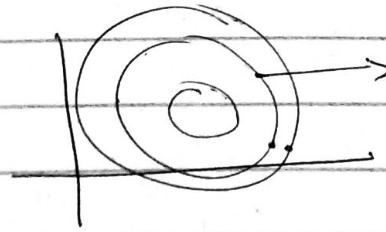
$$p = \begin{bmatrix} \mu \\ \sigma \end{bmatrix} \longrightarrow \begin{bmatrix} 155 \\ 167-155 \end{bmatrix} \longrightarrow \begin{bmatrix} 185 \\ 12 \end{bmatrix} \Big/ \begin{bmatrix} 179 \\ 7 \end{bmatrix}$$

g_p is predicted distribution (data) from a parameter vector p.

Sum of Squared Residual (or Error) is an error metric to measure the quality of your fit.

$\boxed{\text{Important}}$

A constant color on a contour map means that the SSR is zero there



→ so the SSR would be same on the circumference of this circle.

~~$\Delta p =$~~ $\{$

(3) $2(3x - 2y) = (4)\,2$

$\boxed{6x - 4y = 8}$

$6x + 3y = 15$

$-7y = -7$

$\boxed{y = 1}$

$x = \frac{4+2}{3} = 2$

(4) $x = ay + b$

$y = cx + d$

$2y - 2x = 20$

$y - x = 10$

$y - x = 10$

$5x + 3y = 6$

$y = 10 + x$

$5x + 30 + 3x = 6$

$8x + 30 = 6$

$x = \frac{-24}{8}$

-3

$y = 10 + \cancel{12} = 22$

$+(-3) = 7$

$-2x + 2y = 20$

$5x + 3y = 6$

(5) $x + y + z = 2$

$3x - 2y + z = 7$

$6 - 3y - 3z - 2y + z = 7$

$\boxed{6 - 5y - 2z = 7}$

$-5y - 2z = 1$

$x = 2 - y - z$

$3x - 2y + z = 7$

$3x - 2y - z = 3$

$6x - 4y = 10$

$3x - 2y = 5$

$\boxed{y = 4x - 5}$ (1)

$\cancel{2y} = 3x - 5$

$x + y + z = 2$

$3x - 2y - z = 3$

$\boxed{4x - y = 5}$ (1)

$$3x - 2y - z = 3$$
$$x + y + z = 2$$
$$4x - y = 5$$
$$y = 4x - 5$$
$$\boxed{y = -1}$$

$$3x - 2y - z = 3$$
$$3x - 2y + z = 7$$
$$6x - 4y = 10$$

$$6(x) - 4(4x - 5) = 10$$
$$6x - 16x + 20 = 10$$
$$-10x = -10$$
$$\boxed{x = 1}$$

$$1 - 1 + 2 = 2$$

pretty simple stuff
skip if revising.

Vectors Operations and spaces we can apply it to.

Addition = Vector addition is $\boxed{\text{associative(?)}}$ — associative property.
So it doesnt matter the order of addition. The answer is same.

Scaling This also reflects the property of lap.

* Nice way of writing a vector

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$ so pretty *-*

ex = $r - s$

$$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad s = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow r - s$$
$$= r + (-s)$$
$$-s = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} 3 \\ \cancel{2} \end{bmatrix} + \begin{bmatrix} 1 \\ -\cancel{2} \end{bmatrix}$$
$$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

a-c $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $c = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$d = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $-b = \begin{bmatrix} -1 \\ +2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

Next module More operations of a vector / coordinate system of a vector!

Blog Material (Pg 96 of Mathematical Methods in the physical Sciences [MM in PS]

Week 2

Modulus of a vector

Add to blog more about dot product

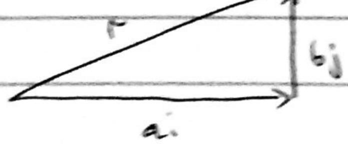
because of notation

$r = a\hat{i} + b\hat{j}$

$\hat{i}$ → unit vector because of notation

Note: the 'hat' notation on i and j refer to them being unit vector.

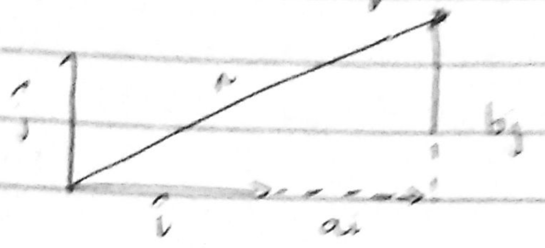
$r = \sqrt{a^2 + b^2}$



Orthogonal

Derivation

Pythagorean theorem and how it's connected to modulus of a vector?



$$r = ai + bj = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\boxed{|r| = \sqrt{a^2 + b^2}}$$

this is also applicable to any dimension of space where r lies.

we derive this from pythagoras theorem where r is the hypotenuse and 'a' is base whereas 'b' is height.

So in 3d space

$$r = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad ai + bj + ck$$

$$|r| = \sqrt{a^2 + b^2 + c^2}$$ this is true for length line segments in 3d space.

and so on for 4d.

$[3 \times -1] + [2 \times 2]$
$-3 + 4$
$\boxed{1}$

Dot product:

$r.s = r_i s_i + r_j s_j$

Properties:

→ Commutative ⇒ Order of operation doesn't matter

$\boxed{r.s = s.r}$

Intuition

Adding $r+s$

$\begin{bmatrix} +2 \\ 4 \end{bmatrix} r.s = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$|r+s| = \sqrt{2^2 + 4^2}$

$\sqrt{4+16}$

$\boxed{\sqrt{20}}$

[Derive the distributive property]

$r \cdot (s+t)$ — ①

$\rightarrow r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix} \quad s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \quad t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_n \end{bmatrix}$

to expand on the above equation — ①

$r_1 \cdot (s_1 + t_1) + r_2(s_2 + t_2)$ ----

$r_1 s_1 + r_1 t_1 + r_2 s_2 + r_2 t_2$ — $r_n s_n + r_n t_n$

$(r_1 s_1 + r_2 s_2 - + r_n s_n) + (r_n t_n + \_ r_1 t_i + r_2 t_2)$

$(r.s) + (r.t)$

$r.(s+t) = r.s + r.t$

→ Association

$r.(as) = a(r.s)$

$\rightarrow r_1 \cdot (a_1 \cdot s_1) + r_2 (a_2 \cdot s_2)$

$\rightarrow (r_1 \cdot s_1) a_1 + (r_2 s_2) a_2$

$a(r_1 s_1 + r_2 s_2)$

$\boxed{a(r s)}$

Data Science   → Mo Sol- → [illegible]

Axis: Bold vs not bold

(Check papers with vector to see how they are written get acclimated to notation)

Remember: that dot product is just a scalar ~~and thats~~

I

A dot product of a vector with itself is basically it modulus^2

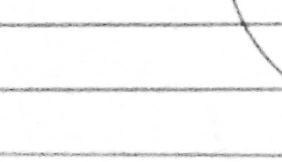
$$\boxed{\begin{aligned} r \cdot r &= r_1 r_1 + r_2 r_2 \\ &= r_1^2 + r_2^2 \\ &= (|r|)^2 \end{aligned}}$$

(This should remind you of the modulus)

Cosine and dot product

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$\cos C$



$b + c = a$

$\boxed{\vec{b} - \vec{a} = c}$

$r - s = \vec{c}$

$-s + r$

$r - s = \vec{c}$

$|r - s|^2 = |r|^2 + |s|^2 + 2|r||s| \cos C$

Dot $\boxed{\text{Pose} \Leftrightarrow \text{data}}_2$ $|r|$

similarity

$\boxed{r.r}\ r^2$

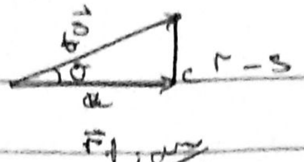
$|r|^2$

$|r| = \sqrt{(r_1)^2 + (r_2)^2}$

$|r|^2\ r_1^2 + r_2^2$

$\boxed{|r|^2}$

10 ms.



$c^2 = a^2 + b^2 - 2ab\cos C$

Derivation

$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$

10 ms

$|r|^2 = r.r$

$\boxed{(r-s)(r-s)} = r.r + s.s - 2|r||s|\cos\theta$

$\boxed{r.r - rs - sr + s.s}$

$|r|^2$ something $|s|^2$ $- 2sr$ ← this

$r.r - 2rs + s.s$
$|r|^2 + |s|^2 - 2rs.$ → should be the right hand side

$\cancel{|r|^2 + |s|^2} - 2rs = \cancel{|r|^2 + |s|^2} - 2|r||s|\cos\theta$

$\cancel{2}rs = \cancel{2}|r||s|\cos\theta$

$r.s = |r||s|\cos\theta$

Dot product is equal to magnitudes mult with $\cos\theta$ and each other

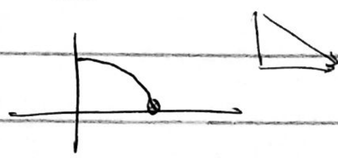
$$\boxed{\frac{r.s}{|r||s|} = \cos\theta}$$

→ And the unknown could be found.

Derive again?

$\boxed{\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = 0}$ $= \theta = 0$

$\cos 90$



Ortogonal is 90°.

$$r.s = |r||s| \cos 90$$

$$\cos 90 = 0$$

$$r.s = 0$$

$$r.s = |r||s| \cos 180$$

$$-1$$

$$r.s = |r||s|(-1)$$

0° between vectors.

$$r.s = |r||s| \cos 0$$

$$r.s = |r||s|$$

when the vectors are in the same direction the dot product = product of the magnitud.

90° between vectors

$$r.s = |r||s| \cdot 0$$

$\boxed{r.s = 0}$ dot product is 0 if the are orthogonal.

→ The dot product in a way is very suggestive of the ortogonality of two vectors

→ indicative is a better word.

180° b/w vect

$$r.s = |r||s| . -1$$

$$\boxed{r.s = -|r||s|}$$

Derivate

$$c^2 = a^2 + b^2 + 2ab \cos$$

$$|(r-s)|^2 = |r|^2 + |s|^2 + 2|r||s| \cos$$

$$(r-s)(r-s)$$

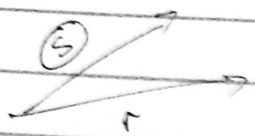
$$r^2 - rs - rs + s^2$$

$$\boxed{|r|^2 + |s|^2 - 2rs} = \cancel{|r|^2 + |s|^2} + 2|r||s| \cos$$

$$+\cancel{2}rs = +\cancel{2}|r||s| \cos \theta$$

$$r.s = |r||s| \cos \theta$$

$$\boxed{\frac{r.s}{|r||s|} = \cos \theta}$$



96 - 108
Section 4
Chapter 3.

Projection → What is the use of projection?

$\cos\theta = \frac{adj}{hyp}$

Very interesting
Talking from the perspective of the dot product. Dot product refers / estimates / indicates the projection of the vector s onto r

$\boxed{r \cdot s = |r|\underbrace{|s|\cos\theta}_{\text{adj side.}}}$

So in the case where $\theta = 90$ and there is no "shadow" the dot product will be equal to 0

if $\theta = 180$
$r \cdot s = |r|\underbrace{(-|s|)}_{\text{adjacent side.}}$
the dot product would be negative also indicating the "shadow" onto $-\vec{r}$

Scalar projection.

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{adj}{|s|}$

$\boxed{|s|\cos\theta = adj.}$

Scalar projection also found here

$r \cdot s = |r|\overbrace{|s|\cos\theta}$

$\frac{r \cdot s}{|r|} = |s|\cos\theta$

$\frac{r \cdot s}{|r|} = |s|\cos\theta$

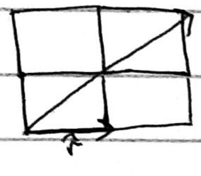
$\boxed{|s|\,0}$

In case of parallel vectors remember that

where r_1 and r_2 are components of r and b: $\frac{r_1}{b_1} = \frac{r_2}{b_2} = \frac{r_3}{b_3}$

Alternatively: the Scalar projection is the dot product of unit vector $\frac{r}{|r|}$ with s.

$\frac{r \cdot s}{|r|} = |s|\cos\theta.$



which is a vector.

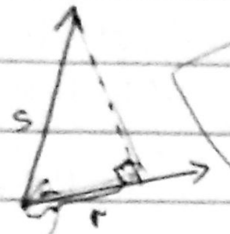
Vector Projection: (Gives information about r whereas Scalar projection talks about S primarily)

$\frac{r.s}{|r|\,|r|}.r$

$r . \frac{r.s}{|r|\,|r|}$ the length of the adjacent side on the shadow of (s).

This shadow is normalised to be of unit length by dividing with r. The encode information about r aswell as its projection from s we ~~must~~ multiply this scalar with r.

at 90° the shadow is will $\cos 90 = 0$



$\frac{r.s}{|r|}$

$\frac{r.s}{|r|} = |s|\cos\theta$

$\cos 1$

projection on the adj

[Quiz]

1) $s = \sqrt{(1)^2 + 5^2 + 4^2 + 2^2}$
$= \sqrt{30}$

2) (5) r 6 -2 r 0
1-2
-1

vect project

$\frac{r.s}{|r|} . \frac{r}{|r|}$ unit vect $\hat{r}$

sc project

(3) $\frac{\pi}{2}$ $\perp$ 90°

$\longrightarrow$

$\boxed{s \text{ onto } r}$

$r = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} \quad s = \begin{bmatrix} 10 \\ 5 \\ -6 \end{bmatrix}$

Scalar proj × unit vector (r)

$\dfrac{r \cdot s}{|r|} \cdot \dfrac{r}{|r|}$ ← r has dimension, has no reference

unit

$\dfrac{r \cdot s}{|r|} = |s| \cos\theta$

$\dfrac{30 - 20 + 0}{\sqrt{9 + 16}} \Rightarrow \dfrac{10}{5} = 2$

$\longleftrightarrow$

$r\left(\dfrac{r \cdot s}{|r|\,|r|}\right) = \dfrac{10}{5^2} = \dfrac{10}{25} = \dfrac{2}{5}$

$|r|^2 = r \cdot r$

$|r|^2$

$|r| \cdot |r| = |r|^2$?

$\begin{pmatrix} 6/5 \\ -8/5 \\ 0 \end{pmatrix}$

c) $|a + b|$ or $|a| + |b|$ larger

$a + b = [3, 5, 16]$

$\sqrt{9 + 25 + 256}$

$\sqrt{290}$

17

$\sqrt{9 + 16} + \sqrt{25 + 144}$

$5 + 13$

18

$a + b > c$

$c < a + b$ is the rule for triangle

$|c| < |a| + |b|$

$|a + b| < |a| + |b|$

Basically a vector projection maintains info about s and r in just one vector.

$(r.r) = \sqrt{|r|^2} = |r|$

$\boxed{r.r}$

$\cos\theta = \frac{r.s}{|r||s|}$

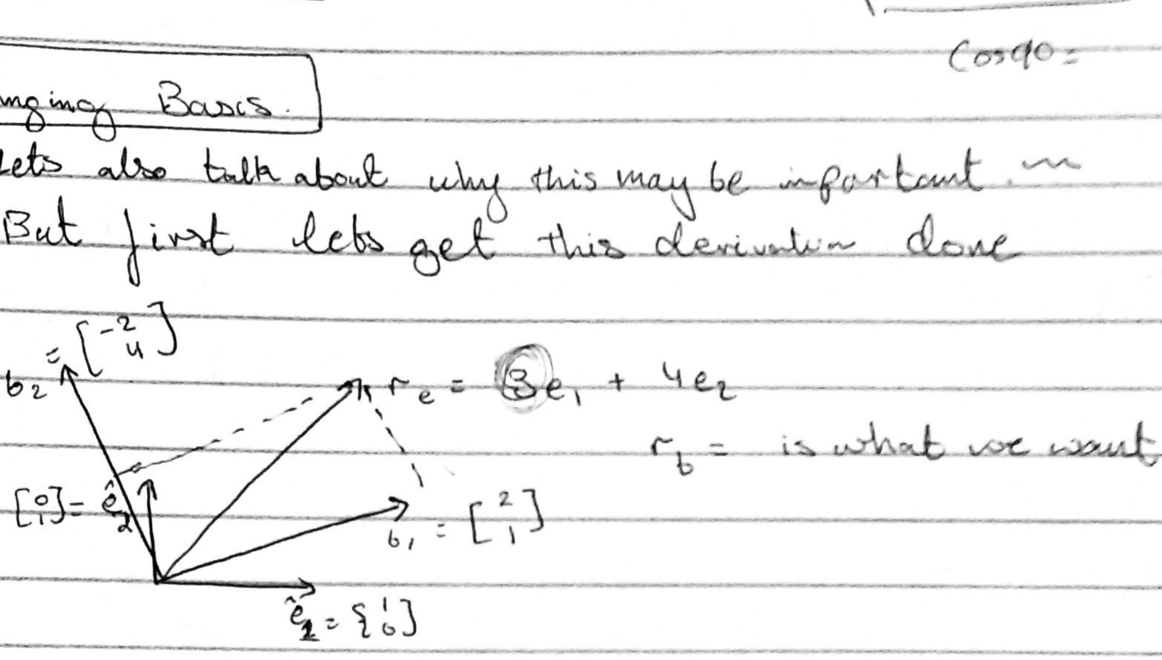
$\theta =$

$\boxed{\cos 0 = 1}$

$\cos 90 =$

Changing Basis

Lets also talk about why this may be important.
But first lets get this derivation done



r_b can be found by changing the basis vector by project r_e onto b_1 and then onto b_2
the ~~vector~~ scalar projection would be the constant beside the vector basis.

$|s|$ $|r|$

$r_e = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\frac{r.s}{|r|} = |s| \cos\theta$

$\frac{r_e . b_1}{|r_e|} = |s| \cos\theta$

$r.s = |r||s| \cos\theta$

$\cos\theta = \frac{adj}{hyp}$

S is projected onto r
so in this example
• r_e is being projected onto $b_{1,e}$

So the formula becomes.

$$\boxed{\frac{b_1 \cdot r_e}{|b_1|}} = |r_e| \cos\theta \qquad \rightarrow \text{hypotenuse}$$

being projected on — adjacent side

$$\frac{\begin{bmatrix}2\\1\end{bmatrix}\cdot\begin{bmatrix}3\\4\end{bmatrix}}{\sqrt{2^2+1^2}} \Rightarrow \frac{6+4}{\sqrt{5}} = \boxed{\frac{10}{\sqrt{5}}}$$ Scalar projection

$\longleftrightarrow$

$$\frac{b_2 \cdot r_e}{|b_2|} = \frac{\begin{bmatrix}-2\\4\end{bmatrix}\cdot\begin{bmatrix}3\\4\end{bmatrix}}{\sqrt{(-2)^2+16}} = \frac{-6+16}{\sqrt{20}} = \frac{10}{\sqrt{20}}$$

$$r_b = \frac{10}{\sqrt{5}} b_1 + \frac{10}{\sqrt{20}} b_2$$

$\frac{1}{2} b_1$

$\frac{1}{2}\begin{bmatrix}3\\4\end{bmatrix} \quad \begin{bmatrix}3/2\\2\end{bmatrix}$

This is only the case where the new basis are orthogonal to each other.

Okay interesting fact:

To change the basis

you need to first normalize the scalar projection with the new basis

Taking from the vector projection formula.

$$\boxed{\frac{r \cdot s}{|r|^2}} \cdot r \rightarrow \text{remember we use this only}$$

$\rightarrow$ This would be the coefficient to the the new basis in direction r.

So. r onto b_1

$$\frac{b_1 \cdot r}{|b_1|^2} = \frac{\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{(\sqrt{5})^2} = \frac{6+4}{5} = 2$$

$$\frac{b_2 \cdot r}{|b_2|^2} = \frac{\begin{bmatrix} -2 \\ 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{(\sqrt{20})^2} = \frac{10}{20} = \frac{1}{2}$$

$$\boxed{r_b = 2b_1 + \frac{1}{2} b_2}$$

$$r_b = \begin{bmatrix} 2 \\ 1/2 \end{bmatrix}$$

$1+0+0+0$

Q1 $v = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ $b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $b_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\frac{b_1 \cdot v}{|b_1|^2} = \frac{5+(-1)}{(\sqrt{1^2+1^2})^2} = \frac{4}{2} = 2$$

$$\frac{b_2 \cdot v}{|b_2|^2} = \frac{6}{2} = 3$$

$$\boxed{V_b = 2b_1 + 3b_2} \qquad v_b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

(2) $v = \begin{bmatrix} 10 \\ -5 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$$\frac{b_1 \cdot v}{|b_1|^2} = \frac{30-20}{(\sqrt{9+16})^2} = \frac{10}{25} = \frac{2}{5}$$

$$\frac{b_2 \cdot v}{|b_2|^2} = \frac{40+15}{(\sqrt{16+9})^2} = \frac{55}{25} = \frac{11}{5}$$

(3)

$$\frac{b_1 \cdot v}{|b_1|^2} = \frac{-6+2}{(\sqrt{9+1})^2} = \frac{-4}{10} = -\frac{2}{5}$$

$$\frac{b_2 \cdot v}{|b_2|^2} = \frac{2+6}{(\sqrt{1^2+9})^2} = \frac{\cancel{8}\,4}{\cancel{10}\,5} \quad \boxed{\frac{4}{5}}$$

(4)

$$\frac{b_1 \cdot v}{|b_1|^2} = \frac{2+1+0}{(\sqrt{2^2+1^2})^2} \quad \frac{3}{5}$$

$$\frac{b_2 \cdot v}{|b_2|^2} = \frac{1-2-1}{1^2+4+1^2} = \frac{-2}{6} = \boxed{-\frac{1}{3}}$$

$$\frac{b_3 \cdot v}{|b_3|^2} = \frac{-1+2\cancel{5}-5}{1+4+25} = \frac{-4}{30} = \boxed{-\frac{2}{15}}$$

Changing basis would be very important
when doing principal component analysis
by finding orthogonal vectors to many r_s where
the new basis would lower the dimensionality of
the basis. ~~That~~ lowering the dimension would [illegible] the
calculations easier and the projection would
lower the amount of data required for storing
data.

For e.g.

$$r_1e = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \qquad r_2e = \begin{bmatrix} 9 \\ 18 \\ 2 \end{bmatrix}$$

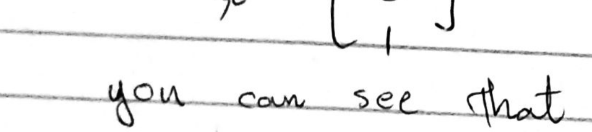
after changing
basis

$$r_{1,b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad r_{2.b} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

you can see that the new basis
introduces a trend in the given data/
vectors where
the third dimension
$r_{1b \to 3}$ is always $= 0$ or close to 0.
we can remove this dimension entirely.

$r_{1,b}$ becomes $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad r_2b$ becomes $= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Why do new basis have to be orthogonal?



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$b_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $b_2 = [$

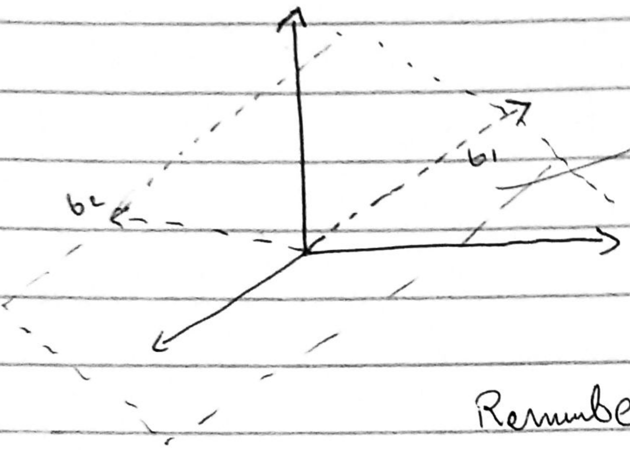
B_1

Linearly Independent:

$b_3 \neq a_1 b_1 + a_2 b_2$

i.e ~~Becasue~~ it is impossible for us to create b_3 with a combination of b_1 and b_2.

So for example if we look at the 3D space if we ~~&~~ have b_2 and b_1 we can only traverse a 2D plane inside that 3D space.



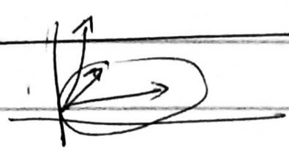
any vector in this 2D plane would not be linearly independent.

Remember: this new space introduced by 'b' basis will still be linearly spaced hence why the addition and other similar vector operation would work.

$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$b = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$b_2 \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$



-1 $\quad$ -3

$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 2 \end{bmatrix} + -3\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

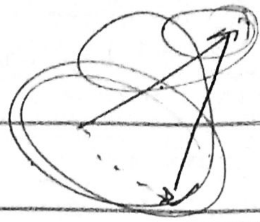
$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$\begin{bmatrix}3\\4\end{bmatrix} \; 2 \begin{bmatrix}-1\\-3\end{bmatrix} \; h$$

$$\begin{bmatrix}3\\4\end{bmatrix} + \begin{bmatrix}-2\\-6\end{bmatrix}$$

$$\begin{bmatrix}1\\-2\end{bmatrix}$$



Same importance

Next up Matrices

18 O68

Lecture 1:

Column

We revisit the concept of linear dependence.

Rank = The number of independent columns.

Note: ~~When~~ I want to do a lecture or two from MIT 18 O65 every week.

The column space is basicall $C(A)$ where $C(A)$ contains vectors from the colums of A which are linearly independent.

$$\begin{bmatrix}2 & 1 & 3\\3 & 1 & 4\\5 & 7 & 12\end{bmatrix} \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = x_1\begin{bmatrix}2\\3\\5\end{bmatrix} + x_2\begin{bmatrix}1\\1\\7\end{bmatrix} + x_3\begin{bmatrix}3\\4\\12\end{bmatrix}$$

3×3 $\quad$ 3×1

$$\begin{bmatrix}2 & 1\\3 & 1\\5 & 7\end{bmatrix} \begin{bmatrix}x_{00} & x_{01} & x_{02}\\x_{10} & x_{11} & x_{12}\end{bmatrix}$$