

# ELEC 4700 Harmonic Wave Equation in 2D FD and Modes

Tom Smy

Feb. 10<sup>rd</sup>, 2022. Due Feb. 12<sup>th</sup> @ midnight.

**Goal** In this PA you should familiarize with yourself with Finite Difference modeling of the harmonic wave equation in 2D and its formulation as a matrix equation and the solution for the modes.

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = \alpha E \quad (1)$$

## Tasks

### 1. Basic formulation

- First write out the finite difference form of the wave equation
- Formulate a node numbering scheme to go from the spatial matrix to the solution vector.
- How will you handle boundary conditions?
- Show this work to one of the TA's

### 2. Coding:

- (a) You are to formulate a wave equation mode solver using a matrix formulation. Set the BC's to be 0 on all sides.
  - i. Create a *sparse* G matrix (nx\*ny,nx\*ny) in size. Initially set nx = ny = 50;
  - ii. For the BC's nodes you will need to set the diagonal value G(m,m) to 1 and all other entries to 0 (G(m,:))
  - iii. For the bulk nodes the G entries are set by the FD equation
  - iv. Plot G using *spy()*
  - v. Use  $[E,D] = \text{eigs}(G,9,'SM')$  to get 9 eigenvectors and values. E is a matrix of vectors and D's diagonal is the eigenvalues.
  - vi. Plot the eigenvalues.
  - vii. Plot the eigenvectors using *surf()*. You will need to remap them onto a matrix that is (nx,ny) in size. Are they as expected?
  - viii. Change nx and ny so they are not equal. What occurs?
  - ix. Change the '-4' in the G matrix diagonal to '-2' for a region of space (say  $i > 10 \ \& \ i < 20 \ \& \ j > 10 \ \& \ j < 20$ ) what happens? Physically what is this doing?

**Checkout** When you are Ready:

1. Create a new repo on your github account called EIPA
2. Clone the repo to your machine
3. Add your code to the repo, commit, and push it back to github
4. Check that it worked, if it did, you're all set