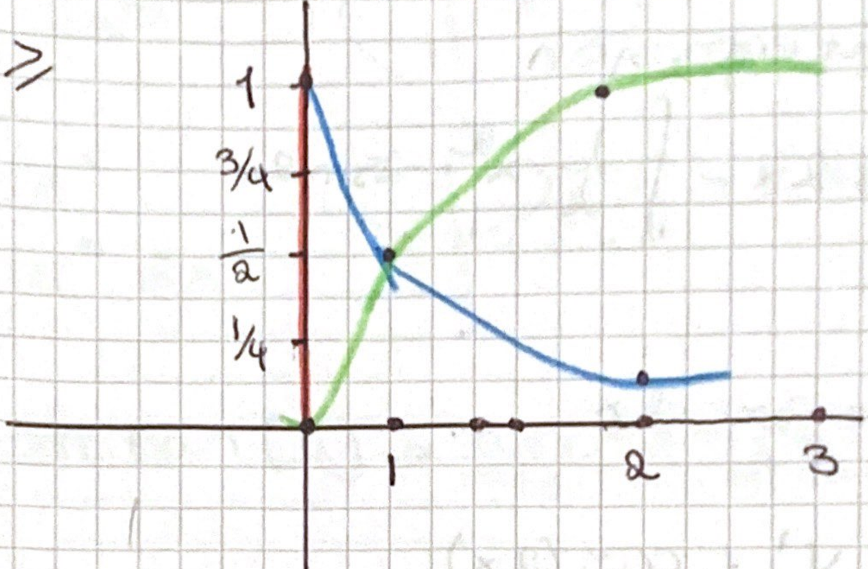
oppgåve 4

a) området R i planet er begrenset av kurvene

$x = 0$ $\quad y_1 = \dfrac{1}{1+x^2}$ og $y_2 = \dfrac{x^2}{1+x^2}$

$x \geq$



$y_1(0) = \dfrac{1}{1+0^2} = 1$ $\qquad y_2(0) = \dfrac{0^2}{1+0^2}$

$y_1(1) = \dfrac{1}{1+1} = \dfrac{1}{2}$ $\qquad y_2(1) = \dfrac{1^2}{1+1^2} = \dfrac{1}{2}$

$y_1(2) = \dfrac{1}{1+2^2} = \dfrac{1}{5}$ $\qquad y_2(2) = \dfrac{2^2}{1+2^2} = \dfrac{4}{5}$

Arealet av R:

$$\int_0^1 \frac{1}{1+x^2} - \int_0^1 \frac{x^2}{1+x^2}$$

① $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx = \int_0^1 \frac{1}{1+\frac{\cos^2 u}{\sin^2 u}} \cdot \frac{1}{\cos^2 u}\,du = \int_0^1 \frac{1}{\frac{\cos^2 u}{\cos^2 u} + \frac{\sin^2 u}{\cos^2 u}} \cdot \frac{1}{\cos^2 u}$

$$x = \tan(u) = \frac{\sin u}{\cos u}$$

$$dx = \frac{\cos u \cdot \cos u - \sin u \cdot \sin u}{\cos^2 u}$$

$$= \int \frac{1}{\frac{\cos^2 u + \sin^2 u}{\cos^2 u}} \cdot \frac{1}{\cos^2 u}\,du = \int \frac{1}{\frac{1}{\cos^2 u}} \cdot \frac{1}{\cos^2 u}\,du$$

$$= \int 1\,du = u = \tan^{-1}(x) = \left[\tan^{-1}(x)\right]_0^1$$

$$x = \tan u$$

$$\tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \underline{\frac{\pi}{4}}$$

② $\displaystyle\int_0^1 \frac{x^2}{1+x^2}\,dx = \int_0^1 \frac{x^2+1-1}{1+x^2}\,dx$

$$= \int_0^1 \underbrace{\frac{1+x^2}{1+x^2}}_{1} - \frac{1}{1+x^2}\,dx = \int_0^1 1\,dx - \int_0^1 \frac{1}{1+x^2}$$

$$= \left[x - \tan^{-1}(x)\right]_0^1 = 1 - \frac{\pi}{4} = \frac{4}{4} - \frac{\pi}{4} = \underline{\frac{(4-\pi)}{4}}$$

arealet: $\displaystyle\frac{\pi}{4} - \frac{(4-\pi)}{4} = \frac{2\pi - 4}{4}$

$$= \underline{\underline{\frac{\pi}{2} - 1}}$$

b) rotasjon om y-aksen

$$V = 2\pi \int_0^1 x\left(\frac{1}{1+x^2} - \frac{x^2}{1+x^2}\right)dx$$

$$V = 2\pi \int_0^1 \frac{x - x^3}{1+x^2}\,dx$$

$$= -\int_0^1 \frac{x^3 - x}{x^2+1}\,dx \qquad u = x^2+1 \qquad \frac{du}{dx} = 2x$$

$$dx = \frac{1}{2x}\,du$$

$$= -\frac{1}{2}\int_0^1 \frac{u-2}{u}\,du$$

$$= \int \left(1 - \frac{2}{u}\right) du$$

$= u - 2\ln u$ sett inn x^2+1

$$= \ln(x^2+1) - \frac{x^2+1}{2} \Rightarrow \ln(x^2+1) - \frac{x^2}{2}$$

$$2\pi\left[\ln(x^2+1) - \frac{x^2}{2}\right]_0^1 = \underline{\underline{2\pi \ln 2 - \pi}}$$

2c ~~[illegible]~~

Skivemetoden

$$V = \pi \int_0^{1/2} \left(\sqrt{\frac{y}{1-y}}\right)^2 dy + \pi \int_{1/2}^{1} \left(\sqrt{\frac{1}{y}-1}\right)^2 dy$$

$$= \pi \int_0^{1/2} \frac{y}{1-y}\, dy = -\pi \int_0^{1/2} \frac{y}{y-1}$$

$u = y-1$

$dy = du$

$$= \int \left(\frac{1}{u} + 1\right) du = \int \frac{1}{u}\, du + \int 1\, du$$

$$= \ln(u) + u$$

$$= \pi\left[-y - \ln|\cancel{\ }y-1|\right]_0^{1/2} = -\pi \ln 1 + \pi \ln 2 - \frac{\pi}{2}$$

$$= \pi \int_{1/2}^{1} \frac{1}{y} - 1\, dy = \int \frac{1}{y}\, dy - \int 1\, dy$$

$$= \pi\left[\ln|y| - y\right]_{1/2}^{1} = \pi\left(-1 - (\ln(1/2) - 1/2)\right)$$

$$= -\frac{\pi}{2} + \pi \ln 2$$

$$= \underline{\underline{2\pi \ln 2 - \pi}}$$