Noora Lindeflaten

Oblig 4

Oppgåve 2a)

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{1}{1+(e^x)^2}\cdot e^x\,dx$$

$u = e^x$

$du = e^x\,dx$

$$= \int \frac{1}{1+u^2} + du$$

$$= \arctan(u) = \underline{\underline{\arctan(e^x) + C}}$$

b) $$\int \frac{1}{\sqrt{x}\ 4x}\,dx$$

$2u\,du = dx$

$u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}\,dx$

$$\int \frac{1}{u^2+u}\cdot 2u\,du = 2\int \frac{u}{u(u+1)}\,du = 2\int \frac{1}{u+1}\,du$$

$$= 2\ln(u+1) = \underline{\underline{2\ln(\sqrt{x}+1) + C}}$$

c) $\int \frac{3x+7}{x^3-1}\,dx$ (for $x \neq 1$)

$$\frac{3x+7}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+x+1)}$$

$$= \frac{A(x^2+x+1) + B(x^2-x) + C(x-1)}{(x-1)(x^2+x+1)}$$

$$3x+7 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$$

$A+B=0$

$A-B+C=3$

$A-C=7$

$AC=7 \Rightarrow A=7+C \Rightarrow C=A-7$

$A+B=0 \Rightarrow B=-A \Rightarrow B=-(7+C)$

$A-B+C=3$

$A=7+C$

$(7+C)-B+C=3$

$7+C-B+C=3$

$2C-B=-4$

$2C+7+C=-4$

$3C=-11$

$C=-\frac{11}{3} \rightarrow A=7+C \Rightarrow A=7-\frac{11}{3}$

$= \frac{10}{3}$

$B=-A \Rightarrow B=-\frac{10}{3}$

Derfor:

$$\int \frac{3x+7}{x^3-1}\,dx = \frac{10}{3}\int \frac{1}{x-1}\,dx - \frac{1}{3}\int \frac{10x+11}{x^2+x+1}$$

$$\frac{3}{10}\int \frac{1}{x-1}\,dx =$$

$$= \frac{10\ln|x-1|}{3} + C$$

$$-\frac{1}{3}\int \frac{10x+11}{x^2+x+1} =$$ ser at derivert av $x^2+x+1 = 2x+1$

$$= -\frac{1}{3}\int \frac{10x+11}{x^2+x+1}\,dx = -\frac{1}{3}\int \left(\frac{5(2x+1)}{x^2+x+1} + \frac{6}{x^2+x+1}\right)$$

$$= -\frac{1}{3}\cdot 5\int \frac{2x+1}{x^2+x+1} = \frac{5}{3}\int \frac{1}{u}\,du$$

$$= \frac{5}{3}\ln|u| + C = \frac{5}{3}\ln|x^2+x+1| + C$$

$$\frac{1}{3}\int \frac{6}{x^2+x+1} = -2\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$$

$$\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}$$

$u = 2x+1$

$du = \frac{2}{\sqrt{3}}\,dx$

$dx = \frac{\sqrt{3}}{2}\,du$

$$= \int \frac{2\sqrt{3}}{3u^2+3}\,du$$

$$= \frac{2}{\sqrt{3}}\int \frac{1}{u^2+1}\,du$$

$$\Rightarrow -2 \cdot \frac{2}{\sqrt{3}} \cdot \tan^{-1}(u)$$

$$= \frac{-4\sqrt{3} \cdot \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3}$$

Så:

$$\int \frac{3x+7}{x^2-1}$$

$$= \frac{5\ln(x^2+x+1)}{3} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{10\ln|x-1|}{\sqrt{3}} + C$$

d) $\int x \cos 2x \; dx$

$u = x \qquad v' = \cos(2x)$

$u' = 1 \qquad v = \frac{1}{2} \sin(2x)$

$\frac{1}{2} x \sin(2x) - \int \frac{1}{2} \sin(2x)$

$\frac{1}{2} x \sin(2x) - \frac{1}{2} \cdot \frac{1}{2} \cos 2x$

$\frac{1}{2} x \sin(2x) - \frac{1}{2} \cdot \frac{1}{2} \cos 2x + C$

$= \frac{1}{2} x \sin(2x) - \frac{1}{4} \cos 2x + C$

2e

$$\int \sqrt{1-x^2} \arccos \sqrt{1-x^2}$$

$u = 1 - x^2$

$x = \sqrt{1-u}$

$\frac{dx}{du} -2x$

$dx = \frac{du}{\sqrt{1-u}}$

$\frac{du}{dx} = -2x$

$$\int \frac{\sqrt{u} \arccos \sqrt{u}}{\sqrt{1-u}}$$

$u = \sqrt{1-x^2}$

$-\frac{x}{\sqrt{1-x^2}} = \frac{dx}{du}$

$u = (1-x^2)^{\frac{1}{2}}$

$\frac{-(\sqrt{1-u})}{u} = \frac{du}{dx}$

$u = 1^{\frac{1}{2}} - x$

$x = 1^{\frac{1}{2}} - u$

$$\int u \arccos u - \left(\frac{\sqrt{1-u}}{u}\right)$$

$$\int u \arccos u - \frac{\sqrt{1-u}}{u}\, du$$

$$\underbrace{\int u \arccos u \, du} + \int \frac{1-u}{u}\, du$$

delvis integral

$f = \arccos u \qquad g' = u$

$f' = -\frac{u}{\sqrt{1-u^2}} \qquad g = \frac{u^2}{u}$

$$\frac{u^2 \arccos u}{2} - \underbrace{\int -\frac{u^2}{2\sqrt{1-u^2}}\, du}$$

$-$

$$-\frac{1}{2} \underbrace{\int \frac{u^2}{\sqrt{1-u^2}}\, du}$$

$\frac{\arcsin(u)}{2} - \frac{u\sqrt{1-u^2}}{2}$

trigonometrisk substitusjon(v

$$\int \frac{\cos(v)\sin^2(v)}{\sqrt{1-\sin^2(v)}}\, dv$$

$$= \int \sin^2 v \, dv$$

$$= \underbrace{-\frac{\cos(v)\sin(v)}{2} + \frac{v}{2}}$$

tar inn løste integral

$$-\frac{1}{2}\int\frac{u^2}{\sqrt{1-u^2}}\,du$$

$$=\frac{u\sqrt{1-u^2}}{4}-\frac{\arcsin(u)}{4}$$

plugger inn løste integral

$$\frac{u^2\arccos u}{4}-\int-\frac{u^2}{2\sqrt{1-u^2}}\,du$$

$$=\frac{\arcsin(u)}{4}+u^2\frac{\arccos(u)}{2}-\frac{u\sqrt{1-u^2}}{4}$$

tilbake til

$$\int\frac{u-1}{u}\,du \qquad \int\left(1-\frac{1}{u}\right)du$$

$$\int 1\,du-\int\frac{1}{u}\,dx$$

$$=u-\ln(u)$$

$$=-\ln(u)+\frac{\arcsin(u)+2u\arccos(u)-u\sqrt{1-u^2}}{4}$$

$$+u+C$$

Lösning

$$-\ln\left|\sqrt{1-x^2}\right| + \frac{\arcsin\left(\sqrt{1-x^2}\right)}{4}$$

$$+ \frac{2(1-x^2)\arccos\left(\sqrt{1-x^2}\right) - \sqrt{1-x^2}\cdot\sqrt{1-\left(\sqrt{1-x^2}\right)}}{4}$$

$$+ \sqrt{1-x^2} + C$$

förenkla

$$-\ln\left|\sqrt{1-x^2}\right| + \frac{\frac{\arcsin\sqrt{1-x^2}}{2} - \frac{\sqrt{1-x^2}\cdot\sqrt{1-\left(\sqrt{1-x^2}\right)}}{2}}{2}$$

$$+ \frac{(1-x^2)\arccos\left(\sqrt{1-x^2}\right)}{2} + \sqrt{1-x^2} + C$$