MNF 130 oblig 3

oppgave 1

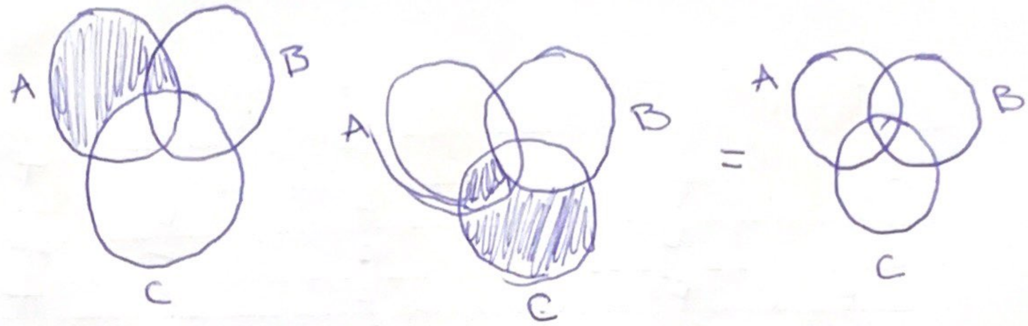
a)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$	$p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T
						T	T

Doora Lindeflaten

oppgave 2

a)



$(A-C) \quad \cap \quad (C-B) \quad = \quad \emptyset$

here we can see that they have no common region, hence $(A-B) \cap (C-B) = \emptyset$

$x \in (A-C) \cap (C-B)$

$x \in (A-C) \wedge x \in (C-B)$

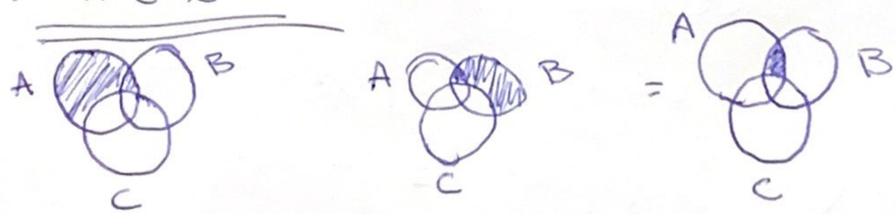
$\rightarrow x \in A \wedge x \notin C \wedge x \in C \wedge x \notin B$

$\rightarrow x \in A \wedge x \notin B \wedge x \in C \wedge x \notin C$

$\rightarrow x \in A \wedge x \notin B \wedge \underline{\text{False}}$

$\rightarrow x \in \emptyset$

b)



$(A-C) \quad \cap \quad (B-C)$

$x \in (A-C) \cap (B-C)$

$x \in (A-C) \wedge x \in (B-C)$

$\rightarrow (x \in A \wedge x \notin C) \wedge (x \in B \wedge x \notin C)$

$\rightarrow (x \in A \wedge x \in B) \wedge (x \notin C \wedge x \notin C)$

$\rightarrow (x \in A \wedge x \in B) \wedge x \notin C$

$\rightarrow \underline{x \in (A \cap B) - C}$

c)

Naora Lindeflaten

$f(x) = x^2$

Checking one-to-one

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

$f(x_1) = f(x_2)$

$x_1 = x_2$ or $x_1 = -x_2$

Since x_1 does not have a unique image

f is not one-to-one

Checking onto

$f(x) = x^2$

Let $f(x) = y$ such that $y \in R$

$x^2 = y$ $\quad x = \pm\sqrt{y}$

y is a real number so it can be negative

$y = -1$

$x = \pm\sqrt{(-1)}$ which is not possible as root of negative number is not real

hence x is not real

so f is not onto

since the function is neither one-to-one or onto it is not a one-to-one correspondence

Noora Lindeflaten

oppgåve 3

a) proving $\frac{(a+1)(a+2)}{3}$ is true

to prove that a is an integer
that is not divisible by 3
a has to leave a remainder in
this case either 1 or 2.

for some integer k
$a = 3k+1$ so that $(a+1)(a+2) =$
$(3k+2)(3k+3) = 3(3k+2)(k+1)$ and
so is divisible by three

we do the same for 2
$a = 3k+2$ such that $(a+1)(a+2)$
$= (3k+3)(3k+4) = 3(k+1)(3k+4)$ and so
is divisible by 3.

b) Using the Euclidean algorithm
to find gcd(252, 356)

$356 = 1 \cdot 252 + 104$
$252 = 2 \cdot 104 + 44$
$104 = 2 \cdot 44 + 16$
$16 = 1 \cdot 12 + 4$
$12 = 3 \cdot 4$

$\underline{\underline{4}}$

c)

Noora Lindeflaten

- $(177 \bmod 31 + 270 \bmod 31) \bmod 31 = 13$

$177 \bmod 31$

$a = 177 = 155 + 22 = 5 \cdot 31 + 22 = 5d + 22$

$177 \bmod 31 = 22$

$270 \bmod 31$

$a = 270 = 248 + 22 = 8 \cdot 31 + 22 = 8d + 22$

$270 \bmod 31 = 22$

$(22 + 22) \bmod 31 = 44 \bmod 31$

$44 - 31 \bmod 31 = 13 \bmod 31 = \underline{\underline{13}}$

- $[5(99^2 \bmod 32)] \bmod 15$

$99^2 \bmod 32$

$99^2 = 9801$

$a = 99^2 = \cancel{}\, 9796 + 5 = 306 \cdot 32 + 5 = 306d + 5$

$(5 \cdot 5) \bmod 15$

$25 \bmod 15 = 25 - 15 \bmod 15$

$10 \bmod 15 = 10$

oppgåve 4

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

step (1) show that it's true for $n=1$

$$1^2 = \frac{1(1+1)(2\cdot1+1)}{6} = \frac{1\cdot2\cdot3}{6} = \underline{\underline{1}}$$

$$\underline{1 = 1} \quad \text{True!}$$

(2) Assume $n=m$ is true

Assume $\sum_{k=1}^{m} k^2 = \frac{1}{6} m(m+1)(2m+1)$ for some $m \in \mathbb{N}$

(3) show that if $n=m$ is true then $n=m+1$ is also true

$$\sum_{k=1}^{m+1} k^2 = \sum_{k=1}^{m} k^2 + (m+1)^2$$

$k = 1, 2, 3 \ldots, m, m+1$

$1^2, 2^2, 3^2 \ldots, + m^2 + (m+1)^2$

$$\sum_{k=1}^{m+1} k^2 = \sum_{k=1}^{m} k^2 + (m+1)^2$$

$$= \frac{1}{6} m (m+1)(2m+1) \cdot A (m+9)^2$$

$$= \frac{1}{6} m (m+1)(2m+1) + \frac{6}{6} (m+1)(m+1)$$

$$= \frac{1}{6} (m+1) [m(2m+1) + 6(m+1)]$$

$$= \frac{1}{6} (m+1)(2m^2 + m + 6m + 6)$$

$$= \frac{1}{6} (m+1)(2m^2 + 7m + 6)$$

$$= \frac{1}{6} (m+1)(2m^2 + 4m + 3m + 6)$$

$$= \frac{1}{6} (m+1) [2m(m+2) + 3(m+2)]$$

$$= \frac{1}{6} (m+1)(m+2)(2m+3)$$

$$= \frac{1}{6} (m+1)[(m+1)+1]\,[2(m+1)+1]$$

the statement is true for $n=1$
and also true for $n=m$ which implied
that $n=m+1$ is also true, the
statement is true Q.E.D

1b)

Noora

$(p \to q) \land (q \to r) \to (p \to r)$ is always true because:

$$\equiv \neg(\neg p \lor q) \land (\neg q \lor r) \lor (\neg p \lor r)$$

$$\equiv \neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor \neg p \lor r$$

$$\equiv (\neg\neg p \land \neg q) \lor (\neg\neg q \land \neg r) \lor \neg p \lor r$$

$$\equiv \neg p \lor (\neg\neg p \land \neg q) \lor (\neg\neg q \land \neg r) \lor r$$

$$\equiv ((\neg p \lor \neg\neg p) \land (\neg p \lor \neg q)) \lor ((\neg\neg q \lor r) \land (\neg r \lor r))$$

$$\equiv (T \land (\neg p \lor \neg q)) \lor ((\neg\neg q \lor r) \land T)$$

$$\equiv (\neg p \lor \neg q) \lor (\neg\neg q \lor r)$$

$$\equiv \neg p \lor (\neg q \lor \neg\neg q) \lor r$$

$$\equiv \neg p \lor T \lor r$$

$$\equiv \underline{\underline{T}}$$

p	q	r	$\neg q$	$(p \wedge \neg q)$	$(p \wedge \neg q) \to r$	$q \vee r$	$p \to (q \to r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	F	F	T	F	T

Noora Lindeltaten

c)

p	q	r	¬q	(p∧¬q)	(p∧¬q)→r	q∨r	p→(q→r)
T	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	F	F	T	F	T

d)