Problem 1

a) we can find the number of leaves and verticies by using induction

- induction step: we know that there is a full binary tree consisting of a single vertex r, this r is a single leaf node

which gives us:

$l(T) = 1$ and $n(T) = 2l(T) - 1 = 2 \cdot 1 - 1$
$= 1$. so the property is true for 1 or a single node.

- Recursive step:

since T_1 and T_2 are disjoint full binary trees if they have the same height h then the tree $T = T_1 \cdot T_2$ is also a full binary tree of height $h+1$

Assume that this is true for T_1 and T_2 then T_1 and T_2 have $n(T)$ number of vertices and $l(T)$ number of leaves if you now form a new binary tree the number of vertices are: $n(T) + n(T) + 1$ in total $2n(T) + 1$ vertices and the new tree contains $l(T) + l(T)$ leaves or $2l(T)$

b) from the task above we know our basis step:

$$n(T) = 1 + n(T_1) + n(T_2)$$
$$l(T) = l(T_1) + l(T_2)$$

if we have $n(T_1) = 2l(T_1) - 1$ and $n(T_2) = 2l(T_2) - 1$

then

$$\begin{aligned} n(T) &= 1 + n(T_1) + n(T_2) \\ &= 1 + 2l(T_1) - 1 + 2l(T_2) - 1 \\ &= 2(l(T_1) + l(T_2)) - 1 \\ &= 2l(T_1, T_2) - 1 \\ &= 2l(T) - 1 \end{aligned}$$

Q.E.D

2) if you first draw four different cards the worst case is that all of these are from different suits and at worst this happens again, but no matter what the 9th card will be a third from one of the suits

so the answer is 9

2b

for each of the 26 black cards we have 26 different parings which means there are 26^2 different parings which contains a red and a black card

c) When dividing the 52 cards among the four players we have 4 options for each card. This is because player 1 can have all the cards and the rest could be zero, so the answer is $\underline{\underline{4^{52}}}$

Problem 3

Let R be a binary relation on the set of integers such that $(a, b) \in R$ if and only if $b - a = 1$

a) $R \circ R = R$? $((R \circ R)(0,1) = (0,1), (R \circ R)(1,2) = (1,2))$

b) $((a,b),(c,d)) \in R$

Reflexive: ~~basis~~ if $a \equiv a$, then

$$b - a = b - a$$

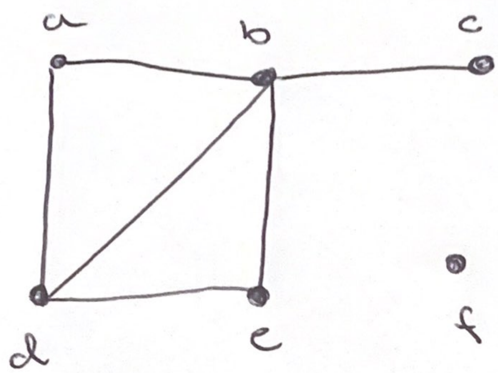
Symetric: if $a \equiv b$, then $b \equiv a$

$\Rightarrow$ if $b - a \equiv d - c$ then $c - d = a - b$

transitive: if $a \equiv b$ and $b \equiv c$, then

$a \equiv c \Rightarrow$?

problem 4



number of vertices = 6

degree of each vertex:

a: 2 b: 4 c: 1

d: 3 e: 2 f: 0

number of edges = 6