$$E_3(x) = \frac{\cos(s)}{4!}(x-0)^4 = \frac{\cos(s)}{24} \cdot x^4$$

$$f(x) = \cos(0) + \sin(0) \cdot (x-0) - \frac{\cos(0)}{2!}(x-0)^2$$

$$+ \frac{\sin(0)}{3!}(x-0)^3$$

$$f(x) = 1 - \frac{1}{2!}(x)^2 = 1 - \frac{1}{2}x^2 = \underline{\underline{1 - \frac{x^2}{2}}}$$

for ein s mellom 0 og x

cos(s) vil alltid ligge mellom -1 og 1

$$0 \leqslant \frac{\cos(s) x^4}{24} \leqslant \frac{x^4}{24}$$

$$0 \leqslant E_3(x) \leqslant \frac{x^4}{24}$$

b) $x < S < a$ der $a = 0$

og $x = \frac{1}{4}$ der $\cos 0 > \cos \frac{1}{4}$

$f^{(4)}(x) < f^{(4)}(S) < f^{(4)}(a)$

$\cos \frac{1}{4} < f''(s) < 1$

$$\frac{\cos \frac{1}{4}}{24} < \frac{f^4(s)}{4!} < \frac{1}{24}$$

$$\frac{\cos \frac{1}{4}}{24}(x-0)^4 < \frac{f^4(s)}{4!}(x-0)^4 < \frac{1}{24}(x-0)^4$$

$$\frac{\cos \frac{1}{4}}{24}x^4 < E_3(x) < \frac{1}{24}x^4$$

$f(x) = \cos x$

$$\int \sqrt{x} \cdot f(x)\, dx \approx \int \sqrt{x} \cdot P_3(x)\, dx$$

$$= \int \sqrt{x}\left(1 - \frac{x^2}{2}\right) dx = \int \sqrt{x} - \frac{1}{2}x^2$$

$$\Rightarrow \left[-\frac{x^{3/2}(3x^2-14)}{21}\right]_0^{1/4}$$

$$x = 1/4 \Rightarrow \frac{2}{3}\left(\frac{1}{4}\right)^{2/3} - \frac{1}{7}\left(\frac{1}{4}\right)^{7/2}$$

$$\Rightarrow \frac{221}{2688}$$

$$\int \sqrt{x} \cdot E_3(x)\, dx$$

$$\int_0^{1/4} \sqrt{x} \cdot \frac{\cos 1/4}{24} \cdot x^4\, dx = \frac{\cos 1/4}{24} \int_0^{1/4} x^{9/2}\, dx$$

$$= \frac{\cos 1/4}{24} \cdot \left[\frac{2}{11} \cdot x^{11/2}\right]$$

$$x = 1/4 \Rightarrow \frac{\cos 1/4}{24} \cdot \frac{2}{11} \cdot \left(\frac{1}{4}\right)^{11/2}$$

$$= 3,58410 \cdot 10^{-6} > 0.0000035$$

$$\int \sqrt{x}\, \frac{1}{24} x^4\, dx = \frac{1}{24} \int x^{9/2}\, dx$$

$$= \frac{1}{24} \left[\frac{2}{11} \cdot x^{11/2}\right]$$

$$x^{1/4} = \frac{1}{24} \cdot \frac{2}{11} \cdot \left(\frac{1}{4}\right)^{11/2} = \frac{1}{270336}$$

$$\cdot \sqrt{x} \cdot E_{3\,min}(x) < \int_0^{1/4} \sqrt{x} \cdot f(x)\, dx - \int \sqrt{x}\, P_3(x)\, dx$$

$$< \int_0^{1/4} \sqrt{x}\, E_3 \max(x)\, dx$$

$$0.0000035 < \int_0^{1/4} \sqrt{x} \cos(x)\, dx - \frac{221}{2688} < \frac{1}{270336} < 0.0000037$$