# Computing Theory

COMP 147 (4 units)

Chapter 1: Regular Languages
Section 1.1: Finite Automata

# Course Segments

- Automata and Languages
  - How can we define abstract models of computers?
- Computability Theory
  - What can (or cannot) be computed?
- Complexity Theory
  - What makes some problems computationally difficult?

# Computational Models

We'll look at three computational models:

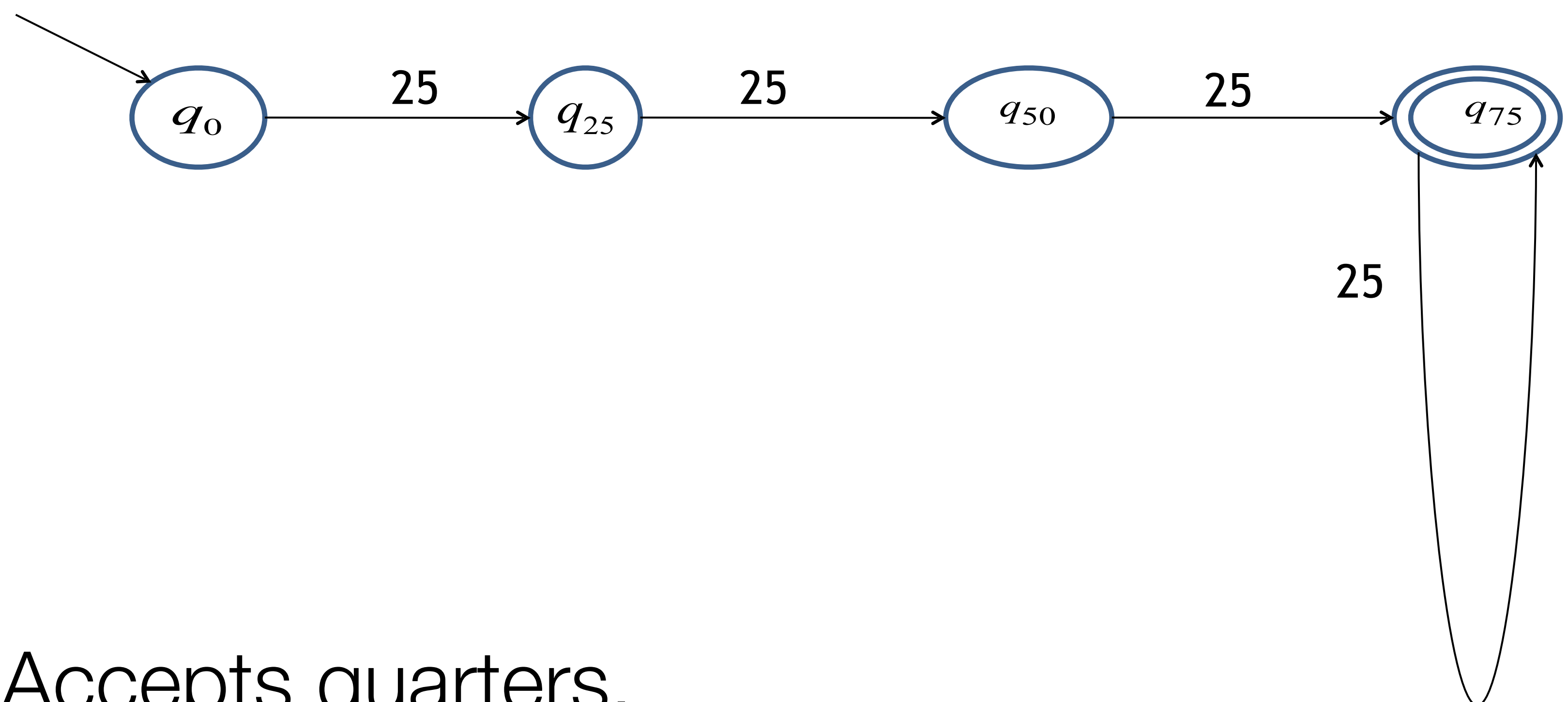
1. Finite Automaton (in short FA) recognize Regular Languages
2. Push Down Automaton (in short PDA) recognize Context Free Languages .
3. Turing Machines (in short TM) recognize Computable Languages .

increasing computational power

# FA: Washing Machine Example

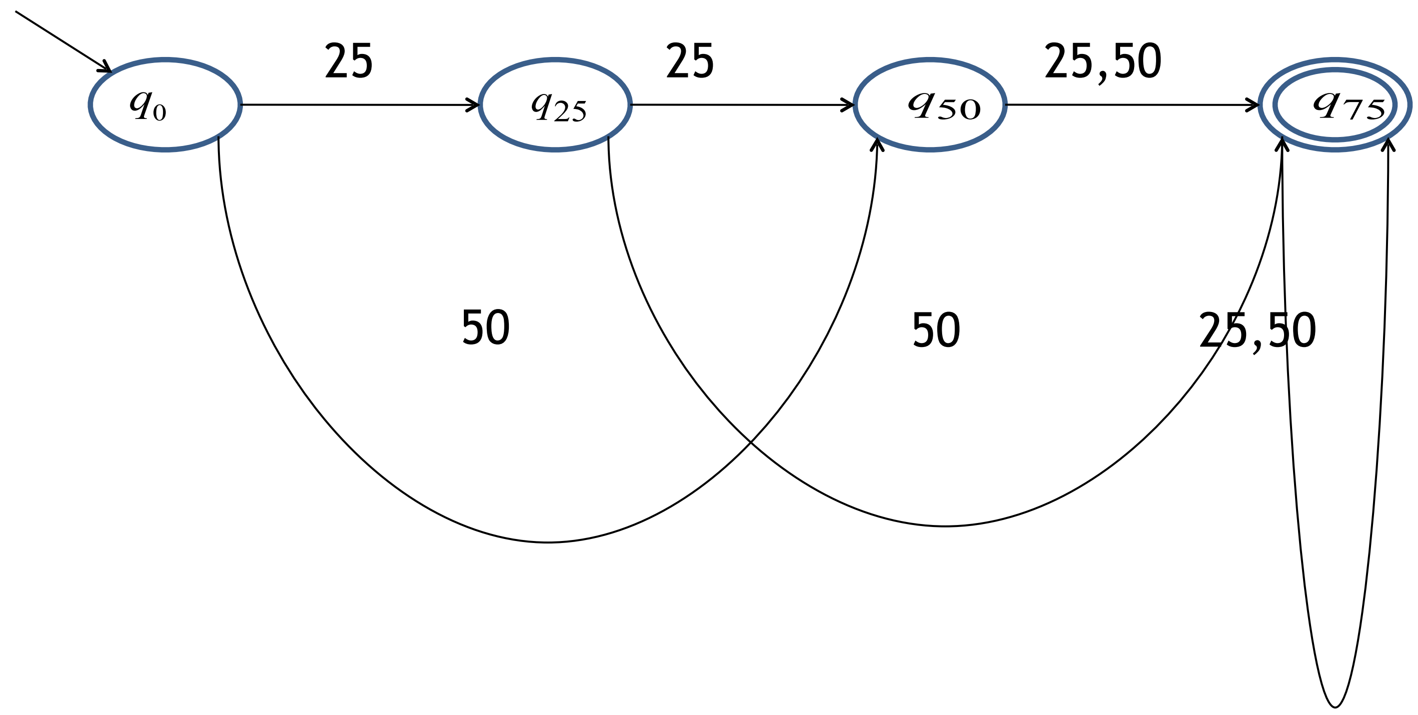
- The control of a washing machine is a very simple example of a finite automaton.
- The most simple washing machine accepts quarters and operation does not start until at least 3 quarters are inserted.

# FA: Washing Machine Example



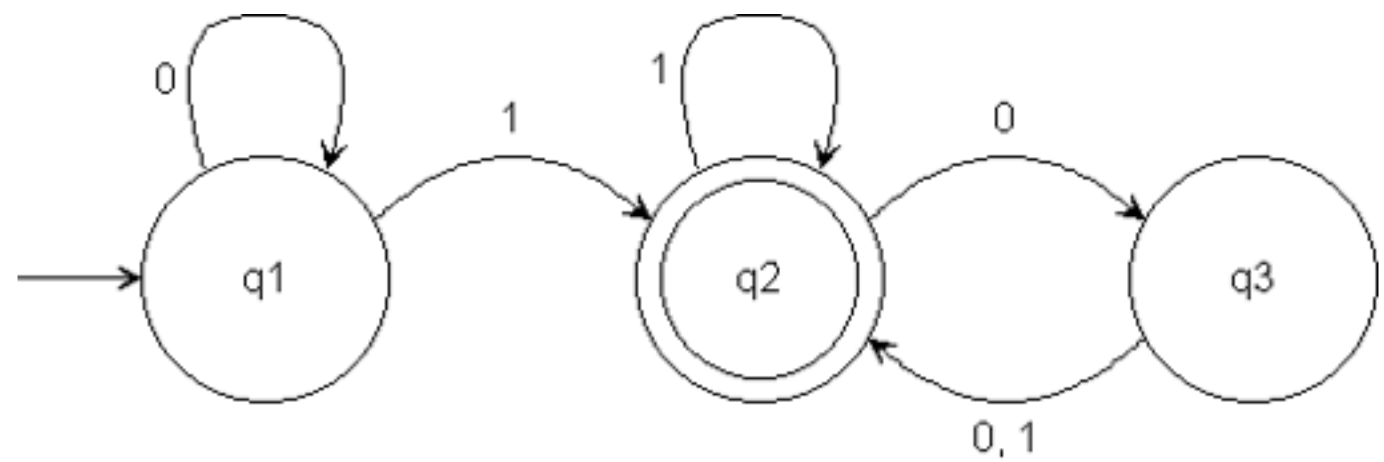

- Accepts quarters.
- Operation starts after at least 3 quarters were inserted.
- Accepted strings: 25,25,25; 25,25,25,25; …

# FA: Washing Machine Example




- A second washing machine also accepts half-dollar coins.
- Accepted strings: 25,25,25; 25,50; …

# FA: Second Example



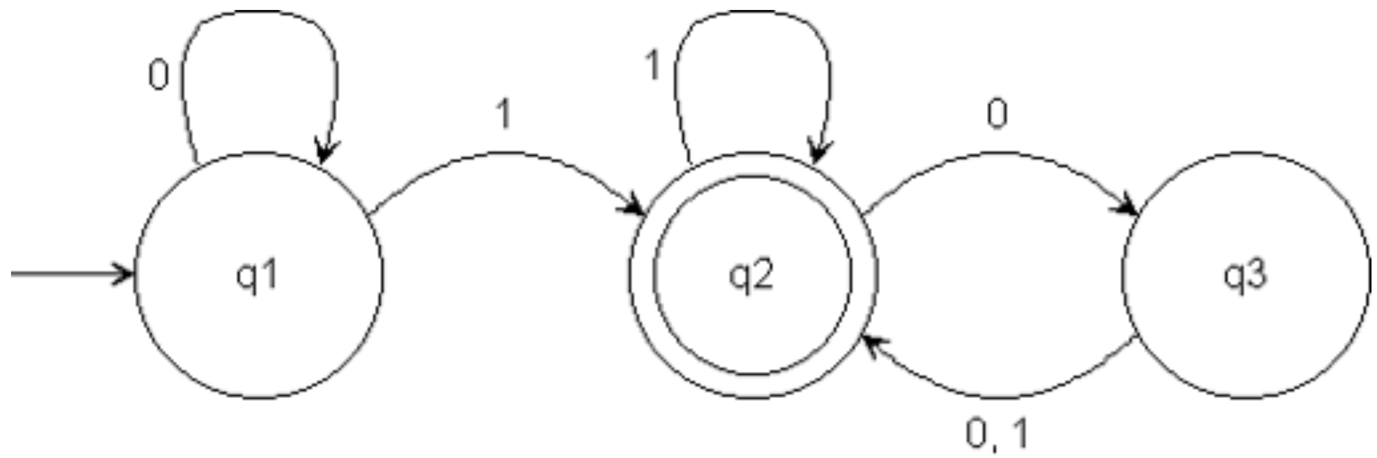

- States: q1, q2, q3
- Start State: q1
- Final State: q2
- Alphabet $\Sigma = \{0, 1\}$
- Transition function:

$$\delta(q_1, 0) = q_1$$

$$\delta(q_1, 1) = q_2$$

$$\ldots$$

# FA: Second Example




Does it accept the following strings:

0101

01110

0100

# Designing FA

- We would like to design a DFA for the following languages (examples on board)
  - L1 = { w | w has even number of 0's}
  - L2 = {w | w has even number of 0's and 1's}
  - L3 ={ w | w start with 00}
  - L4 = { w | w divisible by 4}

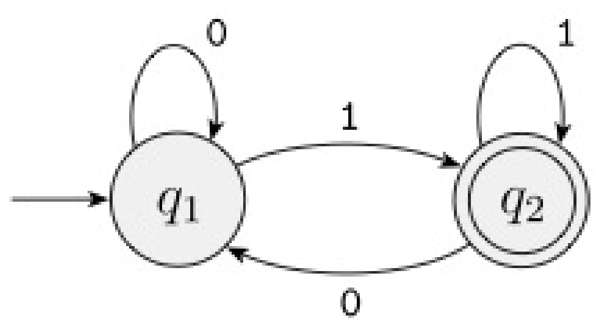
# Formal Definition

A ***finite automaton*** is a 5-tuple $(Q,\Sigma,\delta,q_0,F)$ where:

1. $Q$ is a finite set called the ***states***.
2. $\Sigma$ is a finite set called the ***alphabet.***
3. $\delta : Q \times \Sigma \rightarrow Q$ is the ***transition function.***
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the set of ***accept states***.

# Example

- L1 = { w | w has even number of 0's}




- Formal description

$(\{q_1, q_2\}, \{0,1\}, \delta, q_1, \{q_2\})$ function $\delta$ is

| | 0 | 1 |
|---|---|---|
| $q_1$ | $q_1$ | $q_2$ |
| $q_2$ | $q_1$ | $q_2$. |

# Regular Language

- Definition: A ***language*** is a set of strings over some alphabet.

- The language of an FA, $M$, designated $L(M)$, is the set of strings that $M$ **accepts**

- If L is recognized by some finite automaton, then L is

  a **regular language.**

# Questions

Q1: How do you prove that a language L is regular?

A1: By presenting an FA, $M$, such that $L(M) = L_a$

Q2: Why is this important?

A2: It defines a class of problems that can be solved by a computational device with bounded memory.

Q3: How do you prove that a language L is not regular?

A3: This is more difficult! We'll answer this later.