Computing Theory

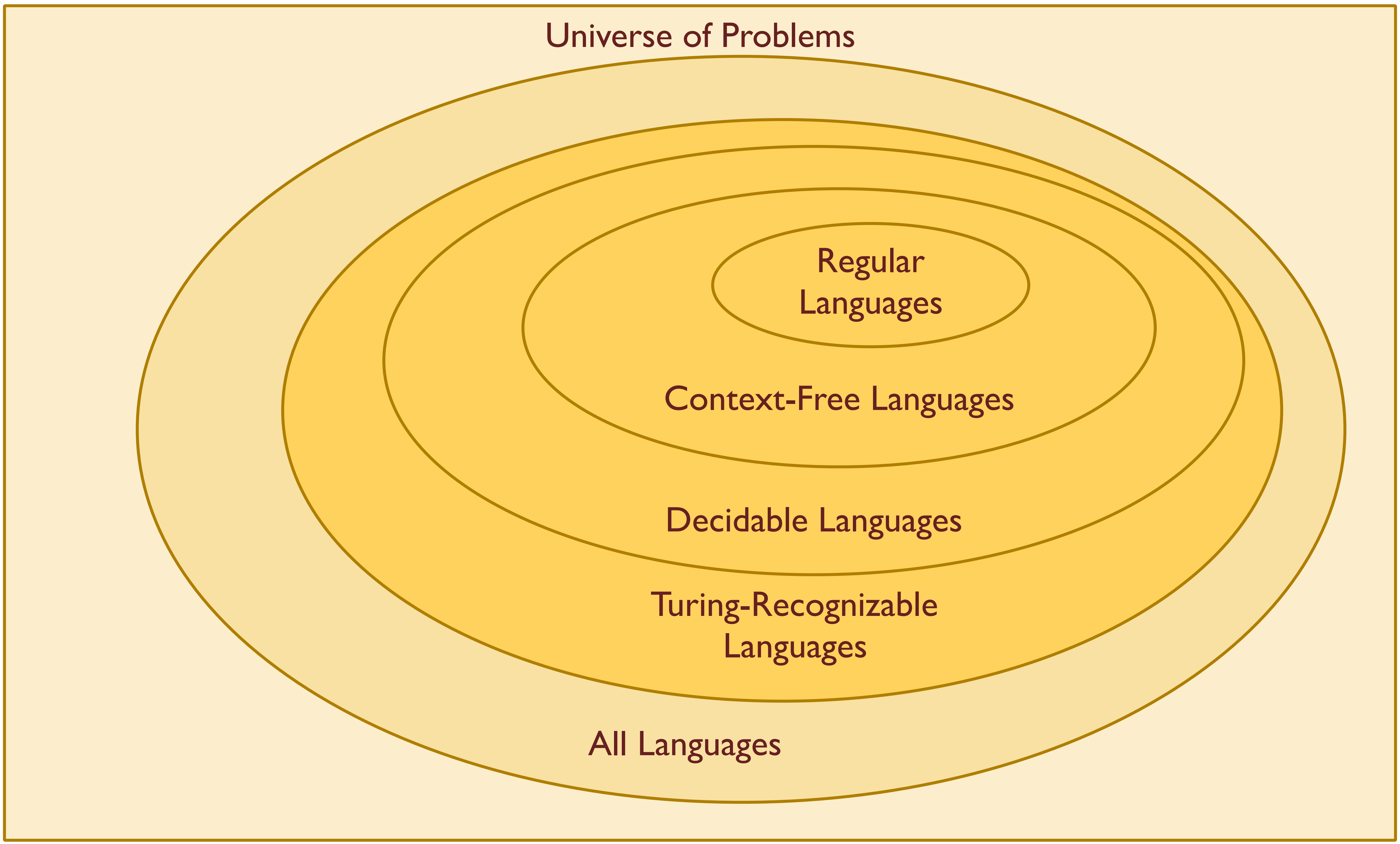
COMP 147 (4 units)

Chapter 3: The Church-Turing Thesis
Section 3.1: Turing Machines

Recap from Last Time

- Turing Machines
  - Definition
  - Examples from Lab
    - $L1 = \{w \mid w \in 01^*0\}$
    - $L2 = \{ w\#w \mid w \in \{0, 1\}^* \}$

The Space of Problems



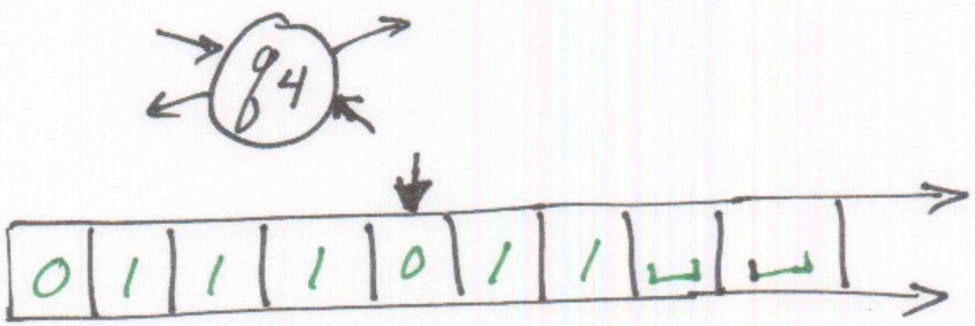
"CONFIGURATION"

GIVES THE ENTIRE STATE OF
THE MACHINE

SNAPSHOT OF EXECUTION AT
SOME STEP.

NEED:

- CONTENTS OF THE TAPE.
- LOCATION OF THE "TAPE HEAD"
- CURRENT STATE.

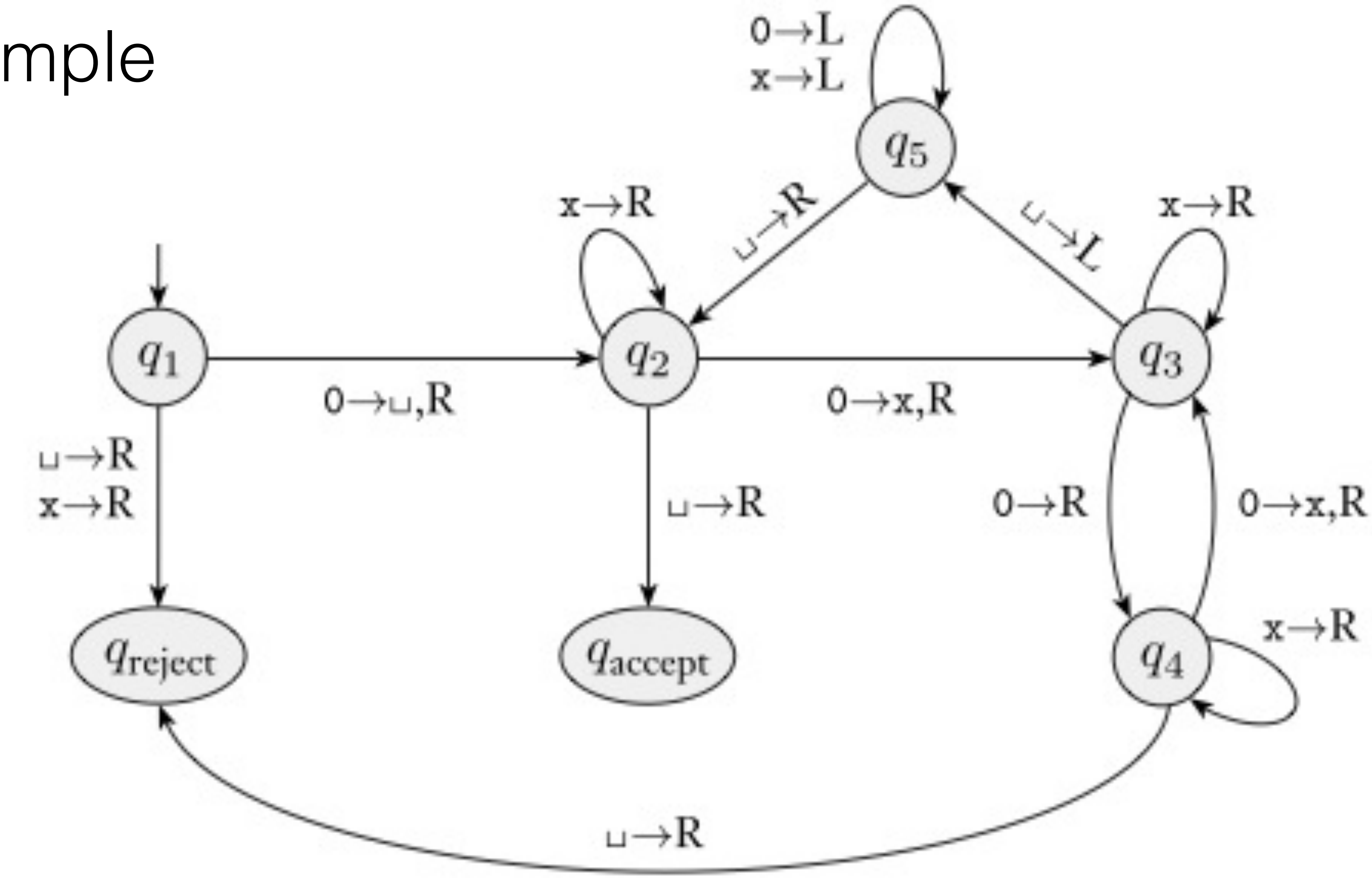


A CONFIGURATION IS A STRING LIKE THIS:

$$0\,1\,1\,1\,q_4\,0\,1\,1$$

A sequence of configurations starting at the initial configuration and ending with an accepting/rejecting configuration provides a "computation history"

Example



$q_1 0000$	$\sqcup q_5 \mathtt{x0x}\sqcup$	$\sqcup \mathtt{x} q_5 \mathtt{xx}\sqcup$
$\sqcup q_2 000$	$q_5 \sqcup \mathtt{x0x}\sqcup$	$\sqcup q_5 \mathtt{xxx}\sqcup$
$\sqcup \mathtt{x} q_3 00$	$\sqcup q_2 \mathtt{x0x}\sqcup$	$q_5 \sqcup \mathtt{xxx}\sqcup$
$\sqcup \mathtt{x0} q_4 0$	$\sqcup \mathtt{x} q_2 \mathtt{0x}\sqcup$	$\sqcup q_2 \mathtt{xxx}\sqcup$
$\sqcup \mathtt{x0x} q_3 \sqcup$	$\sqcup \mathtt{xx} q_3 \mathtt{x}\sqcup$	$\sqcup \mathtt{x} q_2 \mathtt{xx}\sqcup$
$\sqcup \mathtt{x0} q_5 \mathtt{x}\sqcup$	$\sqcup \mathtt{xxx} q_3 \sqcup$	$\sqcup \mathtt{xx} q_2 \mathtt{x}\sqcup$
$\sqcup \mathtt{x} q_5 \mathtt{0x}\sqcup$	$\sqcup \mathtt{xx} q_5 \mathtt{x}\sqcup$	$\sqcup \mathtt{xxx} q_2 \sqcup$
		$\sqcup \mathtt{xxx}\sqcup q_{accept}$

Recognizing vs. Deciding

Turing-recognizable: A language L is "Turing-recognizable" if there exists a TM M such that for all strings w:

- If $w \in L$: eventually M enters q_{accept}.
- If $w \notin L$: either M enters q_{reject} ***or*** M never terminates.

Turing-decidable: A language L is "Turing-decidable" if there exists a TM M such that for all strings w:

- If $w \in L$: eventually M enters q_{accept}.
- If $w \notin L$: eventually M enters q_{reject}.

Closure properties

- Example: **Turing-decidable** languages are closed under union
  - **Proof**: Given turing-decidable L1 and L2 with TMs M1 and M2 respectively, need to show how to construct M3 for the union of L1 and L2
    - on input w
      - Run M1 on w. If M1 accepts then accept
      - If M1 rejects, Run M2 on w. If M2 accepts then accept otherwise reject
- Example: Turing-recognizable languages are closed under union
  - Can I use the same proof as above?

Closure properties

- Example: Turing-recognizable languages are closed under union
    - **Proof**: Given turing decidable L1 and L2 with TMs M1 and M2 respectively, need to show how to construct M3 for the union of L1 and L2
        - on input w
            - Run M1 and M2 alternatively on w step by step. If either accepts, then accept. If both halt and reject then reject.
- Note: if w is either in L1 or L2, one of M1 or M2 will halt and accept and thus M3 accepts. Note if both M1 and M2 loop then M3 will loop as well