STAT 8004 – Statistical Methods II

Spring 2015

Homework Assignment 1 (Due on 1/29/2015 before the end of the day)

- Reading assignment
  - Chapter 7 of R&S.
  - Chapter 2 of the following text book is a another reference of Lecture 2. Full text of the book is available via the Temple library.
    * Christensen, R. (2011). Plane answers to complex questions: the theory of linear models. Springer. Library link
- The following exercises are to be collected. Please upload your homework to the Blackboard. Following the requirement of STAT 8003, please typeset your homework with Latex and upload both the pdf and latex files.

1. Write out the following models of elementary/intermediate statistical analysis in the matrix form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

   (a) A one-variable quadratic polynomial regression model

$$y_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 + \varepsilon_i$$

   for $(i = 1, 2, \ldots, 5)$.

   (b) A two-factor ANCOVA model without interactions

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma(x_{ijk} - \bar{x}) + \varepsilon_{ijk}$$

   for $i = 1, 2$, $j = 1, 2$, and $k = 1, 2$.

2. Use `eigen()` function in R to compute the eigenvalues and eigenvectors of

$$\mathbf{V} = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}.$$

   Then use R to find an "inverse square root" of this matrix. That is, find a symmetric matrix $\mathbf{W}$ such that $\mathbf{W}\mathbf{W} = \mathbf{V}^{-1}$.

3. Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 4 & 4.001 \\ 4.001 & 4.002 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 4 & 4.001 \\ 4.001 & 4.002001 \end{pmatrix}.$$

   Obviously, these matrices are nearly identical. Use R and compute the determinants and inverses of these matrices. (Even though the original two matrices are nearly the same, $\mathbf{A}^{-1} \approx -3\mathbf{B}^{-1}$. This shows that small changes in the in the elements of nearly singular matrices can have big effects on some matrix operations.)

4. Write an R function to conduct projection, e.g. with name `project()`, so that the input is the given design matrix $\mathbf{X}$, and the output is the projection matrix $\mathbf{P}_{\mathbf{X}}$ for projecting a vector onto the column space of $\mathbf{X}$.

5. Consider the (non-full-rank) two-way "effect model" with interactions in the Example (d) in lecture.

   (a) Determine which of the parametric functions below are estimable:

   $$\alpha_1, \alpha_2 - \alpha_1, \mu + \alpha_1 + \beta_1 + \delta_{11}, \delta_{12}, \delta_{12} - \delta_{11} - (\delta_{22} - \delta_{21})$$

   For those that are estimable, find $\mathbf{c}^T(\mathbf{X}^T\mathbf{X})^-\mathbf{X}^T$, such that $\mathbf{c}^T(\mathbf{X}^T\mathbf{X})^-\mathbf{X}^T\mathbf{Y}$ produces the estimate of $\mathbf{c}^T\boldsymbol{\beta}$.

   (b) For the parameter vector $\boldsymbol{\beta}$ written in the order used in class, consider the hypothesis $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$ for

   $$\mathbf{C} = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix}$$

   Is this hypothesis testable? Explain.