Homework 1

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1

$$A*B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 9 \\ -4 & 18 \\ -6 & 27 \end{pmatrix}$$

$$B^T * A = (A*B)^T = \begin{pmatrix} -2 & -4 & -6 \\ 9 & 18 & 27 \end{pmatrix}$$

2

Proof: Since

$$det(I) = det(A*A^{-1}) = det(A)*det(A^{-1}) = 1$$

We have,

$$det(A^{-1}) = [det(A)]^{-1}$$

3

3.1 Part a

$$Q^2 = (I-P)*(I-P) = I-P-P+P^2 = I-P = Q$$

3.2 Part b

$$\begin{aligned} P*P &= X\left(X^T X\right)^{-1} X^T * X\left(X^T X\right)^{-1} X^T \\ &= X\left(X^T X\right)^{-1} X^T \\ &= P \end{aligned}$$

4

$$
\begin{aligned}
\|A-\lambda I\| &= \left\| \begin{array}{ccc} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{array} \right\| \\
&= (2-\lambda)(\lambda-(2+\sqrt{2}))(\lambda-(2-\sqrt{2}))
\end{aligned}
$$

All the eigen values are positive, so matrix A is positive definite.

5

5.1

$$
\begin{aligned}
\tau(\alpha+1) &= \int_0^\infty x^\alpha exp(-x)dx \\
&= -x^\alpha exp(-x)|_0^\infty + \alpha \int_0^\infty x^{\alpha-1} exp(-x)dx \\
&= \alpha \int_0^\infty x^{\alpha-1} exp(-x)dx \\
&= \alpha\tau(\alpha)
\end{aligned}
$$

5.2

$$
\begin{aligned}
\tau(n) &= (n-1)\tau(n-1) \\
&= (n-1)(n-2)\ldots(n-(n-1))\tau(n-(n-1)) \\
&= (n-1)(n-2)\ldots 1\tau(1)
\end{aligned}
$$

and

$$
\tau(1) = \int_0^\infty exp(-x)dx = -e^{-x}|_0^\infty = 1
$$

we have,

$$
\tau(n) = (n-1)!
$$

5.3

Let $t = \dfrac{\beta}{x}$,

$$
\begin{aligned}
&\int_0^\infty x^{-\alpha-1} exp(-\frac{\beta}{x}) dx \\
&\qquad = \int_0^\infty \frac{\beta}{x}^{-\alpha-1} exp(-t) dx \\
&\qquad = \int_0^\infty t^{\alpha+1} \beta^{-\alpha-1} exp(-t) \frac{-\beta}{x}^{t^2} dx \\
&\qquad = \beta^{-\alpha} \int_0^\infty t^{\alpha-1} exp(-t) dx \\
&\qquad = \beta^{-\alpha} \tau(\alpha)
\end{aligned}
$$