STAT 8003, Homework 3

Group # ... (Replace this)
Members: ... (Replace this)

September 16, 2013

Due at 5:30pm on class on Thu., Sep. 26. Please submit one and only one pdf file for your group via blackboard.

Problem 1. (20 points) X and Y are independent random variables with exponential distributions with expectations λ and μ, respectively. Sometimes it is impossible to obtain direct observations of X and Y. Instead, we observe the random variables Z and W, where

$$Z = \min(X, Y) \text{ and } W = \begin{cases} 1 & \text{if } Z = X; \\ 0 & \text{if } Z = Y. \end{cases}$$

(This is a situation that arises, in particular, in medical experiments. The X and Y variables are censored).

a) Find the joint distribution of Z and W.

b) Prove that Z and W are independent. (Hint: show that $\mathbb{P}(Z \leq z \mid W = w) = \mathbb{P}(Z \leq z)$ for $w = 0$ or 1.)

Problem 2. (20 points) Let X and Y have the joint density function

$$f(x, y) = k(x - y), \quad 0 \leq y \leq x \leq 1$$

and 0 elsewhere.

a). Sketch the region over which the density is positive and use it in determining limits of integration to answer the following questions.

b). Find k.

c). Find the marginal densities of X and Y .

d). Find the conditional densities of Y given X and X given Y.

Problem 3. (10 points) A couple decides to continue to have children until a daughter is born. What is expected number of children of this couple?

Problem 4. (20 points) Let X have pdf

$$f(x) = \frac{1}{2}(1 + x), -1 < x < 1.$$

a). Find the pdf of $Y = X^2$.

b). Find $\mathtt{E}(Y)$ and $\mathtt{Var}(Y)$.

Problem 5. (20 points) Suppose that the random variable Y has a binomial distribution with n trials and success probability X, where n is a given constant and X is a $\mathrm{Unif}(0, 1)$ random variable.

a) Find $\mathtt{E}(Y)$ and $\mathtt{Var}(Y)$.

b) Find the joint distribution of X and Y.

c) Find the marginal distribution of Y.

Problem 6. (10 points) Let X, Y and Z be uncorrelated random variables with variances σ_X^2, σ_Y^2 and σ_Z^2, respectively. Let

$$U = Z + X$$
$$V = Z + Y$$

Find $\mathtt{Cov}(U, V)$ and $\rho(U, V)$. Note that $\rho(U, V)$ is defined as

$$\rho(U, V) = \frac{\mathtt{Cov}(U, V)}{\{\mathtt{Var}(U)\mathtt{Var}(V)\}^{1/2}}$$