STAT 8003, Homework 8

Group # ... (Replace this)
Members: ... (Replace this)

October 31, 2013

Due at 5:30pm on class on Thu., Nov. 8. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total points = 100).

Problem 1. We want to know the mean percentage of butterfat in milk produced by a farm by sampling multiple loads of milk. Previous records indicate the average percent butterfat in milk is 3.35 and the standard deviation among loads is 0.15. Now we hope to detect a change of the percent butterfat in milk.

a). Find the rejection region at the significant level $\alpha = 0.05$.

b). Suppose 100 loads of milk are sampled. What is the power for the test for detecting a change of the mean to 3.40.

c). Plot the power as a function of the absolute value of the change of the mean over the standard deviation (which is $|\mu_1 - \mu_0|/\sigma$).

d). Now we hope to detect a change of the percent butterfat in milk to 3.40 with a power 0.8. How many loads do we need to sample?

Problem 2. The Sydney-Hobart yacht race starts from Sydney Harbour on Boxing day (December 26) and finishes several days later in Hobart. It is a 630 nautical mile ocean race. The data give the winning times from 1945 to 1993, as they appeared in the Sydney Morning Herald on 24 December, 1994, plus the winning times for 1994 to 1997. The data file is a tab-delimitated text, named as "yacht.txt", located in the "Data" folder on blackboard.

Variable	Description
Yacht	Name of winning yacht
Year	Year
Days	Days unit of winning time
Hours	Hours unit of winning time
Minutes	Minutes unit of winning time
Time	Winning time in minutes (should match time in Days, Hours and Minutes)

a). Plot histogram of Time and log(Time − 3100). Which one do you think are more likely to follow a normal distribution?

b). Plot a scatter plot log(Time − 3100) *vs.* Year. Do you see any trend? Write out a linear models to study the relationship between log(Time − 3100) and Year. Interprete your two parameters in the model.

c). Use the `lm` function in R to fit the model. Plot the regression line on the scatterplot.

Problem 3. Some matrix practice.

a). Calculate AB by hand and check your result by R, where

$$A = \begin{pmatrix} 3 & 0 & 2 \\ -1 & 2 & 2 \\ 1 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 \\ 0 & -2 \\ -1 & 1 \end{pmatrix}.$$

b). For $a, b > 0$, suppose

$$A = \begin{pmatrix} a & b \\ b & a \end{pmatrix}.$$

What are the eigenvalues of A? What are its eigenvectors? What is the trace? What is the determinant?

c). Follow Problem 3.b. Let $a = 2$, $b = 1$. Calculate A^{-1} by hand and then check your result by R.