Homework Template

Nooreen Dabbish

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1 Write out the following models of elementary/intermediate statistical analysis in the matrix form:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$$

1.1 A one-variable quadratic polynomial regression model

$$y_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 + \epsilon_i$$

for (

$$i = 1, 2, \ldots, 5)$$

.

1.2 A two-factor ANCOVA model without interactions

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma(x_{ijk} - \bar{x}) + \epsilon_{ijk}$$

for $i= 1, 2,$, $j=1,2,$ and $k =1,2.$

2 Use eigen() function in R to compute the eigenvaluse and eigenvectors of

$$\mathbf{V} = (\ )\, 3.00 - 1.001.00 - 1.005.00 - 1.001.00 - 1.003.00$$

Then use R to find and "inverse square root" of this matrix. That is, find a symmetric matrix $\mathbf{W}$ such that $\mathbf{WW} = \mathbf{V}^{-1}$.

3 Consider the matrices

$$\mathbf{A} = (\ )\, 4.004.004.004.00\, and \mathbf{B} = (\ )\, 4.004.004.004.00.$$

Obviously, these matrices are nearly identical. Use R and compute the determinants and inverses of these matrices. (Even though the original two matrices are nearly the same, $\mathbf{A}^{-1} \approx -3\mathbf{B}^{-1}$. This shos that small changes in the elements of nearly singular matrices can have big eggects on some matrix operations.)

4 Write an R function to conduct projection, e.g. with the name project().

The input is the given design matrix $\mathbf{X}$, and the output is the projection matrix $\mathbf{P}_{\mathbf{X}}$ for projecting a vector onto the column space of $\mathbf{X}$.

5 Consider the (non-full-rank) two-way "effect model" with interactions in the Example (d) in lecture.

5.1 Determine which of the parametric functions below are estimable:

α_1,α_2 - α_a, μ + α_1+ β_1+δ_{11}, δ_{12}, δ_{12} - δ_{11} - (δ_{22} - $\delta\{21\}$)

For those that are estimable, find $\mathbf{C}^T(\mathbf{X}^T\mathbf{X})^{-}\mathbf{X}^T$, such that $\mathbf{C}^T(\mathbf{X}^T\mathbf{X})^{-}\mathbf{X}^T\mathbf{Y}$ procudes the extimate os $\mathbf{C}^T\beta$.

5.2 For thte parameter vector β written in the order used in class, consider the hypothesis H_0 : \$$\mathbf{C}\beta = \mathbf{0} for$

$\mathbf{C} = (\ )\, 0.001.00{-}1.000.000.000.000.000.000.000.000.000.000.000.001.00{-}1.00{-}1.001.00$

Is this hypothesis testable? Explain.