STAT 8004, Homework 4

Group # ... (Replace this)
Members: ... (Replace this)

Mar. 13, 2014

This homework is due Thu., 2013/03/20, 5:30pm.

Instructions: Generate a PDF file from it and submit the PDF file to blackboard. Each group should submit one file with file names **hw[number]-[groupnumber].pdf**. For exmaple, "hw01-1.pdf" for homeowrk 1 and group 1. Please also include your R code in the appendix.

Problem 1. (10 points) In prospective study and retrospective study, usually the definition of π_1 and π_2 are different. If we use D to stand for disease and E to stand for exposure, then

$$OR_{\text{Pros}} = \frac{\mathbb{P}(D \mid E)/(1 - \mathbb{P}(D \mid E))}{\mathbb{P}(D \mid NE)/(1 - \mathbb{P}(D \mid NE))}$$
$$OR_{\text{Retro}} = \frac{\mathbb{P}(E \mid D)/(1 - \mathbb{P}(E \mid D))}{\mathbb{P}(E \mid ND)/(1 - \mathbb{P}(E \mid ND))}$$

The nice part of OR is that $OR_{\text{Pros}} = OR_{\text{Retro}}$, and therefore, we can use retrospective study instead of prospective study to study the disease risks. Prove the equation.

Problem 2. (20 points) Consider the Product Binomial Model.

a). (10 points) Please show that

$$\text{Var}(\log(\widehat{OR})) = \frac{1}{n_1\pi_1(1-\pi_1)} + \frac{1}{n_2\pi_2(1-\pi_2)}.$$

b). (10 points) Under $\text{H}_1 : \pi_1 \neq \pi_2$, please show that

$$\widehat{\text{Var}}(\log(\widehat{OR})) = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}.$$

Problem 3. (30 points) Consider $X \sim Binom(n, \pi)$. One convenient way of constructing confidence limits for π contained in the unit interval (0,1) is the log-negative-log transformation $\theta = g(\pi) = \log(-\log \pi)$.

a). (10 points) Use the Delta-method in conjuction with Slutsky's Theorem to show that the asymptotic distribution of $\hat{\theta} = g(\hat{\pi}) = \log(-\log \hat{\pi})$.

b). (10 points) From this result, derive the $(1 - \alpha)$ asymmetric confidence interval for π.

c). (10 points) For the case where $x = 2$ and $n = 46$, compute the 95% confidence limits for π.

Problem 4. (40 points) Thalidomide is a tranquilizer that was prescribed in the late 1950s and early 1960s to pregnant women, with the devastating result of over 12,000 birth defects in 48 countries, before it was banned in 1962 (it was never sold in the United States).

Recently, the drug has reappeared as a possible solution to a very different medical problem. The National Institutes of Health announced on October 31, 1995, the results of 30-hospital study of the effectiveness of thalidomide in healing mouth ulcers in AIDS patients. In the study, which was chaired by Dr. Jeffrey Jacobson of the Bronx Veteran Affairs Medical Center and the Mount SInai School of Medicine in New York, it was found that 14 of 23 patients who received thalidomide had their ulcers heal, compared with 1 of 22 patients who received a placebo. The researchers would like to know whether these results suggest that thalidomide is more effective at healing mouth ulcers than the placebo.

a). (10 points) Display the data as a 2×2 contingency table. Is this study a prospective or retrospective design?

b). (10 points) Based on the question the resarcher is interested, calculate estimates for RD, RR and OR.

c). (10 points) Derive 95% confidence interval for RD, RR and OR.

d). (5 points) Test $\mathrm{H}_0 : \ OR = 1$ based on the Fisher's exact test.

e). (5 points) As a result of these early trial outcomes, the researchers suspended the trial, giving thalidomide to all the patients in the study. Do you agree with this decision?