1.

Methods Homework 4 Notes

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1 Useful Theorems

Theorem 1. *Suppose* $\mathbf{Y} \sim MVN_n(\mu, \mathbf{Sigma})$*,* Σ *positive definite. Also suppose* $\mathbf{A}_{n\times n}$ *symmetric and rank(*$\mathbf{A}$*)* $= k$*.*

If $\mathbf{A\Sigma}$ *idempotent,* $\mathbf{Y'AY} \sim \chi^2_k(\mu'\mathbf{A}\mu)$*.*

Theorem 2. *Suppose* $\mathbf{Y} \sim MVN_n(\mu, \sigma^2\mathbf{I})$*. And the product* $\mathbf{BA} = \mathbf{0}$*, with* A *and* B *of appropriate size.*

Then,

(a) If $\mathbf{A}$ *symmetric,* $\mathbf{Y'AY}$ *and* $\mathbf{BY}$ *are independent.*

(b) If both $\mathbf{B}$ *and* $\mathbf{A}$ *symmetric,* $\mathbf{Y'AY}$ *and* $\mathbf{Y'BY}$ *are independent.*

2 Distributions of interests

2.1 SSE/σ^2

Using theorem 1 above, we can show:

$$\frac{SSE}{\sigma^2} = \frac{(\mathbf{Y} - \hat{\mathbf{Y}})'(\mathbf{Y} - \hat{\mathbf{Y}})}{\sigma^2} \sim \chi^2_{n-rank(X)}$$

Rearranging to find confidence limits for σ gives:

$$P\left(\sqrt{\frac{SSE}{\text{upper}\,\alpha/2\,\text{quantile of}\,\chi^2_{\text{n}-\text{rank(X)}}}} < \sigma < \sqrt{\frac{SSE}{\text{upper}\,\alpha/2\,\text{quantile of}\,\chi^2_{\text{n}-\text{rank(X)}}}}\right) = 1{-}\alpha$$

2.2 Estimable functions c'β

For an estimable **c'**β, we have:

$$\frac{\widehat{\mathbf{c}'\beta} - \mathbf{c}'\beta}{\sqrt{MSE}\sqrt{\mathbf{C}'(\mathbf{X}'\mathbf{X})^{-}\mathbf{C}}} \sim t_{n-\text{rank}(X)}$$

Note that $MSE = \frac{SSE}{n-\text{rank}(X)}$. Rearranging to find 1 - α confidence limits for **c'**β, denoting $\mathrm{t}^{\star}$ = the upper $\alpha/2$ quantile of $\mathrm{t}_{\mathrm{n-rank(X)}}$, we have:

$$P\left(\widehat{\mathbf{c}'\beta} - t^{\star}\sqrt{MSE}\sqrt{\mathbf{C}'(\mathbf{X}'\mathbf{X})^{-}\mathbf{C}} < \mathbf{c}'\beta < \widehat{\mathbf{c}'\beta} + t^{\star}\sqrt{MSE}\sqrt{\mathbf{C}'(\mathbf{X}'\mathbf{X})^{-}\mathbf{C}}\right) = 1-\alpha$$