STAT 8003, Homework 6

Group # ... (Replace this)
Members: ... (Replace this)

October 17, 2013

Due at 5:30pm on class on Thu., Oct. 24. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total points = 100).

Problem 1. A coin is thrown independently 10 times to test the hypothesis that the probability of heads is 1/2 versus the alternative that the probability is not 1/2. The test rejects if either 0 or 10 heads are observed.

a). What is the significance level of the test?

b). If in fact the probability of heads is .1, what is the power of the test?

Problem 2. Suppose that $X \sim Bin(100, p)$. Consider the test that rejects $\mathrm{H}_0 : p = .5$ in favor of $\mathrm{H}_A : p \neq .5$ for $|X - 50| > 10$. Use the normal approximation to the binomial distribution to answer the following:

a). What is α?

b). Graph the power as a function of p.

Problem 3. Suppose that a single observation X is taken from a uniform density on $[0, \theta]$, and consider testing $\mathrm{H}_0 : \ \theta = 1$ versus $\mathrm{H}_1 : \ \theta = 2$.

a). Find a test that has significance level $\alpha = 0$. What is its power?

b). For $0 < \alpha < 1$, consider the test that rejects when $X \in [0, \alpha]$. What is its significance level and power?

c). What is the significance level and power of the test that rejects when $X \in [1 - \alpha, 1]$?

d). Find another test that has the same significance level and power as the previous one.

e). Does the likelihood ratio test determine a unique rejection region?

f). What happens if the null and the alternative hypothesis are interchanged – $\mathrm{H}_0 : \ \theta = 2$ versus $\mathrm{H}_1 : \ \theta = 1$?