STAT 8003, Homework 7

Group # ... (Replace this)
Members: ... (Replace this)

October 24, 2013

Due at 5:30pm on class on Thu., Oct. 31. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total points $= 80$).

Problem 1. Suppose that $X_1, ..., X_{25}$ form a random sample from a normal distribution having a variance of 80. Graph the power of the likelihood ratio test of $H_0 : \mu = \mu_0$ versus $H_A : \mu = \mu_1$ as a function of μ_1, at significance levels .10 and .05. Do the same for a sample size of 100. Compare the graphs and explain what you see.

Problem 2. Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with the density function $f(x \mid \theta) = \theta \exp[-\theta x]$. Set the desired Type I error rate at 5%.

a). Derive a generalized likelihood ratio test and show that the rejection region is of the form $\{\bar{X} \exp(-\theta_0 \bar{X}) \leq c\}$.

b). Suppose $\theta_0 = 1$, $n = 10$. Show that the rejection region in a). is of the form $\{\bar{X} \leq x_0\} \cup \{\bar{X} \geq x_1\}$, where x_0 and x_1 are determined by c.

c). Explain why $\sum_{i=1}^{n} X_i$ and hence $\bar{X}$ follow gamma distributions when $\theta_0 = 1$. How could this knowledge be used to choose c?

Problem 3. Suppose, to be specific, that in Problem 2, the observed data are the following: Suppose your data oberved is:

1.07	0.88	0.66	0.55	1.15	0.65	3.45	3.55	3.51	0.48

a). Based the result in Problem 2, will you reject H_0? What's your p-value?

b). If we start from generalized likelihood ratio test, and use the asymptotical distribution of $2 \log \text{GLR}$, will you reject H_0? What's your p-value?

c). Derive a Wald test of $H_0 : \theta = 1$ versus $H_A : \theta \neq 1$. Do you reject H_0? What's your

p-value?

d). Derive a score test of $H_0 : \theta = 1$ versus $H_A : \theta \neq 1$. Do you reject H_0? What's your *p*-value?