STAT 8004 – Statistical Methods II
Spring 2015
Homework Assignment 4 (Due on 2/19/2015 before the end of the day)

- Reading assignment
  - R&S Chapter 4-5 on multivariate normal distribution.
  - R&S Chapter 8-9.
- The following exercises are to be collected. Please upload your homework to the Blackboard. Following the requirement of STAT 8003, please typeset your homework with Latex and upload both the pdf and latex files.

1. In the context of Problems 2 of Homework Assignment 3, use R matrix calculations to do the following in the (non-full-rank) Gauss-Markov normal linear model
   (a) Find 90% two-sided confidence limits for σ.
   (b) Find 90% two-sided confidence limits for $\mu + \tau_2$.
   (c) Find 90% two-sided confidence limits for $\tau_1 - \tau_2$.
   (d) Find a p-value for testing the null hypothesis $H_0 : \tau_1 - \tau_2 = 0$ vs H_a : not H_0.
   (e) Find 90% two-sided prediction limits for the sample mean of $n = 10$ future observations from the first set of conditions.
   (f) Find 90% two-sided prediction limits for the difference between a pair of future values, one from the first set of conditions (i.e. with mean $\mu+\tau_1$) and one from the second set of conditions (i.e. with mean $\mu + \tau_2$).
   (g) Find a p-value for testing $H_0 : \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \mu \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. What is the practical interpretation of this test?
   (h) Find a p-value for testing $H_0 : \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \mu \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

2. In the following, make use of the data in Problem 4 of Homework Assignment 3. Consider a regression of y on $x_1, x_2, \ldots, x_5$. Use R matrix calculation to do the following in a full rank Gauss-Markov normal linear model.
   (a) Find 90% two-sided confidence limits for σ.

(b) Find 90% two-sided confidence limits for the mean response under the conditions of data point #1.

(c) Find 90% two-sided confidence limits for the difference in mean responses under the conditions of data points #1 and #2.

(d) Find a p-value for testing the hypothesis that the conditions of data points #1 and #2 produce the same mean response.

(e) Find 90% two-sided prediction limits for an additional response for the set of conditions $x_1 = 0.005$, $x_2 = 0.45$, $x_3 = 7$, $x_4 = 45$, and $x_5 = 6$.

(f) Find a p-value for testing the hypothesis that a model including only x_1, x_3 and x_5 is adequate for "explaining" home price. (Hint: write it in the form of $H_0 : \mathbf{C}\boldsymbol{\beta} = 0$).

3. (a) In the context of Problem 1, part g), suppose that in fact $\tau_1 = \tau_2, \tau_3 = \tau_4 = \tau_1 - d\sigma$ What is the distribution of the F statistic?

   (b) Use R to plot the power of an $\alpha = 0.05$ level test as a function of d for $d \in [-5, 5]$, that is plotting $P(F >$ the cut-off value) against d. The R function `pf(q,df1,df2,ncp)` will compute cumulative (non-central) F probabilities for you corresponding to the value `q`, for degrees of freedom `df1` and `df2` when the noncentrality parameter is `ncp`.