STAT 8003, Homework 9

Group # ... (Replace this)
Members: ... (Replace this)

November 7, 2013

Due at 5:30pm on class on Thu., Nov. 14. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total points = 80).

Problem 1. Consider a multiple linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim MVN(\mathbf{0}, \sigma^2\mathbf{I})$$

a). Write out the log-likelihood function of $\boldsymbol{\beta}$ and σ^2.

b). Get MLEs for $\boldsymbol{\beta}$ and σ^2.

Problem 2. Suppose we are interested in studying the relationship between the volume of a cherry tree and its height and diameters. We treat the volume as outcome, and height and diameter as covariates.

a). How to build up a linear regression model to study the relationship between volume and height diameters? Write out your model and interprete all the parameters in your model.

b). The data give the volume (cubic feet), height (feet) and diameter (inches) (at 54 inches above ground) for a sample of 31 black cherry trees in the Allegheny National Forest, Pennsylvania. Please check the tab-separated dataset on blackboard called "cherry.txt". Plot the scatter plots of volume versus height, and volume versus diameter. Do you see any patterns?

For the rest part of the problem, please do not normalize your data when performing the following analysis. Also please do not use "lm" function or any other function which can directly give out results. Use the formula we discussed in class and use R to calculate. You need to attach your R code in the appendix of your homework.

c). Fit the model using the data. What is your LSE estimator? And what's it's variance?

d). Construct a 95% confidence interval for the effect of height on volume? What would be

the result?

e). Test whether the effect of diameter is significant or not, controlling type I error at 5%. What's your p-value. What's your conclusion?

f). Now comes a new tree, with (height, diameter) = $(76, 22)$. Can you predict the volume of the tree? How to construct a 95% CI of the predicted volume?