STAT 8003, Homework 4

Group # ... (Replace this)
Members: ... (Replace this)

September 25, 2013

Due at 5:30pm on class on Thu., Oct. 3. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total 80 points).

Problem 1. In a large traumatic brain injury experiment, put injured and uninjured rats in a Morris water maze and determine whether each animal reaches the platform in 60 seconds. Repeat the experiment until the animal reaches the platform before 60 seconds and record the number of trials up until the first success. We are interested in estimating the success rate p. Now suppose we have an *i.i.d.* sample of size n.

a). Find the method of moments estimate of p.

b). Find the mle of p.

c). After the experiment, we get the following data. Calculate the MOM and MLE with the data.

19	2	2	12	2	1	1	20	0	1

Problem 2. The Poisson distribution has been used by traffic engineers as a model for light traffic, based on the rationale that if the rate is approximately constant and the traffic is light (so the individual cars move independently of each other), the distribution of counts of cars in a given time interval or space area should be nearly Poisson (Gerlough and Schuhl 1955). The following table shows the number of right turns during 300 3-min intervals at a specific intersection.

n	Frequency
0	14
1	30
2	36
3	68
4	43
5	43
6	30
7	14
8	10
9	6
10	4
11	1
12	1
13+	0

a). Fit a Poisson distribution using MLE.

b). Comment on the fit by comparing observed and expected counts. It is useful to know that the 300 intervals were distributed over various hours of the day and various days of the week.

Problem 3. Suppose X is a discrete random variable with $\mathtt{P}(X = 1) = \theta$ and $\mathtt{P}(X = 2) = 1 - \theta$. Three independent observations of X are made: $x_1 = 1$, $x_2 = 2$, $x_3 = 2$.

a). Find the method of moment estimate of θ.

b). What is the likelihood function?

c). What is the MLE of θ?