STAT 8003, Homework 5

Group # ... (Replace this)
Members: ... (Replace this)

October 3, 2013

Due at 5:30pm on class on Thu., Oct. 10. Please submit one and only one pdf file for your group via blackboard. Each sup-problem is 10 points (Total points = 100).

Problem 1. Consider an *i.i.d.* sample of random varialbes with density function

$$f(x \mid \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right).$$

a). Find the method of moments estimate of σ.

b). Find the maximum likelihood estimate of σ.

c). Use the pivot method to construct a $(1-\alpha)\%$ confidence interval of σ.

Problem 2. (Same setting as Problem 2 of Homework 4) The Poisson distribution has been used by traffic engineers as a model for light traffic, based on the rationale that if the rate is approximately constant and the traffic is light (so the individual cars move independently of each other), the distribution of counts of cars in a given time interval or space area should be nearly Poisson (Gerlough and Schuhl 1955). The following table shows the number of right turns during 300 3-min intervals at a specific intersection.

n	Frequency
0	14
1	30
2	36
3	68
4	43
5	43
6	30
7	14
8	10
9	6
10	4
11	1
12	1
13+	0

a). Use the pivot method to construct a $(1-\alpha)\%$ confidence interval of the rate.

b). Use variance stanbilization method to construct a $(1-\alpha)\%$ confidence interval of the rate.

c). Plug in the data and calculate the 95% by both methods. Which one do you prefer? Why?

Problem 3. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* $\mathrm{N}(\mu, sigma^2)$, where μ and σ are unknown. How should the constant c be chosen so that the interval $(-\infty, \bar{X} + c)$ is a 95% confidence interval for μ; that is, c should be chosen so that

$$\mathtt{P}(-\infty < \mu \leq \bar{X} + c) = 0.95.$$

Problem 4. A sample of students from an introductory psychology class were polled regarding the number of hours they spent studying for the last exam. All students anonymously submitted the number of hours on a 3 by 5 card. There were 24 individuals in the one section of the course polled. The data was used to make inferences regarding the other students taking the course. There data are below:

4.5	22	7	14.5	9	9	3.5	8	11	7.5	18	20
7.5	9	10.5	15	19	2.5	5	9	8.5	14	20	8

a). Obtain a confidence interval based on central limit theorem.

b). Obtain a confidence interval based on T-distributions.

c). Plug in the data, are these two intervals similar? What conclusions can you draw based on the confidence intervals.