STAT 8004 – Statistical Methods II

Spring 2015

Homework Assignment 6 (Due on 3/19/2015 before the end of the day)

- Reading assignment
  - All in-class coverage about linear mixed models.
  - R&S Chapter 17 is the related material.
  - Please read material in Chapter 8 of Faraway (2006) on R implementation of linear mixed model.
- The following exercises are to be collected. Please upload your homework to the Blackboard. Following the requirement of STAT 8003, please typeset your homework with Latex and upload both the pdf and latex files.

1. A meat scientist is studying the effect of storage temperature on meat quality. The temperatures of interest are 34, 40, and 46 degrees Fahrenheit. Twelve coolers are available for the study. The three temperatures are randomly assigned to the twelve coolers using a balanced and completely randomized design. Two large cuts of fresh beef are stored in each cooler. After three days, each member of a team of experts independently assigns a quality score to each cut of beef. The experts are not told about the storage conditions of each cut. The scores assigned by the team to each cut of beef are averaged to produce an overall quality score for each cut.

   (a) Write down a model for the overall quality score data. Define your notation thoroughly.

2. Let
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \sigma^2/2 & 0 \\ \sigma^2/2 & \sigma^2 & \sigma^2/2 \\ 0 & \sigma^2/2 & \sigma^2 \end{pmatrix} \right)$$
   where μ_1, μ_2 and σ^2 are unknown parameters. Find the REML of σ^2. Please start with writing it as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, and then try to find $\mathbf{M}$ for calculating the REML.