Set:

Well defined collection of distinct objects is called a set. Well defined, we mean an object that we can separate easily from other objects.

The object in a set are called elements or members of a set Capital letters A, B, C, D, ... are used as names of sets small letters a, b, c, d, ... elements of sets.

Different ways of describing a set

There are three different ways to describe a set.

i. **Descriptive method**: A method by which a set is described in words
For example. N=The set of all natural number.

ii. **Tabular method**: In this form, we have to write all the elements of a set within the brackets. For example; the set of all natural numbers can be written as:

$$N = \{1,2,3,4,5,6,7..........\}$$

iii. **Set-builder form**: In this form, we use a letter or symbol for an arbitrary element of set and also write the property that is common to all element. For example; the set of natural number. Can be written as $N = \{x | x \text{ is any natural numbers}\}$

Some different sets of numbers:

i. N = set of all natural numbers = $\{1,2,3,4..........\}$ = set of all +ve integers = Z^+

ii. W = set of all whole number = $\{0,1,2,3,4..........\}$ = set of non negative integers.

iii. Z = set of all integers = $\{0,\pm1,\pm2,\pm3,..........\}$

iv. Z′= set of all − ve integers = $\{-1,-2,-3,-4..........\}$

v. O= set of all odd integers = $\{\pm1,\pm3,\pm5,..........\}$

vi. E = set of all even integers = $\{0,\pm2,\pm4,..........\}$

vii. Q = set of all rational numbers = $\left\{x \middle| x=\frac{p}{q} \text{ where } p,q\in Z \text{ and } q\neq 0\right\}$

viii. Q′ = set of all irrational numbers = $\left\{x \middle| x\neq\frac{p}{q} \text{ where } p,q\in Z \text{ and } q\neq 0\right\}$

...er of elements in a set is called its order. Lahore 2009

Membership of a set: The symbol used for a member ship of a set is $\in$ is read as "belongs to" Thus $a \in A$ means a is an element of a set A or a belongs to A. If a is not an element of set A. It is written as $a \notin A$.

Equal Sets: Two sets A and B are said to be equal sets if each element of one set is an element of other set, written as **A = B.**

Equivalent sets: Two sets are said to be equivalent if one-to-one correspondence can be established between them

Example: If $A = \{1\ 2\ 3\}$; $B = \{a\ b\ c\}$

Then one–to–one correspondence between **A&B** can be established as under:

$$A = \{1\ 2\ 3\}$$

$$\uparrow\ \uparrow\ \uparrow$$

$$B = \{a\ b\ c\}$$

Singleton Set: A set having one element is called singleton set.

Null Set: A set having zero number of element is called null set or empty set. It is denoted by $\phi = \{\ \}$

Finite Set: A set having finite number of elements.

Infinite Set: A set having infinite numbers of elements.

Sub Set: If each element of set A is also an element set B. Then A is called subset of B written as $A \subseteq B$ and in such a case B is called **SUPER SET** of A.

Note: (i) Empty Set "ϕ" is subset of every set.

(ii) Every Set is sub set of itself.

Power Set:

The set of all subset of set A is called power set of A, defined by **P(A).**

Note: Power Set of empty set is not empty.

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1. **Write the following sets in set builder notation:**

i. $\{1,2,3,......1000\}$

Sol $\{x|x \in N \wedge x \le 1000\}$

ii. $\{0,1,2,........100\}$

Sol $\{x|x \in W \wedge x \le 100\}$

iii. $\{0,\pm1,\pm2,........\pm1000\}$

Sol $\{x|x \in Z \wedge -1000 \le x \le 1000\}$

iv. $\{0,-1,-2,........-500\}$

Sol $\{x|x \in Z \wedge -500 \le x \le 0\}$

v. $\{100,101,102,........400\}$

Sol $\{x|x \in N \ and \ 100 \le x \le 400\}$

vi. $\{-100,-101,-102,........-500\}$

Sol $\{x|x \in Z \wedge -500 \le x \le -100\}$

vii. $\{Peshawar, Lahore, Quetta, Karachi\}$

Sol $\{x|x \ is \ a \ provincial \ capital \ of \ Pakistan\}$

viii. $\{January, June, July\}$

Sol $\{x|x \ is \ month \ of \ Calender \ year \ beginning \ with \ J\}$

ix. The set of all odd natural numbers.

Sol $\{x|x \ is \ an \ odd \ natural \ number\}$

x. The set of all rational numbers.

Sol $\{x|x \in Q\}$

xi. The Set of all real numbers between 1 and 2.

Sol $\{x|x \in \mathbb{R} \wedge 1 < x < 2\}$

xii. The set of all integers between -100 and 1000

Sol $\{x|x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in the descriptive and tabular forms:

i. $\{x | x\ N \wedge x \leq 10\}$

Sol **Tabular Forms:** $\{1, 2, 3, 4, \ldots\ldots\ldots\ldots 10\}$

Des. Form: set of first ten natural numbers.

ii. $\{x | x \in N \wedge 4 < x < 12\}$

Sol **Tabular Forms:** $\{5, 6, 7, \ldots\ldots\ldots\ldots 11\}$

Des. Form: set of natural numbers between 4 and 12.

iii. $\{x | x \in Z \wedge -5 < x < 5\}$

Sol **Tabular Forms:** $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Des. Form: set of all integers between -5 and 5.

iv. $\{x | x \in E \wedge 2 < x \leq 4\}$

Sol **Tabular Forms:** $\{4\}$

Des. Form: set of even numbers between 2 and 5.

v. $\{x | x \in P \wedge x < 12\}$

Sol **Tabular Forms:** $\{2, 3, 5, 7, 11\}$

Des. Form: set of prime numbers between 1 and 12.

vi. $\{x | x \in O \wedge 3 < x < 12\}$

Sol **Tabular Forms:** $\{5, 7, 9, 11\}$

Des. Form: set of odd integers between 3 and 12.

vii. $\{x | x \in E \wedge 4 \leq x \leq 10\}$

Sol **Tabular Forms:** $\{4, 6, 8, 10\}$

Des. Form: The Set of even integers from 4 to 10.

viii. $\{x | x \in E \wedge 4 < x < 6\}$

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xi. $\{x|x \in N \wedge x+4=0\}$

Sol Tabular Forms: $\{\ \}$

Des. Form: The Set of natural numbers x, satisfying x + 4 = 0

xii. $\{x|x \in Q \wedge x^2 = 2\}$ Multan 2010

Sol Tabular Forms: $\{\ \}$

Des. Form: The Set of rational numbers x, satisfying $x^2 = 2$

xiii. $\{x|x \in R \wedge x = x\}$

Sol Tabular Forms: $\mathbb{R}$

Des. Form: The Set of real numbers x, satisfying $x = x$

$x = x$ is satisfying by all real numbers.

xiv. $\{x|x \in Q \wedge x = -x\}$

Sol Tabular Forms: $\{0\}$

Des. Form: The Set of rational numbers satisfying $x = -x$

$\therefore x = -x \Rightarrow 2x = 0 \ or \ x = 0$

xv. $\{x|x \in \mathbb{R} \wedge x \neq 2\}$

Sol Tabular Forms: $\mathbb{R} - \{2\}$

Des. Form: The Set of real numbers x, except 2

xvi. $\{x|x \in \mathbb{R} \wedge x \notin Q\}$

Sol Tabular Forms: Q'

Des. Form: The Set of real numbers x, which are not rational so it will set of irrational numbers.

3. Which of the following sets are finite and which of these are infinite?

i. The set of students of your class.

Sol Finite

ii. The set of all schools in Pakistan.

Sol Finite

iii. The set natural numbers between 3 and 10.

Sol Finite

iv. Set of rational numbers between 3 and 10.

Sol Infinite

v. The set of real numbers between 0 and 1.

Sol Infinite

vi. The set of rationales between 0 and 1.

Sol Infinite

vii. The set of whole numbers between 0 and 1.

Sol Finite

viii. The set of all leaves of trees of Pakistan.

Sol Infinite

ix. $P(N)$:

Sol Infinite

x. $P(a,b,c)$

Sol Finite

xi. $\{1,2,3,4.................\}$

Sol Infinite

xii. $\{1,2,3..........,100,000,0000\}$

Sol Finite

xiii. $\{x|x \in R \wedge x \neq x\}$

Sol Finite

xiv. $\{x|x \in R \wedge x^2 = -16\}$

Sol Finite

xv. $\{x|x \in Q \wedge x^2 = 5\}$

Sol Finite

xvi. $\{x|x \in Q \wedge 0 \leq x \leq 1\}$

Sol Infinite

4. **Write two proper subsets of each of the following sets:**

i. $\{a,b,c\}$

Sol $\{a\},\{b\}$

ii. $\{0,1\}$

Sol $\{0\},\{1\}$

iii. N

Sol $N = \{1,2,............\}$

$\{1\},\{2\}$

iv. Z

Sol $Z = \{0,\pm1,\pm2..........\}$

$\{1\},\{2\}$

v. $\mathbb{R}$

Sol $\mathbb{R} = \textit{set of real numbers}$

$\{1\},\{2\}$

vi. W

Sol $W = \textit{set of whole numbers}$

$\{1\},\{2\}$

vii. $\{x|x \in Q \wedge 0 \leq x \leq 2\}$

Sol $\{1\},\{2\}$

Sol $\{a,b\}$ is a set with o elements and $\{\{a,b\}\}$ is set with one element $\{a,b\}$

7. **Which of the following sentences are true and which of them are false?**

i. $\{1,2\}=\{2,1\}$

Sol True

ii. $\phi\subseteq\{\{2,1\}\}$

Sol True

iii. $\{a\}\subseteq\{\{a\}\}$

Sol False

iv. $\{a\}\in\{\{a\}\}$

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8. What is the number of elements of the power set of the each of the following sets?

i. $\{\ \}$

Sol Power set of $\{\ \}$ has elements $= 2^0 = 1$

ii. $\{0,1\}$

Sol Power set of $\{0,1\}$ has elements $= 2^2 = 4$

iii. $\{1,2,3,4,5,6,7\}$

Sol Power set of $\{1,2,3,4,5,6,7\}$ has elements $= 2^7 = 128$

iv. $\{0,1,2,3,4,5,6,7\}$

Sol Power set of $\{0,1,2,3,4,5,6,7\}$ has elements $= 2^8 = 256$

v. $\{a,\{b,c\}\}$

Sol Power set of $\{a,\{b,c\}\}$ has elements $= 2^2 = 4$

vi. $\{\{a,b\},\{b,c\},\{d,c\}\}$

Sol Power set of $\{\{a,b\},\{b,c\},\{d,c\}\}$ has elements $= 2^3 = 8$

9. Write down the power set of each of the following sets:

Sol (i) $\{9,11\}$ *Power set is* $\{\phi\{9\},\{11\},\{9,11\}\}$

(ii) $\{+,-,\times,+\}$ **Sargodha 2010**

Power set is $\{\phi, \{+\},\{-\},\{\times\},\{+\},\{+,-\},\{+,\times\},\{+,+\},\{-,\times\}\{-,+\},\{\times,+\}$
$\{+,-,\times\},\{+,-,+\},\{+,\times,+\},\{-,\times,+\},\{+,-,\times,+\}\ \}$

(iii) $\{\phi\}$

Sol Power set of $\{\phi\}$ *is* $= \{\phi,\{\phi\}\}$

(iv) $\{a,\{b,c\}\}$ **Sargodha 2009**

Sol Power set $= \{\phi\{a\},\{b,c\},\{a\{b,c\}\}$

10. Which pair of sets are equivalent? Which of them are also/equal?

i. $\{a,b,c\},\{1,2,3\}$

Sol Equivalent

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ii. The set of the first 10 whole numbers, $\{0,1,2,3..........9\}$

Sol Equal

iii. Set of angles of a quadrilateral ABCD, set of the sides of the same quadrilateral

Sol Equivalent

iv. Set of the sides of a hexagon ABCDEF, set of the angles of the same hexagon:

Sol Equivalent

v. $\{1,2,3,4,............\},\{2,4,6,8,.............\}$

Sol Equivalent

vi. $\{1,2,3,4,............\},\left\{1,\frac{1}{2}\frac{1}{3}\frac{1}{4}............\right\}$

Sol Equivalent

vii. $\{5,10,15..........5555\},\{5,10,15,20.............\}$

Sol Neither equivalent nor equal sets.

Union of two Sets:

Union of two sets A and B, denoted by $A\cup B$ is the set of all elements, which belongs to A or B: symbolically;

$$A\cup B=\{x|x\in A\vee x\in B\}$$

Example: If $A=\{1,2,3,\};B=\{2,3,4,5\}$, *then* $A\cup B=\{1,2,3,4,5\}$

Intersection of two sets:

A and B denoted by $A\cap B$, is the set of all elements, which belong to both A and B: symbolically; $A\cap B=\{x|x\in A\wedge x\in B\}$

Example: If $A=\{1,2,3,\};B=\{2,3,4,5\}$, *then* $A\cap B=\{2,3\}$

Disjoint Sets:

If intersection of two set A and B is empty. Then sets A and B are called Disjoint Sets.

Example: $O\cap E=\phi$ Where 'O' is set of odd integers 'E' is even.

Overlapping Sets:

If the intersection of two sets A and B is non-empty but neither is subset of the other, then such sets are called overlapping Sets.

Example: Let $A=\{1,2,3,4\};B=\{3,4,5,6\};A\cap B=overlapping\ set=\{3,4\}$

Complement of a Set:

If U is universal set, then U/A or U – A is called Complement of A, denoted by A' or A^c. Thus $A'=A^c=U-A$

Symbolically $A'=\{x|x\in U\wedge x\notin A\}$

Example: If $U=N$, *then* $E'=O$ *and* $O'=E$

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Difference of Two Sets:

The difference $A-B$ or A/B of two sets A and B is the set of elements which belong to A but do not belong to B.

Symbolically $A-B=A/B=\{x|x\in A\wedge x\notin B\}$

Example: If $A=\{1,2,3,4,5\};B=\{4,5,6,7\};A-B=\{1,2,3\}$

Note: $A-B\neq B-A\ because\ B-A=\{6.7\}$

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i.	$A \cup B = B \cup A;$	Commutative property of union.
ii.	$A \cap B = B \cap A;$	Commutative property of Intersection
iii.	$(A \cup B) \cup C = A \cup (B \cup C);$	Associative property of union
iv.	$(A \cap B) \cap C = A \cap (B \cap C);$	Associative property of Intersection
v.	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C);$	Distributive property of union over Intersection **(Faisalabad 2009)**
vi.	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive property of intersection over union

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1. **Verify the commutative properties of union and intersection for the following pairs of sets:**

i.(a) $A \cup B = B \cup A$

$A = \{1,2,3,4,5\}$, $B = \{4,6,8,10\}$

Sol. $A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\} = \{1,2,3,4,5,6,8,10\} \rightarrow 1$

$B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,8,10\} \rightarrow 2$

From 1 & 2 $A \cup B = B \cup A$

i.(b) $A \cap B = B \cap A$

$A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\} = \{4\} \longrightarrow 1$

$B \cap A = \{4,6,8,10\} \cap \{1,2,3,4,5\} \Rightarrow B \cap A = \{4\} \rightarrow 2$

From 1 and 2

$A \cap B = B \cap A$ proved

ii. N

Sol. N = set of natural numbers Z = set of integers

Given sets are N and Z then

$$Z \cup N = Z$$

$$N \cup Z = Z$$

$$N \cap Z = N$$

$$Z \cap N = N$$

So $N \cup Z = Z \cup N$

and $N \cap Z = Z \cap N$

iii. $A = \{x | x \in \mathbb{R} \wedge x \geq 0\}$ *and* $B = \mathbb{R}$

Sol. $A \cup B = \{x | x \in \mathbb{R} \wedge x \geq 0\} \cup \mathbb{R} = \mathbb{R}$

$B \cup A = \mathbb{R} \cup \{x | x \in \mathbb{R} \wedge x \geq 0\} = \mathbb{R}$

$A \cap B = \{x | x \in \mathbb{R} \wedge x \geq 0\} \cap \mathbb{R}$

$= \{x | x \in \mathbb{R} \wedge x \geq 0\}$

$B \cap A = \mathbb{R} \cap \{x | x \in \mathbb{R} \wedge x \geq 0\}$

$= \{x | x \in \mathbb{R} \wedge x \geq 0\}$

$A \cup B = B \cup A$

and $A \cap B = B \cap A$

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2. Verify the properties for the sets A,B and C given below:

i. Associative Law of Union

$A = \{1,2,3,4\}$

$B = \{3,4,5,6,7,8\}$

$C = \{5,6,7,9,10\}$

Sol. **Associative Law of union** $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S $= A \cup (B \cup C)$

$= \{1,2,3,4\} \cup [\{3,4,5,6,7,8,\} \cup \{5,6,7,9,10\}]$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$

$= \{1,2,3,4,5,\ldots\ldots\ldots\ldots10\} \rightarrow 1$

R.H.S $= (A \cup B) \cup C$

$= [\{1,2,3,4\} \cup \{3,4,5,6,7,8,\}] \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,8,9,10\}$

$= \{1,2,3,4,5,\ldots\ldots\ldots\ldots10\} \rightarrow 2$

From 1 and 2;

$A \cup (B \cup C) = (A \cup B) \cup C$

ii. Associativity of intersection

Sol. $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S $= A \cap (B \cap C)$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,\} \cap \{5,6,7,9,10\}$

$= \{1,2,3,4\} \cap \{5,6,7\}$

$= \{\ \} \rightarrow 1$

R.H.S $= (A \cap B) \cap C$

$= [\{1,2,3,4\} \cap \{3,4,5,6,7,8,\}] \cap \{5,6,7,9,10\}$

$= \{3,4\} \cap \{5,6,7,9,10\} = \{\ \} \rightarrow 2$

From 1 and 2

$A \cap (B \cap C) = (A \cap B) \cap C$

iii. Distributivity of union over intersection

Sol. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= \{1,2,3,4\} \cup [\{3,4,5,6,7,8,\} \cap \{5,6,7,9,10\}]$

$= \{1,2,3,4\} \cup \{5,6,7,\} = \{1,2,3,4,5,6,7\} \rightarrow 1$

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R.H.S $= (A \cup B) \cap (A \cup C)$

$= [\{1,2,3,4\} \cup \{3,4,5,6,7,8,\}] \cap [\{1,2,3,4\} \cup \{5,6,7,9,10\}]$

$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,\ldots\ldots\ldots\ldots 7,9,10\} = \{1,2,3,\ldots\ldots\ldots 7\} \quad \rightarrow 2$

From 1 and 2 L.H.S = R.H.S.

iv. **Distributativty of $\cap$ over $\cup$**

Sol. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= \{1,2,3,4\} \cap [\{3,4,5,6,7,8,\} \cup \{5,6,7,9,10\}]$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$

L.H.S $= \{3,4\} \quad \rightarrow 1$

R.H.S $= (A \cap B) \cup (A \cap C)$

$= [\{1,2,3,4\} \cap \{3,4,5,6,7,8,\}] \cup [\{1,2,3,4\} \cap \{5,6,7,9,10\}]$

$= \{3,4\} \cup \{\ \} = \{3,4\} \quad \rightarrow 2$

From 1 and 2 we get.

L.H.S = R.H.S

Part ii. $A = \phi;\ B = \{0\};\ C = \{0,1,2\}$

Sol. Given $A = \phi;\ B = \{0\};\ C = \{0,1,2\}$ then

(a) **Associativity of union;** $A \cup (B \cup C) = (A \cup B) \cup C \quad \rightarrow I$

Putting value in 1, we get $\phi \cup [\{0\} \cup \{0,1,2\}] = [(\phi \cup \{0\})] \cup \{0,1,2\}$

$\Rightarrow \phi \cup \{0,1,2\} = \{0\} \cup \{0,1,2\}$

$\{0,1,2\} = \{0,1,2\}$

L.H.S = R.H.S

b. **Associtivity of Intersection** $A \cap (B \cap C) = (A \cap B) \cap C \quad \rightarrow I$

Sol. Putting values in 1, we get

$\phi \cap [\{0\} \cap \{0,1,2\}] = \{(\phi \cap \{0\}\} \cap \{0,1,2\}$

$\phi \cap \{0\} = \phi \cap \{0,1,2\}$

$\phi = \phi$

L.H.S = R.H.S

c. **Distributive Law of $\cup$ over $\cap$**

Sol. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= \phi \cup [(\{0\} \cap \{0,1,2\})]$

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$= \phi \cup \{0\} = \{0\} \quad \rightarrow 1$

R.H.S $= (A \cup B) \cap (A \cup C)$

$= [(\{\ \} \cup \{0\})] \cap [(\{\ \} \cup \{0,1,2\})]$

$= \{0\} \cap \{0,1,2\} = \{0\} \quad \rightarrow 2$

From 1 and 2; L.H.S = R.H.S

d. Distributive Law of $\cap$ over $\cup$

Sol. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= \phi \cap [(\{0\} \cup \{0,1,2\})]$

$= \phi \cap \{0,1,2\} = \phi \quad \rightarrow 1$

R.H.S $= (A \cap B) \cup (A \cap C)$

$= [(\phi \cap \{0\})] \cup [(\phi \cap \{0,1,2\})]$

$= \phi \cup \phi = \phi \quad \rightarrow 2$

From 1 and 2

L.H.S = R.H.S

Part-iii. N, Z, Q

Sol. Given $N \leq Z \leq Q$

$N = \{1,2,3,4,.........\}$

$Z = \{0,\pm 1,\pm 2,\pm 3,.........\}$

$Q = Set\ of\ rational\ numbers$

a. Associativity of Union

Sol. $N \cup (Z \cup Q) = (N \cup Z) \cup Q$

$N \cup Q = Z \cup Q (\because N \leq Z \leq Q)$

$Q = Q$

L.H.S = R.H.S ved

b. Associativity of Intersection

Sol. $N \cap (Z \cap Q) = (N \cap Z) \cap Q$

$\Rightarrow N \cap Z = N \cap Q (\because N \leq Z \leq Q)$

$N = N$

$\Rightarrow$ L.H.S = R.H.S proved

c. Distributivity of $\cup$ over $\cap$

Sol. $N \cup (Z \cap Q) = (N \cup Z) \cap (N \cup Q)$

$\Rightarrow N \cup Z = Z \cap Q (\because N \leq Z \leq Q)$

$Z = Z$

$\Rightarrow$ L.H.S = R.H.S proved

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d. **Distributivity of $\cap$ over $\cup$**

Sol. $N \cap (Z \cup Q) = (N \cap Z) \cup (N \cap Q)$

$\Rightarrow N \cap Q = N \cup N (\because N \leq Z \leq Q)$

$N = N$

$\Rightarrow$ L.H.S = R.H.S proved

3. **Verify De Morgan's Laws for the following sets:**

$U = \{1,2,3.........,20\}$, $A = \{2,4,6,.........,20\}$ *and* $B = \{1,3,5,.........,19\}$

Sol.(i) **We have to prove** $(A \cap B)' = A' \cup B'$

L.H.S $= (A \cap B)'$

Where $A \cap B = \{2,4,6,.........,20\} \cap \{1,3,5,.........,19\}$

$A \cap B = \phi = \{\ \}$

$(A \cap B)' = U - (A \cap B) = U - \phi = U \rightarrow 1$

R.H.S $= A' \cup B'$

Where $A' = U - A$

$= \{1,2,3,.........,20\} - \{2,4,6,.........,20\}$

$A' = \{1,3,5,.........,19\}$

$B' = U - B$

$= \{1,2,3,.........,20\} - \{1,3,5,.........,19\}$

$B' = \{2,4,6,.........20\}$

$A' \cup B' = \{1,3,5,.........19\} \cup \{2,4,6,.........20\}$

$A' \cup B' = \{1,2,3,4,.........,20\} = U \rightarrow 2$

From 1 and 2

L.H.S = R.H.S

ii. **We have to prove that $(A \cup B)' = A' \cap B'$**

Sol. L.H.S $= (A \cup B)'$

$A \cup B = \{2,4,6,.........,20\} \cup \{1,3,5,.........19\} = \{1,2,3,......20\} = U$

$(A \cup B)' = U - (A \cup B) = U - U = \phi \rightarrow 1$

R.H.S $= A' \cap B'$

Where $A' = U - A$

$A' = \{1,2,3,4,.........,20\} - \{2,4,6,.........,20\}$

$A' = \{1,3,5,.........,19\}$

$B' = U - B = \{1,2,3,.........,20\} - \{1,3,5,.........19\} = \{2,4,6,.........,20\}$

$A' \cap B' = \phi \rightarrow 2$

From 1 and 2 $\Rightarrow$ L.H.S = R.H.S

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4. Let U = The set of the English alphabet;

$A=\{x|x \text{ is a vowel}\}$, $B=\{y|y \text{ is a consonant}\}$

Verify De Morgan's Laws for these sets.

Sol. We want to prove that

$(A\cup B)'=A'\cap B'$

L.H.S $=(A\cup B)'$

Now $(A\cup B)=\{x|x \text{ is a vowel}\}\cup\{y|y \text{ is a consonant}\}$.

$A\cup B=$ Set of English alphabet $=\cup$

$(A\cup B)'=U-(A\cup B)=U-U=\{\ \}\quad\rightarrow 1$

R.H.S $=A'\cap B'$

Where $A'=U-A=U-\{x|x \text{ is a vowel}\}$

$A'=\{y|y \text{ is a consonant}\}$

and $B'=U-B=U-\{y|y \text{ is a consonant}\}$

$B'=\{x|x \text{ is a vowel}\}$

Then $A'\cap B'=\{y|y \text{ is a consonant}\}\cap\{x|x \text{ is a vowel}\}$

$A'\cap B'=\{\ \}\quad\rightarrow 2$

From 1 and 2

L.H.S = R.H.S

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6. **Taking any set, say $A=\{1,2,3,4,5\}$ verify the following:**

i. $A\cup\varnothing=A$

Sol. L.H.S $=A\cup\varnothing$

$=\{1,2,3,4,5\}\cup\varnothing$

$=\{1,2,3,4,5\}=A=$ R.H.S

ii. $A\cup A=A$

Sol. L.H.S $=A\cup A=A$

$=\{1,2,3,4,5\}\cup\{1,2,3,4,5\}$

$=\{1,2,3,4,5\}=A=$ R.H.S

iii. $A\cap A=A$

Sol. L.H.S $=A\cap A$

$=\{1,2,3,4,5\}\cap\{1,2,3,4,5\}$

$=\{1,2,3,4,5\}=A=$ R.H.S

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7. If $U=\{1,2,3,4,5,\ldots\ldots\ldots,20\}$ and $A=\{1,3,5,\ldots\ldots\ldots,19\}$ verify the following:

i. $A\cup A'=U$ **Mulatan 2008**

Sol. L.H.S $= A\cup A' = \{1,2,3,4,5,\ldots\ldots\ldots,20\}-\{1,3,5,\ldots\ldots\ldots,19\}$

Where $A' = U - A = \{2,4,6,\ldots\ldots\ldots,20\}$

L.H.S $= A\cup A' = \{1,3,5,\ldots\ldots\ldots,19\}\cup\{2,4,\ldots\ldots\ldots,20\}$

$= \{1,2,3,4,5,\ldots\ldots\ldots,20\} = U$

ii. $A\cap U = A$

Sol. L.H.S $= A\cap U$

$= \{1,3,5\ldots\ldots\ldots,19\}\cap\{1,2,3,4,\ldots\ldots\ldots,20\}$

$= \{1,3,5,\ldots\ldots\ldots,19\} = A = R.H.S$

iii. $A\cap A' = \varnothing$ **Faisalabad 2007**

Sol. $A\cap A' = \varnothing$;

L.H.S $= A\cap A'$

$= \{1,3,5,\ldots\ldots\ldots,19\}\cap\{2,4,6,\ldots\ldots\ldots,20\}$

$= \{\ \} = \varnothing = R.H.S$

8. From suitable properties of union and intersection deduce the following results:

i. $A\cap(A\cup B) = A\cup(A\cap B)$

Sol. L.H.S $= A\cap(A\cup B)$

$= (A\cap A)\cup(A\cap B)$ (using Distributive law)

$= A\cup(A\cap B) \because A\cap A = A$

$= A\cup(A\cap B)$

$= R.H.S$

ii. $A\cup(A\cap B) = A\cap(A\cup B)$

Sol. L.H.S $= A\cup(A\cap B)$

$= (A\cup A)\cap(A\cup B)$ (Distributive law)

$= A\cap(A\cup B) \because A\cup A = A$

$= R.H.S$

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