EXERCISE 2.5

Diff of Trigonometry

* $\frac{d}{dx}\sin\theta = \cos\theta\frac{d\theta}{dx}$

* $\frac{d}{dx}\cos\theta = -\sin\theta\frac{d\theta}{dx}$

* $\frac{d}{dx}\tan\theta = \sec^2\theta\frac{d\theta}{dx}$

* $\frac{d}{dx}\sec\theta = \sec\theta\tan\theta\frac{d\theta}{dx}$

* $\frac{d}{dx}\csc\theta = -\csc\theta\cot\theta\frac{d\theta}{dx}$

* $\frac{d}{dx}\cot\theta = -\csc^2\theta\frac{d\theta}{dx}$

(جو c سے شروع ہوں گے تو ان کے diff کے شروع میں - لگتا ہے

اور

آخر میں angle کا diff لازمی لیں)

Question no 02

(i) $x^2 \sec 4x$

let, $y = x^2 \sec 4x$

diff. w.r.t 'x'

$$\frac{d}{dx} y = \frac{d}{dx}(x^2 \cdot \sec 4x)$$

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\sec 4x) + \sec 4x \frac{d}{dx} x^2$$

$$\frac{dy}{dx} = x^2\left(\sec 4x \tan 4x \cdot \frac{d}{dx} 4x\right) + \sec 4x(2x)$$

$$\frac{dy}{dx} = x^2\left(\sec 4x \tan 4x \cdot 4\right) + \sec 4x \cdot 2x$$

$$\frac{dy}{dx} = \sec 4x \cdot 2x\left(1 + 2x \tan 4x\right)$$

$$\frac{dy}{dx} = 2x \sec 4x\left(1 + 2x \tan 4x\right)$$

(ii) $\tan^3\theta \sec^2\theta$

let, $y = \tan^3\theta \sec^2\theta$

diff w.r.t 'θ'

$$\frac{d}{d\theta} y = \frac{d}{d\theta}\left(\tan^3\theta \sec^2\theta\right)$$

$$\frac{dy}{d\theta} = \tan^3\theta \frac{d}{d\theta} \sec^2\theta + \sec^2\theta \frac{d}{d\theta} \tan^3\theta$$

$$\frac{dy}{d\theta} = \tan^3\theta\left(2\sec^{2-1}\theta \frac{d}{d\theta}\sec\theta\right) + \sec^2\theta\left(3\tan^{3-1}\frac{d}{d\theta}\tan\theta\right)$$

$$\frac{dy}{d\theta} = \tan^3\theta\left(2\sec\theta\left(\sec\theta\tan\theta\frac{d}{d\theta}\theta\right)\right) + \sec^2\theta\left(3\tan^2\theta\sec^2\theta\frac{d}{d\theta}\theta\right)$$

$$\frac{dy}{d\theta} = \tan^3\theta . 2\sec\theta . \sec\theta\tan\theta + \sec^2\theta . 3\tan^2\theta\sec^2\theta$$

$$\frac{dy}{d\theta} = \tan^2\theta\sec^2\theta\left(\tan\theta . 2\tan\theta + 3\sec^2\theta\right)$$

$$\frac{dy}{d\theta} = \tan^2\theta\sec^2\theta\left(2\tan^2\theta + 3\sec^2\theta\right)$$

(iii) $(\sin 2\theta - \cos 3\theta)^2$

Let, $y = (\sin 2\theta - \cos 3\theta)^2$

diff. w.r.t 'θ'

$$\frac{d}{d\theta} y = \frac{d}{d\theta}(\sin 2\theta - \cos 3\theta)^2$$

$$\frac{dy}{d\theta} = 2(\sin 2\theta - \cos 3\theta)^{2-1}\frac{d}{d\theta}(\sin 2\theta - \cos 3\theta)$$

$$\frac{dy}{d\theta} = 2(\sin 2\theta - \cos 3\theta)\left(\cos 2\theta\frac{d}{d\theta}2\theta - \left(-\sin 3\theta\frac{d}{d\theta}3\theta\right)\right)$$

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$$\frac{dy}{d\theta} = 2(\sin 2\theta - \cos 3\theta)(2\cos 2\theta + 3\sin 3\theta)$$

(iv) $\cos\sqrt{x} + \sqrt{\sin x}$

let, $y = \cos\sqrt{x} + \sqrt{\sin x}$

diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos\sqrt{x} + \sqrt{\sin x}\right)$$

$$\frac{dy}{dx} = -\sin\sqrt{x}\,\frac{d}{dx}\sqrt{x} + \frac{1}{2\sqrt{\sin x}}\,\frac{d}{dx}\sin x$$

$$\frac{dy}{dx} = -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\,\frac{d}{dx}x + \frac{1}{2\sqrt{\sin x}}\cdot\cos x\,\frac{d}{dx}x$$

$$\frac{dy}{dx} = -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{\sin x}}\cdot\cos x$$

$$\frac{dy}{dx} = \frac{-\sin\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\sin x}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{-\sin\sqrt{x}}{\sqrt{x}} + \frac{\cos x}{\sqrt{\sin x}}\right]$$

Question no. 03

(سب کا 'x' کے مطابق diff لیں گے کیونکہ $\frac{dy}{dx}$ لینا ہے)

(i) $y = x\cos y$

diff w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}(x\cos y)$$

$$\frac{dy}{dx} = x\frac{d}{dx}\cos y + \cos y\frac{d}{dx}x$$

$$\frac{dy}{dx} = x\left(-\sin y\frac{d}{dx}y\right) + \cos y(1)$$

$$\frac{dy}{dx} = -x\sin y\frac{dy}{dx} + \cos y$$

$$\frac{dy}{dx} + x\sin y\frac{dy}{dx} = \cos y$$

$$\frac{dy}{dx}(1 + x\sin y) = \cos y$$

$$\frac{dy}{dx} = \frac{\cos y}{1 + x\sin y}$$

(ii) $x = y\sin y$

diff w.r.t 'x'

$$\frac{d}{dx}x = \frac{d}{dx}y\sin y$$

$$1 = y\frac{d}{dx}\sin y + \sin y\frac{d}{dx}y$$

$$1 = y\left(\cos y\frac{d}{dx}y\right) + \sin y\frac{dy}{dx}$$

$$1 = y\cos y\frac{dy}{dx} + \sin y\frac{dy}{dx}$$

$$1 = \frac{dy}{dx}(y\cos y + \sin y)$$

$$\frac{dy}{dx} = \frac{1}{y\cos y + \sin y}$$

Question no 04

('x' کے مطابق "diff" لیں گے)

(i) $\cos\sqrt{\frac{1+x}{1+2x}}$

let, $y = \cos\sqrt{\frac{1+x}{1+2x}}$

diff. w.r.t 'x'

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$$\frac{d}{dx}y = \frac{d}{dx}\cos\sqrt{\frac{1+x}{1+2x}}$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\ \frac{d}{dx}\sqrt{\frac{1+x}{1+2x}}$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left(\frac{1}{2\sqrt{\frac{1+x}{1+2x}}}\ \frac{d}{dx}\left(\frac{1+x}{1+2x}\right)\right)$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left[\frac{1}{2\sqrt{\frac{1+x}{1+2x}}}\left(\frac{(1+2x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1+2x)}{(1+2x)^2}\right)\right]$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left[\frac{\sqrt{1+2x}}{2\sqrt{1+x}}\left(\frac{(1+2x)1-(1+x)(2)}{(1+2x)^2}\right)\right]$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left[\frac{\sqrt{1+2x}}{2\sqrt{1+x}}\left(\frac{1+\cancel{2x}-2-\cancel{2x}}{(1+2x)^2}\right)\right]$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left[\frac{\sqrt{1+2x}}{2\sqrt{1+x}}\left(\frac{-1}{(1+2x)^2}\right)\right]$$

$$\frac{dy}{dx} = -\sin\sqrt{\frac{1+x}{1+2x}}\left[\frac{-1}{2\sqrt{1+x}\,(1+2x)^{3/2}}\right]$$

$$\frac{dy}{dx} = \frac{\sin\sqrt{\frac{1+x}{1+2x}}}{2\sqrt{1+x}\,(1+2x)^{3/2}}$$

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(ii) $\sin\sqrt{\dfrac{1+2x}{1+x}}$

Let, $y = \sin\sqrt{\dfrac{1+2x}{1+x}}$

diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin\sqrt{\frac{1+2x}{1+x}}\right]$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\ \frac{d}{dx}\sqrt{\frac{1+2x}{1+x}}$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\left[\frac{1}{2\sqrt{\frac{1+2x}{1+x}}}\ \frac{d}{dx}\left(\frac{1+2x}{1+x}\right)\right]$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\left[\frac{\sqrt{1+x}}{2\sqrt{1+2x}}\left(\frac{(1+x)\frac{d}{dx}(1+2x)-(1+2x)\frac{d}{dx}(1+x)}{(1+x)^2}\right)\right]$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\left[\frac{\sqrt{1+x}}{2\sqrt{1+2x}}\left(\frac{(1+x)(2)-(1+2x)(1)}{(1+x)^2}\right)\right]$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\left[\frac{2+\cancel{2x}-1-\cancel{2x}}{2\sqrt{1+2x}\,(1+x)^{3/2}}\right]$$

$$\frac{dy}{dx} = \cos\sqrt{\frac{1+2x}{1+x}}\left[\frac{1}{2\sqrt{1+2x}\,(1+x)^{3/2}}\right]$$

$$\frac{dy}{dx} = \frac{\cos\sqrt{\frac{1+2x}{1+x}}}{2\sqrt{1+2x}\ (1+x)^{3/2}}$$

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Question no 05 (I)

(i) $\sin x$ w.r.t. $\cot x$

Let,

$u = \sin x$, $v = \cot x$

$u = \sin x$

diff. w.r.t 'x'

$$\frac{d}{dx}u = \frac{d}{dx}\sin x$$

$$\frac{du}{dx} = \cos x \frac{d}{dx}x$$

$$\frac{du}{dx} = \cos x (1)$$

$$\frac{du}{dx} = \cos x$$

Then,

$v = \cot x$

diff. w.r.t 'x'

$$\frac{d}{dx}v = \frac{d}{dx}\cot x$$

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x \frac{d}{dx}x$$

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x (1)$$

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x$$

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Chain Rule :-

$$\frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv}$$

$$\frac{du}{dv} = \cos x \cdot \frac{1}{-\operatorname{cosec}^2 x}$$

$$\frac{du}{dv} = -\cos x \sin^2 x \qquad \left(\because \sin\theta = \frac{1}{\operatorname{cosec}\theta}\right)$$

(ii) $\sin^2 x$ w.r.t $\cos^4 x$

let $u = \sin^2 x$, $v = \cos^4 x$

$u = \sin^2 x$

diff. w.r.t 'x'

$$\frac{du}{dx} = \frac{d}{dx} \sin^2 x$$

$$\frac{du}{dx} = 2\sin^{2-1} x \frac{d}{dx} \sin x$$

$$\frac{du}{dx} = 2\sin x \cos x \frac{d}{dx}(x)$$

$$\frac{du}{dx} = 2\sin x \cos x$$

Then,

$v = \cos^4 x$

diff. w.r.t 'x'

$$\frac{d}{dx} v = \frac{d}{dx} \cos^4 x$$

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$$\frac{dv}{dx} = 4\cos^{4-1}x\left(\frac{d}{dx}\cos x\right)$$

$$\frac{dv}{dx} = 4\cos^3 x\left(-\sin x \frac{d}{dx} x\right)$$

$$\frac{dv}{dx} = -4\cos^3 x \sin x$$

Chain Rule:

$$\frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv}$$

$$\frac{du}{dv} = \cancel{2}\cancel{\sin x}\cancel{\cos x} \cdot \frac{1}{-\cancel{4}\cos^{\cancel{3}}x\,\cancel{\sin x}}$$

$$\frac{du}{dv} = \frac{1}{-2\cos^2 x}$$

$$\frac{du}{dv} = -\frac{\sec^2 x}{2} \qquad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$$

Question no 06 (I·L)

(پہلے Simplify کریں پھر diff لیں)

$$\tan y(1+\tan x) = 1-\tan x$$

$$\tan y = \frac{1-\tan x}{1+1.\tan x}$$

we know than $\tan\frac{\pi}{4} = 1$

$$\tan y = \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\tan x}$$

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$\tan y = \dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$

we also know that

$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan y = \tan\left(\frac{\pi}{4} - x\right)$

$y = \frac{\pi}{4} - x$

diff. w.r.t 'x'

$\frac{d}{dx} y = \frac{d}{dx}\left(\frac{\pi}{4} - x\right)$ $\left(\frac{\pi}{4} = \text{constant}\right)$

$\frac{dy}{dx} = 0 - 1$

$\frac{dy}{dx} = -1$

Its proved

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Question no 07 (I)

(جب دونوں طرف sqr کرے گے تو right طرف پھر "y" کی value بن جائے گی)

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x}} + \cdots \infty}$$

Taking squqre on both sides

$$y^2 = \tan x + \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x}} + \cdots \infty}$$

$$y^2 = \tan x + y$$

diff. w.r.t 'x'

$$\frac{d}{dx} y^2 = \frac{d}{dx}(\tan x + y)$$

$$2y^{2-1}\frac{d}{dx}y = \sec^2 x \frac{d}{dx}x + \frac{d}{dx}y$$

$$2y\frac{dy}{dx} = \sec^2 x(1) + \frac{dy}{dx}$$

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x$$

$$\frac{dy}{dx}(2y-1) = \sec^2 x$$

Its proved

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Question no 08 (I)

$x = a\cos^3\theta$, $y = b\sin^3\theta$

$x = a\cos^3\theta$

diff. w.r.t 'θ'

$\frac{d}{d\theta}x = \frac{d}{d\theta}(a\cos^3\theta)$

$\frac{dx}{d\theta} = a(3\cos^{3-1}\theta \frac{d}{d\theta}\cos\theta)$

$\frac{dx}{d\theta} = a(3\cos^2\theta(-\sin\theta \frac{d}{d\theta}\theta))$

$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta(1)$

$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

Then,

$y = b\sin^3\theta$

diff. w.r.t 'θ'

$\frac{d}{d\theta}y = \frac{d}{d\theta}(b\sin^3\theta)$

$\frac{dy}{d\theta} = b(3\sin^{3-1}\theta \frac{d}{d\theta}\sin\theta)$

$\frac{dy}{d\theta} = b(3\sin^2\theta(\cos\theta \frac{d}{d\theta}\theta))$

$\frac{dy}{d\theta} = 3b\sin^2\theta\cos\theta(1)$

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$$\frac{dy}{d\theta} = 3b\sin^2\theta\cos\theta$$

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = \cancel{3}b\sin^{\cancel{2}}\theta\cos\theta \cdot \frac{1}{-\cancel{3}a\cos^{\cancel{2}}\theta\sin\theta}$$

$$\frac{dy}{dx} = -\frac{b\sin\theta}{a\cos\theta}$$

$$a\frac{dy}{dx} = -b\tan\theta \qquad \left(\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$a\frac{dy}{dx} + b\tan\theta = 0$$

Its proved

Question no 09 (I)

$x = a(\cos t + \sin t)$, $y = a(\sin t - t\cos t)$

$x = a(\cos t + \sin t)$

diff. w.r.t 't'

$$\frac{d}{dt}x = \frac{d}{dt}a(\cos t + \sin t)$$

$$\frac{dx}{dt} = a\left(-\sin t\frac{d}{dt}t + \cos t\frac{d}{dt}t\right)$$

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$\frac{dx}{dt} = a(-\sin t(1) + \cos t(1))$

$\frac{dx}{dt} = a(\cos t - \sin t)$

Then,

$y = a(\sin t - t\cos t)$

diff. w.r.t 't'

$\frac{d}{dt} y = \frac{d}{dt} a(\sin t - t\cos t)$

$\frac{dy}{dt} = a\left[\cos t - \left(t\frac{d}{dt}\cos t + \cos t\frac{d}{dt} t\right)\right]$

$\frac{dy}{dt} = a\left[\cos t - \left(t(-\sin t\frac{d}{dt} t) + \cos t(1)\right)\right]$

$\frac{dy}{dt} = a\left[\cos t - \left(t(-\sin t)(1) + \cos t\right)\right]$

$\frac{dy}{dt} = a(\cancel{\cos t} + t\sin t - \cancel{\cos t})$

$\frac{dy}{dt} = at\sin t$

Chain rule :

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$$\frac{dy}{dx} = \cancel{a}t\sin t \cdot \frac{1}{\cancel{a}(\cos t - \sin t)}$$

$$\frac{dy}{dx} = \frac{t\sin t}{\cos t - \sin t}$$

DIFF of INVERSE TRIGONOMETRY

* $\frac{d}{dx}\sin^{-1}\theta = \frac{1}{\sqrt{1-\theta^2}}\frac{d\theta}{dx}$
* $\frac{d}{dx}\cos^{-1}\theta = \frac{-1}{\sqrt{1-\theta^2}}\frac{d\theta}{dx}$
* $\frac{d}{dx}\tan^{-1}\theta = \frac{1}{1+\theta^2}\frac{d\theta}{dx}$
* $\frac{d}{dx}\cot^{-1}\theta = \frac{-1}{1+\theta^2}\frac{d\theta}{dx}$
* $\frac{d}{dx}\sec^{-1}\theta = \frac{1}{\theta\sqrt{\theta^2-1}}\frac{d\theta}{dx}$
* $\frac{d}{dx}\operatorname{cosec}^{-1}\theta = \frac{-1}{\theta\sqrt{\theta^2-1}}\frac{d\theta}{dx}$

(جو "term" "C" سے شروع ہوں گی ان
کے ساتھ - لگائیں اوپر والی term میں)

Question no 10

(i) $\cos^{-1}\frac{x}{a}$, Let, $y = \cos^{-1}\frac{x}{a}$

diff. w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\cos^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\,\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{x^2}{a^2}}}\left[\frac{1}{a}\cdot\frac{d}{dx}(x)\right]$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{\frac{a^2-x^2}{a^2}}}\left[\frac{1}{a}\cdot 1\right]$$

$$\frac{dy}{dx} = \frac{-1}{\frac{\sqrt{a^2-x^2}}{\sqrt{a^2}}}\cdot\frac{1}{a}$$

$$\frac{dy}{dx} = \frac{-\sqrt{a^2}}{\sqrt{a^2-x^2}}\cdot\frac{1}{a}$$

$$\frac{dy}{dx} = \frac{-\cancel{a}}{\sqrt{a^2-x^2}}\cdot\frac{1}{\cancel{a}}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{a^2-x^2}}$$

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(ii) $\cot^{-1}\frac{x}{a}$

Let, $y = \cot^{-1}\frac{x}{a}$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\cot^{-1}\frac{x}{a}$$

$$\frac{dy}{dx} = \frac{-1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$\frac{dy}{dx} = \frac{-1}{\frac{a^2+x^2}{a^2}} \cdot \frac{1}{a}\left(\frac{d}{dx}x\right)$$

$$\frac{dy}{dx} = \frac{-a^2}{a^2+x^2} \cdot \frac{1}{a}(1)$$

$$\frac{dy}{dx} = \frac{-a^2}{a^2+x^2} \cdot \frac{1}{a}$$

$$\frac{dy}{dx} = -\frac{a}{a^2+x^2}$$

(iii) $\frac{1}{a}\sin^{-1}\frac{a}{x}$

let, $y = \frac{1}{a}\sin^{-1}\frac{a}{x}$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\left(\frac{1}{a}\sin^{-1}\frac{a}{x}\right)$$

$$\frac{dy}{dx} = \frac{1}{a}\left(\frac{1}{\sqrt{1-\left(\frac{a}{x}\right)^2}}\cdot\frac{d}{dx}\left(\frac{a}{x}\right)\right)$$

$$\frac{dy}{dx} = \frac{1}{a}\left(\frac{1}{\sqrt{\frac{x^2-a^2}{x^2}}}\right) a.\frac{d}{dx}x^{-1}$$

$$\frac{dy}{dx} = \frac{\cancel{a}x}{\cancel{a}\sqrt{x^2-a^2}}\cdot(-1)x^{-1-1}\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2-a^2}}(-1)x^{-2}(1)$$

$$\frac{dy}{dx} = -\frac{\cancel{x}}{x^{\cancel{2}}\sqrt{x^2-a^2}}$$

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-a^2}}$$

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(iv) $\sin^{-1}\sqrt{1-x^2}$

let, $y = \sin^{-1}\sqrt{1-x^2}$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\sin^{-1}\sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\frac{d}{dx}\sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\cancel{1}-\cancel{1}+x^2}}\cdot\frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}(1-x^2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{0+x^2}}\cdot\frac{1}{2\sqrt{1-x^2}}(0-2x)$$

$$\frac{dy}{dx} = \frac{-\cancel{2x}}{\cancel{(x)2}\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

(v) $\sec^{-1}\left[\frac{x^2+1}{x^2-1}\right]$

let, $y = \sec^{-1}\left[\frac{x^2+1}{x^2-1}\right]$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

$$\frac{dy}{dx} = \frac{1}{\frac{x^2+1}{x^2-1}\left[\sqrt{\left[\frac{x^2+1}{x^2-1}\right]^2 - 1}\right]} \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$

$$\frac{dy}{dx} = \frac{1}{\frac{x^2+1}{x^2-1}\left[\sqrt{\frac{x^4+1+2x^2}{x^4+1-2x^2} - 1}\right]} \cdot \frac{x^2-1\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{x^2+1}{x^2-1}\left[\sqrt{\frac{\cancel{x^4}+\cancel{1}+2x^2-\cancel{x^4}-\cancel{1}+2x^2}{x^4+1-2x^2}}\right]} \cdot \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{x^2+1}{x^2-1}\left[\sqrt{\frac{4x^2}{(x^2-1)^2}}\right]} \cdot \frac{\cancel{2x^3}-2x-\cancel{2x^3}-2x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2-1)\sqrt{(x^2-1)^2}}{(x^2+1)\sqrt{4x^2}} \cdot \frac{-4x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{\cancel{(x^2-1)}\cancel{(x^2-1)}}{(x^2+1)2\cancel{x}} \cdot \frac{-4\cancel{x}}{(x^2-1)^2}$$

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$$\frac{dy}{dx} = \frac{-2}{x^2+1}$$

(vi) $\cot^{-1}\left[\frac{2x}{1-x^2}\right]$

let, $y = \cot^{-1}\left[\frac{2x}{1-x^2}\right]$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\cot^{-1}\left[\frac{2x}{1-x^2}\right]$$

$$\frac{dy}{dx} = \frac{-1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{d}{dx}\left(\frac{2x}{1-x^2}\right)$$

$$\frac{dy}{dx} = \frac{-1}{1+\frac{4x^2}{1+x^4-2x^2}} \cdot \frac{(1-x^2)\frac{d}{dx}(2x)-(2x)\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$\frac{dy}{dx} = \frac{-1}{\frac{1+x^4-2x^2+4x^2}{1+x^4-2x^2}} \cdot \frac{(1-x^2)(2)-(2x)(-2x)}{(1-x^2)^2}$$

$$\frac{dy}{dx} = -\frac{1+x^4-2x^2}{1+x^4+2x^2} \cdot \frac{2-2x^2+4x^2}{(1-x^2)^2}$$

$$\frac{dy}{dx} = -\frac{(1-x^2)^2}{(1+x^2)^2} \cdot \frac{2(1-x^2+2x^2)}{(1-x^2)^2}$$

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$$\frac{dy}{dx} = -\frac{\cancel{(1-x^2)^2}}{(1+x^2)^{\cancel{2}}} \cdot \frac{2\cancel{(1+x^2)}}{\cancel{(1-x^2)^2}}$$

$$\frac{dy}{dx} = -\frac{2}{1+x^2}$$

(vii) $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

let, $y = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{d}{dx}\left[\frac{1-x^2}{1+x^2}\right]$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{1^2+x^4-2x^2}{1^2+x^2+2x^2}}} \cdot \frac{(1+x^2)\frac{d}{dx}(1-x^2)-(1-x^2)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{\frac{\cancel{1}+\cancel{x^4}+2x^2\cancel{-1}\cancel{-x^4}+2x^2}{1+x^4+2x^2}}} \cdot \frac{(1-x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{\frac{4x^2}{1+x^4+2x^2}}} \cdot \frac{-2x+2x^3-2x-2x^3}{(1+x^2)^2}$$

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$$\frac{dy}{dx} = -\frac{\sqrt{1+x^4+2x^2}}{\sqrt{4x^2}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = -\frac{\sqrt{(1+x^2)^2}}{\sqrt{4x^2}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = -\frac{1+x^2}{2x} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

Question no 11 (V.V.I)

$$\frac{y}{x} = \tan^{-1}\frac{x}{y}$$

diff. w.r.t 'x'

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}\left(\tan^{-1}\frac{x}{y}\right)$$

$$\frac{(x)\frac{d}{dx}(y) - (y)\frac{d}{dx}(x)}{x^2} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{y}\right)$$

$$\frac{x\frac{dy}{dx} - y(1)}{x^2} = \frac{1}{1+\frac{x^2}{y^2}} \cdot \frac{(y)\frac{d}{dx}(x) + (x)\frac{d}{dx}(y)}{y^2}$$

$$x\frac{dy}{dx} - y = x^2 \cdot \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{y - x\frac{d}{dx}y}{y^2}$$

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$$x\frac{dy}{dx}-y=\frac{x^2y^{\cancel{2}}}{y^2+x^2}\cdot\frac{y-x\frac{dy}{dx}}{y^{\cancel{2}}}$$

$$x\frac{dy}{dx}-y=\frac{x^2}{y^2+x^2}\left(y-x\frac{dy}{dx}\right)$$

$$x\frac{dy}{dx}-y=\frac{x^2y}{y^2+x^2}-\frac{x^3}{y^2+x^2}\frac{dy}{dx}$$

$$x\frac{dy}{dx}+\frac{x^3}{y^2+x^2}\frac{dy}{dx}=\frac{x^2y}{y^2+x^2}+y$$

$$\frac{dy}{dx}\left(x+\frac{x^3}{y^2+x^2}\right)=\frac{x^2y}{y^2+x^2}+y$$

$$x\frac{dy}{dx}\left(1+\frac{x^2}{y^2+x^2}\right)=y\left(\frac{x^2}{y^2+x^2}+1\right)$$

$$x\frac{dy}{dx}=y\cancel{\left(\frac{x^2}{y^2+x^2}+1\right)}\times\frac{1}{\cancel{\left(1+\frac{x^2}{y^2+x^2}\right)}}$$

$$x\frac{dy}{dx}=y$$

$$\frac{dy}{dx}=\frac{y}{x}$$

Its proved

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Question no 12 (V·I)

$$y = \tan(p\tan^{-1}x)$$

$$\tan^{-1}y = p\tan^{-1}x$$

diff. w.r.t 'x'

$$\frac{d}{dx}(\tan^{-1}y) = \frac{d}{dx}(p\tan^{-1}x)$$

$$\frac{1}{1+y^2}\frac{d}{dx}y = p.\frac{d}{dx}\tan^{-1}x\left(\frac{d}{dx}x\right)$$

$$\frac{1}{1+y^2}\frac{dy}{dx} = p.\frac{1}{1+x^2}(1)$$

$$\left[\frac{dy}{dx} = y_1\right]$$

$$\frac{y_1}{1+y^2} = \frac{p}{1+x^2}$$

Cross Multiply

$$(1+x^2)y_1 = (1+y^2)p$$

$$(1+x^2)y_1 - (1+y^2)p = 0$$

Its proved

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Question 1. (I.L)

(اس کو First principle کے مطابق کرنا ہے
نہ کہ diff کے مطابق)

(i) $\sin 2x$

let, $y = \sin 2x$

$y + \delta y = \sin 2(x + \delta x)$

$y + \delta y = \sin(2x + 2\delta x)$

$\delta y = \sin(2x + 2\delta x) - y$

$\delta y = \sin(2x + 2\delta x) - \sin 2x$

$$\boxed{\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}}$$

$$\delta y = 2\cos\frac{2x + 2\delta x + 2x}{2}\sin\frac{\cancel{2x} + 2\delta x - \cancel{2x}}{2}$$

$$\delta y = 2\cos\frac{2\delta x + 4x}{2}\sin\frac{\cancel{2}\delta x}{\cancel{2}}$$

$$\delta y = 2\cos\frac{\cancel{2}(\delta x + 2x)}{\cancel{2}}\sin\delta x$$

$$\delta y = 2\cos(\delta x + 2x).\sin\delta x$$

Divide by 'δx' on b/s

(جب دو Value آپس میں Multi ہو رہی ہوں تو جس
مرضی Value کے ساتھ divide کر دیتے ہیں)

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$$\frac{\delta y}{\delta x} = 2\cos(\delta x + 2x) \cdot \frac{\sin \delta x}{\delta x}$$

Then, Take limit

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left[2\cos(\delta x + 2x) \cdot \frac{\sin \delta x}{\delta x} \right]$$

$$\boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1} \quad \boxed{\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}}$$

So,

$$\frac{dy}{dx} = 2\cos(0+2x) \cdot 1$$

$$\frac{dy}{dx} = 2\cos 2x$$

(ii) $\tan 3x$

let, $y = \tan 3x$

$$y + \delta y = \tan 3(x + \delta x)$$

$$\delta y = \tan(3x + 3\delta x) - y$$

$$\delta y = \tan(3x + 3\delta x) - \tan 3x$$

$$\delta y = \frac{\sin(3x + 3\delta x)}{\cos(3x + 3\delta x)} - \frac{\sin 3x}{\cos 3x} \quad \left(\because \tan x = \frac{\sin x}{\cos x} \right)$$

$$\delta y = \frac{\sin(3x + 3\delta x)\cos 3x - \cos(3x + 3\delta x)\sin 3x}{\cos(3x + 3\delta x)\cos 3x}$$

$$\boxed{\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha - \beta)}$$

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$$\delta y = \frac{\sin(3x + 3\delta x - 3x)}{\cos(3x+3\delta x)\cos 3x}$$

$$\delta y = \frac{\sin 3\delta x}{\cos(3x+3\delta x)\cos 3x} \Rightarrow \frac{1}{\cos(3x+3\delta x)\cos 3x} \cdot \sin 3\delta x$$

Divide by 'δx' on b/S

$$\frac{\delta y}{\delta x} = \frac{1}{\cos(3x+3\delta x)\cos 3x} \cdot \frac{\sin 3\delta x}{\delta x}$$

Multiply and divide by

$$\frac{\delta y}{\delta x} = \frac{1}{\cos(3x+3\delta x)\cos 3x} \cdot \frac{\sin 3\delta x}{3\delta x}(3)$$

Then, Take limit

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0}\left[\frac{1}{\cos(3x+3\delta x)\cos 3x} \cdot \frac{\sin 3\delta x}{3\delta x}(3)\right]$$

$$\frac{dy}{dx} = \frac{1}{\cos 3x \cdot \cos 3x} \cdot 1(3)$$

$$\frac{dy}{dx} = \frac{1}{\cos^2 3x} \cdot 3$$

$$\frac{dy}{dx} = \sec^2 3x \cdot 3 \quad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$$

$$\frac{dy}{dx} = 3\sec^2 3x$$

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(iii) $\sin 2x + \cos 2x$

Let, $y = \sin 2x + \cos 2x$

$y + \delta y = \sin 2(x+\delta x) + \cos 2(x+\delta x)$

$\delta y = \sin(2x+2\delta x) + \cos(2x+2\delta x) - y$

$\delta y = \sin(2x+2\delta x) + \cos(2x+2\delta x) - \sin 2x - \cos 2x$

$\delta y = [\sin(2x+2\delta x) - \sin 2x] + [\cos(2x+2\delta x) - \cos 2x]$

$$\boxed{\sin P - \sin\theta = 2\cos\frac{P+\theta}{2}\sin\frac{P-\theta}{2}}$$

$$\boxed{\cos P - \cos\theta = -2\sin\frac{P+\theta}{2}\sin\frac{P-\theta}{2}}$$

$$\delta y = 2\cos\left(\frac{2x+2\delta x+2x}{2}\right)\sin\left(\frac{\cancel{2x}+2\delta x-\cancel{2x}}{2}\right)$$

$$+\left[-2\sin\left(\frac{2x+2\delta x+2x}{2}\right)\sin\left(\frac{\cancel{2x}+2\delta x-\cancel{2x}}{2}\right)\right]$$

$$\delta y = 2\cos\left(\frac{4x+2\delta x}{2}\right)\sin\frac{\cancel{2}\delta x}{\cancel{2}}$$

$$-2\sin\left(\frac{4x+2\delta x}{2}\right)\sin\frac{\cancel{2}\delta x}{\cancel{2}}$$

$$\delta y = 2\cos\frac{\cancel{2}(2x+\delta x)}{\cancel{2}}\sin\delta x - 2\sin\frac{\cancel{2}(2x+\delta x)}{\cancel{2}}\sin\delta x$$

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$\delta y = 2\cos(2x+\delta x)\sin\delta x - 2\sin(2x+\delta x)\sin\delta x$

Divide by δx on b/S

$\frac{\delta y}{\delta x} = 2\cos(2x+\delta x)\frac{\sin\delta x}{\delta x} - 2\sin(2x+\delta x)\frac{\sin\delta x}{\delta x}$

Then, Take Limit

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[2\cos(2x+\delta x)\frac{\sin\delta x}{\delta x} - 2\sin(2x+\delta x)\frac{\sin\delta x}{\delta x}\right]$$

$$\frac{dy}{dx} = 2\cos(2x+0)\,1 - 2\sin(2x+0)\,1$$

$$\frac{dy}{dx} = 2\cos 2x - 2\sin 2x$$

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(iv) $\cos x^2$

Let $y = \cos x^2$

$y+\delta y = \cos(x+\delta x)^2$

$y+\delta y = \cos(x^2+\delta x^2+2x\delta x)$

$\delta y = \cos(x^2+\delta x^2+2x\delta x) - y$

$\delta y = \cos(x^2+\delta x^2+2x.\delta x) - \cos x^2$

$$\boxed{\cos P - \cos \theta = -2\sin\frac{P+\theta}{2}\sin\frac{P-\theta}{2}}$$

$$\delta y = -2\sin\frac{(x^2+\delta x^2+2x\delta x+x^2)}{2}\sin\frac{(\cancel{x^2}+\delta x^2+2x\delta x-\cancel{x^2})}{2}$$

$$\delta y = -2\sin\frac{(2x^2+\delta x^2+2x\delta x)}{2}\sin\frac{(\delta x^2+2x\delta x)}{2}$$

$$\delta y = -2\sin\frac{(2x^2+\delta x^2+2x\delta x)}{2}\sin\frac{\delta x(\delta x+2x)}{2}$$

Diride by δx on b/s

$$\frac{\delta y}{\delta x} = -2\sin\frac{(2x^2+\delta x^2+2x\delta x)}{2}\frac{\sin\frac{\delta x(\delta x+2x)}{2}}{\delta x}$$

$$\frac{\delta y}{\delta x} = -2\sin\frac{(2x^2+\delta x^2+2x\delta x)}{2}\frac{\sin\frac{\delta x(\delta x+2x)}{2}}{\frac{\delta x(\delta x+2x)}{2}} \times \frac{\delta x+2x}{2}$$

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Then Taking limit

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[\frac{-2\sin\left(\frac{2x^2+\delta x^2+2x\delta x}{2}\right).\ \frac{\sin\frac{\delta x(\delta x+2x)}{2}}{\frac{\delta x(\delta x+2x)}{2}}.\ \frac{\delta x+2x}{2}}{}\right]$$

$$\frac{dy}{dx} = -2\sin\frac{\cancel{2}x^2}{\cancel{2}}.1.\frac{\cancel{2}x}{\cancel{2}}$$

$$\frac{dy}{dx} = -2\sin x^2.x$$

$$\frac{dy}{dx} = -2x\sin x^2$$

(v) $\tan^2 x$

let, $y = \tan^2 x \Rightarrow (\tan x)^2$

$y+\delta y = \tan(x+\delta x)^2$

$\delta y = \tan(x+\delta x)^2 - y$

$\delta y = \tan(x+\delta x)^2 - (\tan x)^2$

$\delta y = [\tan(x+\delta x)+\tan x][\tan(x+\delta x)-\tan x]$

$\delta y = [\tan(x+\delta x)+\tan x]\left[\frac{\sin(x+\delta x)}{\cos(x+\delta x)} - \frac{\sin x}{\cos x}\right]$

$\delta y = [\tan(x+\delta x)+\tan x]\left[\frac{\sin(x+\delta x)\cos x - \sin x\cos(x+\delta x)}{\cos(x+\delta x)\cos x}\right]$

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$$\boxed{\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)}$$

$$\delta y = [\tan(x+\delta x) + \tan x]\left[\frac{\sin(x+\delta x - x)}{\cos(x+\delta x)\cos x}\right]$$

$$\delta y = [\tan(x+\delta x) + \tan x]\left[\frac{\sin\delta x}{\cos(x+\delta x)\cos x}\right]$$

Diride by δx on b/s

$$\frac{\delta y}{\delta x} = \frac{\tan(x+\delta x)+\tan x}{\cos(x+\delta x)\cos x}\left[\frac{\sin\delta x}{\delta x}\right]$$

Then, Take Limit

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[\frac{\tan(x+\delta x)+\tan x}{\cos(x+\delta x)\cos x} \cdot \frac{\sin\delta x}{\delta x}\right]$$

$$\frac{dy}{dx} = \frac{\tan x+\tan x}{(\cos x)(\cos x)} \cdot 1$$

$$\frac{dy}{dx} = \frac{2\tan x}{\cos^2 x}$$

$$\frac{dy}{dx} = 2\tan x\sec^2 x$$

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(vi) $\sqrt{\tan x}$

Let $y = \sqrt{\tan x}$

$y + \delta y = \sqrt{\tan(x+\delta x)}$

$\delta y = \sqrt{\tan(x+\delta x)} - y$

$\delta y = \sqrt{\tan(x+\delta x)} - \sqrt{\tan x}$

by Rationalization

$$\delta y = \sqrt{\tan(x+\delta x)} - \sqrt{\tan x} \times \frac{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}}$$

$$\delta y = \frac{\left(\sqrt{\tan(x+\delta x)}\right)^2 - \left(\sqrt{\tan x}\right)^2}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}}$$

$$\delta y = \frac{\tan(x+\delta x) - \tan x}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}}$$

$$\delta y = \frac{\tan(x+\delta x) - \tan x}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}}$$

$$\delta y = \frac{1}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}} \tan(x+\delta x) - \tan x$$

$$\delta y = \frac{1}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}} \left[\frac{\sin(x+\delta x)}{\cos(x+\delta x)} - \frac{\sin x}{\cos x}\right]$$

$$\delta y = \frac{1}{\sqrt{\tan(x+\delta x)} + \sqrt{\tan x}} \left[\frac{\sin(x+\delta x)\cos x - \cos(x+\delta x)\sin x}{\cos(x+\delta x)\cos x}\right]$$

$$\boxed{\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)}$$

$$\delta y = \frac{1}{\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}}\left[\frac{\sin(x+\delta x - x)}{\cos(x+\delta x)\cos x}\right]$$

$$\delta y = \frac{1}{\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}}\left[\frac{\sin\delta x}{\cos(x+\delta x)\cos x}\right]$$

Divide by δx on b/s

$$\frac{\delta y}{\delta x} = \frac{1}{\sqrt{\tan(x+\delta x)}+\sqrt{\tan x}}\left[\frac{1}{\cos(x+\delta x)\cos x}\cdot\frac{\sin\delta x}{\delta x}\right]$$

Then, Take limit

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[\frac{1}{\tan\sqrt{x+\delta x}+\sqrt{\tan x}}\cdot\frac{1}{\cos(x+\delta x)\cos x}\cdot\frac{\sin\delta x}{\delta x}\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}+\sqrt{\tan x}}\cdot\frac{1}{\cos x(\cos x)}\cdot 1$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}}\cdot\frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$$

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(vii) V·V·I

$\cos\sqrt{x}$

let, $y = \cos\sqrt{x}$

$y+\delta y = \cos\sqrt{x+\delta x}$

$\delta y = \cos\sqrt{x+\delta x} - y$

$\delta y = \cos\sqrt{x+\delta x} - \cos\sqrt{x}$

$$\boxed{\cos P - \cos Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}}$$

$$\delta y = -2\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}$$

$$\delta y = -2\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}$$

$$\delta y = -2\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}$$

Divide by δx on b/s

$$\frac{\delta y}{\delta x} = -2\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2}\,\frac{\sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}}{\delta x}$$

$$\boxed{\delta x = \left(\sqrt{x+\delta x}-\sqrt{x}\right)\left(\sqrt{x+\delta x}+\sqrt{x}\right)}$$

$$\frac{\delta y}{\delta x} = -2\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2} \cdot \sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}$$

$$\overline{(\sqrt{x+\delta x}-\sqrt{x})(\sqrt{x+\delta x}+\sqrt{x})}$$

$$\frac{\delta y}{\delta x} = -\cancel{2}\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2} \; \frac{\sin\sqrt{x+\delta x}-\sqrt{x}/2}{\frac{(\sqrt{x+\delta x}-\sqrt{x})(\sqrt{x+\delta x}+\sqrt{x})}{2}}$$

$$\frac{\delta y}{\delta x} = \cancel{2}\sin\frac{\sqrt{x+\delta x}+\sqrt{x}}{2} \; \frac{\sin\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}}{\frac{\sqrt{x+\delta x}-\sqrt{x}}{2}} \cdot \frac{1}{\cancel{2}\sqrt{x+\delta x}+\sqrt{x}}$$

Then Take limit

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left[-\sin\frac{(\sqrt{x+\delta x}+\sqrt{x})}{2} \cdot \frac{\sin\sqrt{x+\delta x}-\sqrt{x}/2}{\sqrt{x+\delta x}-\sqrt{x}/2} \cdot \frac{1}{\sqrt{x+\delta x}+\sqrt{x}}\right]$$

$$\frac{dy}{dx} = -\sin\left(\frac{\sqrt{x}+\sqrt{x}/2}{2}\right) \cdot 1 \cdot \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sin\cancel{2}\sqrt{x}/\cancel{2}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$$