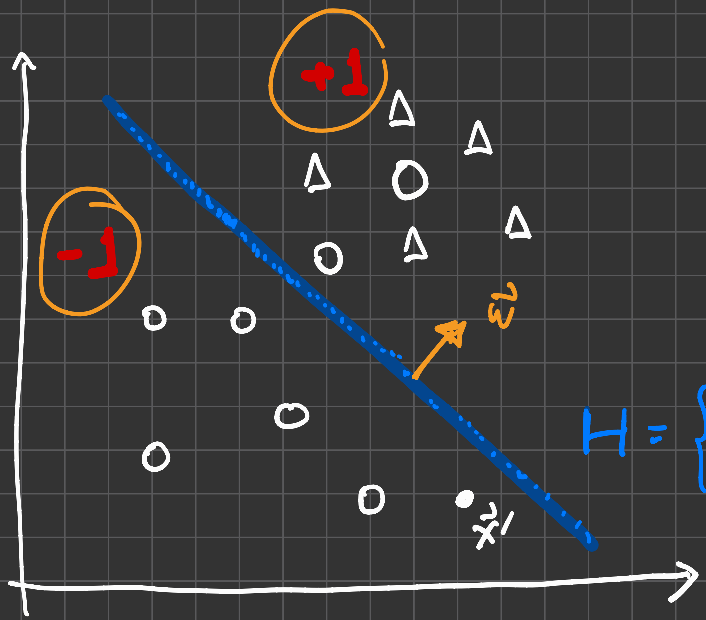


$Y = \{-1, +1\}$

$H = \{\vec{x} \mid \vec{w}^T \vec{x} + b = 0\}$

$O(d)$

Training: learn the vector $\vec{w}$ and offset b

Testing: $\text{sign}(\vec{w}^T \vec{x}' + b)$

$O(d)$

hyperplane

$\parallel$

linear classifier

How to find $\vec{w}$ and b ??

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix} ; \quad \boxed{\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{pmatrix} ; \; b}$$

$$H = \{ \vec{x} \mid \boxed{\vec{w}^T \vec{x} + b} = 0 \}$$

$$\vec{w}'^T \vec{x}' \quad \text{where} \quad \boxed{\vec{w}' = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \\ b \end{pmatrix} , \quad \vec{x}' = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \\ 1 \end{pmatrix}}$$

$$\boxed{\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \\ b \end{pmatrix}}$$

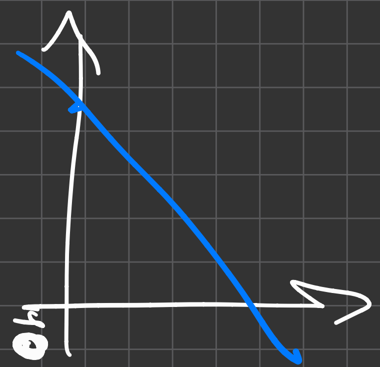
Proof:

$$\vec{w}^T \vec{x} + b = w_1 x_1 + w_2 x_2 + \dots + w_d x_d + b \quad \text{— (A)}$$

$$\vec{w}'^T \vec{x}' = w_1 x_1 + w_2 x_2 + \dots + w_d x_d + b \cdot 1 \quad \text{— (B)}$$

$$(A) = (B) \Rightarrow \boxed{\vec{w}^T \vec{x} + b = \vec{w}'^T \vec{x}'}$$

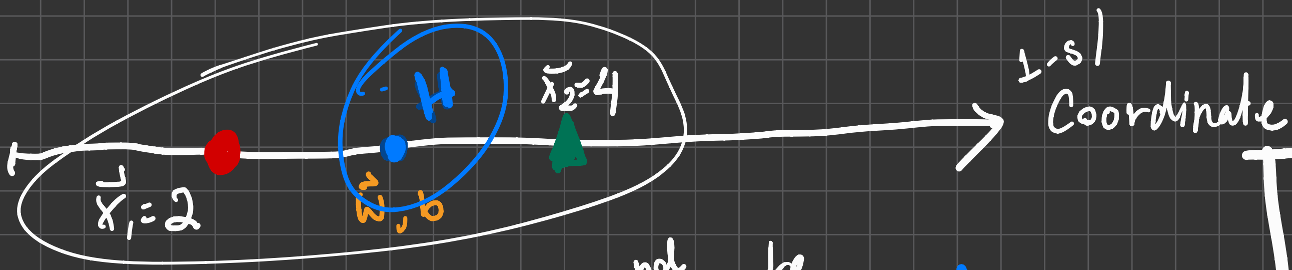
Rule

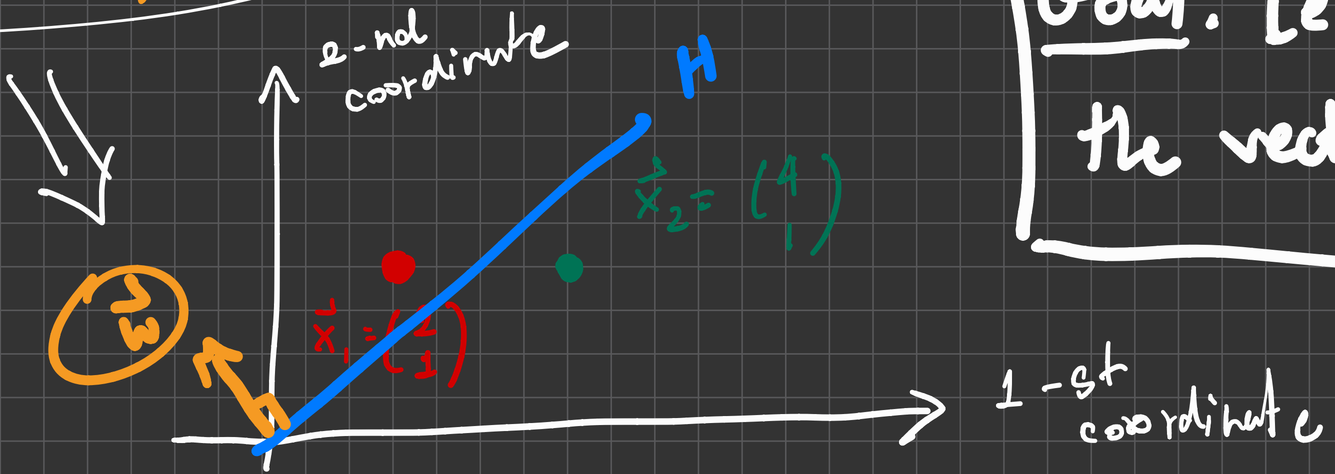


$$\vec{w} \Longrightarrow \boxed{\vec{w}' = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \\ b \end{pmatrix}}$$

$\underline{\underline{d+1}}$

$$\vec{x} \Longrightarrow \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \\ 1 \end{pmatrix}$$





$$\boxed{\text{Goal: Learn only the vector } \vec{w}}$$

The algorithm of the Perceptron:

$\vec{w} \leftarrow \vec{0}$

while true:

  $m \leftarrow 0$

  for $(\vec{x}_i, y_i) \in D$:

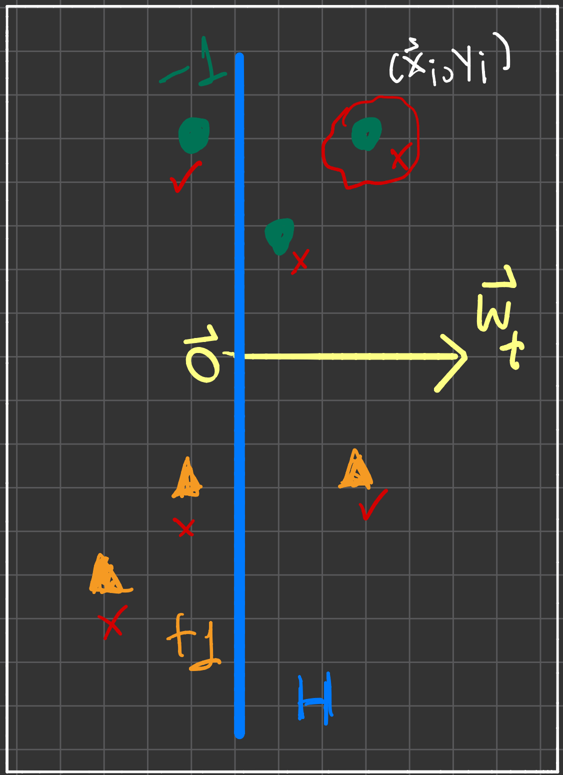
    if $(y_i)(\text{sign}(\vec{w}^T \vec{x})) \leq 0$: // if $(\vec{x}_i, y_i)$ is misclassified

      $m \leftarrow m + 1$ // update counter

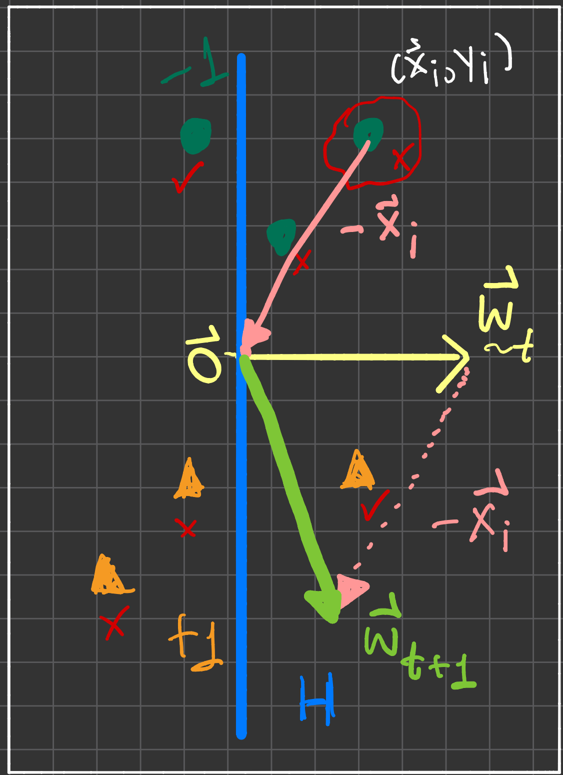
      $\vec{w} \leftarrow \vec{w} + y_i \vec{x}_i$ // fix $\vec{w}$ by using information from (x_i, y_i)

  if $m = 0$:

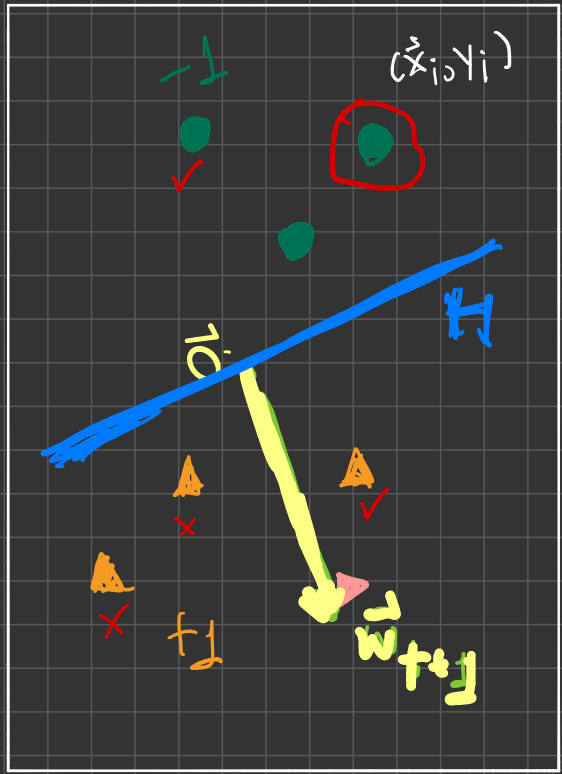
    break



At some iteration t



During iteration t



At iteration $t+1$

$$\vec{w} \leftarrow \vec{w} + (-\vec{x}_i)$$