Generative learning

$D \sim P(X,Y)$

MLE

MAP

$\boxed{P_\theta(X,Y)}$ — Estimate

Binomial distribution

Beta

Gaussian

$P(x,y\,;\theta) \equiv P_\theta(x,y)$

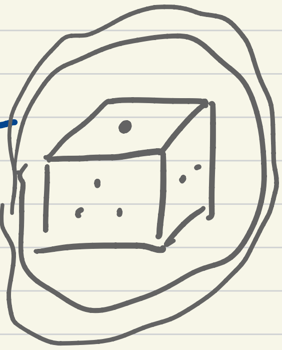
Bayes classifier: return $\boxed{\underset{\forall y}{\arg\max}\ P_\theta(y \mid x)}$

Binomial $\boxed{P(H) = \frac{n_H}{n}}$ MLE

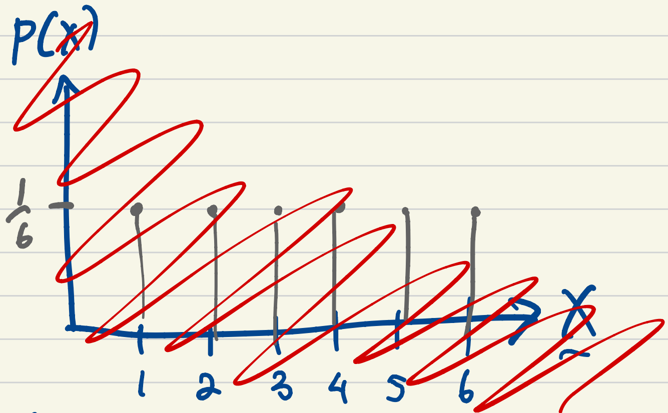
$P_\theta(x)$

$\boxed{H \quad T}$

$P_\theta(X,Y)$



$$I(E) = \begin{cases} 0 & \text{if } E \text{ doesn't occur} \\ 1 & \text{if } E \text{ occurs} \end{cases}$$



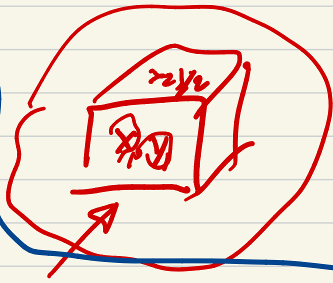
$$P(x) \begin{cases} P(X=1) = \dfrac{n_1}{n} = \dfrac{\sum_{i=1}^{n} I(x_i = 1)}{n} \\ P(X=2) = \dfrac{n_2}{n} = \dfrac{\sum_{i=1}^{n} I(x_i = 2)}{n} \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ P(X=6) = \dfrac{n_6}{n} = \dfrac{\sum_{i=1}^{n} I(x_i = 6)}{n} \end{cases}$$

Estimate $P_\theta(X,Y)$ using $\boxed{\text{Binomial distribution}}$

$$\|$$

$\forall x \forall y$

$$P_\theta(X=x \wedge Y=y) = \frac{\sum_{i=1}^{n} I(X_i = x \text{ and } Y_i = y)}{n}$$

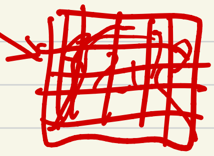


$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix} \quad D \longrightarrow P_\theta(X,Y)$$

$$P_\theta(X=x \wedge Y=y) = P_\theta([X]_1 = x_1, [X]_2 = x_2, \ldots, [X]_d = x_d \wedge Y=y)$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{pmatrix}$$

$$\begin{array}{cl} & \\ \begin{pmatrix} 0 \\ 2 \\ \vdots \\ 6 \end{pmatrix} & \begin{array}{l} a \\ ant \\ \\ zebra \end{array} \end{array}$$



$$P_\theta(X=x \wedge Y=y) \approx \frac{1}{n} = 0$$

$P_\theta(X, Y)$

$$\forall x \forall y \quad P_\theta(X = x \wedge Y = y) = \frac{\sum_{i=1}^{n} \mathbb{I}(X_i = x \wedge Y_i = y)}{n}$$

Binomial

Problem: As $d \to +\infty$, $P_\theta(X = x \wedge Y = y) \approx \frac{1}{n} = 0$

Naive Bayes: Estimate $P_\theta(Y|X)$ insted of $P_\theta(X, Y)$

$$\forall x \forall y \quad P_\theta(Y = y \mid X = x) = \frac{P_\theta(X = x \mid Y = y)\, P_\theta(Y = y)}{P_\theta(X = x)}$$

??

$\sum_{i=1}^{n} \mathbb{I}(Y_i = y)$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$\sum_{i=1}^{n} \mathbb{I}(X_i = x)$

Naive Bayes Assumption: All feature values are independent.

$$P(X = x \mid Y = y) = \prod_{i=1}^{d} P([X]_i = x_i \mid Y = y)$$

Estimate $P_\theta([X]_i = x_i \mid Y = y)$

$$Y = \{\underset{0}{\text{go-out}} / \underset{1}{\text{stay home}}\} = \{0,1\}$$

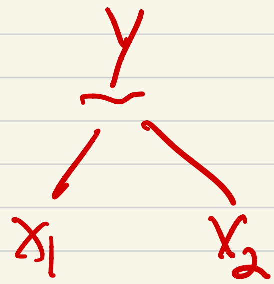
$$\underset{\sim}{X} \begin{cases} X_1 = \{\underset{1}{\text{sunny}} / \underset{0}{\text{rainy}}\} \\ X_2 = \{\underset{1}{\text{car-broken}} / \underset{0}{\text{car working}}\} \end{cases}$$

$$\mathcal{X} = \{0,1\}^2$$

$P_\theta(Y|X)$

Bayes classifier

Estimate $P_\theta(X|Y)$



	X_1	X_2	Y
1	1	1	1
2	0	0	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	0	0
7	0	0	0
8	0	1	0
9	1	0	0
10	0	0	0

$$P(Y=1) = \frac{5}{10} = 0.2$$

$$P(X=x) = \frac{3}{10} = 0.3$$

Binomial

$$P_\theta(Y=1 \mid x=(0,0)) = \frac{P_\theta(x=(0,0) \mid y=1) \times 5}{3} = \frac{\cancel{\frac{1}{25}} \times \cancel{5}}{3} = \frac{1}{15}$$

$$\frac{1}{25}$$

$$\boxed{P_\theta(Y=0 \mid x=(0,0)) = ??}$$

$$= \boxed{P(X_1=0 \mid y=1) \times P(X_2=0 \mid y=1)}$$

$$= \frac{1}{5} \qquad = \frac{1}{5}$$