

ICPC Template Library

postpone | Shu-i64

November 14, 2025

v1.0 

ref: jiangly, skip2004, zeemanz, capps

Attention: 0-indexed, $[l, r)$.

Contents

1	Data Structure	2
1.1	Disjoint Set Union	2
1.2	Fenwick Tree	3
1.3	Lazy Segment Tree	3
1.4	Sparse Table	5
1.5	Mo	6
1.6	Cartesian Tree	7
1.7	Lichao Segment Tree	7
2	Math	8
2.1	Formula	8
2.2	Exgcd	8
2.3	Phi	9
2.4	Sieve	9
2.5	Pollard's Rho	9
2.6	Combination	10
2.7	Linear Basis	10
2.8	Gaussian Elimination	10
2.9	Polynomial	11
2.9.1	Lagrange Interpolation	11
2.9.2	Number Theory Transform	12
2.9.3	Fast Walsh Transform	17
2.9.4	Sum of Subset	17
3	Graph	18
3.1	Tree	18
3.1.1	Lowest Common Ancestor	18
3.1.2	Heavy-Light Decomposition	18
3.1.3	DSU on Tree	20
3.2	Connectivity	20
3.2.1	Strongly Connected Component	20
3.2.2	Block Cut Tree	21
3.3	Two Sat	21

4	String	22
4.1	String Hash	22
4.2	KMP	22
4.3	Z-function	22
4.4	AhoCorasick	22
4.5	Suffix Array	23
4.6	Suffix Automaton	24
4.7	Palindromic Tree	25
5	Geometry	25
6	Util	29
6.1	Mod Integer	29
6.2	Fraction	31
7	Tables	32
7.1	Constant	33

蒋凌宇

1 Data Structure

1.1 Disjoint Set Union

```

1 struct DSU {
2     std::vector<int> f, siz;
3     DSU() {}
4     DSU(int n) {
5         init(n);
6     }
7     void init(int n) {
8         f.resize(n);
9         std::iota(f.begin(), f.end(), 0);
10        siz.assign(n, 1);
11    }
12    int find(int x) {
13        while (x != f[x]) {
14            x = f[x] = f[f[x]];
15        }
16        return x;
17    }
18    bool same(int x, int y) {
19        return find(x) == find(y);
20    }
21    bool merge(int x, int y) {
22        x = find(x);
23        y = find(y);
24        if (x == y) {
25            return false;
26        }
27        siz[x] += siz[y];
28        f[y] = x;
29        return true;
30    }
31    int size(int x) {
32        return siz[find(x)];
33    }
34 };
35
36 // DSU with Rollback
37 struct DSU {
38     std::vector<std::pair<int &, int>> his;
39     std::vector<int> f, siz;
40     DSU() {}
41     DSU(int n) {
42         init(n);
43     }
44     void init(int n) {
45         f.resize(n);
46         std::iota(f.begin(), f.end(), 0);
47         siz.assign(n, 1);

```

```

48     }
49     void set(int &a, int b) {
50         his.emplace_back(a, a);
51         a = b;
52     }
53     int find(int x) {
54         while (x != f[x]) {
55             x = f[x];
56         }
57         return x;
58     }
59     bool merge(int x, int y) {
60         x = find(x);
61         y = find(y);
62         if (x == y) {
63             return false;
64         }
65         if (siz[x] < siz[y]) {
66             std::swap(x, y);
67         }
68         set(siz[x], siz[x] + siz[y]);
69         set(f[y], x);
70         return true;
71     }
72     bool same(int x, int y) {
73         return find(x) == find(y);
74     }
75     int cur() {
76         return his.size();
77     }
78     void rollback(int t) {
79         while (his.size() > t) {
80             auto [x, y] = his.back();
81             x = y;
82             his.pop_back();
83         }
84     }
85 };
86
87 // Maintain whether each connected component is bipartite
88 struct DSU {
89     std::vector<std::pair<int &, int>> his;
90     int n;
91     std::vector<int> f, g, bip;
92     DSU(int n_) : n(n_), f(n, -1), g(n), bip(n, 1) {}
93     std::pair<int, int> find(int x) {
94         if (f[x] < 0) {
95             return {x, 0};
96         }
97         auto [u, v] = find(f[x]);
98         return {u, v ^ g[x]};

```

```

99     }
100 void set(int &a, int b) {
101     his.emplace_back(a, a);
102     a = b;
103 }
104 void merge(int a, int b, int &ans) {
105     auto [u, xa] = find(a);
106     auto [v, xb] = find(b);
107     int w = xa ^ xb ^ 1;
108     if (u == v) {
109         if (bip[u] && w) {
110             set(bip[u], 0);
111             ans--;
112         }
113         return;
114     }
115     if (f[u] > f[v]) {
116         std::swap(u, v);
117     }
118     ans -= bip[u];
119     ans -= bip[v];
120     set(bip[u], bip[u] && bip[v]);
121     set(f[u], f[u] + f[v]);
122     set(f[v], u);
123     set(g[v], w);
124     ans += bip[u];
125 }
126 int cur() {
127     return his.size();
128 }
129 void rollback(int t) {
130     while (his.size() > t) {
131         auto [x, y] = his.back();
132         x = y;
133         his.pop_back();
134     }
135 }
136 };

```

1.2 Fenwick Tree

```

1 template <typename T>
2 struct Fenwick {
3     int n;
4     std::vector<T> a;
5     Fenwick(int n_ = 0) {
6         init(n_);
7     }
8     void init(int n_) {
9         n = n_;

```

```

10     a.assign(n, T{});
11 }
12 void add(int x, const T &v) {
13     for (int i = x + 1; i ≤ n; i += i & -i) {
14         a[i - 1] = a[i - 1] + v;
15     }
16 }
17 T sum(int x) {
18     T ans{};
19     for (int i = x; i > 0; i -= i & -i) {
20         ans = ans + a[i - 1];
21     }
22     return ans;
23 }
24 T rangeSum(int l, int r) {
25     return sum(r) - sum(l);
26 }
27 int select(const T &k) {
28     int x = 0;
29     T cur{};
30     for (int i = 1 << std::__lg(n); i; i /= 2) {
31         if (x + i ≤ n && cur + a[x + i - 1] ≤ k) {
32             x += i;
33             cur = cur + a[x - 1];
34         }
35     }
36     return x;
37 }
38 };

```

1.3 Lazy Segment Tree

```

1 template <class Info, class Tag>
2 struct LazySegmentTree {
3     int n;
4     std::vector<Info> info;
5     std::vector<Tag> tag;
6
7     LazySegmentTree() = delete;
8     LazySegmentTree(int n_, const Info &v_ = {}) { init(std::vector<Info>(n_, v_)); }
9     template <class T>
10     LazySegmentTree(const std::vector<T> &data) { init(data); }
11
12     template <class T>
13     void init(const std::vector<T> &data) {
14         n = data.size();
15         info.assign(4 << std::__lg(n), {});
16         tag.assign(4 << std::__lg(n), {});
17

```

```

18     auto build = [&](auto self, int p, int l, int r) → void {
19         if (r - l == 1) {
20             info[p] = data[l];
21             return;
22         }
23         int m = (l + r) / 2;
24         self(self, 2 * p, l, m);
25         self(self, 2 * p + 1, m, r);
26         pull(p);
27     };
28     build(build, 1, 0, n);
29 }
30
31 void pull(int p) {
32     info[p] = info[2 * p] + info[2 * p + 1];
33 }
34 void apply(int p, const Tag &v) {
35     info[p].apply(v);
36     tag[p].apply(v);
37 }
38 void push(int p) {
39     apply(2 * p, tag[p]);
40     apply(2 * p + 1, tag[p]);
41     tag[p] = {};
42 }
43 void modify(int p, int l, int r, int x, const Info &v) {
44     if (r - l == 1) {
45         info[p] = v;
46         return;
47     }
48     int m = (l + r) / 2;
49     push(p);
50     if (x < m) {
51         modify(2 * p, l, m, x, v);
52     } else {
53         modify(2 * p + 1, m, r, x, v);
54     }
55     pull(p);
56 }
57 void modify(int p, const Info &v) {
58     modify(1, 0, n, p, v);
59 }
60 Info rangeQuery(int p, int l, int r, int x, int y) {
61     if (l ≥ y || r ≤ x) {
62         return {};
63     }
64     if (l ≥ x && r ≤ y) {
65         return info[p];
66     }
67     int m = (l + r) / 2;
68     push(p);

```

```

69     return rangeQuery(2 * p, l, m, x, y) + rangeQuery(2 * p + 1, m, r, x,
70     y);
71 }
72 Info rangeQuery(int l, int r) {
73     return rangeQuery(1, 0, n, l, r);
74 }
75 void rangeApply(int p, int l, int r, int x, int y, const Tag &v) {
76     if (l ≥ y || r ≤ x) {
77         return;
78     }
79     if (l ≥ x && r ≤ y) {
80         apply(p, v);
81         return;
82     }
83     int m = (l + r) / 2;
84     push(p);
85     rangeApply(2 * p, l, m, x, y, v);
86     rangeApply(2 * p + 1, m, r, x, y, v);
87     pull(p);
88 }
89 void rangeApply(int l, int r, const Tag &v) {
90     return rangeApply(1, 0, n, l, r, v);
91 }
92 template <class F>
93 int findFirst(int p, int l, int r, int x, int y, const F &pred) {
94     if (l ≥ y || r ≤ x) {
95         return -1;
96     }
97     if (l ≥ x && r ≤ y && !pred(info[p])) {
98         return -1;
99     }
100     if (r - l == 1) {
101         return l;
102     }
103     push(p);
104     int m = (l + r) / 2;
105     int res = findFirst(2 * p, l, m, x, y, pred);
106     if (res == -1) {
107         res = findFirst(2 * p + 1, m, r, x, y, pred);
108     }
109     return res;
110 }
111 template <class F>
112 int findFirst(int l, int r, const F &pred) {
113     return findFirst(1, 0, n, l, r, pred);
114 }
115 template <class F>
116 int findLast(int p, int l, int r, int x, int y, const F &pred) {
117     if (l ≥ y || r ≤ x) {
118         return -1;

```

```

119     if (l ≥ x && r ≤ y && !pred(info[p])) {
120         return -1;
121     }
122     if (r - l == 1) {
123         return l;
124     }
125     push(p);
126     int m = (l + r) / 2;
127     int res = findLast(2 * p + 1, m, r, x, y, pred);
128     if (res == -1) {
129         res = findLast(2 * p, l, m, x, y, pred);
130     }
131     return res;
132 }
133 template <class F>
134 int findLast(int l, int r, const F &pred) {
135     return findLast(1, 0, n, l, r, pred);
136 }
137 };
138
139 struct Tag {
140     void apply(const Tag &t) {
141     }
142 };
143
144 struct Info {
145     void apply(const Tag &t) {
146     }
147 };
148
149 Info operator+(const Info &a, const Info &b) {
150 }

```

1.4 Sparse Table

```

1 // O(n log n) - O(1)
2 // template <class T, T e, T (*F)(T, T)>, if cpp < 20
3 template <class T, T e, auto F>
4 struct SparseTable {
5     int n;
6     std::vector<std::vector<T>> a;
7     SparseTable(const std::vector<T> &v = {}) {
8         init(v);
9     }
10    void init(const std::vector<T> &v) {
11        n = v.size();
12        if (n == 0) {
13            return;
14        }
15        const int m = std::__lg(n);

```

```

16        a.assign(m + 1, std::vector<T>(n));
17        a[0] = v;
18        for (int j = 0; j < m; j++) {
19            for (int i = 0; i + (2 << j) ≤ n; i++) {
20                a[j + 1][i] = F(a[j][i], a[j][i + (1 << j)]);
21            }
22        }
23    }
24    T operator()(int l, int r) const {
25        if (l ≥ r) {
26            return e;
27        } else {
28            const int k = std::__lg(r - l);
29            return F(a[k][l], a[k][r - (1 << k)]);
30        }
31    }
32 };
33
34 // O(n) - O(1)
35 // only for minmax
36 template<class T, T e, class Cmp = std::less<>>
37 struct SparseTable {
38     const Cmp cmp = Cmp();
39     static constexpr unsigned B = 64;
40     int n;
41     std::vector<std::vector<T>> a;
42     std::vector<T> pre, suf, ini;
43     std::vector<u64> stk;
44     SparseTable(const std::vector<T> &v = {}) {
45         init(v);
46     }
47     void init(const std::vector<T> &v) {
48         n = v.size();
49         if (n == 0) {
50             return;
51         }
52         pre = suf = ini = v;
53         stk.resize(n);
54         const int M = (n - 1) / B + 1;
55         const int lg = std::__lg(M);
56         a.assign(lg + 1, std::vector<T>(M));
57         for (int i = 0; i < M; i++) {
58             a[0][i] = v[i * B];
59             for (int j = 1; j < B && i * B + j < n; j++) {
60                 a[0][i] = std::min(a[0][i], v[i * B + j], cmp);
61             }
62         }
63         for (int i = 1; i < n; i++) {
64             if (i % B) {
65                 pre[i] = std::min(pre[i], pre[i - 1], cmp);
66             }

```

```

67     }
68     for (int i = n - 2; i ≥ 0; i--) {
69         if (i % B ≠ B - 1) {
70             suf[i] = std::min(suf[i], suf[i + 1], cmp);
71         }
72     }
73     for (int j = 0; j < lg; j++) {
74         for (int i = 0; i + (2 << j) ≤ M; i++) {
75             a[j + 1][i] = std::min(a[j][i], a[j][i + (1 << j)], cmp);
76         }
77     }
78     for (int i = 0; i < M; i++) {
79         const int l = i * B;
80         const int r = std::min(1U * n, l + B);
81         u64 s = 0;
82         for (int j = l; j < r; j++) {
83             while (s && cmp(v[j], v[std::__lg(s) + l])) {
84                 s ^= 1ULL << std::__lg(s);
85             }
86             s |= 1ULL << (j - l);
87             stk[j] = s;
88         }
89     }
90 }
91 T operator()(int l, int r) {
92     if (l ≥ r) {
93         return e;
94     }
95     if (l / B ≠ (r - 1) / B) {
96         T ans = std::min(suf[l], pre[r - 1], cmp);
97         l = l / B + 1;
98         r = r / B;
99         if (l < r) {
100             int k = std::__lg(r - l);
101             ans = std::min({ans, a[k][l], a[k][r - (1 << k)]}, cmp);
102         }
103         return ans;
104     } else {
105         int x = B * (l / B);
106         return ini[__builtin_ctzll(stk[r - 1] >> (l - x)) + l];
107     }
108 }
109 };

```

1.5 Mo

```

1 {
2     std::vector<std::array<int, 3>> ask(q);
3     // ...
4     std::ranges::sort(ask, [&](auto i, auto j) {

```

```

5         if (i[0] / B ≠ j[0] / B) {
6             return i[0] < j[0];
7         }
8         return (i[0] / B) % 2 ? i[1] > j[1] : i[1] < j[1];
9     });
10    // ...
11    // [l, r)
12    int L = 0, R = 0;
13    for (auto [l, r, i] : ask) {
14        while (L > l) {
15            add(--L);
16        }
17        while (R < r) {
18            add(R++);
19        }
20        while (L < l) {
21            del(L++);
22        }
23        while (R > r) {
24            del(--R);
25        }
26        // ans[i] = ?
27    }
28 }
29
30 // Mo With Modify
31 // B = n ^ (2 / 3)
32 {
33     // ...
34     std::vector<std::array<int, 4>> ask;
35     std::vector<std::pair<int, int>> mod;
36     for (int i = 0; i < m; i++) {
37         char o;
38         int x, y;
39         std::cin >> o >> x >> y;
40         x--;
41         if (modify) {
42             mod.emplace_back(x, y);
43         } else {
44             ask.push_back({x, y, (int)mod.size(), (int)ask.size()});
45         }
46     }
47     std::ranges::sort(ask, [&](auto i, auto j) {
48         if (i[0] / B ≠ j[0] / B)
49             return i[0] < j[0];
50         if (i[1] / B ≠ j[1] / B)
51             return i[1] < j[1];
52         return (i[1] / B) & 1 ? i[2] < j[2] : i[2] > j[2];
53     });
54     auto add = [&](int c) {
55         // ...

```

```

56 };
57 auto del = [&](int c) {
58     // ...
59 };
60 auto modify = [&](int p, int l, int r) {
61     // if (l ≤ x and x < r) then del(ori) and add(cur)
62
63     // first modify : x → y
64     // second modify : y → x
65     // using swap
66 };
67 // [l, r)
68 int L = 0, R = 0, T = 0;
69 for (auto [l, r, t, i] : que) {
70     while (l < L) {
71         add(a[--L]);
72     }
73     while (R < r) {
74         add(a[R++]);
75     }
76     while (L < l) {
77         del(a[L++]);
78     }
79     while (R > r) {
80         del(a[--R]);
81     }
82     while (T < t) {
83         modify(T++, l, r);
84     }
85     while (T > t) {
86         modify(--T, l, r);
87     }
88     // ans[i] = ?
89 }
90 }

```

1.6 Cartesian Tree

```

1 // root = rangeMin
2 vector<int> lc(n, -1), rc(n, -1);
3 vector<int> stk;
4 for (int i = 0; i < n; i++) {
5     while (not stk.empty() and p[i] < p[stk.back()]) {
6         int x = stk.back();
7         stk.pop_back();
8
9         rc[x] = lc[i];
10        lc[i] = x;
11    }
12    stk.push_back(i);

```

```

13 }
14 while (stk.size() > 1) {
15     int x = stk.back();
16     stk.pop_back();
17     rc[stk.back()] = x;
18 }

```

1.7 Lichao Segment Tree

```

1 template <class T, T inf, class C = std::less<>>
2 struct LiChaoSegmentTree {
3     static constexpr C cmp = {};
4     struct Line {
5         int i;
6         T k, b;
7         constexpr Line(int i = std::min(-1, 1, cmp), T k = 0, T b = std::max(
8             (-inf, inf, cmp)) : i{i}, k{k}, b{b} {}
9         constexpr std::pair<T, int> operator()(int x) const {
10             return {k * x + b, i};
11         }
12     };
13     struct Node {
14         Node *l, *r;
15         Line f;
16         Node() : l{}, r{}, f{} {}
17     };
18     int n;
19     Node *t;
20     LiChaoSegmentTree(int n = 0) {
21         init(n);
22     }
23     void init(int n) {
24         this->n = n;
25         t = nullptr;
26     }
27     void insert(Node *&p, int l, int r, int x, int y, Line f) {
28         if (l ≥ y || r ≤ x) {
29             return;
30         }
31         if (p == nullptr) {
32             p = new Node();
33         }
34         int m = (l + r) / 2;
35         if (l ≥ x && r ≤ y) {
36             if (cmp(f(m), p->f(m))) {
37                 std::swap(f, p->f);
38             }
39             if (r - l == 1) {
40                 return;

```

```

41     if (cmp(f(l), p→f(l))) {
42         insert(p→l, l, m, x, y, f);
43     } else {
44         insert(p→r, m, r, x, y, f);
45     }
46 } else {
47     insert(p→l, l, m, x, y, f);
48     insert(p→r, m, r, x, y, f);
49 }
50 }
51 void insert(int l, int r, Line f) {
52     insert(t, 0, n, l, r, f);
53 }
54 void insert(Line f) {
55     insert(t, 0, n, 0, n, f);
56 }
57 // {val, id}
58 std::pair<T, int> query(Node *p, int l, int r, int x) {
59     if (p == nullptr) {
60         return Line()(x);
61     }
62     if (r - l == 1) {
63         return p→f(x);
64     }
65     int m = (l + r) / 2;
66     if (x < m) {
67         return std::min(p→f(x), query(p→l, l, m, x), cmp);
68     } else {
69         return std::min(p→f(x), query(p→r, m, r, x), cmp);
70     }
71 }
72 std::pair<T, int> query(int x) {
73     return query(t, 0, n, x);
74 }
75 };

```

2 Math

2.1 Formula

- Ex Euler's Theorem:

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(m)}, & \gcd(a, b) = 1 \\ a^{(b \bmod \varphi(m)) + \varphi(m)}, & \gcd(a, b) \neq 1, b \geq \varphi(m) \end{cases} \pmod{m}$$

- Euclidean algorithm: $\gcd(a, b) = \gcd(b, a \bmod b)$

- Binomial Inversion

$$f(n) = |\bigcap_{1 \leq i \leq n} A_i|, g(n) = |\bigcap_{1 \leq i \leq n} A_i^C|$$

$$f(n) = \sum_{i=0}^n (-1)^i \binom{n}{i} g(i)$$

$$g(n) = \sum_{i=0}^n (-1)^i \binom{n}{i} f(i)$$

$$f(n) = \sum_{i=0}^n \binom{n}{i} h(i)$$

$$h(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$$

$f(n)$: select at least n . \leftrightarrow $g(n)$: select exactly n .

$$f(n) = \sum_{i=n}^m \binom{i}{n} g(i)$$

$$g(n) = \sum_{i=n}^m (-1)^{i-n} \binom{i}{n} f(i)$$

2.2 Exgcd

```

1 // ax + by = gcd(a, b)
2 // if a ≠ 0, b ≠ 0 then -b ≤ x ≤ b, -a ≤ y ≤ a
3 int exgcd(int a, int b, int &x, int &y) {
4     if (b == 0) {
5         x = 1;
6         y = 0;
7         return a;
8     }
9     int g = exgcd(b, a % b, y, x);
10    y -= a / b * x;
11    return g;
12 }
13
14 // {x, mod} → {_, inv(x)}
15 constexpr std::pair<i64, i64> invGcd(i64 a, i64 b) {
16     a %= b;
17     if (a < 0) {
18         a += b;
19     }
20     if (a == 0) {
21         return {b, 0};
22     }
23     i64 s = b, t = a;
24     i64 m0 = 0, m1 = 1;
25     while (t) {
26         i64 u = s / t;

```



```

27     s -= t * u;
28     m0 -= m1 * u;
29
30     std::swap(s, t);
31     std::swap(m0, m1);
32 }
33 if (m0 < 0) {
34     m0 += b / s;
35 }
36 return {s, m0};
37 }

```

2.3 Phi

```

1 int phi(int n) {
2     int res = n;
3     for (int i = 2; i * i ≤ n; i++) {
4         if (n % i == 0) {
5             while (n % i == 0) {
6                 n /= i;
7             }
8             res = res / i * (i - 1);
9         }
10    }
11    if (n > 1) {
12        res = res / n * (n - 1);
13    }
14    return res;
15 }

```

2.4 Sieve

```

1 vector<int> minp, primes;
2 vector<int> phi, mu;
3
4 void sieve(int n) {
5     minp.assign(n + 1, 0);
6     phi.assign(n + 1, 0);
7     mu.assign(n + 1, 0);
8     primes.clear();
9
10    phi[1] = 1;
11    mu[1] = 1;
12
13    for (int i = 2; i ≤ n; i++) {
14        if (minp[i] == 0) {
15            minp[i] = i;
16            primes.push_back(i);
17            phi[i] = i - 1;

```

```

18        mu[i] = -1;
19    }
20
21    for (auto p : primes) {
22        if (i * p > n) {
23            break;
24        }
25        minp[i * p] = p;
26        if (p == minp[i]) {
27            phi[i * p] = phi[i] * p;
28            mu[i * p] = 0;
29            break;
30        } else {
31            phi[i * p] = phi[i] * (p - 1);
32            mu[i * p] = -mu[i];
33        }
34    }
35 }
36
37 bool isprime(int n) {
38     return minp[n] == n;
39 }
40 }

```

2.5 Pollard's Rho

```

1 std::mt19937_64 rng(std::chrono::steady_clock::now().time_since_epoch().count
2   ());
3
4 i64 factor(i64 n) {
5     if (n % 2 == 0) {
6         return 2;
7     }
8     if (isPrime(n)) {
9         return n;
10    }
11    i64 m = 2;
12    while (true) {
13        i64 c = (rng() % (n - 1)) + 1;
14        auto f = [&](i64 x) { return (mul(x, x, n) + c) % n; };
15        i64 d = 1, x = m, y = m, p = 1, q = 0, v = 1;
16        while (d == 1) {
17            y = f(y);
18            q++;
19            v = mul(v, std::abs(x - y), n);
20            if (q % 127 == 0) {
21                d = std::gcd(v, n);
22                v = 1;
23            }
24            if (p == q) {

```

```

24         x = y;
25         p *= 2;
26         q = 0;
27         d = std::gcd(v, n);
28         v = 1;
29     }
30 }
31 if (d != n) {
32     return d;
33 }
34 m++;
35 }
36 }
37
38 std::vector<i64> factorize(i64 n) {
39     std::vector<i64> p;
40     auto dfs = [&](auto &&self, i64 n) → void {
41         if (isPrime(n)) {
42             p.push_back(n);
43             return;
44         }
45         i64 d = factor(n);
46         self(self, d);
47         self(self, n / d);
48     };
49     dfs(dfs, n);
50     std::sort(p.begin(), p.end());
51     return p;
52 }

```

2.6 Combination

```

1 {
2     vector<int> fac(n + 1), invfac(n + 1);
3     fac[0] = 1;
4     for (int i = 1; i ≤ n; i++) {
5         fac[i] = mul(fac[i - 1], i);
6     }
7     invfac[n] = power(fac[n], P - 2);
8     for (int i = n; i ≥ 1; i--) {
9         invfac[i - 1] = mul(invfac[i], i);
10    }
11    auto binom = [&](int n, int m) → int {
12        if (n < m or m < 0) {
13            return 0;
14        }
15        return i64(fac[n]) * invfac[m] % P * invfac[n - m] % P;
16    };
17 }

```

2.7 Linear Basis

```

1 // a : base
2 // k : dimension
3 {
4     auto insert = [&](int x) {
5         for (int d = k - 1; d ≥ 0 and x != 0; d--) {
6             if (x >> d & 1) {
7                 if (a[d] == 0) {
8                     a[d] = x;
9                 }
10                x ^= a[d];
11            }
12        }
13    };
14    // ...
15    int ans = 0;
16    for (int d = k - 1; d ≥ 0; d--) {
17        ans = max(ans, ans ^ a[d]);
18    }
19 }
20 // t : time stamp
21 {
22     auto insert = [&](int x, int i) {
23         for (int d = k - 1; d ≥ 0; d--) {
24             if (x >> d & 1) {
25                 if (i > t[d]) {
26                     swap(i, t[d]);
27                     swap(x, a[d]);
28                 }
29                 x ^= a[d];
30             }
31         }
32     };
33     auto query = [&](int i) {
34         int res = 0;
35         for (int d = k - 1; d ≥ 0; d--) {
36             if (t[d] ≥ i) {
37                 res = max(res, res ^ a[d]);
38             }
39         }
40         return res;
41     };
42 }

```

2.8 Gaussian Elimination

```

1 std::vector<int> operator*(const std::vector<int> &lhs, const std::vector<int>
2 > &rhs) {
3     std::vector<int> res(lhs.size() + rhs.size() - 1);

```

```

3   for (int i = 0; i < int(lhs.size()); ++i)
4       for (int j = 0; j < int(rhs.size()); ++j)
5           res[i + j] = (res[i + j] + 1ll * lhs[i] * rhs[j]) % P;
6   return res;
7 }
8 std::vector<int> operator%(const std::vector<int> &lhs, const std::vector<int>
9 > &rhs) {
10     auto res = lhs;
11     int m = rhs.size() - 1;
12     int inv = power(rhs.back(), P - 2);
13     for (int i = res.size() - 1; i ≥ m; --i) {
14         int x = 1ll * inv * res[i] % P;
15         for (int j = 0; j < m; ++j)
16             res[i - m + j] = (res[i - m + j] + 1ll * (P - x) * rhs[j]) % P;
17     }
18     if (int(res.size()) > m)
19         res.resize(m);
20     return res;
21 }
22 std::vector<int> gauss(std::vector<std::vector<int>> a, std::vector<int> b) {
23     int n = a.size();
24     for (int i = 0; i < n; ++i) {
25         int r = i;
26         while (a[r][i] == 0)
27             ++r;
28         std::swap(a[i], a[r]);
29         std::swap(b[i], b[r]);
30         int inv = power(a[i][i], P - 2);
31         for (int j = i; j < n; ++j)
32             a[i][j] = 1ll * a[i][j] * inv % P;
33         b[i] = 1ll * b[i] * inv % P;
34         for (int j = 0; j < n; ++j) {
35             if (i == j)
36                 continue;
37             int x = a[j][i];
38             for (int k = i; k < n; ++k)
39                 a[j][k] = (a[j][k] + 1ll * (P - x) * a[i][k]) % P;
40             b[j] = (b[j] + 1ll * (P - x) * b[i]) % P;
41         }
42     }
43     return b;
44 }
45
46 std::vector<double> gauss(std::vector<std::vector<double>> a, std::vector<
47 double> b) {
48     int n = a.size();
49     for (int i = 0; i < n; ++i) {
50         double x = a[i][i];
51         for (int j = i; j < n; ++j) a[i][j] /= x;
52         b[i] /= x;

```

```

53     for (int j = 0; j < n; ++j) {
54         if (i == j) continue;
55         x = a[j][i];
56         for (int k = i; k < n; ++k) a[j][k] -= a[i][k] * x;
57         b[j] -= b[i] * x;
58     }
59     return b;
60 }

```

2.9 Polynomial

2.9.1 Lagrange Interpolation

```

1 // n points → n - 1 polynomial      O(n ^ 2)
2 for (int i = 0; i < n; i++) {
3     Z num = y[i];
4     Z den = 1;
5     for (int j = 0; j < n; j++) {
6         if (i == j) {
7             continue;
8         }
9         num *= (k - x[j]); // f(k)
10        den *= (x[i] - x[j]);
11    }
12    ans += num / den;
13 }
14
15 // Continuous x      O(n)
16 vector<Z> fac(n + 1);
17 fac[0] = 1;
18 for (int i = 1; i ≤ n; i++) {
19     fac[i] = fac[i - 1] * i;
20 }
21
22 vector<Z> pre(n + 1);
23 pre[0] = 1;
24 for (int i = 0; i < n; i++) {
25     pre[i + 1] = pre[i] * (k - i);
26 }
27
28 vector<Z> suf(n + 1);
29 suf[n] = 1;
30 for (int i = n - 1; i ≥ 0; i--) {
31     suf[i] = suf[i + 1] * (k - i);
32 }
33
34 Z ans = 0;
35 for (int i = 0; i < n; i++) {
36     Z res = y[i];

```

```

37     res *= pre[i] * suf[i + 1];
38     res /= ((n - 1 - i) % 2 ? -1 : 1) * fac[i] * fac[n - 1 - i];
39
40     ans += res;
41 }

```

2.9.2 Number Theory Transform

```

1 constexpr int P = 998244353;
2
3 int power(int a, int b) {
4     int res = 1;
5     for (; b; b /= 2, a = 1LL * a * a % P) {
6         if (b % 2) {
7             res = 1LL * res * a % P;
8         }
9     }
10    return res;
11 }
12
13 std::vector<int> rev, roots {0, 1};
14
15 void dft(std::vector<int> &a) {
16     int n = a.size();
17     if (int(rev.size()) != n) {
18         int k = __builtin_ctz(n) - 1;
19         rev.resize(n);
20         for (int i = 0; i < n; i++) {
21             rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
22         }
23     }
24     for (int i = 0; i < n; i++) {
25         if (rev[i] < i) {
26             std::swap(a[i], a[rev[i]]);
27         }
28     }
29     if (roots.size() < n) {
30         int k = __builtin_ctz(roots.size());
31         roots.resize(n);
32         while ((1 << k) < n) {
33             int e = power(31, 1 << (__builtin_ctz(P - 1) - k - 1));
34             for (int i = 1 << (k - 1); i < (1 << k); i++) {
35                 roots[2 * i] = roots[i];
36                 roots[2 * i + 1] = 1LL * roots[i] * e % P;
37             }
38             k++;
39         }
40     }
41
42     for (int k = 1; k < n; k *= 2) {

```

```

43         for (int i = 0; i < n; i += 2 * k) {
44             for (int j = 0; j < k; j++) {
45                 int u = a[i + j];
46                 int v = 1LL * a[i + j + k] * roots[k + j] % P;
47                 a[i + j] = (u + v) % P;
48                 a[i + j + k] = (u - v) % P;
49             }
50         }
51     }
52 }
53
54 void idft(std::vector<int> &a) {
55     int n = a.size();
56     std::reverse(a.begin() + 1, a.end());
57     dft(a);
58     int inv = (1 - P) / n;
59     for (int i = 0; i < n; i++) {
60         a[i] = 1LL * a[i] * inv % P;
61     }
62 }
63
64 std::vector<int> mul(std::vector<int> a, std::vector<int> b) {
65     int n = 1, tot = a.size() + b.size() - 1;
66     while (n < tot) {
67         n *= 2;
68     }
69     a.resize(n);
70     b.resize(n);
71     dft(a);
72     dft(b);
73     for (int i = 0; i < n; i++) {
74         a[i] = 1LL * a[i] * b[i] % P;
75     }
76     idft(a);
77     a.resize(tot);
78     return a;
79 }

```

```

1 // with ModIntBase
2 std::vector<int> rev;
3 template <u32 P>
4 std::vector<ModInt<P>> roots{0, 1};
5
6 template <u32 P>
7 constexpr ModInt<P> findPrimitiveRoot() {
8     ModInt<P> i = 2;
9     int k = __builtin_ctz((int)P - 1);
10    while (true) {
11        if (power(i, (P - 1) / 2) != 1) {
12            break;
13        }

```

```

14     i += 1;
15 }
16 return power(i, (P - 1) >> k);
17 }
18
19 template <u32 P>
20 constexpr ModInt<P> primitiveRoot = findPrimitiveRoot<P>();
21
22 template <>
23 constexpr ModInt<998244353> primitiveRoot<998244353>{31};
24
25 template <u32 P>
26 constexpr void dft(std::vector<ModInt<P>> &a) {
27     int n = a.size();
28
29     if (int(rev.size()) != n) {
30         int k = __builtin_ctz(n) - 1;
31         rev.resize(n);
32         for (int i = 0; i < n; i++) {
33             rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
34         }
35     }
36
37     for (int i = 0; i < n; i++) {
38         if (rev[i] < i) {
39             std::swap(a[i], a[rev[i]]);
40         }
41     }
42
43     if (roots<P>.size() < n) {
44         int k = __builtin_ctz(roots<P>.size());
45         roots<P>.resize(n);
46         while ((1 << k) < n) {
47             auto e = power(primitiveRoot<P>, 1 << (__builtin_ctz(P - 1) - k - 1));
48             for (int i = 1 << (k - 1); i < (1 << k); i++) {
49                 roots<P>[2 * i] = roots<P>[i];
50                 roots<P>[2 * i + 1] = roots<P>[i] * e;
51             }
52             k++;
53         }
54     }
55     for (int k = 1; k < n; k *= 2) {
56         for (int i = 0; i < n; i += 2 * k) {
57             for (int j = 0; j < k; j++) {
58                 ModInt<P> u = a[i + j];
59                 ModInt<P> v = a[i + j + k] * roots<P>[k + j];
60                 a[i + j] = u + v;
61                 a[i + j + k] = u - v;
62             }
63         }

```

```

64     }
65 }
66
67 template <u32 P>
68 constexpr void idft(std::vector<ModInt<P>> &a) {
69     int n = a.size();
70     std::reverse(a.begin() + 1, a.end());
71     dft(a);
72     ModInt<P> inv = (1 - (int)P) / n;
73     for (int i = 0; i < n; i++) {
74         a[i] *= inv;
75     }
76 }
77
78 template <int P = 998244353>
79 struct Poly : public std::vector<ModInt<P>> {
80     using V = ModInt<P>;
81
82     Poly() : std::vector<V>() {}
83     explicit constexpr Poly(int n) : std::vector<V>(n) {}
84
85     explicit constexpr Poly(const std::vector<V> &a) : std::vector<V>(a) {}
86     constexpr Poly(const std::initializer_list<V> &a) : std::vector<V>(a) {}
87
88     template <class InputIt, class = std::_RequireInputIter<InputIt>>
89     explicit constexpr Poly(InputIt first, InputIt last) : std::vector<V>(
90         first, last) {}
91
92     template <class F>
93     explicit constexpr Poly(int n, F f) : std::vector<V>(n) {
94         for (int i = 0; i < n; i++) {
95             (*this)[i] = f(i);
96         }
97     }
98
99     constexpr Poly shift(int k) const {
100         if (k ≥ 0) {
101             auto b = *this;
102             b.insert(b.begin(), k, 0);
103             return b;
104         } else if (this->size() ≤ -k) {
105             return Poly();
106         } else {
107             return Poly(this->begin() + (-k), this->end());
108         }
109     }
110
111     constexpr Poly trunc(int k) const {
112         Poly f = *this;
113         f.resize(k);
114         return f;
115     }

```

```

114 constexpr friend Poly operator+(const Poly &a, const Poly &b) {
115     Poly res(std::max(a.size(), b.size()));
116     for (int i = 0; i < a.size(); i++) {
117         res[i] += a[i];
118     }
119     for (int i = 0; i < b.size(); i++) {
120         res[i] += b[i];
121     }
122     return res;
123 }
124 constexpr friend Poly operator-(const Poly &a, const Poly &b) {
125     Poly res(std::max(a.size(), b.size()));
126     for (int i = 0; i < a.size(); i++) {
127         res[i] += a[i];
128     }
129     for (int i = 0; i < b.size(); i++) {
130         res[i] -= b[i];
131     }
132     return res;
133 }
134 constexpr friend Poly operator-(const Poly &a) {
135     std::vector<V> res(a.size());
136     for (int i = 0; i < int(res.size()); i++) {
137         res[i] = -a[i];
138     }
139     return Poly(res);
140 }
141 constexpr friend Poly operator*(Poly a, Poly b) {
142     if (a.size() == 0 || b.size() == 0) {
143         return Poly();
144     }
145     if (a.size() < b.size()) {
146         std::swap(a, b);
147     }
148     int n = 1, tot = a.size() + b.size() - 1;
149     while (n < tot) {
150         n *= 2;
151     }
152     if (((P - 1) & (n - 1)) != 0 || b.size() < 128) {
153         Poly c(a.size() + b.size() - 1);
154         for (int i = 0; i < a.size(); i++) {
155             for (int j = 0; j < b.size(); j++) {
156                 c[i + j] += a[i] * b[j];
157             }
158         }
159         return c;
160     }
161     a.resize(n);
162     b.resize(n);
163     dft<P>(a);
164     dft<P>(b);

```

```

165     for (int i = 0; i < n; ++i) {
166         a[i] *= b[i];
167     }
168     idft<P>(a);
169     a.resize(tot);
170     return a;
171 }
172 constexpr friend Poly operator*(V a, Poly b) {
173     for (int i = 0; i < int(b.size()); i++) {
174         b[i] *= a;
175     }
176     return b;
177 }
178 constexpr friend Poly operator*(Poly a, V b) {
179     for (int i = 0; i < int(a.size()); i++) {
180         a[i] *= b;
181     }
182     return a;
183 }
184 constexpr friend Poly operator/(Poly a, V b) {
185     for (int i = 0; i < int(a.size()); i++) {
186         a[i] /= b;
187     }
188     return a;
189 }
190 constexpr Poly &operator+=(Poly b) {
191     return (*this) = (*this) + b;
192 }
193 constexpr Poly &operator-=(Poly b) {
194     return (*this) = (*this) - b;
195 }
196 constexpr Poly &operator*=(Poly b) {
197     return (*this) = (*this) * b;
198 }
199 constexpr Poly &operator*=(V b) {
200     return (*this) = (*this) * b;
201 }
202 constexpr Poly &operator/=(V b) {
203     return (*this) = (*this) / b;
204 }
205 constexpr Poly deriv() const {
206     if (this->empty()) {
207         return Poly();
208     }
209     Poly res((int)this->size() - 1);
210     for (int i = 0; i < this->size() - 1; ++i) {
211         res[i] = (i + 1) * (*this)[i + 1];
212     }
213     return res;
214 }
215 constexpr Poly integr() const {

```

```

216     Poly res(this->size() + 1);
217     for (int i = 0; i < this->size(); ++i) {
218         res[i + 1] = (*this)[i] / (i + 1);
219     }
220     return res;
221 }
222 constexpr Poly inv(int m) const {
223     Poly x{(*this)[0].inv()};
224     int k = 1;
225     while (k < m) {
226         k *= 2;
227         x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
228     }
229     return x.trunc(m);
230 }
231 constexpr Poly log(int m) const {
232     return deriv() * inv(m)).integr().trunc(m);
233 }
234 constexpr Poly exp(int m) const {
235     Poly x{1};
236     int k = 1;
237     while (k < m) {
238         k *= 2;
239         x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
240     }
241     return x.trunc(m);
242 }
243 constexpr Poly pow(int k, int m) const {
244     int i = 0;
245     while (i < this->size() && (*this)[i] == 0) {
246         i++;
247     }
248     if (i == this->size() || 1LL * i * k ≥ m) {
249         return Poly(m);
250     }
251     V v = (*this)[i];
252     auto f = shift(-i) * v.inv();
253     return (f.log(m - i * k) * k).exp(m - i * k).shift(i * k) * power(v,
254         k);
255 }
256 constexpr Poly sqrt(int m) const {
257     Poly x{1};
258     int k = 1;
259     while (k < m) {
260         k *= 2;
261         x = (x + (trunc(k) * x.inv(k)).trunc(k)) * CInv<2, P>;
262     }
263     return x.trunc(m);
264 }
265 constexpr Poly mulT(Poly b) const {
266     if (b.size() == 0) {

```

```

266         return Poly();
267     }
268     int n = b.size();
269     std::reverse(b.begin(), b.end());
270     return ((*this) * b).shift(-(n - 1));
271 }
272 constexpr std::vector<V> eval(std::vector<V> x) const {
273     if (this->size() == 0) {
274         return std::vector<V>(x.size(), 0);
275     }
276     const int n = std::max(x.size(), this->size());
277     std::vector<Poly> q(4 * n);
278     std::vector<V> ans(x.size());
279     x.resize(n);
280     std::function<void(int, int, int)> build = [&](int p, int l, int r) {
281         if (r - l == 1) {
282             q[p] = Poly{1, -x[l]};
283         } else {
284             int m = (l + r) / 2;
285             build(2 * p, l, m);
286             build(2 * p + 1, m, r);
287             q[p] = q[2 * p] * q[2 * p + 1];
288         }
289     };
290     build(1, 0, n);
291     std::function<void(int, int, int, const Poly &)> work = [&](int p,
292         int l, int r, const Poly &num) {
293         if (r - l == 1) {
294             if (l < int(ans.size())) {
295                 ans[l] = num[0];
296             }
297         } else {
298             int m = (l + r) / 2;
299             work(2 * p, l, m, num.mulT(q[2 * p + 1]).trunc(m - l));
300             work(2 * p + 1, m, r, num.mulT(q[2 * p]).trunc(r - m));
301         }
302     };
303     work(1, 0, n, mulT(q[1].inv(n)));
304     return ans;
305 }
306 };
307 template <int P = 998244353>
308 Poly<P> berlekampMassey(const Poly<P> &s) {
309     Poly<P> c;
310     Poly<P> oldC;
311     int f = -1;
312     for (int i = 0; i < s.size(); i++) {
313         auto delta = s[i];
314         for (int j = 1; j ≤ c.size(); j++) {
315             delta -= c[j - 1] * s[i - j];

```

```

316     }
317     if (delta == 0) {
318         continue;
319     }
320     if (f == -1) {
321         c.resize(i + 1);
322         f = i;
323     } else {
324         auto d = oldC;
325         d *= -1;
326         d.insert(d.begin(), 1);
327         ModInt<P> df1 = 0;
328         for (int j = 1; j ≤ d.size(); j++) {
329             df1 += d[j - 1] * s[f + 1 - j];
330         }
331         assert(df1 ≠ 0);
332         auto coef = delta / df1;
333         d *= coef;
334         Poly<P> zeros(i - f - 1);
335         zeros.insert(zeros.end(), d.begin(), d.end());
336         d = zeros;
337         auto temp = c;
338         c += d;
339         if (i - temp.size() > f - oldC.size()) {
340             oldC = temp;
341             f = i;
342         }
343     }
344 }
345 c *= -1;
346 c.insert(c.begin(), 1);
347 return c;
348 }
349
350 template <int P = 998244353>
351 ModInt<P> linearRecurrence(Poly<P> p, Poly<P> q, i64 n) {
352     int m = q.size() - 1;
353     while (n > 0) {
354         auto newq = q;
355         for (int i = 1; i ≤ m; i += 2) {
356             newq[i] *= -1;
357         }
358         auto newp = p * newq;
359         newq = q * newq;
360         for (int i = 0; i < m; i++) {
361             p[i] = newp[i * 2 + n % 2];
362         }
363         for (int i = 0; i ≤ m; i++) {
364             q[i] = newq[i * 2];
365         }
366         n ≠ 2;

```

```

367     }
368     return p[0] / q[0];
369 }
370
371 struct Comb {
372     int n;
373     std::vector<Z> _fac;
374     std::vector<Z> _invfac;
375     std::vector<Z> _inv;
376
377     Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
378     Comb(int n) : Comb() {
379         init(n);
380     }
381
382     void init(int m) {
383         m = std::min(m, (int)Z::mod() - 1);
384         if (m ≤ n)
385             return;
386         _fac.resize(m + 1);
387         _invfac.resize(m + 1);
388         _inv.resize(m + 1);
389
390         for (int i = n + 1; i ≤ m; i++) {
391             _fac[i] = _fac[i - 1] * i;
392         }
393         _invfac[m] = _fac[m].inv();
394         for (int i = m; i > n; i--) {
395             _invfac[i - 1] = _invfac[i] * i;
396             _inv[i] = _invfac[i] * _fac[i - 1];
397         }
398         n = m;
399     }
400
401     Z fac(int m) {
402         if (m > n)
403             init(2 * m);
404         return _fac[m];
405     }
406     Z invfac(int m) {
407         if (m > n)
408             init(2 * m);
409         return _invfac[m];
410     }
411     Z inv(int m) {
412         if (m > n)
413             init(2 * m);
414         return _inv[m];
415     }
416     Z binom(int n, int m) {
417         if (n < m || m < 0)

```



```

418     return 0;
419     return fac(n) * invfac(m) * invfac(n - m);
420 }
421 } comb;
422
423 template <int P = 998244353>
424 Poly<P> get(int n, int m) {
425     if (m == 0) {
426         return Poly(n + 1);
427     }
428     if (m % 2 == 1) {
429         auto f = get(n, m - 1);
430         Z p = 1;
431         for (int i = 0; i ≤ n; i++) {
432             f[n - i] += comb.binom(n, i) * p;
433             p *= m;
434         }
435         return f;
436     }
437     auto f = get(n, m / 2);
438     auto fm = f;
439     for (int i = 0; i ≤ n; i++) {
440         fm[i] *= comb.fac(i);
441     }
442     Poly pw(n + 1);
443     pw[0] = 1;
444     for (int i = 1; i ≤ n; i++) {
445         pw[i] = pw[i - 1] * (m / 2);
446     }
447     for (int i = 0; i ≤ n; i++) {
448         pw[i] *= comb.invfac(i);
449     }
450     fm = fm.mulT(pw);
451     for (int i = 0; i ≤ n; i++) {
452         fm[i] *= comb.invfac(i);
453     }
454     return f + fm;
455 }

```

2.9.3 Fast Walsh Transform

```

1 void andFwt(auto &a, int t) {
2     int n = a.size();
3     for (int i = 1; i < n; i *= 2) {
4         for (int s = 0; s < n; s++) {
5             if (~s & i) {
6                 a[s] += t * a[s | i];
7             }
8         }
9     }

```

```

10 }
11 void orFwt(auto &a, int t) {
12     int n = a.size();
13     for (int i = 1; i < n; i *= 2) {
14         for (int s = 0; s < n; s++) {
15             if (~s & i) {
16                 a[s | i] += t * a[s];
17             }
18         }
19     }
20 }
21 void xorFwt(auto &a) {
22     int n = a.size();
23     for (int i = 1; i < n; i *= 2) {
24         for (int j = 0; j < n; j += 2 * i) {
25             for (int k = 0; k < i; k++) {
26                 auto u = a[j + k], v = a[i + j + k];
27                 a[j + k] = u + v;
28                 a[i + j + k] = u - v;
29             }
30         }
31     }
32 }
33 auto xorConv(auto a, auto b) {
34     int n = a.size();
35     xorFwt(a);
36     xorFwt(b);
37
38     for (int i = 0; i < n; i++) {
39         a[i] *= b[i];
40     }
41     xorFwt(a);
42     auto invn = Z(n).inv();
43     for (int i = 0; i < n; i++) {
44         a[i] *= invn;
45     }
46     // if not module:
47     // for (int i = 0; i < n; i++) {
48     //     a[i] >>= __lg(n);
49     // }
50     return move(a);
51 }

```

2.9.4 Sum of Subset

```

1 {
2     std::vector<int> f(1 << n);
3     // ...
4
5     // prefix

```

```

6   for (int i = 0; i < n; i++) {
7       for (int s = 0; s < 1 << n; s++) {
8           if (s >> i & 1) {
9               f[s] += f[s ^ (1 << i)];
10          }
11      }
12  }
13  // suffix
14  for (int i = 0; i < n; i++) {
15      for (int s = 0; s < 1 << n; s++) {
16          if (~s >> i & 1) {
17              f[s] += f[s ^ (1 << i)];
18          }
19      }
20  }
21 }

```

```

28     if (dep[x] < dep[y]) {
29         std::swap(x, y);
30     }
31     while (dep[x] > dep[y]) {
32         x = p[std::__lg(dep[x] - dep[y])][x];
33     }
34     if (x == y) {
35         return x;
36     }
37     for (int i = std::__lg(dep[x]); i ≥ 0; i--) {
38         if (p[i][x] ≠ p[i][y]) {
39             x = p[i][x];
40             y = p[i][y];
41         }
42     }
43     return p[0][x];
44 };
45 }

```

3 Graph

3.1 Tree

3.1.1 Lowest Common Ancestor

```

1  // Binary Lifting  $O(n \log n) - O(\log n)$ 
2  {
3      const int logn = std::__lg(n);
4
5      std::vector<int> dep(n);
6      std::vector p(logn + 1, std::vector<int>(n));
7      auto dfs = [&](auto &&self, int x) → void {
8          for (auto y : adj[x]) {
9              if (y == p[0][x]) {
10                 continue;
11             }
12             p[0][y] = x;
13             dep[y] = dep[x] + 1;
14             self(self, y);
15         }
16     };
17
18     p[0][s] = s;
19     dfs(dfs, s);
20
21     for (int j = 0; j < logn; j++) {
22         for (int i = 0; i < n; i++) {
23             p[j + 1][i] = p[j][p[j][i]];
24         }
25     }
26
27     auto lca = [&](int x, int y) {

```

3.1.2 Heavy-Light Decomposition

```

1  struct HLD {
2      int n;
3      std::vector<int> siz, top, dep, parent, in, out, seq;
4      std::vector<std::vector<int>> adj;
5      int cur;
6
7      HLD() {}
8      HLD(int n) {
9          init(n);
10     }
11     void init(int n) {
12         this→n = n;
13         siz.resize(n);
14         top.resize(n);
15         dep.resize(n);
16         parent.resize(n);
17         in.resize(n);
18         out.resize(n);
19         seq.resize(n);
20         cur = 0;
21         adj.assign(n, {});
22     }
23     void addEdge(int u, int v) {
24         adj[u].push_back(v);
25         adj[v].push_back(u);
26     }
27     void work(int root = 0) {
28         top[root] = root;
29         dep[root] = 0;

```

```

30     parent[root] = -1;
31     dfs1(root);
32     dfs2(root);
33 }
34 void dfs1(int u) {
35     if (parent[u] ≠ -1) {
36         adj[u].erase(find(adj[u].begin(), adj[u].end(), parent[u]));
37     }
38
39     siz[u] = 1;
40     for (auto &v : adj[u]) {
41         parent[v] = u;
42         dep[v] = dep[u] + 1;
43         dfs1(v);
44         siz[u] += siz[v];
45         if (siz[v] > siz[adj[u][0]]) {
46             std::swap(v, adj[u][0]);
47         }
48     }
49 }
50 void dfs2(int u) {
51     in[u] = cur++;
52     seq[in[u]] = u;
53     for (auto v : adj[u]) {
54         top[v] = v == adj[u][0] ? top[u] : v;
55         dfs2(v);
56     }
57     out[u] = cur;
58 }
59 int lca(int u, int v) {
60     while (top[u] ≠ top[v]) {
61         if (dep[top[u]] > dep[top[v]]) {
62             u = parent[top[u]];
63         } else {
64             v = parent[top[v]];
65         }
66     }
67     return dep[u] < dep[v] ? u : v;
68 }
69 int dist(int u, int v) {
70     return dep[u] + dep[v] - 2 * dep[lca(u, v)];
71 }
72 bool isAncestor(int fa, int son) {
73     return in[fa] ≤ in[son] && in[son] < out[fa];
74 }
75 // [u, v]
76 auto getPath(int u, int v) {
77     std::vector<std::pair<int, int>> ret;
78     while (top[u] ≠ top[v]) {
79         if (dep[top[u]] > dep[top[v]]) {
80             ret.push_back({in[top[u]], in[u]});

```

```

81         u = parent[top[u]];
82     } else {
83         ret.push_back({in[top[v]], in[v]});
84         v = parent[top[v]];
85     }
86 }
87 if (dep[u] > dep[v]) {
88     ret.push_back({in[v], in[u]});
89 } else {
90     ret.push_back({in[u], in[v]});
91 }
92 return std::move(ret);
93 }
94 // [u, v]
95 std::pair<int, int> getTree(int u) {
96     return pair(in[u], out[u]);
97 }
98 int jump(int u, int k) {
99     if (dep[u] < k) {
100         return -1;
101     }
102     int d = dep[u] - k;
103     while (dep[top[u]] > d) {
104         u = parent[top[u]];
105     }
106     return seq[in[u] - dep[u] + d];
107 }
108 int rootedParent(int u, int v) {
109     std::swap(u, v);
110     if (u == v) {
111         return u;
112     }
113     if (!isAncestor(u, v)) {
114         return parent[u];
115     }
116     auto it = std::upper_bound(adj[u].begin(), adj[u].end(), v, [&](int x
117         , int y) {
118             return in[x] < in[y];
119         }) - 1;
120     return *it;
121 }
122 int rootedSize(int u, int v) {
123     if (u == v) {
124         return n;
125     }
126     if (!isAncestor(v, u)) {
127         return siz[v];
128     }
129     return n - siz[rootedParent(u, v)];
130 }
131 int rootedLca(int a, int b, int c) {

```

```

131     return lca(a, b) ^ lca(b, c) ^ lca(c, a);
132 }
133 };

```

```

45     addv(color[x], 1);
46     // ans[x] = ?
47 };
48 // dfs(0);
49 }

```

3.1.3 DSU on Tree

```

1 {
2     int n;
3     std::vector<std::vector<int>> adj(n);
4     // ...
5     std::vector<int> siz(n);
6     [&](this auto &&self, int x, int p) → void {
7         if (p ≠ -1) {
8             adj[x].erase(find(adj[x].begin(), adj[x].end(), p));
9         }
10        siz[x] = 1;
11        // &y
12        for (auto &y : adj[x]) {
13            self(y, x);
14            siz[x] += siz[y];
15            if (siz[y] > siz[adj[x][0]]) {
16                swap(adj[x][0], y);
17            }
18        }
19    } (0, -1);
20
21    auto addv = [&](int color, int t) {
22        // freq[color] += t
23    };
24    auto add = [&](auto &&self, int x, int t) → void {
25        // addv(color[x], t);
26        for (auto y : adj[x]) {
27            self(self, y, t);
28        }
29    };
30    auto dfs = [&](auto &&self, int x) → void {
31        for (auto y : adj[x]) {
32            if (y ≠ adj[x][0]) {
33                self(self, y);
34                add(add, y, -1);
35            }
36        }
37        if (not adj[x].empty()) {
38            self(self, adj[x][0]);
39            for (auto y : adj[x]) {
40                if (y ≠ adj[x][0]) {
41                    add(add, y, 1);
42                }
43            }
44        }

```

3.2 Connectivity

3.2.1 Strongly Connected Component

```

1 struct SCC {
2     int n;
3     std::vector<std::vector<int>> adj;
4     std::vector<int> stk;
5     std::vector<int> dfn, low, bel;
6     int cur, cnt;
7
8     SCC() {}
9     SCC(int n) {
10         init(n);
11     }
12     void init(int n) {
13         this→n = n;
14         adj.assign(n, {});
15         dfn.assign(n, -1);
16         low.resize(n);
17         bel.assign(n, -1);
18         stk.clear();
19         cur = cnt = 0;
20     }
21     void addEdge(int u, int v) {
22         adj[u].push_back(v);
23     }
24     void dfs(int x) {
25         dfn[x] = low[x] = cur++;
26         stk.push_back(x);
27         for (auto y : adj[x]) {
28             if (dfn[y] == -1) {
29                 dfs(y);
30                 low[x] = std::min(low[x], low[y]);
31             } else if (bel[y] == -1) {
32                 low[x] = std::min(low[x], dfn[y]);
33             }
34         }
35         if (dfn[x] == low[x]) {
36             int y;
37             do {
38                 y = stk.back();
39                 bel[y] = cnt;
40                 stk.pop_back();

```

```

41         } while (y ≠ x);
42         cnt++;
43     }
44 }
45 std::vector<int> work() {
46     for (int i = 0; i < n; i++) {
47         if (dfn[i] = -1) {
48             dfs(i);
49         }
50     }
51     return bel;
52 }
53 };

```

3.2.2 Block Cut Tree

```

1 struct BlockCutTree {
2     int n;
3     std::vector<std::vector<int>> adj;
4     std::vector<int> dfn, low, stk;
5     int cnt, cur;
6     std::vector<std::pair<int, int>> edges;
7     BlockCutTree() {}
8     BlockCutTree(int n) {
9         init(n);
10    }
11    void init(int n) {
12        this->n = n;
13        adj.assign(n, {});
14        dfn.assign(n, -1);
15        low.resize(n);
16        stk.clear();
17        cnt = cur = 0;
18        edges.clear();
19    }
20    void addEdge(int u, int v) {
21        adj[u].push_back(v);
22        adj[v].push_back(u);
23    }
24    void dfs(int x) {
25        stk.push_back(x);
26        dfn[x] = low[x] = cur++;
27        for (auto y : adj[x]) {
28            if (dfn[y] = -1) {
29                dfs(y);
30                low[x] = std::min(low[x], low[y]);
31                if (low[y] = dfn[x]) {
32                    int v;
33                    do {
34                        v = stk.back();

```

```

35            stk.pop_back();
36            edges.emplace_back(n + cnt, v);
37        } while (v ≠ y);
38        edges.emplace_back(x, n + cnt);
39        cnt++;
40    }
41    } else {
42        low[x] = std::min(low[x], dfn[y]);
43    }
44 }
45 }
46 std::pair<int, std::vector<std::pair<int, int>>> work() {
47     for (int i = 0; i < n; i++) {
48         if (dfn[i] = -1) {
49             stk.clear();
50             dfs(i);
51         }
52     }
53     return {cnt, edges};
54 }
55 };

```

3.3 Two Sat

```

1 struct TwoSat {
2     int n;
3     std::vector<std::vector<int>> e;
4     std::vector<bool> ans;
5     TwoSat(int n) : n(n), e(2 * n), ans(n) {}
6     void addClause(int u, bool f, int v, bool g) {
7         e[2 * u + !f].push_back(2 * v + g);
8         e[2 * v + !g].push_back(2 * u + f);
9     }
10    bool satisfiable() {
11        std::vector<int> id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
12        std::vector<int> stk;
13        int now = 0, cnt = 0;
14        std::function<void(int)> tarjan = [&](int u) {
15            stk.push_back(u);
16            dfn[u] = low[u] = now++;
17            for (auto v : e[u]) {
18                if (dfn[v] = -1) {
19                    tarjan(v);
20                    low[u] = std::min(low[u], low[v]);
21                } else if (id[v] = -1) {
22                    low[u] = std::min(low[u], dfn[v]);
23                }
24            }
25            if (dfn[u] = low[u]) {
26                int v;

```

```

27         do {
28             v = stk.back();
29             stk.pop_back();
30             id[v] = cnt;
31         } while (v != u);
32         ++cnt;
33     }
34 };
35 for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
36 for (int i = 0; i < n; ++i) {
37     if (id[2 * i] == id[2 * i + 1]) return false;
38     ans[i] = id[2 * i] > id[2 * i + 1];
39 }
40 return true;
41 }
42 std::vector<bool> answer() { return ans; }
43 };

```

4 String

4.1 String Hash

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 using i64 = long long;
5 using Hash = array<int, 2>;
6
7 constexpr int P = 998244353;
8 constexpr Hash base = {567, 1234}; // any
9
10 int main() {
11     ios::sync_with_stdio(false);
12     cin.tie(nullptr);
13
14     string s;
15     cin >> s;
16
17     const int n = s.size();
18
19     vector<Hash> h(n + 1), p(n + 1);
20     p[0] = {1, 1};
21     for (int j = 0; j < 2; ++j) {
22         for (int i = 0; i < n; ++i) {
23             h[i + 1][j] = (i64(base[j]) * h[i][j] + s[i]) % P;
24             p[i + 1][j] = (i64(base[j]) * p[i][j]) % P;
25         }
26     }
27

```

```

28     auto get = [&](int l, int r) {
29         Hash res{};
30         for (int i = 0; i < 2; ++i) {
31             res[i] = (h[r][i] + 1ll * (P - h[l][i]) * p[r - l][i]) % P;
32         }
33         return res;
34     };
35
36     return 0;
37 }

```

4.2 KMP

```

1 std::vector<int> kmp(std::string s) {
2     int n = s.size();
3     std::vector<int> f(n + 1);
4     for (int i = 1, j = 0; i < n; ++i) {
5         while (j && s[i] != s[j]) {
6             j = f[j];
7         }
8         j += (s[i] == s[j]);
9         f[i + 1] = j;
10    }
11    return f;
12 }

```

4.3 Z-function

```

1 std::vector<int> Z(std::string s) {
2     int n = s.size();
3     std::vector<int> z(n + 1);
4     z[0] = n;
5     for (int i = 1, j = 1; i < n; ++i) {
6         z[i] = std::max(0, std::min(j + z[j] - i, z[i - j]));
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
8             z[i]++;
9         }
10        if (i + z[i] > j + z[j]) {
11            j = i;
12        }
13    }
14    return z;
15 }

```

4.4 AhoCorasick

```

1 struct AhoCorasick {

```

```

2  static constexpr int ALPHABET = 26;
3  struct Node {
4      int len;
5      int link;
6      std::array<int, ALPHABET> next;
7      Node() : len{0}, link{0}, next{} {}
8  };
9
10 std::vector<Node> t;
11
12 AhoCorasick() {
13     init();
14 }
15
16 void init() {
17     t.assign(2, Node());
18     t[0].next.fill(1);
19     t[0].len = -1;
20 }
21
22 int newNode() {
23     t.emplace_back();
24     return t.size() - 1;
25 }
26
27 int add(const std::string &a) {
28     int p = 1;
29     for (auto c : a) {
30         int x = c - 'a';
31         if (t[p].next[x] == 0) {
32             t[p].next[x] = newNode();
33             t[t[p].next[x]].len = t[p].len + 1;
34         }
35         p = t[p].next[x];
36     }
37     return p;
38 }
39
40 void work() {
41     std::queue<int> q;
42     q.push(1);
43
44     while (!q.empty()) {
45         int x = q.front();
46         q.pop();
47
48         for (int i = 0; i < ALPHABET; i++) {
49             if (t[x].next[i] == 0) {
50                 t[x].next[i] = t[t[x].link].next[i];
51             } else {
52                 t[t[x].next[i]].link = t[t[x].link].next[i];

```

```

53         q.push(t[x].next[i]);
54     }
55 }
56 }
57 }
58
59 int next(int p, int x) {
60     return t[p].next[x];
61 }
62
63 int link(int p) {
64     return t[p].link;
65 }
66
67 int len(int p) {
68     return t[p].len;
69 }
70
71 int size() {
72     return t.size();
73 }
74 };

```

4.5 Suffix Array

```

1  struct SuffixArray {
2      int n;
3      std::vector<int> sa, rk, lc;
4      SuffixArray(const std::string &s) {
5          n = s.length();
6          sa.resize(n);
7          lc.resize(n - 1);
8          rk.resize(n);
9          std::iota(sa.begin(), sa.end(), 0);
10         std::sort(sa.begin(), sa.end(),
11             [&](int a, int b) {
12                 return s[a] < s[b];
13             });
14         rk[sa[0]] = 0;
15         for (int i = 1; i < n; i++) {
16             rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
17         }
18         int k = 1;
19         std::vector<int> tmp, cnt(n);
20         tmp.reserve(n);
21         while (rk[sa[n - 1]] < n - 1) {
22             tmp.clear();
23             for (int i = 0; i < k; i++) {
24                 tmp.push_back(n - k + i);
25             }

```

```

26     for (auto i : sa) {
27         if (i ≥ k) {
28             tmp.push_back(i - k);
29         }
30     }
31     std::fill(cnt.begin(), cnt.end(), 0);
32     for (int i = 0; i < n; i++) {
33         cnt[rk[i]]++;
34     }
35     for (int i = 1; i < n; i++) {
36         cnt[i] += cnt[i - 1];
37     }
38     for (int i = n - 1; i ≥ 0; i--) {
39         sa[--cnt[rk[tmp[i]]]] = tmp[i];
40     }
41     std::swap(rk, tmp);
42     rk[sa[0]] = 0;
43     for (int i = 1; i < n; i++) {
44         rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] ||
45             sa[i - 1] + k = n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
46     }
47     k *= 2;
48     for (int i = 0, j = 0; i < n; i++) {
49         if (rk[i] == 0) {
50             j = 0;
51         } else {
52             for (j -= (j > 0); i + j < n && sa[rk[i] - 1] + j < n && s[i
53                 + j] == s[sa[rk[i] - 1] + j]; j++)
54             ;
55             lc[rk[i] - 1] = j;
56         }
57     }
58 };

```

4.6 Suffix Automaton

```

1 struct SAM {
2     static constexpr int ALPHABET_SIZE = 26;
3     struct Node {
4         int len;
5         int link;
6         std::array<int, ALPHABET_SIZE> next;
7         Node() : len{}, link{}, next{} {}
8     };
9     std::vector<Node> t;
10    SAM() {
11        init();
12    }

```

```

13    void init() {
14        t.assign(2, Node());
15        t[0].next.fill(1);
16        t[0].len = -1;
17    }
18    int newNode() {
19        t.emplace_back();
20        return t.size() - 1;
21    }
22    int extend(int p, int c) {
23        if (t[p].next[c]) {
24            int q = t[p].next[c];
25            if (t[q].len == t[p].len + 1) {
26                return q;
27            }
28            int r = newNode();
29            t[r].len = t[p].len + 1;
30            t[r].link = t[q].link;
31            t[r].next = t[q].next;
32            t[q].link = r;
33            while (t[p].next[c] == q) {
34                t[p].next[c] = r;
35                p = t[p].link;
36            }
37            return r;
38        }
39        int cur = newNode();
40        t[cur].len = t[p].len + 1;
41        while (!t[p].next[c]) {
42            t[p].next[c] = cur;
43            p = t[p].link;
44        }
45        t[cur].link = extend(p, c);
46        return cur;
47    }
48    int extend(int p, char c, char offset = 'a') {
49        return extend(p, c - offset);
50    }
51
52    int next(int p, int x) {
53        return t[p].next[x];
54    }
55
56    int next(int p, char c, char offset = 'a') {
57        return next(p, c - 'a');
58    }
59
60    int link(int p) {
61        return t[p].link;
62    }
63

```



```

64 int len(int p) {
65     return t[p].len;
66 }
67
68 int size() {
69     return t.size();
70 }
71 };

```

4.7 Palindromic Tree

```

1 struct PAM {
2     static constexpr int ALPHABET_SIZE = 26;
3     struct Node {
4         int len;
5         int link;
6         int cnt;
7         std::array<int, ALPHABET_SIZE> next;
8         Node() : len{}, link{}, cnt{}, next{} {}
9     };
10    std::vector<Node> t;
11    int suff;
12    std::string s;
13    PAM() {
14        init();
15    }
16    void init() {
17        t.assign(2, Node());
18        t[0].len = -1;
19        suff = 1;
20        s.clear();
21    }
22    int newNode() {
23        t.emplace_back();
24        return t.size() - 1;
25    }
26    bool add(char c) {
27        int pos = s.size();
28        s += c;
29        int let = c - 'a';
30        int cur = suff, curlen = 0;
31        while (true) {
32            curlen = t[cur].len;
33            if (pos - 1 - curlen ≥ 0 && s[pos - 1 - curlen] == s[pos]) {
34                break;
35            }
36            cur = t[cur].link;
37        }
38        if (t[cur].next[let]) {
39            suff = t[cur].next[let];

```

```

40        return false;
41    }
42    int num = newNode();
43    suff = num;
44    t[num].len = t[cur].len + 2;
45    t[cur].next[let] = num;
46    if (t[num].len == 1) {
47        t[num].link = 1;
48        t[num].cnt = 1;
49        return true;
50    }
51    while (true) {
52        cur = t[cur].link;
53        curlen = t[cur].len;
54        if (pos - 1 - curlen ≥ 0 && s[pos - 1 - curlen] == s[pos]) {
55            t[num].link = t[cur].next[let];
56            break;
57        }
58    }
59    t[num].cnt = 1 + t[t[num].link].cnt;
60    return true;
61 }
62 int next(int p, int x) {
63     return t[p].next[x];
64 }
65 int link(int p) {
66     return t[p].link;
67 }
68 int len(int p) {
69     return t[p].len;
70 }
71 int size() {
72     return t.size();
73 }
74 };

```

5 Geometry

```

1 template<class T>
2 struct Point {
3     T x;
4     T y;
5     Point(const T &x_ = 0, const T &y_ = 0) : x(x_), y(y_) {}
6
7     template<class U>
8     operator Point<U>() {
9         return Point<U>(U(x), U(y));
10    }
11    Point &operator+=(const Point &p) & {

```

```

12     x += p.x;
13     y += p.y;
14     return *this;
15 }
16 Point &operator--=(const Point &p) & {
17     x -= p.x;
18     y -= p.y;
19     return *this;
20 }
21 Point &operator*=(const T &v) & {
22     x *= v;
23     y *= v;
24     return *this;
25 }
26 Point &operator/=(const T &v) & {
27     x /= v;
28     y /= v;
29     return *this;
30 }
31 Point operator-() const {
32     return Point(-x, -y);
33 }
34 friend Point operator+(Point a, const Point &b) {
35     return a += b;
36 }
37 friend Point operator-(Point a, const Point &b) {
38     return a -= b;
39 }
40 friend Point operator*(Point a, const T &b) {
41     return a *= b;
42 }
43 friend Point operator/(Point a, const T &b) {
44     return a /= b;
45 }
46 friend Point operator*(const T &a, Point b) {
47     return b *= a;
48 }
49 friend bool operator==(const Point &a, const Point &b) {
50     return a.x == b.x && a.y == b.y;
51 }
52 friend std::istream &operator>>(std::istream &is, Point &p) {
53     return is >> p.x >> p.y;
54 }
55 friend std::ostream &operator<<(std::ostream &os, const Point &p) {
56     return os << "(" << p.x << ", " << p.y << ")";
57 }
58 };
59
60 template<class T>
61 struct Line {
62     Point<T> a;

```

```

63     Point<T> b;
64     Line(const Point<T> &a_ = Point<T>(), const Point<T> &b_ = Point<T>()) :
65         a(a_), b(b_) {}
66 };
67 template<class T>
68 T dot(const Point<T> &a, const Point<T> &b) {
69     return a.x * b.x + a.y * b.y;
70 }
71
72 template<class T>
73 T cross(const Point<T> &a, const Point<T> &b) {
74     return a.x * b.y - a.y * b.x;
75 }
76
77 template<class T>
78 T square(const Point<T> &p) {
79     return dot(p, p);
80 }
81
82 template<class T>
83 double length(const Point<T> &p) {
84     return std::sqrt(square(p));
85 }
86
87 template<class T>
88 double length(const Line<T> &l) {
89     return length(l.a - l.b);
90 }
91
92 template<class T>
93 Point<T> normalize(const Point<T> &p) {
94     return p / length(p);
95 }
96
97 template<class T>
98 bool parallel(const Line<T> &l1, const Line<T> &l2) {
99     return cross(l1.b - l1.a, l2.b - l2.a) == 0;
100 }
101
102 template<class T>
103 double distance(const Point<T> &a, const Point<T> &b) {
104     return length(a - b);
105 }
106
107 template<class T>
108 double distancePL(const Point<T> &p, const Line<T> &l) {
109     return std::abs(cross(l.a - l.b, l.a - p)) / length(l);
110 }
111
112 template<class T>

```

```

113 double distancePS(const Point<T> &p, const Line<T> &l) {
114     if (dot(p - l.a, l.b - l.a) < 0) {
115         return distance(p, l.a);
116     }
117     if (dot(p - l.b, l.a - l.b) < 0) {
118         return distance(p, l.b);
119     }
120     return distancePL(p, l);
121 }
122
123 template<class T>
124 Point<T> rotate(const Point<T> &a) {
125     return Point(-a.y, a.x);
126 }
127
128 template<class T>
129 int sgn(const Point<T> &a) {
130     return a.y > 0 || (a.y == 0 && a.x > 0) ? 1 : -1;
131 }
132
133 template<class T>
134 bool pointOnLineLeft(const Point<T> &p, const Line<T> &l) {
135     return cross(l.b - l.a, p - l.a) > 0;
136 }
137
138 template<class T>
139 Point<T> lineIntersection(const Line<T> &l1, const Line<T> &l2) {
140     return l1.a + (l1.b - l1.a) * (cross(l2.b - l2.a, l1.a - l2.a) / cross(l2
        .b - l2.a, l1.a - l1.b));
141 }
142
143 template<class T>
144 bool pointOnSegment(const Point<T> &p, const Line<T> &l) {
145     return cross(p - l.a, l.b - l.a) == 0 && std::min(l.a.x, l.b.x) ≤ p.x &&
        p.x ≤ std::max(l.a.x, l.b.x)
146         && std::min(l.a.y, l.b.y) ≤ p.y && p.y ≤ std::max(l.a.y, l.b.y);
147 }
148
149 template<class T>
150 bool pointInPolygon(const Point<T> &a, const std::vector<Point<T>> &p) {
151     int n = p.size();
152     for (int i = 0; i < n; i++) {
153         if (pointOnSegment(a, Line(p[i], p[(i + 1) % n]))) {
154             return true;
155         }
156     }
157
158     int t = 0;
159     for (int i = 0; i < n; i++) {
160         auto u = p[i];
161         auto v = p[(i + 1) % n];

```

```

162         if (u.x < a.x && v.x ≥ a.x && pointOnLineLeft(a, Line(v, u))) {
163             t ^= 1;
164         }
165         if (u.x ≥ a.x && v.x < a.x && pointOnLineLeft(a, Line(u, v))) {
166             t ^= 1;
167         }
168     }
169
170     return t == 1;
171 }
172
173 // 0 : not intersect
174 // 1 : strictly intersect
175 // 2 : overlap
176 // 3 : intersect at endpoint
177 template<class T>
178 std::tuple<int, Point<T>, Point<T>> segmentIntersection(const Line<T> &l1,
    const Line<T> &l2) {
179     if (std::max(l1.a.x, l1.b.x) < std::min(l2.a.x, l2.b.x)) {
180         return {0, Point<T>(), Point<T>()};
181     }
182     if (std::min(l1.a.x, l1.b.x) > std::max(l2.a.x, l2.b.x)) {
183         return {0, Point<T>(), Point<T>()};
184     }
185     if (std::max(l1.a.y, l1.b.y) < std::min(l2.a.y, l2.b.y)) {
186         return {0, Point<T>(), Point<T>()};
187     }
188     if (std::min(l1.a.y, l1.b.y) > std::max(l2.a.y, l2.b.y)) {
189         return {0, Point<T>(), Point<T>()};
190     }
191     if (cross(l1.b - l1.a, l2.b - l2.a) == 0) {
192         if (cross(l1.b - l1.a, l2.a - l1.a) ≠ 0) {
193             return {0, Point<T>(), Point<T>()};
194         } else {
195             auto maxx1 = std::max(l1.a.x, l1.b.x);
196             auto minx1 = std::min(l1.a.x, l1.b.x);
197             auto maxy1 = std::max(l1.a.y, l1.b.y);
198             auto miny1 = std::min(l1.a.y, l1.b.y);
199             auto maxx2 = std::max(l2.a.x, l2.b.x);
200             auto minx2 = std::min(l2.a.x, l2.b.x);
201             auto maxy2 = std::max(l2.a.y, l2.b.y);
202             auto miny2 = std::min(l2.a.y, l2.b.y);
203             Point<T> p1(std::max(minx1, minx2), std::max(miny1, miny2));
204             Point<T> p2(std::min(maxx1, maxx2), std::min(maxy1, maxy2));
205             if (!pointOnSegment(p1, l1)) {
206                 std::swap(p1.y, p2.y);
207             }
208             if (p1 == p2) {
209                 return {3, p1, p2};
210             } else {
211                 return {2, p1, p2};

```

```

212     }
213 }
214 }
215 auto cp1 = cross(l2.a - l1.a, l2.b - l1.a);
216 auto cp2 = cross(l2.a - l1.b, l2.b - l1.b);
217 auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
218 auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);
219
220 if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2 < 0) || (cp3 > 0 && cp4 > 0)
    || (cp3 < 0 && cp4 < 0)) {
221     return {0, Point<T>(), Point<T>()};
222 }
223
224 Point p = lineIntersection(l1, l2);
225 if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) {
226     return {1, p, p};
227 } else {
228     return {3, p, p};
229 }
230 }
231
232 template<class T>
233 double distanceSS(const Line<T> &l1, const Line<T> &l2) {
234     if (std::get<0>(segmentIntersection(l1, l2)) != 0) {
235         return 0.0;
236     }
237     return std::min({distancePS(l1.a, l2), distancePS(l1.b, l2), distancePS(
        l2.a, l1), distancePS(l2.b, l1)});
238 }
239
240 template<class T>
241 bool segmentInPolygon(const Line<T> &l, const std::vector<Point<T>> &p) {
242     int n = p.size();
243     if (!pointInPolygon(l.a, p)) {
244         return false;
245     }
246     if (!pointInPolygon(l.b, p)) {
247         return false;
248     }
249     for (int i = 0; i < n; i++) {
250         auto u = p[i];
251         auto v = p[(i + 1) % n];
252         auto w = p[(i + 2) % n];
253         auto [t, p1, p2] = segmentIntersection(l, Line(u, v));
254
255         if (t == 1) {
256             return false;
257         }
258         if (t == 0) {
259             continue;
260         }

```

```

261     if (t == 2) {
262         if (pointOnSegment(v, l) && v != l.a && v != l.b) {
263             if (cross(v - u, w - v) > 0) {
264                 return false;
265             }
266         }
267     } else {
268         if (p1 != u && p1 != v) {
269             if (pointOnLineLeft(l.a, Line(v, u))
                || pointOnLineLeft(l.b, Line(v, u))) {
270                 return false;
271             }
272         }
273         if (p1 == v) {
274             if (l.a == v) {
275                 if (pointOnLineLeft(u, l)) {
276                     if (pointOnLineLeft(w, l)
                        && pointOnLineLeft(w, Line(u, v))) {
277                         return false;
278                     }
279                 }
280             } else {
281                 if (pointOnLineLeft(w, l)
                    || pointOnLineLeft(w, Line(u, v))) {
282                     return false;
283                 }
284             }
285         }
286         if (l.b == v) {
287             if (pointOnLineLeft(u, Line(l.b, l.a))) {
288                 if (pointOnLineLeft(w, Line(l.b, l.a))
                    && pointOnLineLeft(w, Line(u, v))) {
289                     return false;
290                 }
291             }
292             if (pointOnLineLeft(w, Line(l.b, l.a))
                || pointOnLineLeft(w, Line(u, v))) {
293                 return false;
294             }
295         }
296     }
297 } else {
298     if (pointOnLineLeft(u, l)) {
299         if (pointOnLineLeft(w, Line(l.b, l.a))
            || pointOnLineLeft(w, Line(u, v))) {
300             return false;
301         }
302     }
303 } else {
304     if (pointOnLineLeft(w, l)
        || pointOnLineLeft(w, Line(u, v))) {
305         return false;
306     }
307 }
308 }
309 }
310 }
311 }

```

```

312     }
313 }
314 return true;
315 }
316
317 template<class T>
318 std::vector<Point<T>> hp(std::vector<Line<T>> lines) {
319     std::sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
320         auto d1 = l1.b - l1.a;
321         auto d2 = l2.b - l2.a;
322
323         if (sgn(d1) != sgn(d2)) {
324             return sgn(d1) == 1;
325         }
326
327         return cross(d1, d2) > 0;
328     });
329
330     std::deque<Line<T>> ls;
331     std::deque<Point<T>> ps;
332     for (auto l : lines) {
333         if (ls.empty()) {
334             ls.push_back(l);
335             continue;
336         }
337
338         while (!ps.empty() && !pointOnLineLeft(ps.back(), l)) {
339             ps.pop_back();
340             ls.pop_back();
341         }
342
343         while (!ps.empty() && !pointOnLineLeft(ps[0], l)) {
344             ps.pop_front();
345             ls.pop_front();
346         }
347
348         if (cross(l.b - l.a, ls.back().b - ls.back().a) == 0) {
349             if (dot(l.b - l.a, ls.back().b - ls.back().a) > 0) {
350
351                 if (!pointOnLineLeft(ls.back().a, l)) {
352                     assert(ls.size() == 1);
353                     ls[0] = l;
354                 }
355                 continue;
356             }
357             return {};
358         }
359
360         ps.push_back(lineIntersection(ls.back(), l));
361         ls.push_back(l);
362     }

```

```

363
364     while (!ps.empty() && !pointOnLineLeft(ps.back(), ls[0])) {
365         ps.pop_back();
366         ls.pop_back();
367     }
368     if (ls.size() ≤ 2) {
369         return {};
370     }
371     ps.push_back(lineIntersection(ls[0], ls.back()));
372
373     return std::vector(ps.begin(), ps.end());
374 }
375
376 using P = Point<double>;

```

6 Util

6.1 Mod Integer

```

1 constexpr int P = 998244353;
2 using i64 = long long;
3 // assume -P ≤ x < 2P
4 int norm(int x) {
5     if (x < 0) {
6         x += P;
7     }
8     if (x ≥ P) {
9         x -= P;
10    }
11    return x;
12 }
13 template<class T>
14 T power(T a, i64 b) {
15     T res = 1;
16     for (; b; b /= 2, a *= a) {
17         if (b % 2) {
18             res *= a;
19         }
20     }
21     return res;
22 }
23 struct Z {
24     int x;
25     Z(int x = 0) : x(norm(x)) {}
26     Z(i64 x) : x(norm(x % P)) {}
27     int val() const {
28         return x;
29     }
30     Z operator-() const {

```

```

31     return Z(norm(P - x));
32 }
33 Z inv() const {
34     assert(x ≠ 0);
35     return power(*this, P - 2);
36 }
37 Z &operator*=(const Z &rhs) {
38     x = i64(x) * rhs.x % P;
39     return *this;
40 }
41 Z &operator+=(const Z &rhs) {
42     x = norm(x + rhs.x);
43     return *this;
44 }
45 Z &operator-=(const Z &rhs) {
46     x = norm(x - rhs.x);
47     return *this;
48 }
49 Z &operator/=(const Z &rhs) {
50     return *this *= rhs.inv();
51 }
52 friend Z operator*(const Z &lhs, const Z &rhs) {
53     Z res = lhs;
54     res *= rhs;
55     return res;
56 }
57 friend Z operator+(const Z &lhs, const Z &rhs) {
58     Z res = lhs;
59     res += rhs;
60     return res;
61 }
62 friend Z operator-(const Z &lhs, const Z &rhs) {
63     Z res = lhs;
64     res -= rhs;
65     return res;
66 }
67 friend Z operator/(const Z &lhs, const Z &rhs) {
68     Z res = lhs;
69     res /= rhs;
70     return res;
71 }
72 friend std::istream &operator>>(std::istream &is, Z &a) {
73     i64 v;
74     is >> v;
75     a = Z(v);
76     return is;
77 }
78 friend std::ostream &operator<<(std::ostream &os, const Z &a) {
79     return os << a.val();
80 }
81 };

```

```

1 template <class T>
2 constexpr T power(T a, u64 b, T res = 1) {
3     for (; b ≠ 0; b /= 2, a *= a) {
4         if (b & 1) {
5             res *= a;
6         }
7     }
8     return res;
9 }
10 template <u32 P>
11 constexpr u32 mulMod(u32 a, u32 b) {
12     return u64(a) * b % P;
13 }
14 template <u64 P>
15 constexpr u64 mulMod(u64 a, u64 b) {
16     u64 res = a * b - u64(1.L * a * b / P - 0.5L) * P;
17     res %= P;
18     return res;
19 }
20 template <std::unsigned_integral U, U P>
21 struct ModIntBase {
22 public:
23     constexpr ModIntBase() : x(0) {}
24     template <std::unsigned_integral T>
25     constexpr ModIntBase(T x_) : x(x_ % mod()) {}
26     template <std::signed_integral T>
27     constexpr ModIntBase(T x_) {
28         using S = std::make_signed_t<U>;
29         S v = x_ % S(mod());
30         if (v < 0) {
31             v += mod();
32         }
33         x = v;
34     }
35     constexpr static U mod() {
36         return P;
37     }
38     constexpr U val() const {
39         return x;
40     }
41     constexpr ModIntBase operator-() const {
42         ModIntBase res;
43         res.x = (x == 0 ? 0 : mod() - x);
44         return res;
45     }
46     constexpr ModIntBase inv() const {
47         return power(*this, mod() - 2);
48     }
49     constexpr ModIntBase pow(u64 b) const {
50         return power(*this, b);

```

```

51 }
52 constexpr ModIntBase &operator*=(const ModIntBase &rhs) & {
53     x = mulMod<mod()>(x, rhs.val());
54     return *this;
55 }
56 constexpr ModIntBase &operator+=(const ModIntBase &rhs) & {
57     x += rhs.val();
58     if (x ≥ mod()) {
59         x -= mod();
60     }
61     return *this;
62 }
63 constexpr ModIntBase &operator-=(const ModIntBase &rhs) & {
64     x -= rhs.val();
65     if (x ≥ mod()) {
66         x += mod();
67     }
68     return *this;
69 }
70 constexpr ModIntBase &operator/=(const ModIntBase &rhs) & {
71     return *this *= rhs.inv();
72 }
73
74 friend constexpr ModIntBase operator*(ModIntBase lhs, const ModIntBase &
75     rhs) {
76     lhs *= rhs;
77     return lhs;
78 }
79 friend constexpr ModIntBase operator+(ModIntBase lhs, const ModIntBase &
80     rhs) {
81     lhs += rhs;
82     return lhs;
83 }
84 friend constexpr ModIntBase operator-(ModIntBase lhs, const ModIntBase &
85     rhs) {
86     lhs -= rhs;
87     return lhs;
88 }
89 friend constexpr ModIntBase operator/(ModIntBase lhs, const ModIntBase &
90     rhs) {
91     lhs /= rhs;
92     return lhs;
93 }
94
95 friend constexpr std::istream &operator>>(std::istream &is, ModIntBase &a
96     ) {
97     i64 i;
98     is >> i;
99     a = i;
100     return is;
101 }

```

```

97 friend constexpr std::ostream &operator<<(std::ostream &os, const
100     ModIntBase &a) {
101     return os << a.val();
102 }
103
104 friend constexpr bool operator==(const ModIntBase &lhs, const ModIntBase
105     &rhs) {
106     return lhs.val() == rhs.val();
107 }
108 friend constexpr std::strong_ordering operator<=>(const ModIntBase &lhs,
109     const ModIntBase &rhs) {
110     return lhs.val() <=> rhs.val();
111 }
112
113 private:
114     U x;
115 };
116
117 template <u32 P>
118 using ModInt = ModIntBase<u32, P>;
119 template <u64 P>
120 using ModInt64 = ModIntBase<u64, P>;

```

6.2 Fraction

```

1 struct Frac {
2     i64 num, den;
3     constexpr Frac(i64 x = 0) : Frac(x, 1) {}
4     constexpr Frac(i64 num, i64 den) : num{num}, den{den} {
5         norm();
6     }
7     constexpr void norm() {
8         if (den < 0) {
9             num = -num;
10            den = -den;
11        }
12    }
13    constexpr Frac operator-() const {
14        return Frac(-num, den);
15    }
16    constexpr Frac &operator+=(const Frac &rhs) {
17        num = num * rhs.den + rhs.num * den;
18        den *= rhs.den;
19        norm();
20        return *this;

```

```

21 }
22 constexpr Frac &operator==(const Frac &rhs) {
23     return *this += -rhs;
24 }
25 constexpr Frac &operator*=(const Frac &rhs) {
26     num *= rhs.num;
27     den *= rhs.den;
28     norm();
29     return *this;
30 }
31 constexpr Frac &operator/=(const Frac &rhs) {
32     num *= rhs.den;
33     den *= rhs.num;
34     norm();
35     return *this;
36 }
37 friend constexpr Frac operator+(const Frac &lhs, const Frac &rhs) {
38     Frac res = lhs;
39     res += rhs;
40     return res;
41 }
42 friend constexpr Frac operator-(const Frac &lhs, const Frac &rhs) {
43     Frac res = lhs;
44     res -= rhs;
45     return res;
46 }
47 friend constexpr Frac operator*(const Frac &lhs, const Frac &rhs) {
48     Frac res = lhs;
49     res *= rhs;
50     return res;
51 }
52 friend constexpr Frac operator/(const Frac &lhs, const Frac &rhs) {
53     Frac res = lhs;
54     res /= rhs;
55     return res;
56 }
57 friend constexpr bool operator<(const Frac &lhs, const Frac &rhs) {
58     return static_cast<__int128>(lhs.num) * rhs.den < static_cast<__
        int128>(rhs.num) * lhs.den;
59 }
60 friend constexpr bool operator>(const Frac &lhs, const Frac &rhs) {
61     return static_cast<__int128>(lhs.num) * rhs.den > static_cast<__
        int128>(rhs.num) * lhs.den;
62 }
63 friend std::ostream &operator<<(std::ostream &os, const Frac &rhs) {
64     return os << rhs.num << "/" << rhs.den;
65 }
66 };

```

7 Tables

7.1 Constant

n	$\log_{10} n$	$n!$	$C(n, n/2)$	$\text{LCM}(1 \dots n)$	P_n
2	0.30102999	2	2	2	2
3	0.47712125	6	3	6	3
4	0.60205999	24	6	12	5
5	0.69897000	120	10	60	7
6	0.77815125	720	20	60	11
7	0.84509804	5040	35	420	15
8	0.90308998	40320	70	840	22
9	0.95424251	362880	126	2520	30
10	1	3628800	252	2520	42
11	1.04139269	39916800	462	27720	56
12	1.07918125	479001600	924	27720	77
15	1.17609126	1.31×10^{12}	6435	360360	176
20	1.30103000	2.43×10^{18}	184756	232792560	627
25	1.39794001	1.55×10^{25}	5200300	26771144400	1958
30	1.47712125	2.65×10^{32}	155117520	1.444×10^{14}	5604

$n \leq$	10	100	10^3	10^4	10^5	10^6
$\max \omega(n)$	2	3	4	5	6	7
$\max d(n)$	4	12	32	64	128	240
$\pi(n)$	4	25	168	1229	9592	78498
$n \leq$	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
$\max \omega(n)$	8	8	9	10	10	11
$\max d(n)$	448	768	1344	2304	4032	6720
$\pi(n)$	664579	5761455	5.08×10^7	4.55×10^8	4.12×10^9	3.7×10^{10}
$n \leq$	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}
$\max \omega(n)$	12	12	13	13	14	15
$\max d(n)$	10752	17280	26880	41472	64512	103680

Prime number theorem: $\pi(x) \sim x/\log(x)$