*In the name of God,*
*the merciful, the compassionate*

دانشگاه صنعتی امیرکبیر
( پلی تکنیک تهران )

# HOMEWORK 8
# (TRACKING, DISTURBANCE AND LQR)

**By**

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# Problem 1 Description

### Theory and Simulation

1. Solve Friedland Problem 8.4 Inverted pendulum on cart: compensator design.

   Bonus: Implement the compensator in Simulink. Consider initial conditions as $[0, 0, 10°, 0]^T$ (ref = 0). Compare the system responses and control efforts and explain your results. Also, implement the compensator on the nonlinear system and compare the results.

# Solution

1. Solve Friedland problem 8.4. Compensator design

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

$$\xrightarrow{u=-K\hat{x}} \dot{\hat{x}} = (A - BK - LC)\hat{x} + Ly \longrightarrow \hat{X}(s) = (sI - A + BK + LC)^{-1} L Y(s)$$

$$U(s) = -K\hat{X}(s) = -K(sI - A + BK + LC)^{-1} L Y(s) = -D(s)Y(s)$$

$$D(s) = K(sI - A + BK + LC)^{-1} L$$

$$K = [-326.53 \quad -226.3265 \quad -812.0906 \quad -242.3265]^T$$

$$L = [-11.935 \quad 39451.3 \quad -452.0058 \quad -1193.6188]^T$$

$$\Rightarrow D(s) = \frac{7.65\times10^5 s^3 + 2.15\times10^7 s^2 + 5.96\times10^7 s - 2.04\times10^5}{s^4 + 21.065 s^3 + 232.655 s^2 - 3.81 s - 1.2\times10^6}$$

# Bonus

Systems are implemented in MATLAB Simulink and the results for both linear and nonlinear system is as follows,

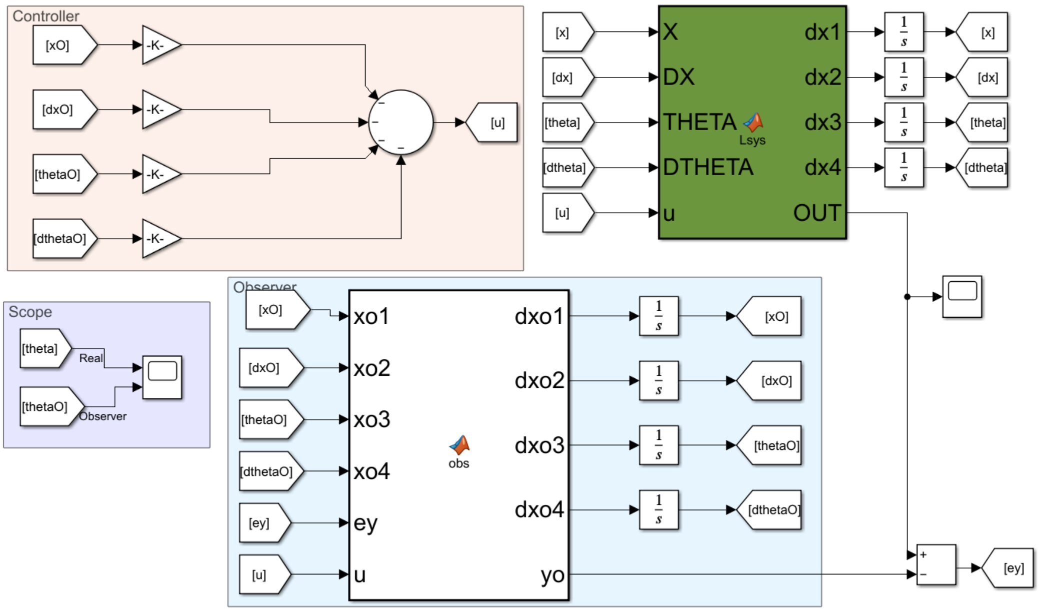



Figure 1 Linear system with compensator.

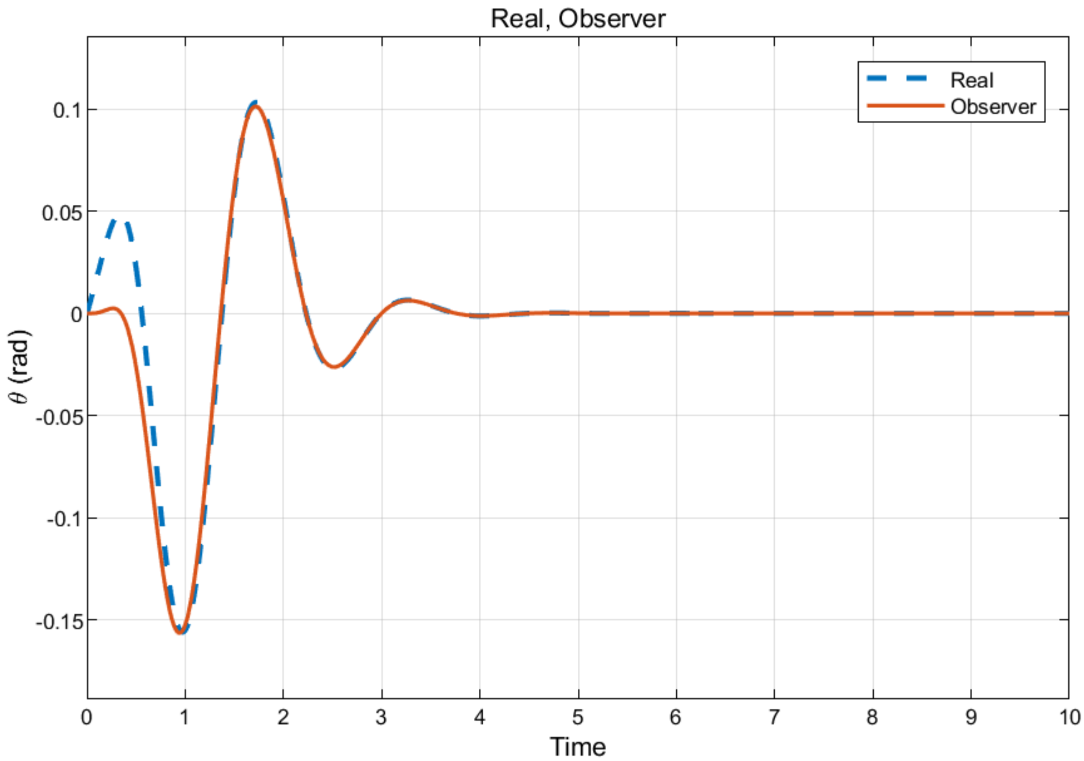



Figure 2 Linear system result.

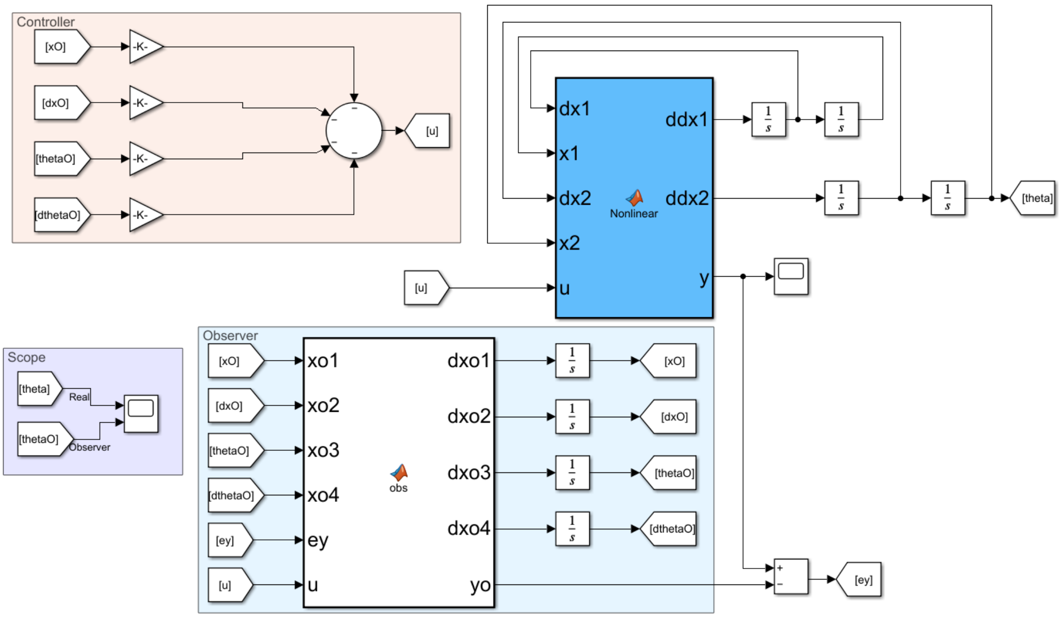



Figure 3 Nonlinear system with compensator.

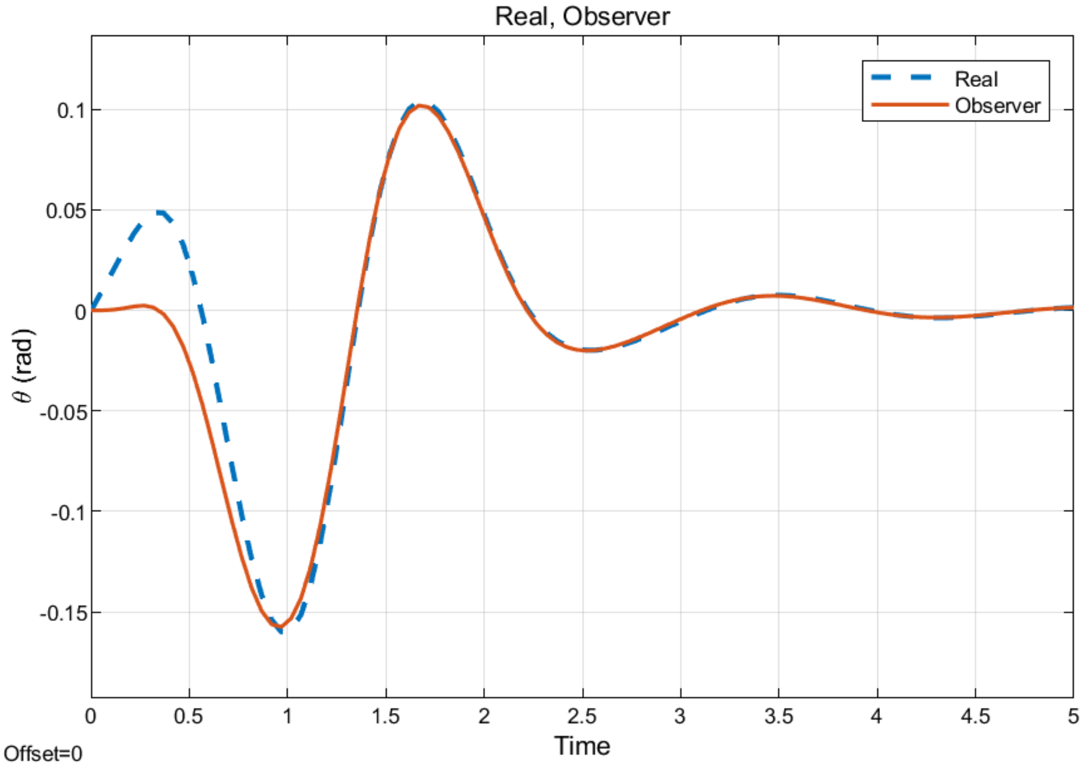



Figure 4 Nonlinear system result.

# Problem 2 Description

2. Consider a system with state matrices

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

# Solution

2. $A=\begin{bmatrix}-2 & 1\\ 0 & -3\end{bmatrix}$ $B=\begin{bmatrix}1\\1\end{bmatrix}$ $C=\begin{bmatrix}1 & 3\end{bmatrix}$

a) $u=-kx(t)+\bar{N}r(t)$

$\rightarrow$ poles @ $-3\pm 3j$

$$\rightarrow A-BK=\begin{bmatrix}-2 & 1\\ 0 & -3\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}k_1 & k_2\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 0 & -3\end{bmatrix}-\begin{bmatrix}k_1 & k_2\\ k_1 & k_2\end{bmatrix}=\begin{bmatrix}-2-k_1 & 1-k_2\\ -k_1 & -3-k_2\end{bmatrix}$$

$$\Rightarrow \left|sI-(A-BK)\right| = s^2+(k_1+k_2+5)s+4k_1+2k_2+6 \equiv s^2+6s+18$$

$$\begin{cases} k_1+k_2+5=6 \\ 4k_1+2k_2+6=18 \end{cases} \rightarrow k_1=5 \quad k_2=-4 \rightarrow \begin{bmatrix}5 & -4\end{bmatrix}$$

b) $\dot{x}_{1ss}=-2x_{1ss}+x_{2ss}-k_1x_{1ss}-k_2x_{2ss}+\bar{N}r_{ss}=0$

$-7x_{1ss}+5x_{2ss}+\bar{N}r_{ss}=0$ *

$\dot{x}_{2ss}=-3x_{2ss}-k_1x_{1ss}-k_2x_{2ss}+\bar{N}r_{ss}=0$

$-5x_{1ss}+x_{2ss}+\bar{N}r_{ss}=0$ **

$\Rightarrow x_{1ss}=\frac{2}{9}\bar{N}r_{ss}$ , $x_{2ss}=\frac{1}{9}\bar{N}r_{ss}$

$y_{ss}=Cx_{ss}=x_{1ss}+3x_{2ss}$

$=\frac{5}{9}\bar{N}r_{ss}=r_{ss}$

$\Rightarrow \boxed{N=\frac{9}{5}}$

c) $\dot{x} = Ax + Bu = Ax + B(-Kx + \bar{N}r) = (A - BK)x + B\bar{N}r$

$SX = (A - BK)X + \bar{N}R; \quad X = (SI - A + BK)^{-1} B\bar{N}R$

$y = Cx \rightarrow Y = CX = C(SI - A + BK)^{-1} B\bar{N}R$

$G_{cl}(S) = \dfrac{Y}{R} = C(SI - A + BK)^{-1} B\bar{N}$

$G_{cl}(0) = C(-A + BK)^{-1} B\bar{N}$

تغییرات $A \rightarrow A_N = A + \delta A = \begin{bmatrix} -2+\delta_{11} & 1+\delta_{12} \\ \delta_{21} & -3+\delta_{22} \end{bmatrix} \rightarrow (-A + BK)^{-1} =$

$$\frac{1}{\underbrace{18 + \delta_{11} + 5\delta_{12} - 5\delta_{21} - 7\delta_{22} + \delta_{11}\delta_{22} - \delta_{12}\delta_{21}}_{Z}} \begin{bmatrix} -1-\delta_{22} & 5-\delta_{21} \\ -5-\delta_{12} & 7-\delta_{11} \end{bmatrix}$$

$G_{cl}(0) = \dfrac{1.8}{Z}(10 - 3\delta_{11} - 3\delta_{12} - \delta_{21} - \delta_{22}) \neq 1.8$

d) integrator

$\dot{x}_I = Cx - r \rightarrow \begin{Bmatrix} \dot{x} \\ \dot{x}_I \end{Bmatrix} = \left[\begin{array}{c|c} A & 0 \\ \hline C & 0 \end{array}\right] \begin{Bmatrix} x \\ \hline x_I \end{Bmatrix} + \left[\begin{array}{c} B \\ \hline 0 \end{array}\right] u + \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right] r$

$\Rightarrow A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 0 \\ 1 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \Rightarrow |SI - A| = S^3 + 5S^2 + 6S$

$\rightarrow a = [5 \ 6 \ 0]$

$W = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

desired $\rightarrow S^3 + 7S^2 + 19S + 21$

$\alpha = [7 \ 19 \ 21]$

$\alpha - a = [2 \ 13 \ 21]^T$

$Q = [B \ \ AB \ \ A^2B] = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -3 & 9 \\ 0 & 4 & -10 \end{bmatrix} \quad QW = \begin{bmatrix} 1 & 4 & 0 \\ 1 & 2 & 0 \\ 0 & 4 & 10 \end{bmatrix}$

$\Rightarrow K^T = [(QW)^T]^{-1}(\alpha - a) = [0{,}3 \ \ 1{,}7 \ \ 2.1]^T$

e) $\dot{x} = Ax + Bu \rightarrow SX = AX + BU = AX + B(-KX - K_I X_I)$

$\dot{x}_I = Cx + r \rightarrow SX_I = CX + R \rightarrow X_I = S^{-1}C + S^{-1}R$

$\rightarrow SX = AX + B(-KX - K_I S^{-1} CX - K_I S^{-1} R)$

$\rightarrow X = (SI - A + BK + BK_I S^{-1} C)^{-1}(-BK_I S^{-1} R)$

$Y = CX \rightarrow G_{cl}(S) = -C(SI - A + BK + BK_I S^{-1} C)^{-1}(BK_I S^{-1})$

$= -C[(S^2 I - S(A + BK) + K_I BC) S^{-1}]^{-1}(BK_I S^{-1})$

$= -C[S^2 I - S(A + BK) + K_I BC]^{-1} S K_I S^{-1} B$

$= -C(S^2 I - S(A + BK) + K_I BC]^{-1} K_I B$

$(0) = -C(K_I BC)^{-1} B K_I = \boxed{1}$ تعقیب صحیح = خطای صفر

# Bonus

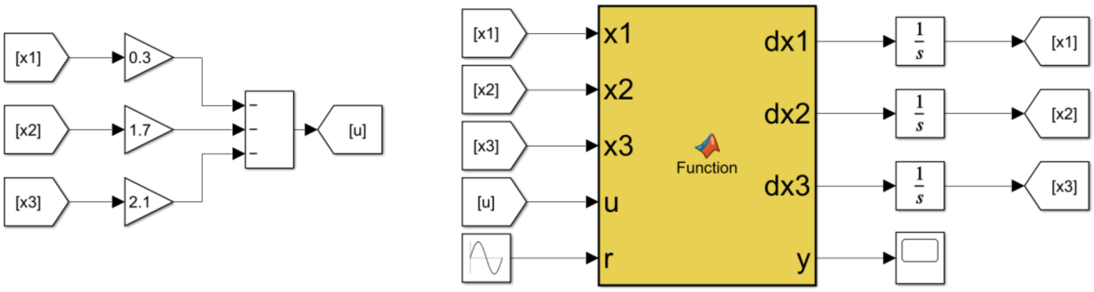



Figure 5 system with sine wave as input.

Final response of the system follows the sine wave!

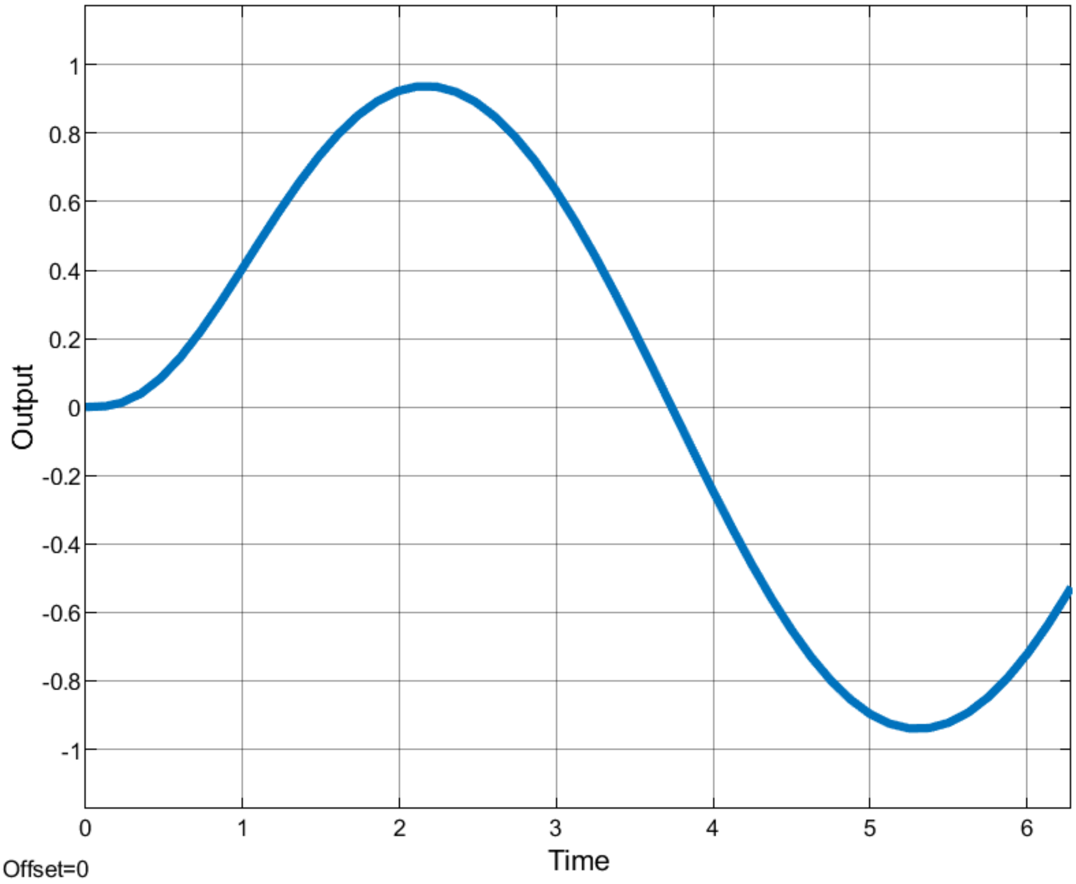



Figure 6 system result with sine wave as input.

# Problem 3 Description (Bonus)

Bonus: Consider the transfer function: (you have found its full state feedback gains in HW6)

$$G(s) = \frac{-2(0.1s+1)(s+1)}{(s-2)^2(s^3+2s^2+s)}$$

# Solution

$$y(t) = Cx(t)$$

at $\infty$ $\longrightarrow$ $0 = Ax(\infty) + Bu(\infty)$

$$r = y(\infty) = Cx(\infty)$$

$\longrightarrow \dot{x}(t) = A[x(t) - x(\infty)] + B[u(t) - u(\infty)]$

$y(t) - r = C[x(t) - x(\infty)]$

$\longrightarrow u(t) = -Kx(t) + u_a = -Kx + u(\infty) + Kx(\infty)$

$u_a = -C(a-bK)^{-1} r$

$u(t) = -Kx(t) + [G_d(0)]^{-1} r$

$\longrightarrow K = [11.1 \quad 29.15 \quad 27.275 \quad 11.375 \quad 1.25]^T$

$\Rightarrow u(t) = -Kx - Kr$

$\longrightarrow$ integral control $\quad \dot{x}(t) = Ax(t) + Bu(t)$

$$y(t) = Cx(t)$$

$$\dot{q} = r - y(t) = r - Cx(t)$$

$\Rightarrow A = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \quad B = \begin{bmatrix} B \\ 0 \end{bmatrix}$

ماتریس کنترل پذیری $\rightarrow \begin{bmatrix} 1 & 2 & 7 & 16 & 41 & 94 \\ 0 & 1 & 2 & 7 & 16 & 41 \\ 0 & 0 & 1 & 2 & 7 & 16 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \det \neq 0 \quad$ full rank

$\Rightarrow K = [37.5 \quad 4545 \quad 2605.5 \quad 6913.5 \quad 4725 \quad 2250]^T$

# Problem 4 Description

Solve Friedland Problem 9.10 Inverted pendulum on cart: optimal gains.

# Solution

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -25 & -0.98 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 25 & 10.78 & 0 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ -0.5 \end{bmatrix} u$$

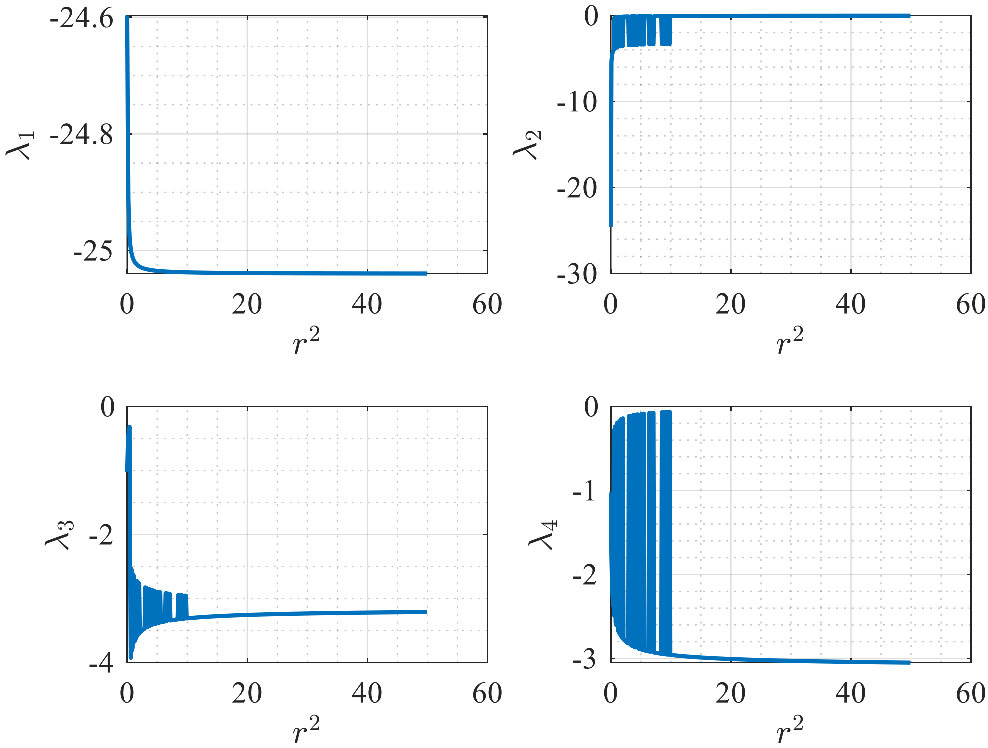
$$J = \int_0^\infty (q_1^2 x_1^2 + q_3^2 x_3^2 + r^2 u^2)\,dt \longrightarrow Q = \begin{bmatrix} q_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q_3^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

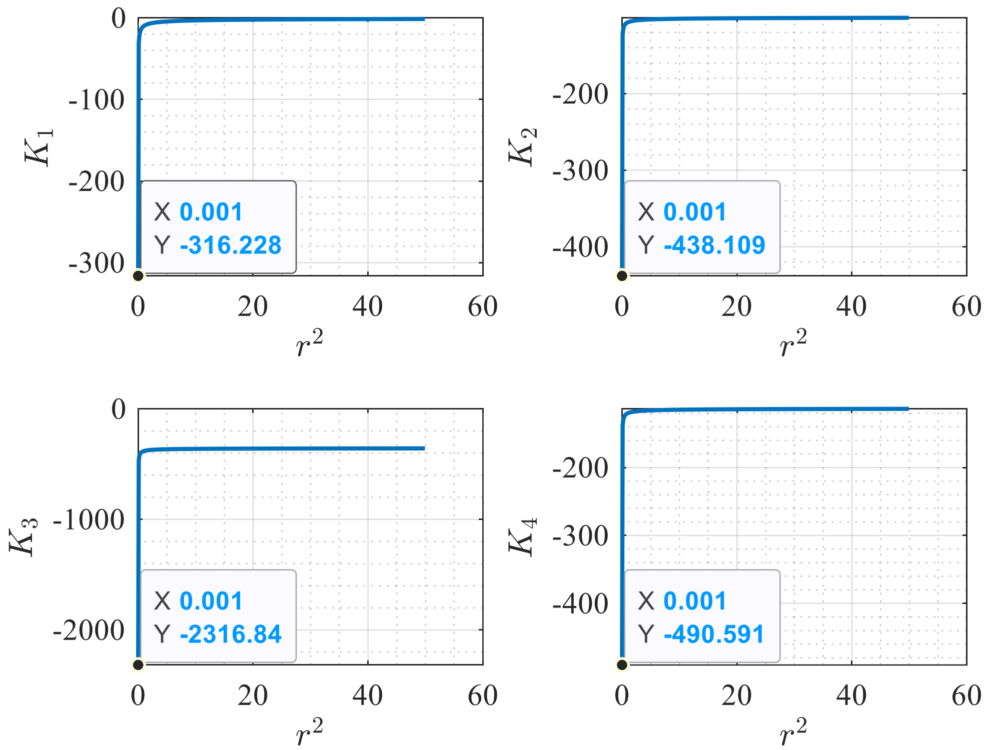
$R = r^2$

$\longrightarrow$ ریکاتی $\rightarrow -PA - A^T P + PBR^{-1}B^T P = Q$

$\longrightarrow$ MATLAB

با افزایش ۲ قطب‌ها به صفر نزدیکتر می‌شوند، تلاش کنترلی کاهش پیدا می‌کند
در حالی که افزایش $q_1$ باعث کاهش Cost function می‌شود و منجر به
افزایش ضریب کنترلی می‌شود.




```
clc
clear
close all
M = 1;
```

```
m = 0.1;
l = 1;
g = 9.8;
k = 1;
R = 100;
r = 0.02;
A = [0 1 0 0;...
0 -k^2/(M*r^2*R) -m*g/M 0;...
0 0 0 1;...
0 k^2/(M*r^2*R*l) (M+m)*g/(M*l) 0];
B = [0;k/(M*R*r);0;-k/(M*R*r*l)];
q1 = 100;
q3 = 3000;
Q = [q1 0 0 0;0 0 0 0;0 0 q3 0;0 0 0 0];
R = 0.001:0.1:50;
for i = 1:numel(R)
    K(i,:) = lqr(A,B,Q,R(i));
    Ac = A-B*K(i,:);
    landa(:,i) = real(eig(Ac));
end
figure(1)
subplot(4,1,1)
plot(R,K(:,1),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$K_{1}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,2)
plot(R,K(:,2),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$K_{2}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,3)
plot(R,K(:,3),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$K_{3}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,4)
plot(R,K(:,4),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$K_{4}$','Interpreter','latex')
```

```
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
figure(2)
subplot(4,1,1)
plot(R,landa(1,:),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$\lambda_{1}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,2)
plot(R,landa(2,:),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$\lambda_{2}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,3)
plot(R,landa(3,:),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$\lambda_{3}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
subplot(4,1,4)
plot(R,landa(4,:),'linewidth',1.6)
grid on
grid minor
xlabel('$r^{2}$','Interpreter','latex')
ylabel('$\lambda_{4}$','Interpreter','latex')
set(gca,'FontSize',12)
set(gca,'fontname','Times New Roman')
```

# Problem 5 Description

Consider the system below with $\omega_n = 2, \zeta = 0.5$:

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$J = \int_0^\infty [y(t)^T Q y(t) + u(t) R u(t)] dt, \qquad Q = \begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix}, \qquad R = r$$

a. Design an optimal feedback controller for the system with $q = 0.1, 1, 10, 100$, and $r = 1$. Plot the system response (states) and control effort and explain your results.

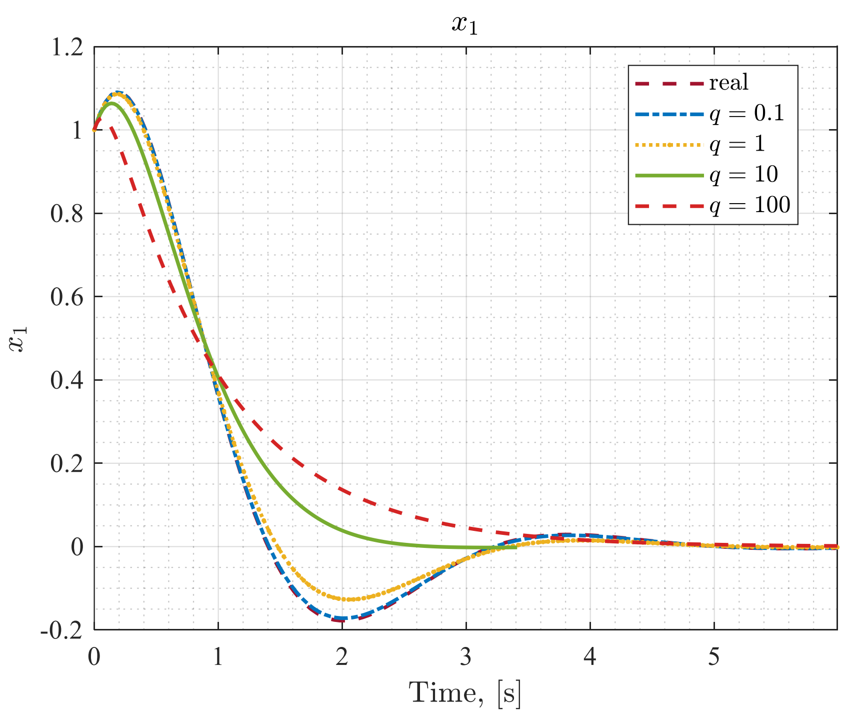
b. Design an optimal feedback controller for the system with $r = 0.1, 1, 10, 100$, and $q = 1$. Plot the system response (states) and control effort and explain your results.

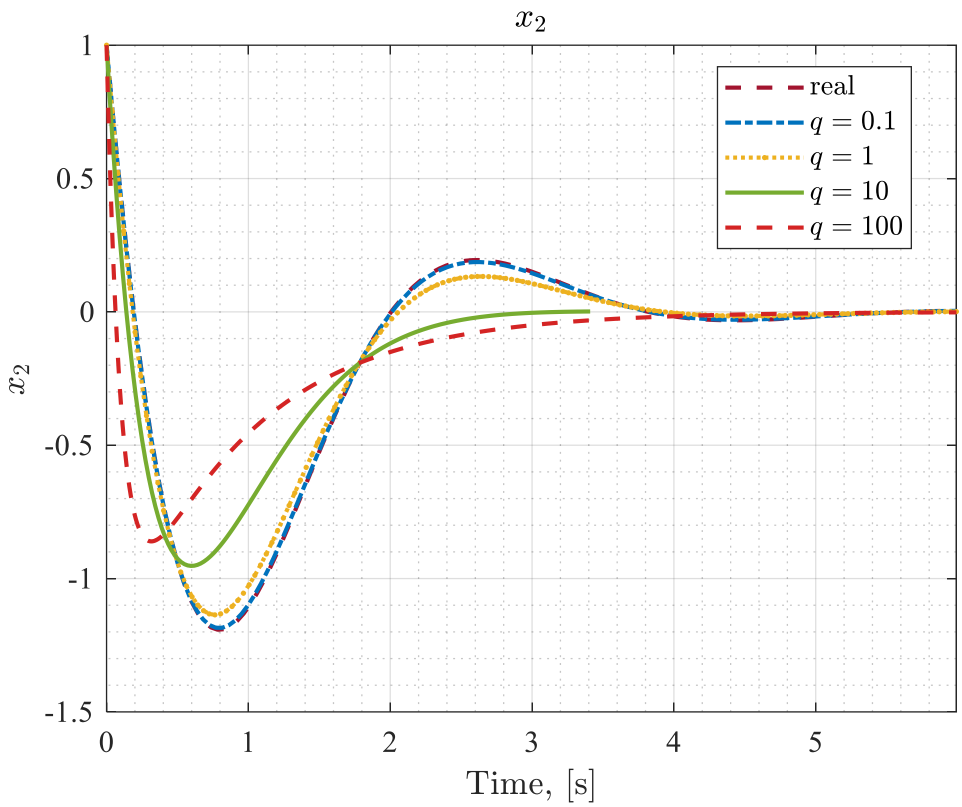
c. How can we change the cost function to design an optimal feedback controller for tracking a constant value $y_d$?

This link explains how you can implement LQR controller in MATLAB.
https://nl.mathworks.com/help/control/ref/lti.lqr.html#d126e97612

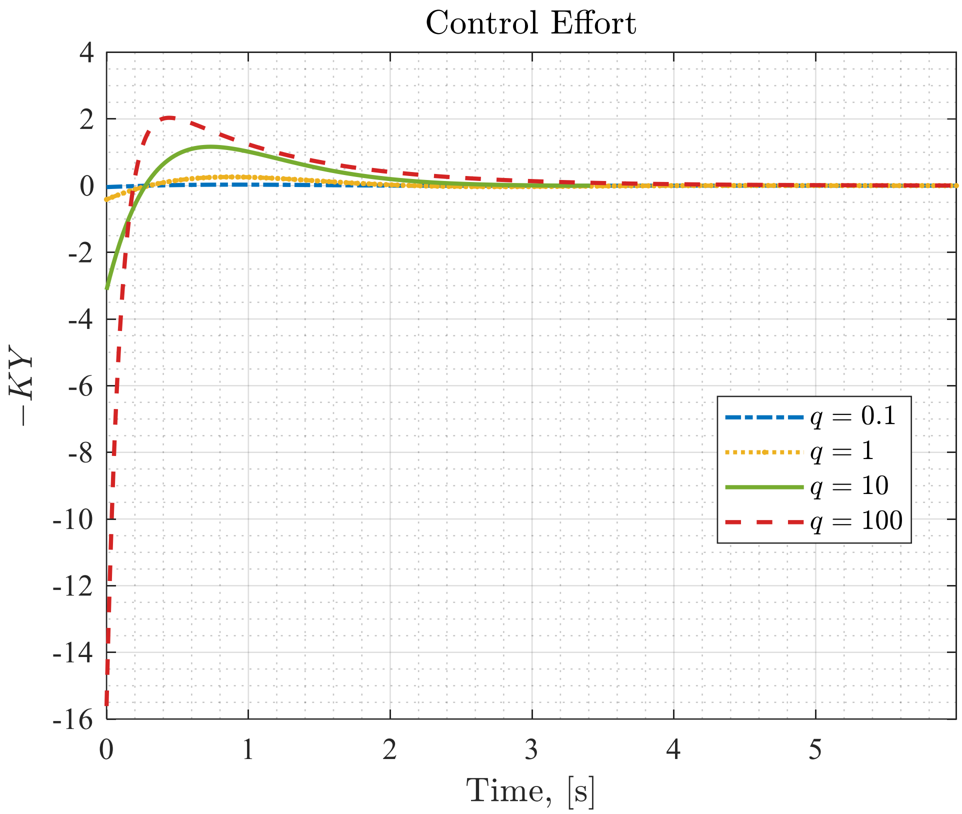
# Solution

a) Increasing $q$ value, results in increasing the control effort, as placing the poles far from the origin.



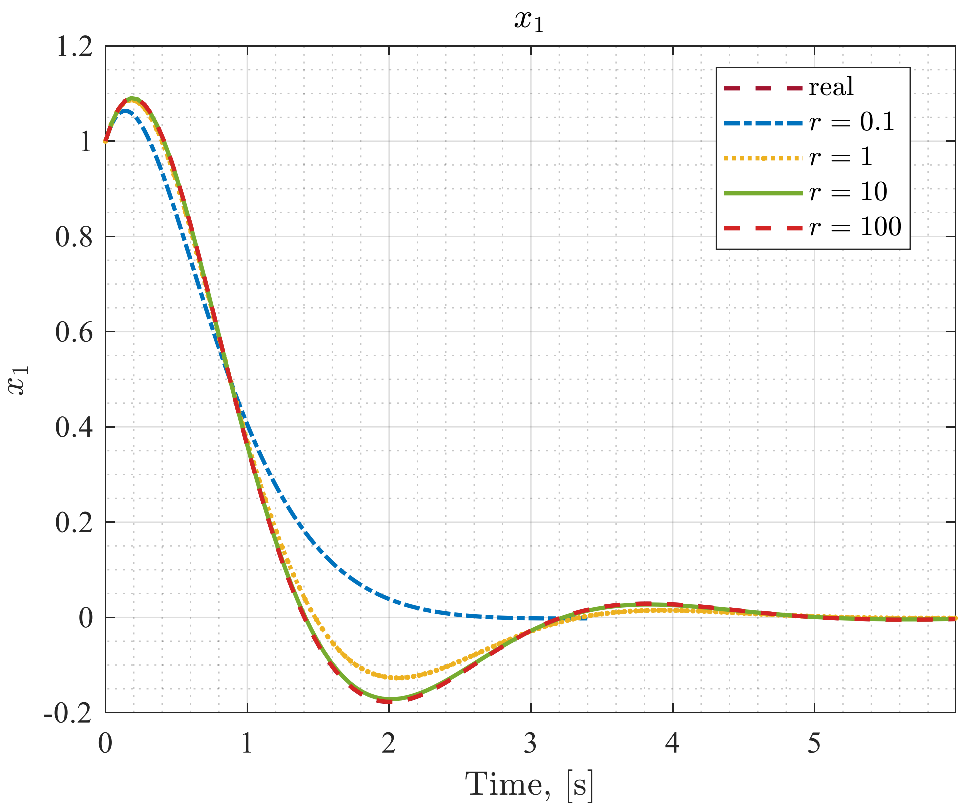


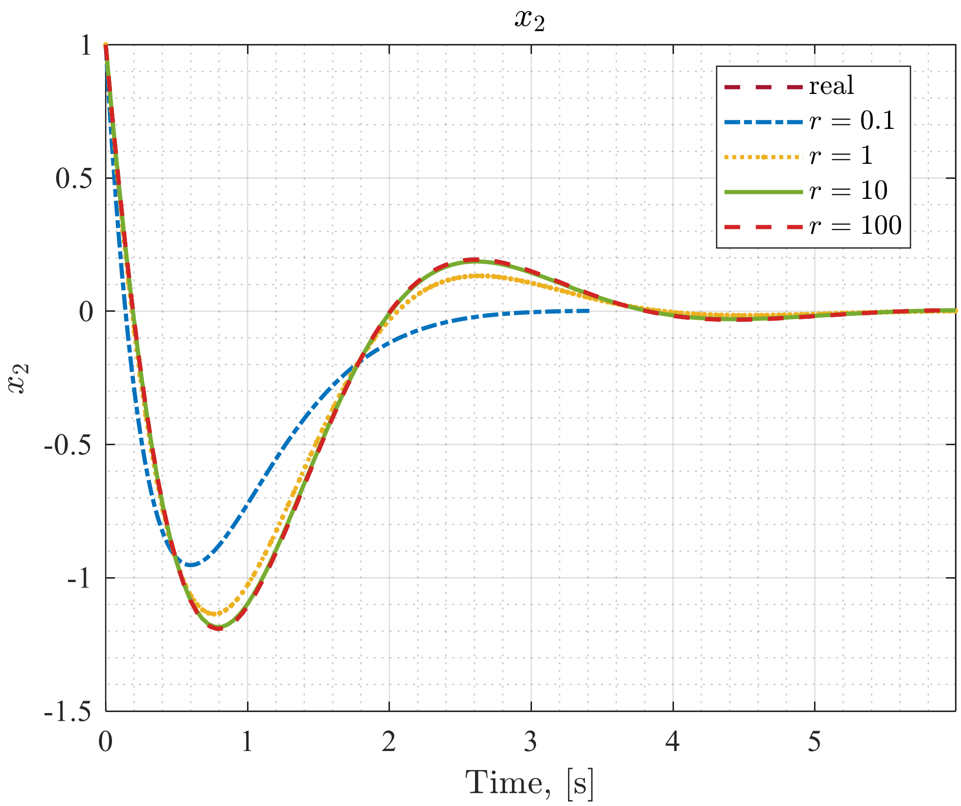
$x_2$
real
$q = 0.1$
$q = 1$
$q = 10$
$q = 100$
$x_2$
Time, [s]

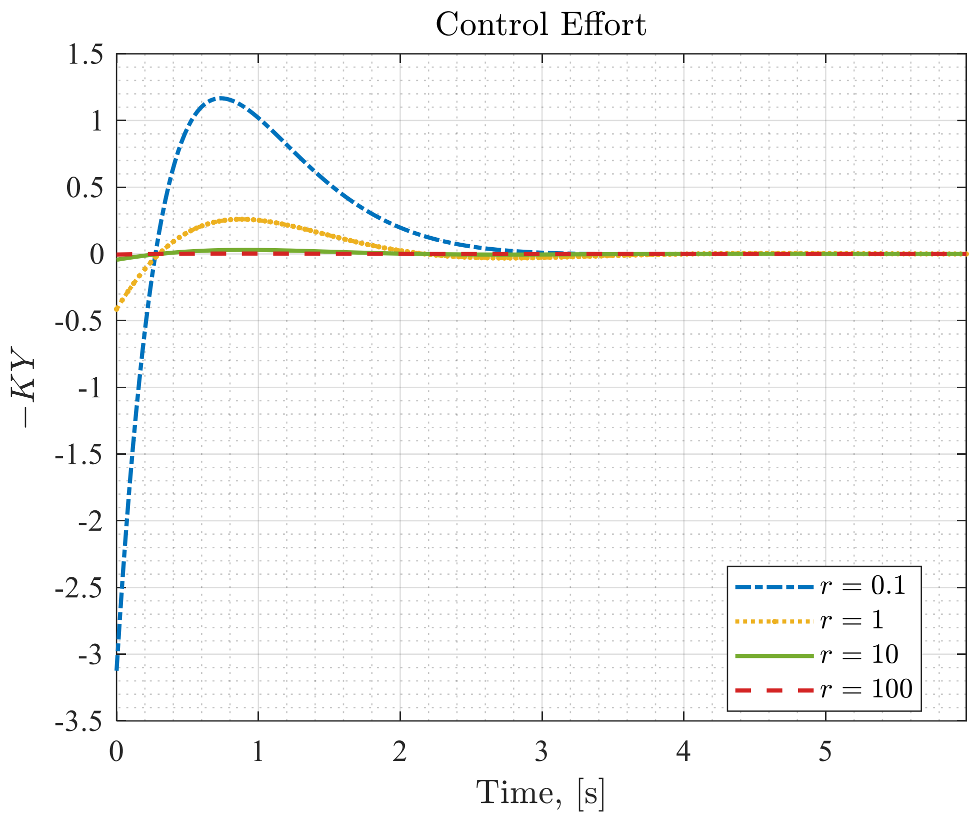



Control Effort
$q = 0.1$
$q = 1$
$q = 10$
$q = 100$
$-KY$
Time, [s]

b) Increasing $r$ value, results in decreasing the control effort, as placing the poles near the origin.






c) For feedback control, usually an integral controller would be added by increasing the states of the system, consequently, an increase in $Q$, matrix is inevitable! $Q$ can be diagonally $1/\max(states)$.

# Problem 6 Description

6. For the system shown, determine the feedback matrix K based on EESA, which assigns the closed-loop eigenvalues.
   a. at {-4,-5,-6} with two different choice of Eigen vectors.
   b. at {-1+j,-1-j,-2}

$$\dot{x} = \begin{bmatrix} -2 & 1 & 2 \\ -1 & -2 & 2 \\ -2 & 0 & 2 \end{bmatrix} x + B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u$$

Plot the time response of all three systems with zero input, and $x(0) = [1 \quad 0 \quad 1]^T$.

# Solution

$$6.\quad A=\begin{bmatrix}-2 & 1 & 2\\ -1 & -2 & 2\\ -2 & 0 & 2\end{bmatrix}\qquad B=\begin{bmatrix}0 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix}$$

$$eig(A)\longrightarrow (i),(-i),(-2) \leftarrow \lambda_i \quad \text{marginestable}$$

$$\mu_d=\{-4\ \ -5\ \ -6\}$$

$$\mu_1=-4\longrightarrow \left[(A-\mu_1 I)\quad B\right]\Rightarrow\begin{bmatrix}2 & 1 & 2 & 0 & 0\\ -1 & 2 & 2 & 0 & 1\\ -2 & 0 & 6 & 1 & 0\end{bmatrix}\begin{Bmatrix}a\\ b\\ c\\ d\\ e\end{Bmatrix}=\phi$$

$$\mu_2=-5\Rightarrow \left[(A-\mu_2 I)\quad B\right]\Rightarrow\begin{bmatrix}3 & 1 & 2 & 0 & 0\\ -1 & 3 & 2 & 0 & 1\\ -2 & 0 & 7 & 1 & 0\end{bmatrix}\begin{Bmatrix}a\\ b\\ c\\ d\\ e\end{Bmatrix}=\phi$$

$$\mu_3=-6\Rightarrow \left[(A-\mu_3 I)\quad B\right]\Rightarrow\begin{bmatrix}4 & 1 & 2 & 0 & 0\\ -1 & 4 & 2 & 0 & 1\\ -2 & 0 & 8 & 1 & 0\end{bmatrix}\begin{Bmatrix}a\\ b\\ c\\ d\\ e\end{Bmatrix}=\phi$$

$$\rightarrow 1\rightarrow\begin{cases}2a+b+2c=0\\ -a+2b+2c+e=0\\ -2a+6c+d=0\end{cases}\rightarrow S_1=\left\langle\begin{Bmatrix}1\\ 0\\ -1\\ 8\\ 3\end{Bmatrix}\begin{Bmatrix}0\\ 1\\ -1/2\\ 3\\ -1\end{Bmatrix}\right\rangle$$

$$\rightarrow 2\rightarrow\begin{cases}3a+b+2c=0\\ -a+3b+2c+e=0\\ -2a+7c+d=0\end{cases}\rightarrow S_2=\left\langle\begin{Bmatrix}1\\ 0\\ -3/2\\ 12.5\\ 4\end{Bmatrix},\begin{Bmatrix}0\\ 1\\ -1/2\\ 7/2\\ -2\end{Bmatrix}\right\rangle$$

$$\rightarrow 3\rightarrow\begin{cases}4a+b+2c=0\\ -a+4b+2c+e=0\\ -2a+8c+d=0\end{cases}\rightarrow S_3=\left\langle\begin{Bmatrix}1\\ 0\\ -2\\ 18\\ 5\end{Bmatrix}\begin{Bmatrix}0\\ 1\\ -0.5\\ 4\\ 5\end{Bmatrix}\right\rangle$$

$$\Rightarrow\left[A-\mu_i I\quad B\right]\begin{bmatrix}v^{(i)}\\ q^{(i)}=-Kv^{(i)}\end{bmatrix}\rightarrow K=-\left[q^{(1)}\ q^{(2)}\ q^{(3)}\right]\left[v^{(1)}\ v^{(2)}\ v^{(3)}\right]^{-1}$$

$$\rightarrow K=-\begin{bmatrix}3 & 12.5 & 18\\ -1 & 4 & 5\end{bmatrix}\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 0\\ 0.5 & -1.5 & -2\end{bmatrix}^{-1}=\begin{bmatrix}4 & 2.5 & 11\\ -1 & 2 & 2\end{bmatrix}$$

سوال 6 $\to$ b) $\to \mu_1=-1+j$

$$\to [A-\mu_1 I \;\; B]=\begin{bmatrix}-1-j & 1 & 2 & 0 & 0\\ -1 & -1-j & 2 & 0 & 1\\ -2 & 0 & 3-j & 1 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\\ e\end{bmatrix}=\emptyset$$

$$\to \begin{cases}(-1-j)a+b+2c=0\\ -a+(-1-j)b+2c+e=0\\ -2a+(3-j)c+d=0\end{cases}$$

$$S_1=\left\langle \begin{bmatrix}1\\ 0\\ 0.5+0.5j\\ -3-4j\\ -j\end{bmatrix}, \begin{bmatrix}0\\ 1\\ -0.5\\ 3-j\\ -j\end{bmatrix}\right\rangle$$ برابر $S_2$

$$\to \mu_3=-2 \to [A-\mu_3 I \;\; B]=$$

$$\begin{bmatrix}0 & 1 & 2 & 0 & 0\\ -1 & 0 & 2 & 0 & 1\\ -2 & 0 & 4 & 1 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\\ e\end{bmatrix}=\emptyset$$

$$\Rightarrow \begin{cases}b+2c=0\\ -a+2c+e=0\\ -2a+4c+d=0\end{cases} \to S_3=\left\langle \begin{bmatrix}1\\ 0\\ 0\\ 2\\ 1\end{bmatrix}, \begin{bmatrix}0\\ 1\\ -0.5\\ 2\\ 1\end{bmatrix}\right\rangle$$

$$\to K=-\left[q^{(1)}\; q^{(2)}\; q^{(3)}\right]\left[v^{(1)}\; v^{(2)}\; v^{(3)}\right]^{-1}$$

اعداد مختلط

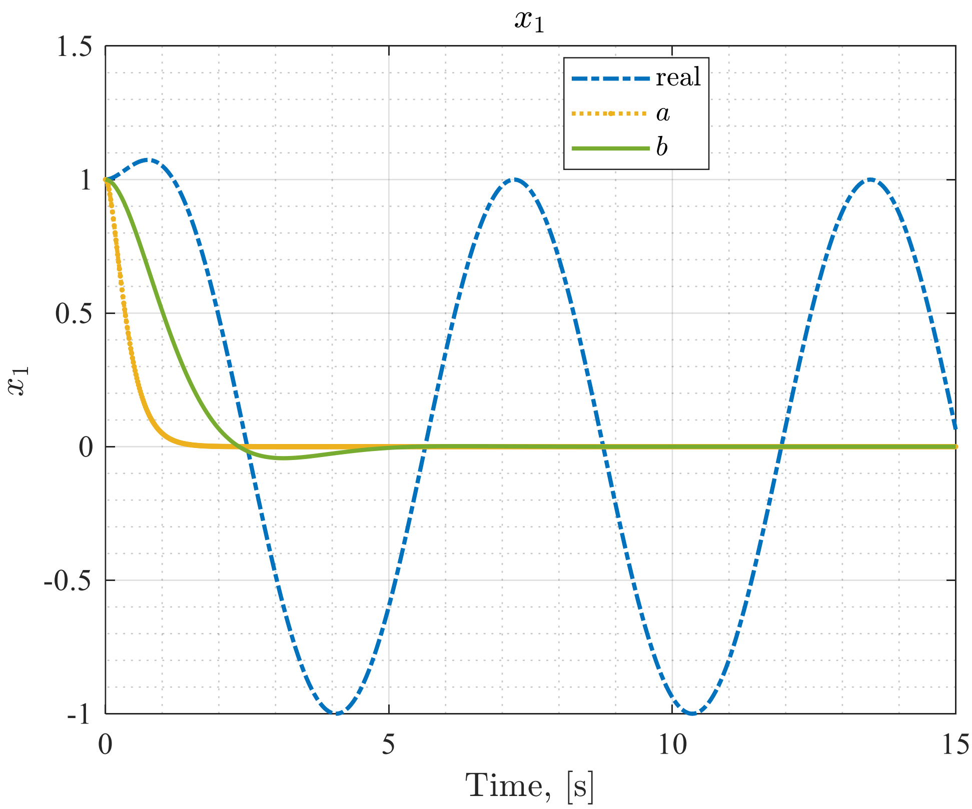
$$\to K=-\left[q^{(1)}\;\; Real(q^{(2)})\;\; Im(q^{(2)})\right]\left[v^{(1)}\;\; Real(v^{(2)})\;\; Im(v^{(2)})\right]^{-1}$$

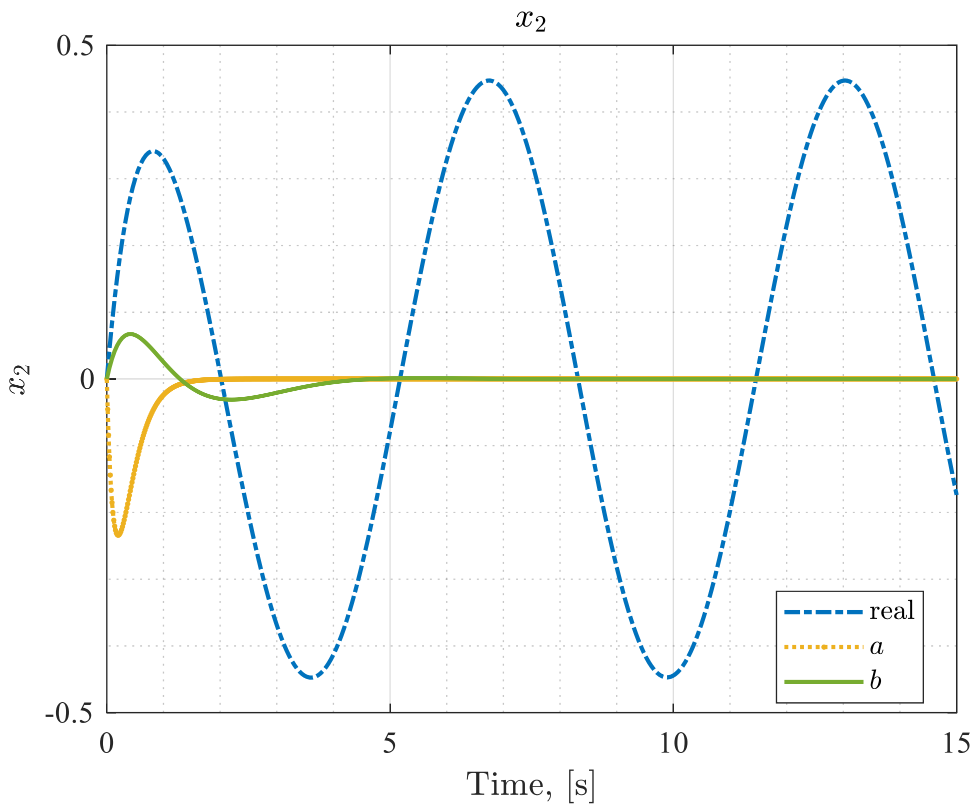
$$K=-\begin{bmatrix}2 & -3 & -4\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ -0.5 & 0.5 & 0.5\end{bmatrix}^{-1}=\begin{bmatrix}-1 & 2 & 8\\ -1 & 0 & 2\end{bmatrix}$$

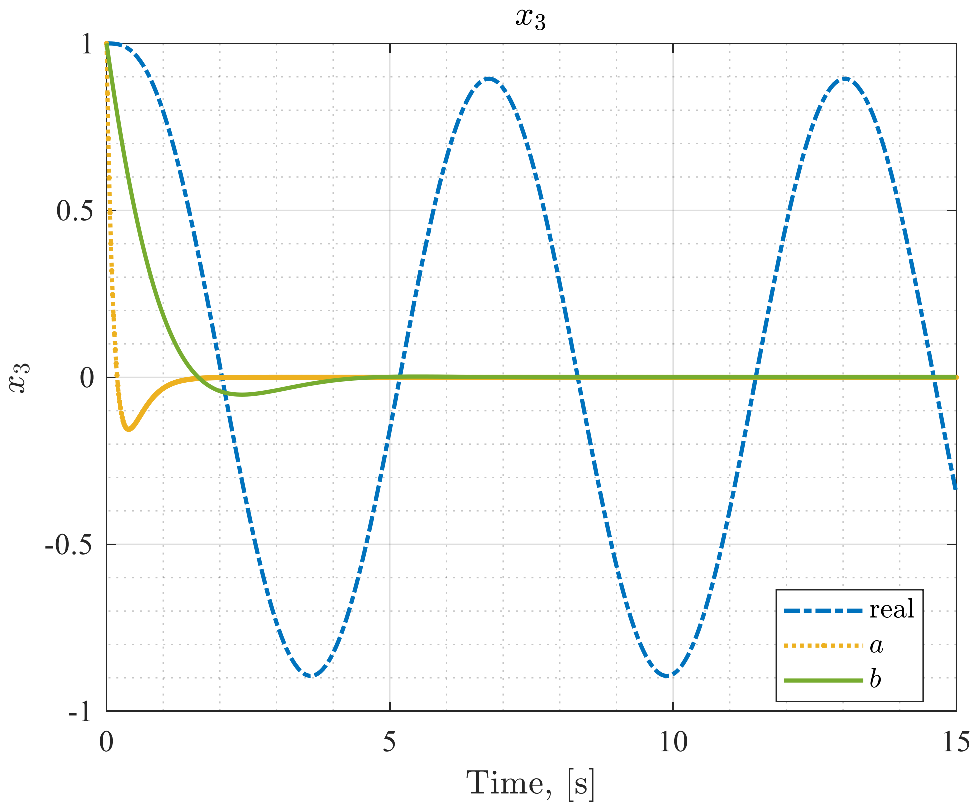
با دستور null متلب نیز بررسی شد، که جوابها متفاوت بود :_)

```
clc
clear
A=[-2 1 2;-1 -2 2;-2 0 2];
B=[0 0;0 1;1 0];
m1=[-4 -5 -6].';
A1=[A-m1(1)*eye(3) B];
A1=null(A1)
q1=real(A1(4:end,1))
v1=real(A1(1:3,1))
A2=[A-m1(2)*eye(3) B];
A2=null(A2)
q2=real(A2(4:end,2))
v2=real(A2(1:3,2))
A3=[A-m1(3)*eye(3) B];
A3=null(A3)
q3=real(A3(4:end,1))
v3=real(A3(1:3,1))
```

```
k=-[q1 q2 q3]*[v1 v2 v3]^-1


place(A,B,m1)

%% 2
A=[-2 1 2;-1 -2 2;-2 0 2];
B=[0 0;0 1;1 0];
m1=[-1+1i;-1-1i;-2].';
A1=[A-m1(1)*eye(3) B];
A1=null(A1)
q1=real(A1(4:end,1))
v1=real(A1(1:3,1))
A2=[A-m1(2)*eye(3) B];
A2=null(A2)
q2=imag(A1(4:end,1))
v2=imag(A1(1:3,1))
A3=[A-m1(3)*eye(3) B];
A3=null(A3)
q3=real(A3(4:end,2))
v3=real(A3(1:3,2))

k=-[q1 q2 q3]*inv([v1 v2 v3])

place(A,B,m1)
```

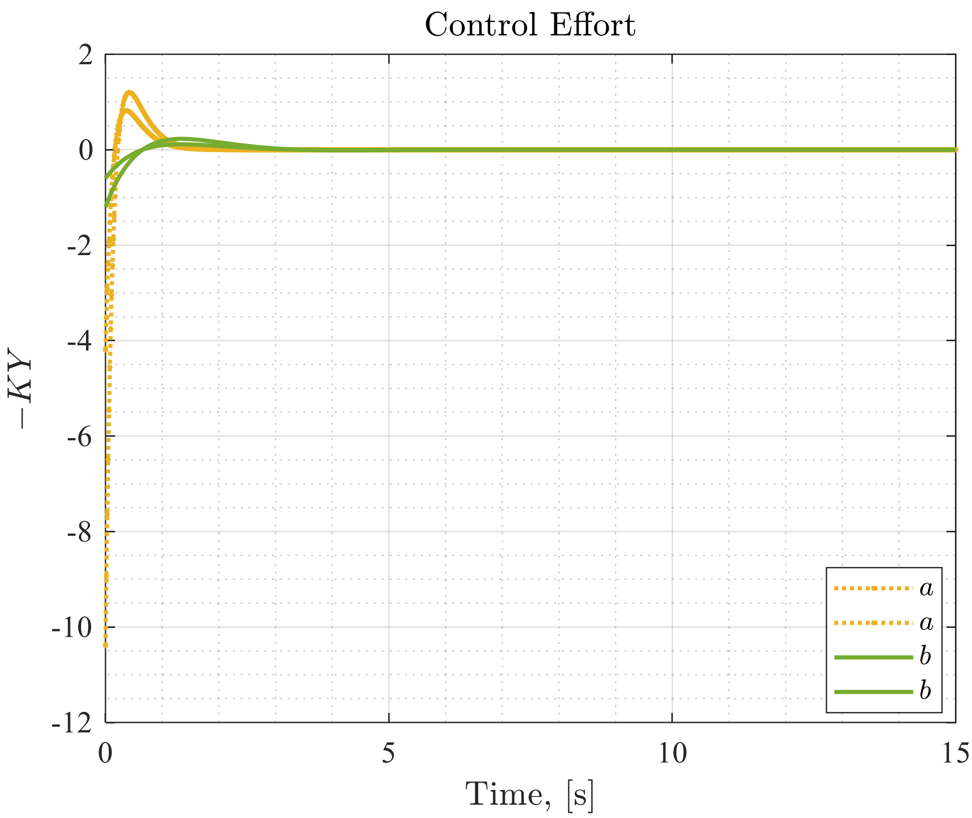



$x_2$
0.5
0
-0.5
$x_2$
0
5
10
15
Time, [s]
real
$a$
$b$


$x_3$
1
0.5
0
-0.5
-1
$x_3$
0
5
10
15
Time, [s]
real
$a$
$b$

در حالت **a**، که قطبها دورتر از مرکز هستند، جواب زودتر به حالت پایدار خودش بازمیگردد، زیرا تلاش کنترلی آن، همانطور که نشان داده شده است، بیشتر خواهد بود.

در حالت **real** به دلیل وجود قطب در روی محور موهومی، جواب به صورت **marginally stable** هست.




به دلیل وجود دو سطر (دو ضریب کنترلی)، دو مقدار برای هر قسمت محاسبه شده است.