STAT PHYS SCIENCE Midterm Cheat Sheet

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1 Measure of Central Tendency

1.1 Mean

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} \tag{1}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{2}$$

1.2 Median

$$M = x_{(\frac{N+1}{2})} \tag{3}$$

2 Quantiles

2.1 Quartiles

$$Q_i = x_{(i(\frac{N+1}{4}))} \tag{4}$$

2.2 Deciles

$$D_i = x_{(i(\frac{N+1}{10}))} \tag{5}$$

2.3 Percentiles

$$P_i = x_{(i(\frac{N+1}{100}))} \tag{6}$$

3 Measurement of Variation or Dispersion

3.1 Range

$$R = x_{max} - x_{min} \tag{7}$$

3.2 Average Deviation (A.D.)

$$A.D. = \frac{\sum_{i=1}^{n} |x_i - \mu|}{n} \tag{8}$$

3.3 Standard Deviation (S.D.)

3.3.1 Standard Deviation (Population)

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}} = \sqrt{\frac{\sum_{i=1}^{N} x_i - N\mu^2}{N}} \tag{9}$$

3.3.2 Standard Deviation (Sample)

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} x_i - N\mu^2}{n-1}} \tag{10}$$

3.4 Quatile Deviation

$$Q.D. = \frac{Q_3 - Q_1}{2} \tag{11}$$

3.5 Skewness

3.5.1 Skewness (Population)

$$S_k = \sum_{i=1}^{N} \frac{[x_i - \mu]^3}{\sigma^3 N} \tag{12}$$

if $S_k = 0$ the data is normal
else if $S_k > 0$ the data is skwed right
else if $S_k < 0$ the data is skwed left

3.5.2 Skewness (Sample)

$$s_k = \sum_{i=1}^{n} \frac{[x_i - \bar{x}]^3}{s^3 n} \tag{13}$$

if $-1 \leq s_k \leq 1$ the data is normal
else if $s_k > 1$ the data is skwed right
else if $s_k < -1$ the data is skwed left

3.6 Relative Kurtosis

A measure of the peakedness of a distribution

3.6.1 Relative Kurtosis (Population)

$$K = \sum_{i=1}^{N} \frac{[x_i - \mu]^4}{\sigma^4 N} - 3 \tag{14}$$

if $K = 0$ the data is normal
else if $K > 0$ the data is higher than normal
else if $K < 0$ the data is lower than normal

3.6.2 Relative Kurtosis (Sample)

$$k = \frac{n^2(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^{n} \frac{[x_i - x]^4}{s^4 n} - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{15}$$

if $-1 \leq k \leq 1$ the data is normal
else if $k > 1$ the data is higher than normal
else if $k < -1$ the data is lower than normal

4 Coefficient of Variation

$$C.V. = \frac{\sigma}{\mu} \times 100 \tag{16}$$

$$C.V. = \frac{s}{\bar{x}} \times 100 \tag{17}$$

5 Outlier by Box Plot

5.1 Inter Quartile Range

$$IQR = Q_3 - Q_1 \tag{18}$$

5.2 Whisker

Whisker is the highest or lowest value not reaching the Inner Fence.

5.3 Fence

Outlier is the value lower or higher than the Outer Fence.
Suspected Outlier is the value between the Inner Fence and the Outer Fence.

$$Outer\ Fence = Q_1 - 3(IQR) \tag{19}$$

$$Inner\ Fence = Q_1 - 1.5(IQR) \tag{20}$$

$$Inner\ Fence = Q_3 + 1.5(IQR) \tag{21}$$

$$Outer\ Fence = Q_3 + 3(IQR) \tag{22}$$

6 Probability

6.1 Permutation

$${}^nP_r = \frac{n!}{(n-r)!} \tag{23}$$

6.2 Combination

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \tag{24}$$

6.3 Properties of Probability

- $0 \leq P(A) \leq 1$
- $P(\emptyset) = 0$
- $P(S) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A) = 1 - P(A)'$

6.4 Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{25}$$

6.5 Joint Probability

$$P(A \cap B) = P(A)P(B|A) \tag{26}$$

$$P(A \cap B) = P(B)P(A|B) \tag{27}$$

$$P(A \cap B \cap C) = P(A \cap B)P(C|(A \cap B)) \tag{28}$$

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|(A \cap B)) \tag{29}$$

7 Probability Distribution

7.1 Probability Mass Function

$$f(x) \geq 0;\ for\ all\ x \tag{30}$$

$$\sum_{all x} f(x) = 1 \tag{31}$$

$$F(x) = P(X \leq x) = \sum_{all\ \ x \in [-\infty, x]} f(x) \tag{32}$$

$$\mu = E(X) = \sum_{all\ \ x} x f(x) \tag{33}$$

$$\sigma^2 = V(X) = \sum_{all\ \ x} x^2 f(x) - \mu^2 \tag{34}$$

7.2 Probability Density Function

$$f(x) \geq 0;\ for\ all\ x \tag{35}$$

$$\int_{-\infty}^{\infty} f(x) = 1 \tag{36}$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)dy \tag{37}$$

$$\mu = E(X) = \int_{-\infty} \infty x f(x) \tag{38}$$

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2 \tag{39}$$

8 Probability Function

8.1 Uniform Distribution

$$f(x) = \frac{1}{k} \tag{40}$$

$$\mu = \frac{1}{k}\sum_{i=1}^{k} x_i \quad \sigma^2 = \frac{1}{k}\sum_{i=1}^{k}(x_i - \mu)^2 \tag{41}$$

8.2 Binomial Distribution

$$f(x) = {}^{n}C_x p^x (1-p)^{1-x}; x = 0, 1, 2, ..., n \tag{42}$$

$$\mu = np \quad \sigma^2 = np(1-p) \tag{43}$$

8.3 Negative Binomial Distribution

$$f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r \tag{44}$$

$$x = r, r+1, r+2, ...$$

$$\mu = E(X) = \frac{r}{p} \tag{45}$$

$$\sigma^2 = V(X) = \frac{r(1-p)}{p^2} \tag{46}$$

Geometric Distribution ($r = 1$)

$$f(x) = p(1-p)^{x-1}; x = 1, 2, ... \tag{47}$$

$$\mu = E(X) = \frac{1}{p} \tag{48}$$

$$\sigma^2 = V(X) = \frac{(1-p)}{p^2} \tag{49}$$

8.4 Poisson Distribution

$$f(x) = \frac{e^{-\mu}\mu^x}{x!} \tag{50}$$

$$\sigma^2 = \mu \tag{51}$$

8.5 Hypergeometric Distribution

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \tag{52}$$

$$\mu = E(X) = np \tag{53}$$

$$\sigma^2 = V(X) = np(1-p)\frac{N-n}{N-1} \tag{54}$$