Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

If A and B are independent $P(A \cap B) = P(A)P(B)$

Expected Value

$$E[x] = \int_{-\infty}^{\infty} xp(x)\,dx \qquad E[g(x)] = \int_{-\infty}^{\infty} g(x)p(x)\,dx$$

$E[a] = a; a$ is a constant $\qquad E[aX+b] = aE[x]+b$

$E[X+Y] = E[X] + E[y]$

Variance

$$Var[x] = E[(x - E[x])^2] = \sigma^2 = \int_{-\infty}^{\infty} (x - E[x])^2 p(x)\,dx$$

$E((x-E[x])^2) = E[x^2] - (E[x])^2$

$Var[a] = 0; a$ is a constant $\qquad Var[aX+b] = a^2 Var[X]$

Covariance

$$\begin{aligned} cov(X_1, X_2) &= E[(X_1 - m_1)(X_2 - m_2)] \\ &= E[(X_1)(X_2)] - m_1 m_2 \end{aligned}$$

Correlation

$$\rho = \frac{cov(X_1, X_2)}{\sqrt{V(X_1)V(X_2)}}$$

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Algebra of Random Variable

$Z = Y + X$

$P_Z(Z_0) = P_Y(y) * P_X(x)$

Probability Distribution

Cumulative Distribution Function (CDF)

Normal

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad E[X] = \mu \qquad Var[X] = \sigma^2$$

Exponential

$$\lambda e^{-\lambda x} \qquad E[X] = \frac{1}{\lambda} \qquad Var[X] = \frac{1}{\lambda^2}$$

Uniform

$$\begin{cases} \frac{1}{b-a} & \text{for } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{2}(a+b) \qquad Var[X] = \frac{1}{12}(b-a)^2$$

Bernoulli

Success (p), Fail $(1-p)$

$$\begin{cases} q = 1-p & \text{if } k = 0 \\ p & \text{if } k = 1 \end{cases} \qquad E[X] = p \quad Var[X] = p(1-p)$$

Binomial

n Bernoulli trials

$$\binom{n}{k} p^k (1-p)^{n-k} \qquad E[X] = np \qquad Var[X] = np(1-p)$$

Poisson

$$\frac{\lambda^k e^{-\lambda}}{k!} \qquad E[X] = \lambda \qquad Var[X] = \lambda$$

Pareto

$$\frac{\alpha x_m^a}{x^{\alpha+1}} \qquad E[x] = \begin{cases} \infty & \text{for } \alpha \le 1 \\ \frac{\alpha x_m}{\alpha - 1} & \text{for } \alpha > 1 \end{cases}$$

$$Var[X] = \begin{cases} \infty & \text{for } \alpha \le 1 \\ \frac{x_m^2 \alpha}{(\alpha-1)^2(\alpha-2)} & \text{for } \alpha > 1 \end{cases}$$

MLE

To find the MLE given data

1. The likelihood function $P(data|\lambda)$, λ is the parameter
2. $\frac{d}{d\lambda}$(log likelihood) $= 0$, Find λ

LLN & CLT

LLN: As n grows,the probability that X_n is close to $\mu \rightarrow 1$.
CLT: As n grows,the distribution of X_n converges to the normal distribution $N\ (\mu, \sigma^2/n)$.

Confidence Interval (Polling)

95% Confidence Interval $\bar{x} \pm \frac{1}{\sqrt{n}}$

Null Hypothesis Significance Testing

Errors

		True State of Nature	
		H_0	H_A
Our Decision	Reject H_0	Type-I Error	correct decision
	Reject H_0	correct decision	Type-II Error

P-value

We usually do testing by specifying significance level and do testing using p-values. **If p-value is less than the significance level we reject H_0**
P-value - Probability assuming Null of seeing data at least as extreme as the experiment data.

Problems with P-value

- **P-hacking** Do experiment multiple times until the results is what we want
- **Base rate fallacy** Low base rate $\rightarrow$More chance for false positive
- **Low power experiments** If the test has low power (underpowered study), no significant is likely to be due to not enough samples t detect small differences.

Significance level and power

$$\begin{aligned}\text{Significance level} &= P(reject H_0|H_0)\\ &= \text{probability we incorrectly reject } H_0\\ &= P(\text{type I error})\end{aligned}$$

$$\begin{aligned}\text{Power} &= \text{probability we correctly reject } H_0\\ &= P(reject H_0|H_A)\\ &= 1 - P(\text{type II error})\end{aligned}$$

One Sample z-test

Use when the **Variance** (σ^2) of the data is known

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

One Sample t-test

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{where } s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad df = n - 1$$

Two Sample z-test

$$z = \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Two Sample t-test with Equal Variance

Assume that the data have $\sigma_1 = \sigma_2$

$$t = \frac{\bar{x} - \bar{y}}{s_p} \qquad s_p^2 = \frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}\left(\frac{1}{n} + \frac{1}{m}\right)$$

Two Sample t-test with Unequal Variance

Assume that the data have $\sigma_1 \neq \sigma_2$

$$t = \frac{\bar{x} - \bar{y} - \mu_0}{s_P} \qquad s_P^2 = \frac{s_x^2}{n} + \frac{s_y^2}{n}$$

$$df = \frac{(s_x^2/n + s_y^2/m)^2}{(s_x^2/n)^2/(n-1) + (s_y^2/m)^2/(m-1)}$$

Paired two-sample t-test

$$t = \frac{\bar{w} - \mu_0}{s/\sqrt{n}} \quad w_i = x_i - y_i \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(w_i - \bar{w})^2$$

One-way ANOVA (F-tesst for equal means)

Test if the population means from n group are all the same
Data for each group is an independent normal sample drawn from distributions with (possibly) different means but the same variance.

$$w = \frac{MS_B}{MS_w} \qquad \bar{x}_i = \text{ mean of group i} \qquad \bar{x} = \text{ mean of all data}$$
$$s_i^2 = \frac{1}{m-1}\sum_{j=1} m(x_{i,j} - \bar{x}_i)^2$$

$$\begin{aligned}MS_B &= \text{ between group variance}\\ &= m \times \text{sample variance of group means}\\ &= \frac{m}{n-1}\sum_{i=1}^{n}(\bar{x}_i - \bar{x})^2\\ MS_w &= \text{ average within group variance}\\ &= m \times \text{sample means } s_1^2, \ldots, s_n^2\\ &= \frac{s_1^2 + s_2^2 + \ldots + s_n^2}{n}\end{aligned}$$

A/B Testing

Steps

1. Define relevant metrics
2. Split samples into comparable groups
3. Choose statistical tests and validate their assumptions
4. Decide on stopping criteria
5. Run and monitor the experiment
6. Analyze results and suggest actions

Possible event probabilities

1. # checkout events / # hits - double-count on page refreshes
2. # checkout events/ # sessions - double-count on inactive visits; good to see which products get bought within fewer visits
3. # checkout events / # cookies on product page - "users" as denominator; includes both logins and non-logins; different browsers/devices double-counts
4. # payment events / # cookies on product page - captures successful purchases
5. # payment events / # user ids - non-logins count as one user id
6. # payment events / # people - who are people?

Attribution Period

Conversion rate of August

conversions within August / number of users that visited in August

Conversion rate of August cohort

conversions within X days / number of users that visited in August

What Frequentist Hypothesis Tests Are NOT Saying

1. The p-value is not the probability that the null hypothesis is true, or the probability that the alternative hypothesis is false.
2. The p-value is not the probability that the observed effects were produced by random chance alone.
3. The 0.05 significance level is merely a convention.
4. The p-value does not indicate the size or importance of the observed effect.
5. The p-value is not the observed false positive rate; that depends on the prevalence of the data.

MDE

Sample Size

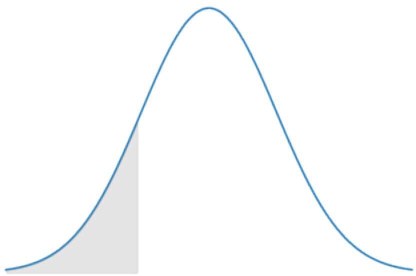
m is the split rate (50:50 = 1, 80:20 = 4)

$$n = \frac{m+1}{m}\left(\frac{(Z_\alpha + Z_\beta)\sigma}{MDE}\right)^2$$

When not to do an A/B test

- Things that cannot be summarized into one or a few metrics
- Totally new things
- Delayed results
- One-off events
- Cannot split group independently

Z-score table (Area to the left of the z score)



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414