

## Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

If A and B are independent  $P(A \cap B) = P(A)P(B)$

## Expected Value

$$E[x] = \int_{-\infty}^{\infty} xp(x) dx \quad E[g(x)] = \int_{-\infty}^{\infty} g(x)p(x) dx$$

## Variance

$$Var[x] = E[(x - E[x])^2] = \sigma^2 = \int_{-\infty}^{\infty} (x - E[x])^2 p(x) dx$$

$$E((x - E[x])^2) = E[x^2] - (E[x])^2$$

## Covariance

$$\begin{aligned} cov(X_1, X_2) &= E[(X_1 - m_1)(X_2 - m_2)] \\ &= E[(X_1)(X_2)] - m_1 m_2 \end{aligned}$$

## Correlation

$$\rho = \frac{cov(X_1, X_2)}{\sqrt{V(X_1)V(X_2)}}$$

## Probability Distribution

### Normal

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad E[X] = \mu \quad Var[X] = \sigma^2$$

### Exponential

$$\lambda e^{-\lambda x} \quad E[X] = \frac{1}{\lambda} \quad Var[X] = \frac{1}{\lambda^2}$$

### Uniform

$$\begin{cases} \frac{1}{b-a} & \text{for } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{2}(a+b) \quad Var[X] = \frac{1}{12}(b-a)^2$$

### Bernoulli

$$\begin{cases} q = 1-p & \text{if } k=0 \\ p & \text{if } k=1 \end{cases} \quad E[X] = p \quad Var[X] = p(1-p)$$

### Binomial

$$\binom{n}{k} p^k (1-p)^{n-k} \quad E[X] = np \quad Var[X] = np(1-p)$$

## Poisson

$$\frac{\lambda^k e^{-\lambda}}{k!} \quad E[X] = \lambda \quad Var[X] = \lambda$$

## Pareto

$$\frac{\alpha x_m^\alpha}{x^{\alpha+1}} \quad E[x] = \begin{cases} \infty & \text{for } \alpha \leq 1 \\ \frac{\alpha x_m}{\alpha-1} & \text{for } \alpha > 1 \end{cases}$$
$$Var[X] = \begin{cases} \infty & \text{for } \alpha \leq 1 \\ \frac{x_m^2 \alpha}{(\alpha-1)^2(\alpha-2)} & \text{for } \alpha > 1 \end{cases}$$

## MLE

To find the MLE given data

1. The likelihood function  $P(data|\lambda)$ ,  $\lambda$  is the parameter
2.  $\frac{d}{d\lambda}(\log \text{likelihood}) = 0$ , Find  $\lambda$

## Hypothesis Testing

### Errors

	True State of Nature	
	$H_0$	$H_A$
Our Decision	Reject $H_0$	Type-I Error
	Accept $H_0$	correct decision
	Reject $H_0$	correct decision
	Accept $H_0$	Type-II Error

### One Sample z-test

Use when the **Variance** ( $\sigma^2$ ) of the data is known

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

### Two Sample z-test

### One Sample t-test

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{where } s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad df = n-1$$

### Two Sample t-test with Equal Variance

Assume that the data have  $\sigma_1 = \sigma_2$

### Two Sample t-test with Unequal Variance

Assume that the data have  $\sigma_1 \neq \sigma_2$

## A/B Testing

### MDE

### Sample Size

$m$  is the split rate (50:50 = 1, 80:20 = 4)

$$n = \frac{m+1}{m} \left( \frac{(Z_\alpha + Z_\beta)\sigma}{MDE} \right)^2$$

## When not to do an A/B test

- Things that cannot be summarized into one or a few metrics
- Totally new things
- Delayed results
- One-off events
- Cannot split group independently