Exercise session: Filtering

This session is about linear filters using both spatial convolution and frequency multiplication. You will learn which convolution masks correspond to which MTF (Modulation Transfer Function) and what the effects are. In addition non-linear filtering will be investigated.

Most exercises begin with a theoretical part which does not require Matlab. However, you can verify your results by using Matlab. Most of the programming has already been done for you. You will only have to fill in some blanks, change small parts of code, set some parameters etc. You can verify the purpose of each command in Matlab using -help-

All necessary images and Matlab code can be found at: Toledo.

1 Noise filtering

Suppose you get an image that contains lot of noise. Smoothing the image is one option to reduce the noise. We're going to apply two filters (separately) in the frequency domain:

1. a block filter (rectangular disk MTF)
2. a Gaussian filter (Gaussian MTF)

Which do you think will have the nicest results? What is the undesired side-effect of the other one?
You can see this effect using the `ex1.m` Matlab script.

2 Edge detection

There exist many methods for edge detection. In the following we will consider a simple linear filter.

- Starting from the Gaussian filter from exercise 1, how could you turn it into an edge-filter? (Edges are regions with high-frequency content, try with $sigma_f = 10$, remember, Gaussian filter did the opposite we are trying to do here)

- How does the filter's spatial convolution mask look like?
- What is its response to an abrupt edge?

Change the `ex1.m` script code at the `TODO` line to allow it to detect edges in the image. Look at the response near Lena's shoulder. Is this what you expected?

The noise inherent in the image affects the edge detection.

- Explain why.
- Although it's intrinsic to the problem, you can reduce the effects. How?

Incorporate the necessary changes in the `ex1.m` script and examine the effect.

3 Linearity and shift-invariance

Which of the following operators are linear and shift-invariant?

$$x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} \tag{1}$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2}$$

$$\left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 \tag{3}$$

Could you see this intuitively?
A short reminder:

$$\begin{aligned} O\{\cdot\}\text{is linear} \quad &\Longleftrightarrow \quad O\{af(x,y) + bg(x,y)\} = aO\{f(x,y)\} + bO\{g(x,y)\} \\ O\{\cdot\}\text{is shift} - \text{invariant} \quad &\Longleftrightarrow \quad O\{f(x-x_0, y-y_0)\} = O\{f(u,v)\}|_{u=x-x_0, v=y-y_0} \end{aligned}$$

4 Median filtering

Smoothing an image looses a lot of its edge information. Median filtering can overcome this problem to a certain extent, since it removes noise, while preserving edges.

- Is median filtering a linear operation?

- Will running a 3x3 median operation twice yield the same result as one 5x5 median operation? Give an example (1D).

You can see the effects of median filtering and compare the two cases of question 2 with the `ex2.m` script. What is the effect of increasing the number of runs?

5 Matched filters

Suppose the following convolution mask:

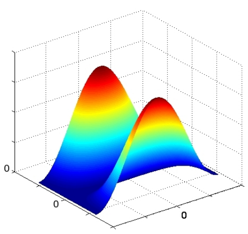
$$F = \begin{bmatrix} 1 & 2 & 0 & -2 & -1 \\ 2 & 4 & 0 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & -4 & 0 & 4 & 2 \\ -1 & -2 & 0 & 2 & 1 \end{bmatrix} \tag{4}$$

- Is this filter separable? Try to separate it as much as possible. Remember that the mask was created with the multiplication of several parts/small masks, (smoothing, vertical edges and horizontal edges).
- Looking at all the components, what is its MTF?
- What's the purpose of this filter?

You can verify your results with the `ex3.m` script. Use it on the `./images/skyscraper.jpg` image.

6 The other way around

Let's try it the other way around. Given a MTF, what is the corrsponding convolution mask?



$$H(u,v) \quad = \quad (2 + 2\cos(2\pi u)) \cdot (2 - 2\cos(2\pi v))$$

- What filter coefficients result in such a MTF spectrum?
- What would this filter do to an image?

Verify your results again with `ex3.m` script, but change the convolution mask to the one you calculated. Does the MTF correspond to the figure above?

7 Isotropy

Let's compare the following two operators:

$$\sqrt{\left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2} \tag{5}$$

$$\left|\frac{\partial}{\partial x}\right| + \left|\frac{\partial}{\partial y}\right| \tag{6}$$

- What are their correspondences?
- What are their differences?
- How would one implement such filters? Hint: *One possible implementation was proposed by Sobel:*

$$\frac{\partial}{\partial x} \quad \leftarrow \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial}{\partial y} \quad \leftarrow \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Implement the two operators one by one by changing the `ex4.m` script. Run them on isotropy.bmp, fractal.jpg, eiffel.jpg images. What difference do you see?