

Find the solutions of $\left(\frac{dy}{dx}\right)^2 + y = 0$.

■ SOLUTION: $\frac{dy}{dx} = \sqrt{-y} \Rightarrow \frac{dy}{\sqrt{-y}} = dx \Rightarrow 2\sqrt{-y} = x + c$

$$y = -\frac{1}{4} x^2$$

■ $i = \text{square root of } -1$

■ By Mathematica we have

In[78]:= **sol = DSolve[(y'[x])^2 + y[x] == 0, y[x], x]**

Out[78]=

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 - 2 i x c_1 + c_1^2) \right\}, \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 + 2 i x c_1 + c_1^2) \right\} \right\}$$

In[76]:= $\left\{ \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 - 2 i x c_1 + c_1^2) \right\}, \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 + 2 i x c_1 + c_1^2) \right\} \right\}$

Out[76]=

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 - 2 i x c_1 + c_1^2) \right\}, \left\{ y[x] \rightarrow \frac{1}{4} (-x^2 + 2 i x c_1 + c_1^2) \right\} \right\}$$

specificSol =

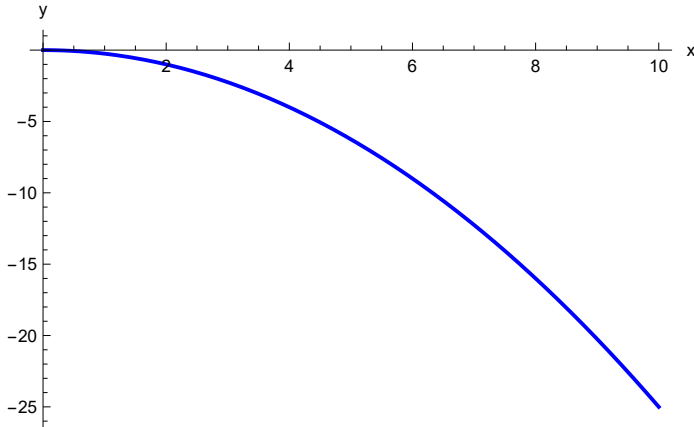
**sol /. C[1] → 0 (* Let the constant C
of integration be equal to 0 *)**

Out[77]=

$$\left\{ \left\{ y[x] \rightarrow -\frac{x^2}{4} \right\}, \left\{ y[x] \rightarrow -\frac{x^2}{4} \right\} \right\}$$

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In[83]:= Plot[-(x^2)/4, {x, 0, 10}, PlotRange -> All,  
AxesLabel -> {"x", "y"}, PlotStyle -> Blue]
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Out[83]=



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In[81]:= Table[{x, -(x^2)/4}, {x, 0, 10}] // N
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Out[81]=

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{ {0., 0.}, {1., -0.25}, {2., -1.}, {3., -2.25},  
  {4., -4.}, {5., -6.25}, {6., -9.}, {7., -12.25},  
  {8., -16.}, {9., -20.25}, {10., -25.} }
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In[82]:= Grid[Table[{x, -(x^2) / 4},
                    {x, 0, 10}], Frame → All]
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Out[82]=

0	0
1	$-\frac{1}{4}$
2	-1
3	$-\frac{9}{4}$
4	-4
5	$-\frac{25}{4}$
6	-9
7	$-\frac{49}{4}$
8	-16
9	$-\frac{81}{4}$
10	-25