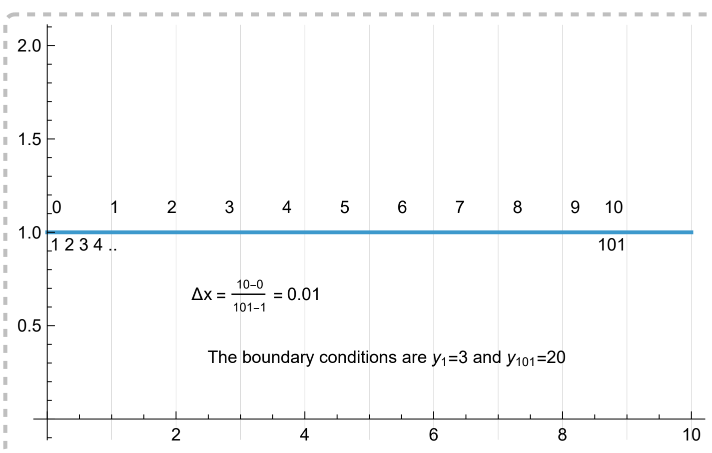
- This is the ordinary differential equation (ODE) to be solved.
- $\frac{dy^2}{dx^2} + 3\frac{dy}{dx} + y = 0$. $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} + \mathcal{O}(h^2)$ by Taylor's series approximation
- $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$ by Taylor's series approximation.
- $\frac{f(x+h)-2f(x)+f(x-h)}{h^2} + 3\frac{f(x+h)-f(x-h)}{2h} + f(x) = 0$
- Let f(x) = y_i, f(x+h) = y_{i+1}, and f(x-h) = y_{i-1}. So by substitution we have
- $\frac{y_{i+1}-2y_i+y_{i-1}}{h^2} + 3\frac{y_{i+1}-y_{i-1}}{2h} + y_i = 0$ or (by grouping like terms)
- $\left(1+\frac{3}{2}h\right)y_{i+1} + \left(-2+h^2\right)y_i + \left(1-\frac{3}{2}h\right)y_{i-1}\right) = 0$. For $h = 0.01$ we have
- $0.985\, y_{i-1} - 1.9999\, y_i + 1.015\, y_{i+1} = 0$
  - For i=2: $0.985\, y_1 - 1.9999\, y_2 + 1.015\, y_3 = 0$
  - For i =3: $0.985\, y_2 - 1.9999\, y_3 + 1.015\, y_4 = 0$

- If we continue this pattern up to y_{101} we have the the last equation
  - $0.985\, y_{99} - 1.9999\, y_{100} + 1.015\, y_{101} = 0$
- Our boundary conditions are: $y_1 = 3$ and $y_{101} = 20$. So our first and last equations are, respectively,
  - $0.985\,(3) - 1.9999\, y_2 + 1.015\, y_3 = 0$ and
    $0.985\, y_{99} - 1.9999\, y_{100} + 1.015\,(20) = 0$
-
- Our step size is $h = \frac{10-0}{101-1} = 0.01$
-
- Let us first generate all the 100 equations using Mathematica:

In[ ]:=

```
Plot[1, {x, 0, 10}, GridLines →
  {{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {0}},
 GridLinesStyle → Directive[Red, Dashed]]
```



In[ ]:=
```
(*Generate equations from 0.985y_0-1.9999y_1+
 1.015y_2 to 0.985y_99-1.9999y_100+1.015y_101*)
eqns = Table[0.985 Subscript[y, n] -
     1.9999 Subscript[y, n + 1] +
     1.015 Subscript[y, n + 2] == 0, {n, 0, 99}];

(*Display the equations*)
eqns
```

Out[ ]=

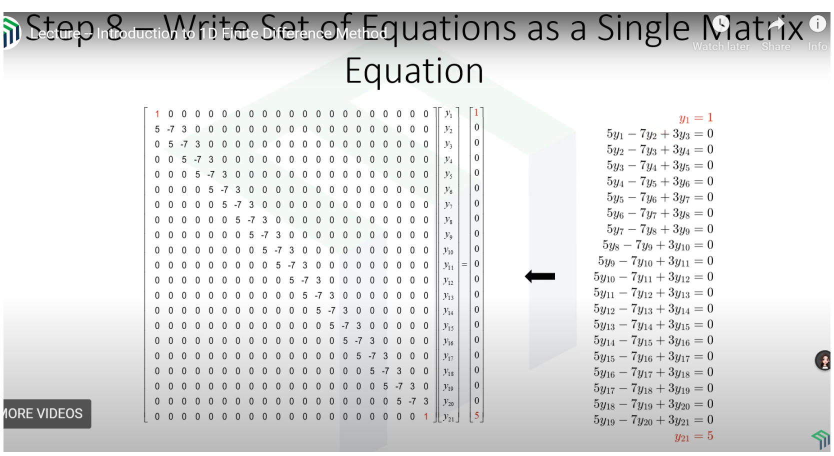
$\{0.985\,y_0 - 1.9999\,y_1 + 1.015\,y_2 == 0,$
$0.985\,y_1 - 1.9999\,y_2 + 1.015\,y_3 == 0,$
$0.985\,y_2 - 1.9999\,y_3 + 1.015\,y_4 == 0,$
$0.985\,y_3 - 1.9999\,y_4 + 1.015\,y_5 == 0,$
$0.985\,y_4 - 1.9999\,y_5 + 1.015\,y_6 == 0,$
$0.985\,y_5 - 1.9999\,y_6 + 1.015\,y_7 == 0,$
$0.985\,y_6 - 1.9999\,y_7 + 1.015\,y_8 == 0,$
$0.985\,y_7 - 1.9999\,y_8 + 1.015\,y_9 == 0,$
$0.985\,y_8 - 1.9999\,y_9 + 1.015\,y_{10} == 0,$
$0.985\,y_9 - 1.9999\,y_{10} + 1.015\,y_{11} == 0,$
$0.985\,y_{10} - 1.9999\,y_{11} + 1.015\,y_{12} == 0,$
$0.985\,y_{11} - 1.9999\,y_{12} + 1.015\,y_{13} == 0,$
$0.985\,y_{12} - 1.9999\,y_{13} + 1.015\,y_{14} == 0,$
$0.985\,y_{13} - 1.9999\,y_{14} + 1.015\,y_{15} == 0,$

$$0.985\,y_{14} - 1.9999\,y_{15} + 1.015\,y_{16} == 0,$$
$$0.985\,y_{15} - 1.9999\,y_{16} + 1.015\,y_{17} == 0,$$
$$0.985\,y_{16} - 1.9999\,y_{17} + 1.015\,y_{18} == 0,$$
$$0.985\,y_{17} - 1.9999\,y_{18} + 1.015\,y_{19} == 0,$$
$$0.985\,y_{18} - 1.9999\,y_{19} + 1.015\,y_{20} == 0,$$
$$0.985\,y_{19} - 1.9999\,y_{20} + 1.015\,y_{21} == 0,$$
$$0.985\,y_{20} - 1.9999\,y_{21} + 1.015\,y_{22} == 0,$$
$$0.985\,y_{21} - 1.9999\,y_{22} + 1.015\,y_{23} == 0,$$
$$0.985\,y_{22} - 1.9999\,y_{23} + 1.015\,y_{24} == 0,$$
$$0.985\,y_{23} - 1.9999\,y_{24} + 1.015\,y_{25} == 0,$$
$$0.985\,y_{24} - 1.9999\,y_{25} + 1.015\,y_{26} == 0,$$
$$0.985\,y_{25} - 1.9999\,y_{26} + 1.015\,y_{27} == 0,$$
$$0.985\,y_{26} - 1.9999\,y_{27} + 1.015\,y_{28} == 0,$$
$$0.985\,y_{27} - 1.9999\,y_{28} + 1.015\,y_{29} == 0,$$
$$0.985\,y_{28} - 1.9999\,y_{29} + 1.015\,y_{30} == 0,$$
$$0.985\,y_{29} - 1.9999\,y_{30} + 1.015\,y_{31} == 0,$$
$$0.985\,y_{30} - 1.9999\,y_{31} + 1.015\,y_{32} == 0,$$
$$0.985\,y_{31} - 1.9999\,y_{32} + 1.015\,y_{33} == 0,$$
$$0.985\,y_{32} - 1.9999\,y_{33} + 1.015\,y_{34} == 0,$$
$$0.985\,y_{33} - 1.9999\,y_{34} + 1.015\,y_{35} == 0,$$
$$0.985\,y_{34} - 1.9999\,y_{35} + 1.015\,y_{36} == 0,$$
$$0.985\,y_{35} - 1.9999\,y_{36} + 1.015\,y_{37} == 0,$$
$$0.985\,y_{36} - 1.9999\,y_{37} + 1.015\,y_{38} == 0,$$
$$0.985\,y_{37} - 1.9999\,y_{38} + 1.015\,y_{39} == 0,$$
$$0.985\,y_{38} - 1.9999\,y_{39} + 1.015\,y_{40} == 0,$$

$$0.985\,y_{39} - 1.9999\,y_{40} + 1.015\,y_{41} == 0,$$
$$0.985\,y_{40} - 1.9999\,y_{41} + 1.015\,y_{42} == 0,$$
$$0.985\,y_{41} - 1.9999\,y_{42} + 1.015\,y_{43} == 0,$$
$$0.985\,y_{42} - 1.9999\,y_{43} + 1.015\,y_{44} == 0,$$
$$0.985\,y_{43} - 1.9999\,y_{44} + 1.015\,y_{45} == 0,$$
$$0.985\,y_{44} - 1.9999\,y_{45} + 1.015\,y_{46} == 0,$$
$$0.985\,y_{45} - 1.9999\,y_{46} + 1.015\,y_{47} == 0,$$
$$0.985\,y_{46} - 1.9999\,y_{47} + 1.015\,y_{48} == 0,$$
$$0.985\,y_{47} - 1.9999\,y_{48} + 1.015\,y_{49} == 0,$$
$$0.985\,y_{48} - 1.9999\,y_{49} + 1.015\,y_{50} == 0,$$
$$0.985\,y_{49} - 1.9999\,y_{50} + 1.015\,y_{51} == 0,$$
$$0.985\,y_{50} - 1.9999\,y_{51} + 1.015\,y_{52} == 0,$$
$$0.985\,y_{51} - 1.9999\,y_{52} + 1.015\,y_{53} == 0,$$
$$0.985\,y_{52} - 1.9999\,y_{53} + 1.015\,y_{54} == 0,$$
$$0.985\,y_{53} - 1.9999\,y_{54} + 1.015\,y_{55} == 0,$$
$$0.985\,y_{54} - 1.9999\,y_{55} + 1.015\,y_{56} == 0,$$
$$0.985\,y_{55} - 1.9999\,y_{56} + 1.015\,y_{57} == 0,$$
$$0.985\,y_{56} - 1.9999\,y_{57} + 1.015\,y_{58} == 0,$$
$$0.985\,y_{57} - 1.9999\,y_{58} + 1.015\,y_{59} == 0,$$
$$0.985\,y_{58} - 1.9999\,y_{59} + 1.015\,y_{60} == 0,$$
$$0.985\,y_{59} - 1.9999\,y_{60} + 1.015\,y_{61} == 0,$$
$$0.985\,y_{60} - 1.9999\,y_{61} + 1.015\,y_{62} == 0,$$
$$0.985\,y_{61} - 1.9999\,y_{62} + 1.015\,y_{63} == 0,$$
$$0.985\,y_{62} - 1.9999\,y_{63} + 1.015\,y_{64} == 0,$$
$$0.985\,y_{63} - 1.9999\,y_{64} + 1.015\,y_{65} == 0,$$

$$0.985\, y_{64} - 1.9999\, y_{65} + 1.015\, y_{66} == 0,$$
$$0.985\, y_{65} - 1.9999\, y_{66} + 1.015\, y_{67} == 0,$$
$$0.985\, y_{66} - 1.9999\, y_{67} + 1.015\, y_{68} == 0,$$
$$0.985\, y_{67} - 1.9999\, y_{68} + 1.015\, y_{69} == 0,$$
$$0.985\, y_{68} - 1.9999\, y_{69} + 1.015\, y_{70} == 0,$$
$$0.985\, y_{69} - 1.9999\, y_{70} + 1.015\, y_{71} == 0,$$
$$0.985\, y_{70} - 1.9999\, y_{71} + 1.015\, y_{72} == 0,$$
$$0.985\, y_{71} - 1.9999\, y_{72} + 1.015\, y_{73} == 0,$$
$$0.985\, y_{72} - 1.9999\, y_{73} + 1.015\, y_{74} == 0,$$
$$0.985\, y_{73} - 1.9999\, y_{74} + 1.015\, y_{75} == 0,$$
$$0.985\, y_{74} - 1.9999\, y_{75} + 1.015\, y_{76} == 0,$$
$$0.985\, y_{75} - 1.9999\, y_{76} + 1.015\, y_{77} == 0,$$
$$0.985\, y_{76} - 1.9999\, y_{77} + 1.015\, y_{78} == 0,$$
$$0.985\, y_{77} - 1.9999\, y_{78} + 1.015\, y_{79} == 0,$$
$$0.985\, y_{78} - 1.9999\, y_{79} + 1.015\, y_{80} == 0,$$
$$0.985\, y_{79} - 1.9999\, y_{80} + 1.015\, y_{81} == 0,$$
$$0.985\, y_{80} - 1.9999\, y_{81} + 1.015\, y_{82} == 0,$$
$$0.985\, y_{81} - 1.9999\, y_{82} + 1.015\, y_{83} == 0,$$
$$0.985\, y_{82} - 1.9999\, y_{83} + 1.015\, y_{84} == 0,$$
$$0.985\, y_{83} - 1.9999\, y_{84} + 1.015\, y_{85} == 0,$$
$$0.985\, y_{84} - 1.9999\, y_{85} + 1.015\, y_{86} == 0,$$
$$0.985\, y_{85} - 1.9999\, y_{86} + 1.015\, y_{87} == 0,$$
$$0.985\, y_{86} - 1.9999\, y_{87} + 1.015\, y_{88} == 0,$$
$$0.985\, y_{87} - 1.9999\, y_{88} + 1.015\, y_{89} == 0,$$
$$0.985\, y_{88} - 1.9999\, y_{89} + 1.015\, y_{90} == 0,$$

$0.985\, y_{89} - 1.9999\, y_{90} + 1.015\, y_{91} == 0,$
$0.985\, y_{90} - 1.9999\, y_{91} + 1.015\, y_{92} == 0,$
$0.985\, y_{91} - 1.9999\, y_{92} + 1.015\, y_{93} == 0,$
$0.985\, y_{92} - 1.9999\, y_{93} + 1.015\, y_{94} == 0,$
$0.985\, y_{93} - 1.9999\, y_{94} + 1.015\, y_{95} == 0,$
$0.985\, y_{94} - 1.9999\, y_{95} + 1.015\, y_{96} == 0,$
$0.985\, y_{95} - 1.9999\, y_{96} + 1.015\, y_{97} == 0,$
$0.985\, y_{96} - 1.9999\, y_{97} + 1.015\, y_{98} == 0,$
$0.985\, y_{97} - 1.9999\, y_{98} + 1.015\, y_{99} == 0,$
$0.985\, y_{98} - 1.9999\, y_{99} + 1.015\, y_{100} == 0,$
$0.985\, y_{99} - 1.9999\, y_{100} + 1.015\, y_{101} == 0\}$

- Similar to the matrix equation, we can express the system of linear equations above in terms of matrix shown below. Notice the boundary conditions y_1=1 and y_{21} = 5 in the example below.
- 

- ==
- Example of A[[1,1]]

In[ ]:=
```
A = {{0, 0}, {0, 0}};
(*Initialize a 2×2 matrix*)
A[[1, 1]] = 1; (*Set the first element to 1*)
A (*Output the modified matrix*)
```

Out[ ]=
```
{{1, 0}, {0, 0}}
```

■ ++

In[ ]:=
```
A = SparseArray[{}, {3, 3}];
(*Creates a 3×3 zero matrix*)
A[[1, 1]] = 1;
Normal[A]
(*Convert back to normal list format*)
```

Out[ ]=
```
{{1, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

In[ ]:=
```
ConstantArray[0, {3, 3}]
(* Example of ConstantArray *)
```

Out[ ]=
```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

■ ==

In[ ]:=
```
Do[Print["Iteration: ", i], {i, 1, 5}]
```

```
Iteration: 1
Iteration: 2
Iteration: 3
Iteration: 4
Iteration: 5
```

In[ ]:=
```
Do[Print["i = ", i], {i, 1, 10, 2}]
```

```
i = 1
i = 3
i = 5
i = 7
i = 9
```

In[ ]:=
```
Do[Print["i = ", i, ", j = ", j],
  {i, 1, 3}, {j, 1, 2}]
```

```
i = 1, j = 1
i = 1, j = 2
i = 2, j = 1
i = 2, j = 2
i = 3, j = 1
i = 3, j = 2
```

In[ ]:=
```
sum = 0;
Do[sum += i, {i, 1, 100}]
sum
```

Out[ ]=
```
5050
```

- ____-
- So we have also

In[ ]:=
```
(*Define the number of equations*)
numEqns = 101;

(*Initialize an empty matrix
 of size numEqns x numEqns*)
A = ConstantArray[0, {numEqns, numEqns}];

(*Fill the first row
  for boundary condition y1=3*)
A[[1, 1]] = 1;

(*Fill the last row for
  boundary condition y101=20*)
A[[numEqns, numEqns]] = 1;

(*Fill the coefficient matrix
```

```
 for the system of equations*)
Do[A[[n, n - 1]] = 0.985;
  A[[n, n]] = -1.9999;
  A[[n, n + 1]] = 1.015, {n, 2, numEqns - 1}];

(*Define column matrix Y
 (unknown variables y1 to y101)*)
Y = Table[Subscript[y, n], {n, 1, numEqns}];

(*Define right-hand side matrix B*)
B = ConstantArray[0, numEqns];

(*Set boundary condition values*)
B[[1]] = 3;  (*y1=3*)
B[[numEqns]] = 20; (*y101=20*)

(*Display the system in matrix form*)
MatrixForm[A].MatrixForm[Y] == MatrixForm[B]
```

Out[ ]=

$$\left(\begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0 \\
0.985 & -1.9999 & 1.015 & 0 & 0 & 0 \\
0 & 0.985 & -1.9999 & 1.015 & 0 & 0 \\
0 & 0 & 0.985 & -1.9999 & 1.015 & 0 \\
0 & 0 & 0 & 0.985 & -1.9999 & 1.015 \\
0 & 0 & 0 & 0 & 0.985 & -1.9999 \\
0 & 0 & 0 & 0 & 0 & 0.985
\end{array}\right.$$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

In[ ]:=

```
Reduce[{0.985 * 3 - 1.9999 * y1 + 1.015 * y2 == 0,
  0.985 * y1 - 1.9999 * y2 + 1.015 * y3 == 0,
  0.985 * y2 - 1.9999 * y3 + 1.015 * y4 == 0,
  0.985 * y3 - 1.9999 * y4 + 1.015 * y5 == 0,
  0.985 * y4 - 1.9999 * y5 + 1.015 * y6 == 0,
  0.985 * y5 - 1.9999 * y6 + 1.015 * y7 == 0,
  0.985 * y6 - 1.9999 * y7 + 1.015 * y8 == 0,
  0.985 * y7 - 1.9999 * y8 + 1.015 * y9 == 0,
  0.985 * y8 - 1.9999 * y9 + 1.015 * y10 == 0,
  0.985 * y9 - 1.9999 * y10 + 1.015 * y11 == 0,
  0.985 * y10 - 1.9999 * y11 + 1.015 * y12 == 0,
  0.985 * y11 - 1.9999 * y12 + 1.015 * y13 == 0,
  0.985 * y12 - 1.9999 * y13 + 1.015 * y14 == 0,
  0.985 * y13 - 1.9999 * y14 + 1.015 * y15 == 0,
  0.985 * y14 - 1.9999 * y15 + 1.015 * y16 == 0,
  0.985 * y15 - 1.9999 * y16 + 1.015 * y17 == 0,
  0.985 * y16 - 1.9999 * y17 + 1.015 * y18 == 0,
  0.985 * y17 - 1.9999 * y18 + 1.015 * y19 == 0,
```

$0.985 * y_{18} - 1.9999 * y_{19} + 1.015 * y_{20} == 0,$
$0.985 * y_{19} - 1.9999 * y_{20} + 1.015 * y_{21} == 0,$
$0.985 * y_{20} - 1.9999 * y_{21} + 1.015 * y_{22} == 0,$
$0.985 * y_{21} - 1.9999 * y_{22} + 1.015 * y_{23} == 0,$
$0.985 * y_{22} - 1.9999 * y_{23} + 1.015 * y_{24} == 0,$
$0.985 * y_{23} - 1.9999 * y_{24} + 1.015 * y_{25} == 0,$
$0.985 * y_{24} - 1.9999 * y_{25} + 1.015 * y_{26} == 0,$
$0.985 * y_{25} - 1.9999 * y_{26} + 1.015 * y_{27} == 0,$
$0.985 * y_{26} - 1.9999 * y_{27} + 1.015 * y_{28} == 0,$
$0.985 * y_{27} - 1.9999 * y_{28} + 1.015 * y_{29} == 0,$
$0.985 * y_{28} - 1.9999 * y_{29} + 1.015 * y_{30} == 0,$
$0.985 * y_{29} - 1.9999 * y_{30} + 1.015 * y_{31} == 0,$
$0.985 * y_{30} - 1.9999 * y_{31} + 1.015 * y_{32} == 0,$
$0.985 * y_{31} - 1.9999 * y_{32} + 1.015 * y_{33} == 0,$
$0.985 * y_{32} - 1.9999 * y_{33} + 1.015 * y_{34} == 0,$
$0.985 * y_{33} - 1.9999 * y_{34} + 1.015 * y_{35} == 0,$
$0.985 * y_{34} - 1.9999 * y_{35} + 1.015 * y_{36} == 0,$
$0.985 * y_{35} - 1.9999 * y_{36} + 1.015 * y_{37} == 0,$
$0.985 * y_{36} - 1.9999 * y_{37} + 1.015 * y_{38} == 0,$
$0.985 * y_{37} - 1.9999 * y_{38} + 1.015 * y_{39} == 0,$
$0.985 * y_{38} - 1.9999 * y_{39} + 1.015 * y_{40} == 0,$
$0.985 * y_{39} - 1.9999 * y_{40} + 1.015 * y_{41} == 0,$
$0.985 * y_{40} - 1.9999 * y_{41} + 1.015 * y_{42} == 0,$
$0.985 * y_{41} - 1.9999 * y_{42} + 1.015 * y_{43} == 0,$
$0.985 * y_{42} - 1.9999 * y_{43} + 1.015 * y_{44} == 0,$

$0.985 * y_{43} - 1.9999 * y_{44} + 1.015 * y_{45} == 0,$
$0.985 * y_{44} - 1.9999 * y_{45} + 1.015 * y_{46} == 0,$
$0.985 * y_{45} - 1.9999 * y_{46} + 1.015 * y_{47} == 0,$
$0.985 * y_{46} - 1.9999 * y_{47} + 1.015 * y_{48} == 0,$
$0.98 * y_{47} - 1.9999 * y_{48} + 1.015 * y_{49} == 0,$
$0.985 * y_{48} - 1.9999 * y_{49} + 1.015 * y_{50} == 0,$
$0.985 * y_{49} - 1.9999 * y_{50} + 1.015 * y_{51} == 0,$
$0.985 * y_{50} - 1.9999 * y_{51} + 1.015 * y_{52} == 0,$
$0.985 * y_{51} - 1.9999 * y_{52} + 1.015 * y_{53} == 0,$
$0.985 * y_{52} - 1.9999 * y_{53} + 1.015 * y_{54} == 0,$
$0.985 * y_{53} - 1.9999 * y_{54} + 1.015 * y_{55} == 0,$
$0.985 * y_{54} - 1.9999 * y_{55} + 1.015 * y_{56} == 0,$
$0.985 * y_{55} - 1.9999 * y_{56} + 1.015 * y_{57} == 0,$
$0.985 * y_{56} - 1.9999 * y_{57} + 1.015 * y_{58} == 0,$
$0.985 * y_{57} - 1.9999 * y_{58} + 1.015 * y_{59} == 0,$
$0.985 * y_{58} - 1.9999 * y_{59} + 1.015 * y_{60} == 0,$
$0.985 * y_{59} - 1.9999 * y_{60} + 1.015 * y_{61} == 0,$
$0.985 * y_{60} - 1.9999 * y_{61} + 1.015 * y_{62} == 0,$
$0.985 * y_{61} - 1.9999 * y_{62} + 1.015 * y_{63} == 0,$
$0.985 * y_{62} - 1.9999 * y_{63} + 1.015 * y_{64} == 0,$
$0.985 * y_{63} - 1.9999 * y_{64} + 1.015 * y_{65} == 0,$
$0.985 * y_{64} - 1.9999 * y_{65} + 1.015 * y_{66} == 0,$
$0.985 * y_{65} - 1.9999 * y_{66} + 1.015 * y_{67} == 0,$
$0.985 * y_{66} - 1.9999 * y_{67} + 1.015 * y_{68} == 0,$
$0.985 * y_{67} - 1.9999 * y_{68} + 1.015 * y_{69} == 0,$

$0.985 * y_{68} - 1.9999 * y_{69} + 1.015 * y_{70} == 0,$
$0.985 * y_{69} - 1.9999 * y_{70} + 1.015 * y_{71} == 0,$
$0.985 * y_{70} - 1.9999 * y_{71} + 1.015 * y_{72} == 0,$
$0.985 * y_{71} - 1.9999 * y_{72} + 1.015 * y_{73} == 0,$
$0.985 * y_{72} - 1.9999 * y_{73} + 1.015 * y_{74} == 0,$
$0.985 * y_{73} - 1.9999 * y_{74} + 1.015 * y_{75} == 0,$
$0.985 * y_{74} - 1.9999 * y_{75} + 1.015 * y_{76} == 0,$
$0.985 * y_{75} - 1.9999 * y_{76} + 1.015 * y_{77} == 0,$
$0.985 * y_{76} - 1.9999 * y_{77} + 1.015 * y_{78} == 0,$
$0.985 * y_{77} - 1.9999 * y_{78} + 1.015 * y_{79} == 0,$
$0.985 * y_{78} - 1.9999 * y_{79} + 1.015 * y_{80} == 0,$
$0.985 * y_{79} - 1.9999 * y_{80} + 1.015 * y_{81} == 0,$
$0.985 * y_{80} - 1.9999 * y_{81} + 1.015 * y_{82} == 0,$
$0.985 * y_{81} - 1.9999 * y_{82} + 1.015 * y_{83} == 0,$
$0.985 * y_{82} - 1.9999 * y_{83} + 1.015 * y_{84} == 0,$
$0.985 * y_{83} - 1.9999 * y_{84} + 1.015 * y_{85} == 0,$
$0.985 * y_{84} - 1.9999 * y_{85} + 1.015 * y_{86} == 0,$
$0.985 * y_{85} - 1.9999 * y_{86} + 1.015 * y_{87} == 0,$
$0.985 * y_{86} - 1.9999 * y_{87} + 1.015 * y_{88} == 0,$
$0.985 * y_{87} - 1.9999 * y_{88} + 1.015 * y_{89} == 0,$
$0.985 * y_{88} - 1.9999 * y_{89} + 1.015 * y_{90} == 0,$
$0.985 * y_{89} - 1.9999 * y_{90} + 1.015 * y_{91} == 0,$
$0.985 * y_{90} - 1.9999 * y_{91} + 1.015 * y_{92} == 0,$
$0.985 * y_{91} - 1.9999 * y_{92} + 1.015 * y_{93} == 0,$
$0.985 * y_{92} - 1.9999 * y_{93} + 1.015 * y_{94} == 0,$

```
0.985 * y93 - 1.9999 * y94 + 1.015 * y95 == 0,
0.985 * y94 - 1.9999 * y95 + 1.015 * y96 == 0,
0.985 * y95 - 1.9999 * y96 + 1.015 * y97 == 0,
0.985 * y96 - 1.9999 * y97 + 1.015 * y98 == 0,
0.985 * y97 - 1.9999 * y98 + 1.015 * y99 == 0,
0.985 * y98 - 1.9999 * y99 + 1.015 * y100 == 0,
0.985 * y99 - 1.9999 * y100 + 1.015 * 20 == 0},
{y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12,
y13, y14, y15, y16, y17, y18, y19, y20, y21, y22, y23,
y24, y25, y26, y27, y28, y29, y30, y31, y32, y33, y34,
y35, y36, y37, y38, y39, y40, y41, y42, y43, y44, y45,
y46, y47, y48, y49, y50, y51, y52, y53, y54, y55, y56,
y57, y58, y59, y60, y61, y62, y63, y64, y65, y66, y67,
y68, y69, y70, y71, y72, y73, y74, y75, y76, y77, y78,
y79, y80, y81, y82, y83, y84, y85, y86, y87, y88, y89,
y90, y991, y92, y93, y94, y95, y96, y97, y98, y99, y100}]
```

Out[ ]=

```
y91 == 20.0552 && y1 == 3.56199 &&
y2 == 4.10703 && y3 == 4.63555 && y4 == 5.14799 &&
y5 == 5.64478 && y6 == 6.12632 && y7 == 6.59304 &&
y8 == 7.0453 && y9 == 7.48351 && y10 == 7.90803 &&
y11 == 8.31922 && y12 == 8.71744 &&
y13 == 9.10302 && y14 == 9.47632 && y15 == 9.83765 &&
y16 == 10.1873 && y17 == 10.5257 && y18 == 10.853 &&
y19 == 11.1695 && y20 == 11.4756 && y21 == 11.7716 &&
y22 == 12.0576 && y23 == 12.334 && y24 == 12.6009 &&
```

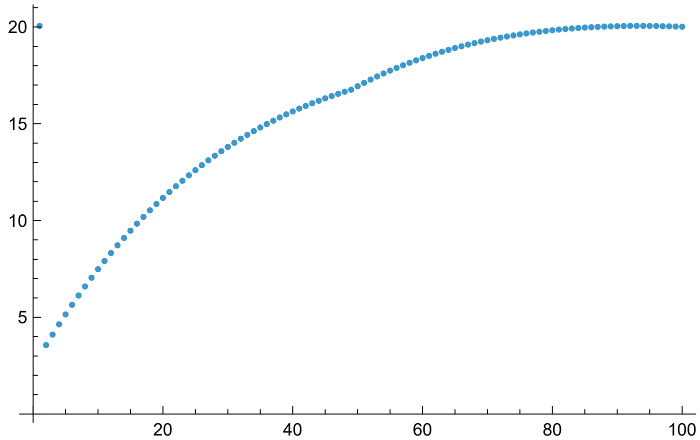
y_{25} == 12.8588 && y_{26} == 13.1078 && y_{27} == 13.3481 &&
y_{28} == 13.58 && y_{29} == 13.8037 && y_{30} == 14.0195 &&
y_{31} == 14.2274 && y_{32} == 14.4279 && y_{33} == 14.621 &&
y_{34} == 14.8069 && y_{35} == 14.9859 && y_{36} == 15.1581 &&
y_{37} == 15.3237 && y_{38} == 15.483 && y_{39} == 15.636 &&
y_{40} == 15.7829 && y_{41} == 15.9239 && y_{42} == 16.0592 &&
y_{43} == 16.1889 && y_{44} == 16.3132 && y_{45} == 16.4322 &&
y_{46} == 16.5461 && y_{47} == 16.655 && y_{48} == 16.759 &&
y_{49} == 16.9403 && y_{50} == 17.1147 && y_{51} == 17.2821 &&
y_{52} == 17.4429 && y_{53} == 17.5973 && y_{54} == 17.7453 &&
y_{55} == 17.8873 && y_{56} == 18.0232 && y_{57} == 18.1534 &&
y_{58} == 18.2779 && y_{59} == 18.397 && y_{60} == 18.5107 &&
y_{61} == 18.6193 && y_{62} == 18.7228 && y_{63} == 18.8214 &&
y_{64} == 18.9152 && y_{65} == 19.0044 && y_{66} == 19.089 &&
y_{67} == 19.1693 && y_{68} == 19.2454 && y_{69} == 19.3173 &&
y_{70} == 19.3851 && y_{71} == 19.4491 && y_{72} == 19.5092 &&
y_{73} == 19.5657 && y_{74} == 19.6185 && y_{75} == 19.6679 &&
y_{76} == 19.7138 && y_{77} == 19.7565 && y_{78} == 19.7959 &&
y_{79} == 19.8322 && y_{80} == 19.8655 && y_{81} == 19.8959 &&
y_{82} == 19.9234 && y_{83} == 19.9481 && y_{84} == 19.9702 &&
y_{85} == 19.9896 && y_{86} == 20.0065 && y_{87} == 20.0209 &&
y_{88} == 20.0329 && y_{89} == 20.0425 && y_{90} == 20.0499 &&
y_{92} == 20.0582 && y_{93} == 20.0593 && y_{94} == 20.0583 &&
y_{95} == 20.0553 && y_{96} == 20.0505 && y_{97} == 20.0439 &&
y_{98} == 20.0354 && y_{99} == 20.0253 && y_{100} == 20.0135

In[ ]:=

```
ListPlot[{20.055157866789195, 3.5619947009578463,
  4.107027785660687, 4.635546884826925,
  5.14798802570403, 5.644775932069919,
  6.1263243164809476, 6.593036164966873,
  7.045304014367996, 7.4835102225046155,
  7.908027231364044, 8.319217823485621,
  8.717435371719516, 9.103024082530563,
  9.476319233013943, 9.8376474017852,
  10.187326693902943, 10.525666959978398,
  10.852970009622068, 11.169529819373842,
  11.475632735259122, 11.771557670109834,
  12.057576295785635, 12.333953230427099,
  12.600946220869265, 12.858806320340639,
  13.107778061569475, 13.348099625416022,
  13.580003005146375, 13.803714166460546,
  14.019453203384497, 14.227434490133023,
  14.427866829047586, 14.620953594710578,
  14.806892874334789, 14.985877604524356,
  15.158095704500974, 15.323730205886706,
  15.482959379132376, 15.635956856678261,
  15.782891752931487, 15.923928781142457,
  16.059228367260378, 16.18894676084602,
  16.313236143117713, 16.432244732204712,
  16.546116885680053, 16.65499320044325,
  16.759010610021285, 16.940346780834663,
```

```
17.11465426218746, 17.28212362544886,
17.442940483027172, 17.59728561669244,
17.74533510250388, 17.887260432566954,
18.02322863362003, 18.15340238255985,
18.277940118981, 18.39699615480655,
18.51072078108506, 18.61926037202715,
18.722757486353018, 18.821350966020354,
18.915176032400378, 19.004364379967946,
19.08904426756997, 19.169340607334735,
19.24537505128307, 19.317266075700786,
19.385129063330226, 19.449076383437284,
19.509217469808817, 19.56565889673392,
19.618504453021163, 19.667855214102573,
19.71380961227378, 19.75646350511852,
19.79591024216439, 19.832240729815585,
19.865543494607156, 19.895904744824136,
19.92340843052782, 19.94813630203036,
19.970167966857744, 19.989580945240295,
20.006450724168655, 20.020850810052412,
20.03285278001743, 20.042526331877074,
20.049939332811615, 20.0582462807608 5,
20.0592672296613 5, 20.05828172024659,
20.05534915379776, 20.050527367721422,
20.0438726760741 7, 20.035439909039532,
20.025282451384335, 20.013452279920777}]
```

Out[ ]=



■ ---

■ Solving the given differential equation by Mathematica

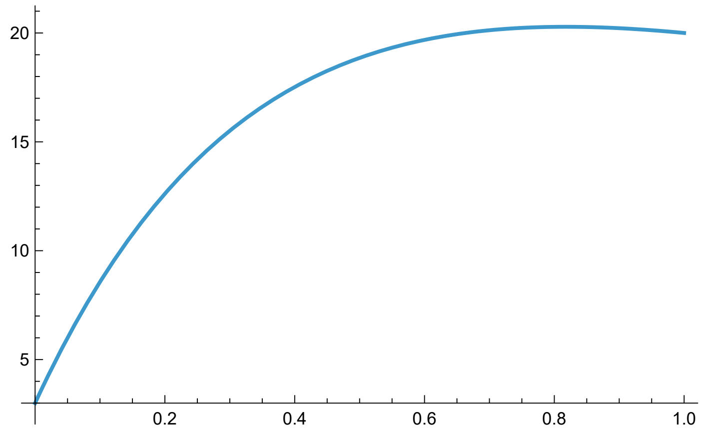
In[ ]:=
```
sol = NDSolve[{y''[x] + 3 y'[x] + y[x] == 0,
   y[0] == 3, y[1] == 20}, y, {x, 0, 1}]
```

In[ ]:=
```
{{y → InterpolatingFunction[ Domain: {{0., 1.}} Output: scalar ]}}
```

Out[ ]=
```
{{y → InterpolatingFunction[ Domain: {{0., 1.}} Output: scalar ]}}
```

```
Plot[Evaluate[y[x] /. sol], {x, 0, 1}]
(* Δx=0.01 and 1= 100(0.01)= 1 *)
```

Out[•]=



- ==

- Let us plot all the data points:

In[•]:=

```
(*Define the data points*)
data = {20.055157866789195, 3.5619947009578463,
   4.107027785660687, 4.635546884826925,
   5.14798802570403, 5.644775932069919,
   6.126324316480947 6, 6.593036164966873,
   7.045304014367996, 7.483510222504615 5,
   7.908027231364044, 8.319217823485621,
   8.717435371719516, 9.103024082530563,
   9.476319233013943, 9.8376474017852,
   10.187326693902943, 10.525666959978398,
   10.852970009622068, 11.169529819373842,
   11.475632735259122, 11.771557670109834,
```

12.057576295785635, 12.333953230427099,
12.600946220869265, 12.858806320340639,
13.107778061569475, 13.348099625416022,
13.580003005146375, 13.80371416646054 6,
14.019453203384497, 14.227434490133023,
14.427866829047586, 14.620953594710578,
14.806892874334789, 14.985877604524356,
15.158095704500974, 15.323730205886706,
15.482959379132376, 15.635956856678261,
15.782891752931487, 15.923928781142457,
16.059228367260378, 16.18894676084602,
16.313236143117713, 16.432244732204712,
16.546116885680053, 16.65499320044325,
16.759010610021285, 16.940346780834663,
17.11465426218746, 17.28212362544488 6,
17.442940483027172, 17.59728561669244,
17.74533510250388, 17.887260432566954,
18.02322863362003, 18.15340238255985,
18.277940118981, 18.39699615480655,
18.51072078108506, 18.61926037202715,
18.722757486353018, 18.821350966020354,
18.915176032400378, 19.004364379967946,
19.0890442675699 7, 19.169340607334735,
19.24537505128307, 19.317266075700786,
19.385129063330226, 19.449076383437284,

```
    19.509217469808817, 19.56565889673392,
    19.618504453021163, 19.667855214102573,
    19.71380961227378, 19.75646350511852,
    19.79591024216439, 19.832240729815585,
    19.865543494607156, 19.895904744824136,
    19.92340843052782, 19.94813630203036,
    19.970167966857744, 19.989580945240295,
    20.006450724168655, 20.020850810052412,
    20.03285278001743, 20.042526331877074,
    20.049939332811615, 20.0582462807608 5,
    20.0592672296613 5, 20.05828172024659,
    20.05534915379776, 20.050527367721422,
    20.04387267607417, 20.035439909039532,
    20.025282451384335, 20.013452279920777};

(*Fit a polynomial equation
 (try quadratic and cubic)*)
quadraticFit = Fit[data, {1, x, x^2}, x]
cubicFit = Fit[data, {1, x, x^2, x^3}, x]

(*Plot data points and the fitted curves*)
Show[ListPlot[data,
  PlotStyle → {Red, PointSize[Medium]},
  AxesLabel → {"x", "y"}],
 Plot[quadraticFit, {x, 0, 8},
```

```
PlotStyle → {Blue, Dashed}], Plot[cubicFit,
 {x, 0, 8}, PlotStyle → {Green, Thick}],
PlotLabel → "Data Points and Fitted Curves",
PlotLegends →
 {"Data", "Quadratic Fit", "Cubic Fit"}]
```

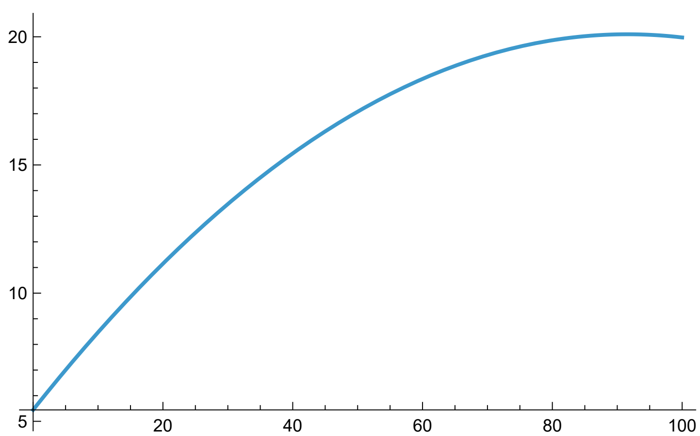
In[ ]:=
```
5.445289487539549` + 0.3200887935891954` x - 0.001748120361804426` x^2
```

Out[ ]=

$$5.44529 + 0.320089\, x - 0.00174812\, x^2$$

In[ ]:=
```
Plot[5.445289487539549 + 0.3200887935891954 * x -
  0.001748120361804426 * x^2, {x, 0, 100}]
```

Out[ ]=



- The solution of the differential equation is approximately given by y = **5.445289487539549 + 0.3200887935891954 * x - 0.001748120361804426 * x^2**

In[ ]:=

```
NSolve[{0.985 * 3 - 1.9999 * y1 + 1.015 * y2 == 0,
  0.985 * y1 - 1.9999 * y2 + 1.015 * y3 == 0,
  0.985 * y2 - 1.9999 * y3 + 1.015 * y4 == 0,
  0.985 * y3 - 1.9999 * y4 + 1.015 * y5 == 0,
  0.985 * y4 - 1.9999 * y5 + 1.015 * y6 == 0,
  0.985 * y5 - 1.9999 * y6 + 1.015 * y7 == 0,
  0.985 * y6 - 1.9999 * y7 + 1.015 * y8 == 0,
  0.985 * y7 - 1.9999 * y8 + 1.015 * y9 == 0,
  0.985 * y8 - 1.9999 * y9 + 1.015 * y10 == 0,
  0.985 * y9 - 1.9999 * y10 + 1.015 * y11 == 0,
  0.985 * y10 - 1.9999 * y11 + 1.015 * y12 == 0,
  0.985 * y11 - 1.9999 * y12 + 1.015 * y13 == 0,
  0.985 * y12 - 1.9999 * y13 + 1.015 * y14 == 0,
  0.985 * y13 - 1.9999 * y14 + 1.015 * y15 == 0,
  0.985 * y14 - 1.9999 * y15 + 1.015 * y16 == 0,
  0.985 * y15 - 1.9999 * y16 + 1.015 * y17 == 0,
  0.985 * y16 - 1.9999 * y17 + 1.015 * y18 == 0,
  0.985 * y17 - 1.9999 * y18 + 1.015 * y19 == 0,
  0.985 * y18 - 1.9999 * y19 + 1.015 * y20 == 0,
  0.985 * y19 - 1.9999 * y20 + 1.015 * 20 == 0},
 {y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11,
  y12, y13, y14, y15, y16, y17, y18, y19, y20}]
 (* For only 20 points *)
```

Out[#]=

$\{\{y_1 \to 4.0861, y_2 \to 5.1397,$
$y_3 \to 6.16166, y_4 \to 7.1528, y_5 \to 8.11394,$
$y_6 \to 9.04587, y_7 \to 9.94937, y_8 \to 10.8252,$
$y_9 \to 11.674, y_{10} \to 12.4967, y_{11} \to 13.2937,$
$y_{12} \to 14.0659, y_{13} \to 14.8139, y_{14} \to 15.5384,$
$y_{15} \to 16.2399, y_{16} \to 16.919, y_{17} \to 17.5764,$
$y_{18} \to 18.2127, y_{19} \to 18.8283, y_{20} \to 19.4239\}\}$

In[ ]:=

$$\{\{y_1 \to 4.086104065119888\text{`}, y_2 \to 5.1397039604268615\text{`}, y_3 \to 6.161656597354277\text{`}, y_4 \to 7.152796677860453\text{`}, y_5 \to 8.113937268629712\text{`}, y_6 \to 9.045870360433513\text{`}, y_7 \to 9.949367413035198\text{`}, y_8 \to 10.825179886011899\text{`}, y_9 \to 11.674039755857663\text{`}, y_{10} \to 12.496660019722198\text{`}, y_{11} \to 13.293735186130657\text{`}, y_{12} \to 14.065941753021026\text{`}, y_{13} \to 14.813938673426655\text{`}, y_{14} \to 15.53836780912341\text{`}, y_{15} \to 16.23985437255237\text{`}, y_{16} \to 16.919007357321107\text{`}, y_{17} \to 17.57641995757872\text{`}, y_{18} \to 18.212669976552117\text{`}, y_{19} \to 18.828320224523686\text{`}, y_{20} \to 19.42391890652324\text{`}\}\}$$

Out[ ]=

$$\{\{y_1 \to 4.0861, y_2 \to 5.1397, y_3 \to 6.16166, y_4 \to 7.1528, y_5 \to 8.11394, y_6 \to 9.04587, y_7 \to 9.94937, y_8 \to 10.8252, y_9 \to 11.674, y_{10} \to 12.4967, y_{11} \to 13.2937, y_{12} \to 14.0659, y_{13} \to 14.8139, y_{14} \to 15.5384, y_{15} \to 16.2399, y_{16} \to 16.919, y_{17} \to 17.5764, y_{18} \to 18.2127, y_{19} \to 18.8283, y_{20} \to 19.4239\}\}$$

In[ ]:=

```
data = {4.086104065119888, 5.139703960426861 5,
  6.161656597354277, 7.152796677860453,
  8.113937268629712, 9.045870360433513,
  9.949367413035198, 10.825179886011899,
  11.674039755857663, 12.496660019722198,
  13.293735186130657, 14.065941753021026,
  14.813938673426655, 15.53836780912341,
  16.23985437255237, 16.919007357321107,
  17.57641995757872, 18.212669976552117,
  18.82832022452368 6, 19.42391890652324}
```

Out[ ]=

```
{4.0861, 5.1397, 6.16166, 7.1528, 8.11394,
 9.04587, 9.94937, 10.8252, 11.674, 12.4967,
 13.2937, 14.0659, 14.8139, 15.5384, 16.2399,
 16.919, 17.5764, 18.2127, 18.8283, 19.4239}
```

In[ ]:=

```
quadraticFit = Fit[data, {1, x, x^2}, x]
cubicFit = Fit[data, {1, x, x^2, x^3}, x]
```

Out[ ]=

$$3.06293 + 1.06974\,x - 0.0126645\,x^2$$

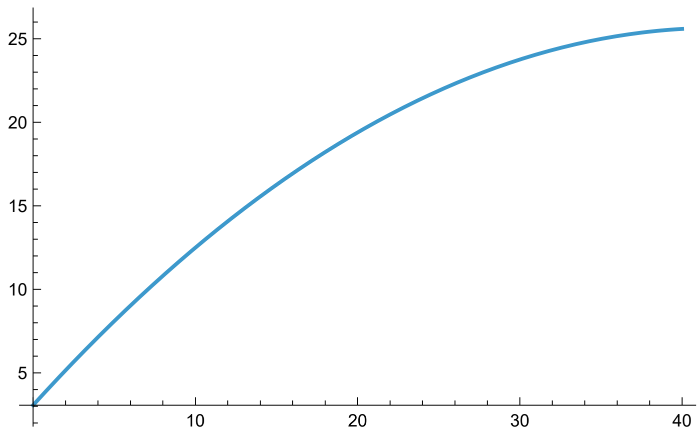
Out[ ]=

$$3.00281 + 1.10042\,x - 0.0162292\,x^2 + 0.000113164\,x^3$$

In[ ]:=

```
Plot[3.0629320049862114 + 1.0697424491320113 * x -
  0.012664482298342118 * x^2, {x, 0, 40}]
```

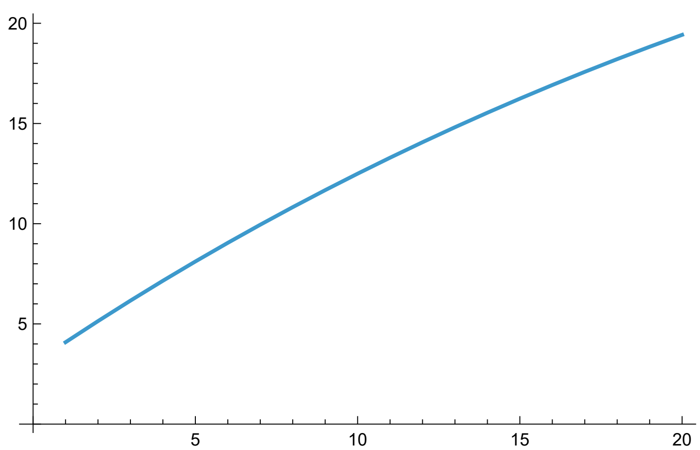
Out[ ]=



In[ ]:=

```
ListLinePlot[
 {4.086104065119888, 5.1397039604268615,
  6.161656597354277, 7.152796677860453,
  8.113937268629712, 9.045870360433513,
  9.949367413035198, 10.825179886011899,
  11.674039755857663, 12.496660019722198,
  13.293735186130657, 14.065941753021026,
  14.813938673426655, 15.53836780912341,
  16.23985437255237, 16.919007357321107,
  17.57641995757872, 18.212669976552117,
  18.82832024523686, 19.42391890652324}]
```

Out[ ]=



```
sol = NDSolve[{y''[x] + 3 y'[x] + y[x] == -40,
   y[0] == 3, y[1] == 20}, y, {x, 0, 1}]
  (* The problem is solving
```

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=-40$$

```
*)
```

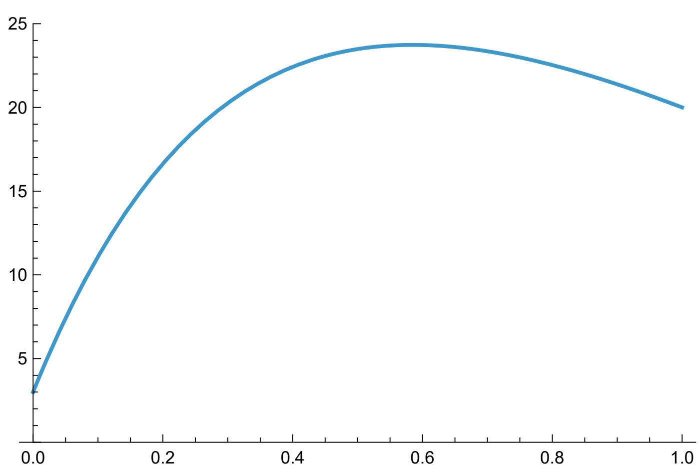
Out[ ]=

```
{{y →
   InterpolatingFunction[ Domain: {{0., 1.}}
                          Output: scalar ]}}
```

In[ ]:=

```
Plot[Evaluate[y[x] /. sol],
  {x, 0, 1}, PlotRange → {0, 25}]
```

Out[ ]=



■ Examples of solutions of systems of linear equations

In[ ]:=

```
Solve[{y1 + y2 == 5, 2 * y1 - y2 == 3}, {y1, y2}]
```

Out[ ]=

$$\left\{\left\{y1 \to \frac{8}{3},\ y2 \to \frac{7}{3}\right\}\right\}$$

In[ ]:=

```
(*Define the system of equations*)
eqs = {Subscript[y, 1] + Subscript[y, 2] == 5,
   2 Subscript[y, 1] - Subscript[y, 2] == 3};

(*Define boundary conditions (if applicable)*)
boundaryConditions = {};
(*No additional conditions in this example*)

(*Solve the system*)
solution =
 Solve[Flatten@Join[eqs, boundaryConditions],
  Table[Subscript[y, n], {n, 1, 2}]]

(*Display the result*)
solution
```

Out[ ]=

$$\left\{\left\{y_1 \to \frac{8}{3},\ y_2 \to \frac{7}{3}\right\}\right\}$$

Out[ ]=

$$\left\{\left\{y_1 \to \frac{8}{3},\ y_2 \to \frac{7}{3}\right\}\right\}$$

In[]:=
```
list1 = {a, b, c};
list2 = {d, e, f};
Join[list1, list2]
```

Out[]=
```
{a, b, c, d, e, f}
```

In[]:=
```
nestedList = {{a, b}, {c, d}, {e, f}};
Flatten[nestedList]
```

Out[]=
```
{a, b, c, d, e, f}
```

In[]:=
```
eqs = {{x + y == 3}, {x - y == 1}};
boundaryConditions = {};
(*No extra conditions here*)

Flatten@Join[eqs, boundaryConditions]
```

Out[]=
```
{x + y == 3, x - y == 1}
```

- ===
=

In[ ]:=
```
(*Define the number of equations*)numEqs = 100;

(*Generate the system of equations*)
eqs = Table[0.985 Subscript[y, n - 1] - 1.9999
      Subscript[y, n] + 1.015 Subscript[y, n + 1] ==
    -40 * 0.01^2, {n, 1, numEqs}];

(*Define boundary conditions*)
boundaryConditions =
  {Subscript[y, 0] → 3, Subscript[y, 101] → 20};

(*Solve the system*)
solution =
  Solve[Flatten@Join[eqs, boundaryConditions],
   Table[Subscript[y, n], {n, 1, numEqs}]];

(*Convert to numerical form*)
numericalSolution = N[solution];

(*Display the result*)
numericalSolution
```

Solve: ≪1≫ is not a quantified system of equations and inequalities.

Solve: ≪1≫ is not a quantified system of equations and inequalities.

Out[ ]=

$\text{Solve}[\{0.985\, y_0 - 1.9999\, y_1 + 1.015\, y_2 == -0.004,$

$0.985\, y_1 - 1.9999\, y_2 + 1.015\, y_3 == -0.004,$
$0.985\, y_2 - 1.9999\, y_3 + 1.015\, y_4 == -0.004,$
$0.985\, y_3 - 1.9999\, y_4 + 1.015\, y_5 == -0.004,$
$0.985\, y_4 - 1.9999\, y_5 + 1.015\, y_6 == -0.004,$
$0.985\, y_5 - 1.9999\, y_6 + 1.015\, y_7 == -0.004,$
$0.985\, y_6 - 1.9999\, y_7 + 1.015\, y_8 == -0.004,$
$0.985\, y_7 - 1.9999\, y_8 + 1.015\, y_9 == -0.004,$
$0.985\, y_8 - 1.9999\, y_9 + 1.015\, y_{10} == -0.004,$
$0.985\, y_9 - 1.9999\, y_{10} + 1.015\, y_{11} == -0.004,$
$0.985\, y_{10} - 1.9999\, y_{11} + 1.015\, y_{12} == -0.004,$
$0.985\, y_{11} - 1.9999\, y_{12} + 1.015\, y_{13} == -0.004,$
$0.985\, y_{12} - 1.9999\, y_{13} + 1.015\, y_{14} == -0.004,$
$0.985\, y_{13} - 1.9999\, y_{14} + 1.015\, y_{15} == -0.004,$
$0.985\, y_{14} - 1.9999\, y_{15} + 1.015\, y_{16} == -0.004,$
$0.985\, y_{15} - 1.9999\, y_{16} + 1.015\, y_{17} == -0.004,$
$0.985\, y_{16} - 1.9999\, y_{17} + 1.015\, y_{18} == -0.004,$
$0.985\, y_{17} - 1.9999\, y_{18} + 1.015\, y_{19} == -0.004,$
$0.985\, y_{18} - 1.9999\, y_{19} + 1.015\, y_{20} == -0.004,$
$0.985\, y_{19} - 1.9999\, y_{20} + 1.015\, y_{21} == -0.004,$
$0.985\, y_{20} - 1.9999\, y_{21} + 1.015\, y_{22} == -0.004,$
$0.985\, y_{21} - 1.9999\, y_{22} + 1.015\, y_{23} == -0.004,$
$0.985\, y_{22} - 1.9999\, y_{23} + 1.015\, y_{24} == -0.004,$
$0.985\, y_{23} - 1.9999\, y_{24} + 1.015\, y_{25} == -0.004,$
$0.985\, y_{24} - 1.9999\, y_{25} + 1.015\, y_{26} == -0.004,$
$0.985\, y_{25} - 1.9999\, y_{26} + 1.015\, y_{27} == -0.004,$

$0.985\,y_{26} - 1.9999\,y_{27} + 1.015\,y_{28} == -0.004,$
$0.985\,y_{27} - 1.9999\,y_{28} + 1.015\,y_{29} == -0.004,$
$0.985\,y_{28} - 1.9999\,y_{29} + 1.015\,y_{30} == -0.004,$
$0.985\,y_{29} - 1.9999\,y_{30} + 1.015\,y_{31} == -0.004,$
$0.985\,y_{30} - 1.9999\,y_{31} + 1.015\,y_{32} == -0.004,$
$0.985\,y_{31} - 1.9999\,y_{32} + 1.015\,y_{33} == -0.004,$
$0.985\,y_{32} - 1.9999\,y_{33} + 1.015\,y_{34} == -0.004,$
$0.985\,y_{33} - 1.9999\,y_{34} + 1.015\,y_{35} == -0.004,$
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$0.985\,y_{50} - 1.9999\,y_{51} + 1.015\,y_{52} == -0.004,$

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$0.985\,y_{53} - 1.9999\,y_{54} + 1.015\,y_{55} == -0.004,$
$0.985\,y_{54} - 1.9999\,y_{55} + 1.015\,y_{56} == -0.004,$
$0.985\,y_{55} - 1.9999\,y_{56} + 1.015\,y_{57} == -0.004,$
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$0.985\,y_{60} - 1.9999\,y_{61} + 1.015\,y_{62} == -0.004,$
$0.985\,y_{61} - 1.9999\,y_{62} + 1.015\,y_{63} == -0.004,$
$0.985\,y_{62} - 1.9999\,y_{63} + 1.015\,y_{64} == -0.004,$
$0.985\,y_{63} - 1.9999\,y_{64} + 1.015\,y_{65} == -0.004,$
$0.985\,y_{64} - 1.9999\,y_{65} + 1.015\,y_{66} == -0.004,$
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$0.985\,y_{75} - 1.9999\,y_{76} + 1.015\,y_{77} == -0.004,$

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$0.985\, y_{94} - 1.9999\, y_{95} + 1.015\, y_{96} == -0.004,$
$0.985\, y_{95} - 1.9999\, y_{96} + 1.015\, y_{97} == -0.004,$
$0.985\, y_{96} - 1.9999\, y_{97} + 1.015\, y_{98} == -0.004,$
$0.985\, y_{97} - 1.9999\, y_{98} + 1.015\, y_{99} == -0.004,$
$0.985\, y_{98} - 1.9999\, y_{99} + 1.015\, y_{100} == -0.004,$
$0.985\, y_{99} - 1.9999\, y_{100} + 1.015\, y_{101} == -0.004,$
$y_0 \to 3.,\ y_{101} \to 20.\}$,

```
{y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12,
 y13, y14, y15, y16, y17, y18, y19, y20, y21, y22, y23,
 y24, y25, y26, y27, y28, y29, y30, y31, y32, y33, y34,
 y35, y36, y37, y38, y39, y40, y41, y42, y43, y44, y45,
 y46, y47, y48, y49, y50, y51, y52, y53, y54, y55, y56,
 y57, y58, y59, y60, y61, y62, y63, y64, y65, y66, y67,
 y68, y69, y70, y71, y72, y73, y74, y75, y76, y77, y78,
 y79, y80, y81, y82, y83, y84, y85, y86, y87, y88, y89,
 y90, y91, y92, y93, y94, y95, y96, y97, y98, y99, y100}]
```

In[ ]:=

```
Reduce[{0.985 * 3 - 1.9999` y1 + 1.015` y2 == -0.004`,
  0.985` y1 - 1.9999` y2 + 1.015` y3 == -0.004`,
  0.985` y2 - 1.9999` y3 + 1.015` y4 == -0.004`,
  0.985` y3 - 1.9999` y4 + 1.015` y5 == -0.004`,
  0.985` y4 - 1.9999` y5 + 1.015` y6 == -0.004`,
  0.985` y5 - 1.9999` y6 + 1.015` y7 == -0.004`,
  0.985` y6 - 1.9999` y7 + 1.015` y8 == -0.004`,
  0.985` y7 - 1.9999` y8 + 1.015` y9 == -0.004`,
  0.985` y8 - 1.9999` y9 + 1.015` y10 == -0.004`,
  0.985` y9 - 1.9999` y10 + 1.015` y11 == -0.004`,
  0.985` y10 - 1.9999` y11 + 1.015` y12 == -0.004`,
  0.985` y11 - 1.9999` y12 + 1.015` y13 == -0.004`,
  0.985` y12 - 1.9999` y13 + 1.015` y14 == -0.004`,
  0.985` y13 - 1.9999` y14 + 1.015` y15 == -0.004`,
  0.985` y14 - 1.9999` y15 + 1.015` y16 == -0.004`,
```

$0.985` y_{15} - 1.9999` y_{16} + 1.015` y_{17} == -0.004`,$
$0.985` y_{16} - 1.9999` y_{17} + 1.015` y_{18} == -0.004`,$
$0.985` y_{17} - 1.9999` y_{18} + 1.015` y_{19} == -0.004`,$
$0.985` y_{18} - 1.9999` y_{19} + 1.015` y_{20} == -0.004`,$
$0.985` y_{19} - 1.9999` y_{20} + 1.015` y_{21} == -0.004`,$
$0.985` y_{20} - 1.9999` y_{21} + 1.015` y_{22} == -0.004`,$
$0.985` y_{21} - 1.9999` y_{22} + 1.015` y_{23} == -0.004`,$
$0.985` y_{22} - 1.9999` y_{23} + 1.015` y_{24} == -0.004`,$
$0.985` y_{23} - 1.9999` y_{24} + 1.015` y_{25} == -0.004`,$
$0.985` y_{24} - 1.9999` y_{25} + 1.015` y_{26} == -0.004`,$
$0.985` y_{25} - 1.9999` y_{26} + 1.015` y_{27} == -0.004`,$
$0.985` y_{26} - 1.9999` y_{27} + 1.015` y_{28} == -0.004`,$
$0.985` y_{27} - 1.9999` y_{28} + 1.015` y_{29} == -0.004`,$
$0.985` y_{28} - 1.9999` y_{29} + 1.015` y_{30} == -0.004`,$
$0.985` y_{29} - 1.9999` y_{30} + 1.015` y_{31} == -0.004`,$
$0.985` y_{30} - 1.9999` y_{31} + 1.015` y_{32} == -0.004`,$
$0.985` y_{31} - 1.9999` y_{32} + 1.015` y_{33} == -0.004`,$
$0.985` y_{32} - 1.9999` y_{33} + 1.015` y_{34} == -0.004`,$
$0.985` y_{33} - 1.9999` y_{34} + 1.015` y_{35} == -0.004`,$
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$0.985` y_{35} - 1.9999` y_{36} + 1.015` y_{37} == -0.004`,$
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$0.985` y_{41} - 1.9999` y_{42} + 1.015` y_{43} == -0.004`,$
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$0.985` y_{44} - 1.9999` y_{45} + 1.015` y_{46} == -0.004`,$
$0.985` y_{45} - 1.9999` y_{46} + 1.015` y_{47} == -0.004`,$
$0.985` y_{46} - 1.9999` y_{47} + 1.015` y_{48} == -0.004`,$
$0.985` y_{47} - 1.9999` y_{48} + 1.015` y_{49} == -0.004`,$
$0.985` y_{48} - 1.9999` y_{49} + 1.015` y_{50} == -0.004`,$
$0.985` y_{49} - 1.9999` y_{50} + 1.015` y_{51} == -0.004`,$
$0.985` y_{50} - 1.9999` y_{51} + 1.015` y_{52} == -0.004`,$
$0.985` y_{51} - 1.9999` y_{52} + 1.015` y_{53} == -0.004`,$
$0.985` y_{52} - 1.9999` y_{53} + 1.015` y_{54} == -0.004`,$
$0.985` y_{53} - 1.9999` y_{54} + 1.015` y_{55} == -0.004`,$
$0.985` y_{54} - 1.9999` y_{55} + 1.015` y_{56} == -0.004`,$
$0.985` y_{55} - 1.9999` y_{56} + 1.015` y_{57} == -0.004`,$
$0.985` y_{56} - 1.9999` y_{57} + 1.015` y_{58} == -0.004`,$
$0.985` y_{57} - 1.9999` y_{58} + 1.015` y_{59} == -0.004`,$
$0.985` y_{58} - 1.9999` y_{59} + 1.015` y_{60} == -0.004`,$
$0.985` y_{59} - 1.9999` y_{60} + 1.015` y_{61} == -0.004`,$
$0.985` y_{60} - 1.9999` y_{61} + 1.015` y_{62} == -0.004`,$
$0.985` y_{61} - 1.9999` y_{62} + 1.015` y_{63} == -0.004`,$
$0.985` y_{62} - 1.9999` y_{63} + 1.015` y_{64} == -0.004`,$
$0.985` y_{63} - 1.9999` y_{64} + 1.015` y_{65} == -0.004`,$
$0.985` y_{64} - 1.9999` y_{65} + 1.015` y_{66} == -0.004`,$

$0.985` y_{65} - 1.9999` y_{66} + 1.015` y_{67} == -0.004`$,
$0.985` y_{66} - 1.9999` y_{67} + 1.015` y_{68} == -0.004`$,
$0.985` y_{67} - 1.9999` y_{68} + 1.015` y_{69} == -0.004`$,
$0.985` y_{68} - 1.9999` y_{69} + 1.015` y_{70} == -0.004`$,
$0.985` y_{69} - 1.9999` y_{70} + 1.015` y_{71} == -0.004`$,
$0.985` y_{70} - 1.9999` y_{71} + 1.015` y_{72} == -0.004`$,
$0.985` y_{71} - 1.9999` y_{72} + 1.015` y_{73} == -0.004`$,
$0.985` y_{72} - 1.9999` y_{73} + 1.015` y_{74} == -0.004`$,
$0.985` y_{73} - 1.9999` y_{74} + 1.015` y_{75} == -0.004`$,
$0.985` y_{74} - 1.9999` y_{75} + 1.015` y_{76} == -0.004`$,
$0.985` y_{75} - 1.9999` y_{76} + 1.015` y_{77} == -0.004`$,
$0.985` y_{76} - 1.9999` y_{77} + 1.015` y_{78} == -0.004`$,
$0.985` y_{77} - 1.9999` y_{78} + 1.015` y_{79} == -0.004`$,
$0.985` y_{78} - 1.9999` y_{79} + 1.015` y_{80} == -0.004`$,
$0.985` y_{79} - 1.9999` y_{80} + 1.015` y_{81} == -0.004`$,
$0.985` y_{80} - 1.9999` y_{81} + 1.015` y_{82} == -0.004`$,
$0.985` y_{81} - 1.9999` y_{82} + 1.015` y_{83} == -0.004`$,
$0.985` y_{82} - 1.9999` y_{83} + 1.015` y_{84} == -0.004`$,
$0.985` y_{83} - 1.9999` y_{84} + 1.015` y_{85} == -0.004`$,
$0.985` y_{84} - 1.9999` y_{85} + 1.015` y_{86} == -0.004`$,
$0.985` y_{85} - 1.9999` y_{86} + 1.015` y_{87} == -0.004`$,
$0.985` y_{86} - 1.9999` y_{87} + 1.015` y_{88} == -0.004`$,
$0.985` y_{87} - 1.9999` y_{88} + 1.015` y_{89} == -0.004`$,
$0.985` y_{88} - 1.9999` y_{89} + 1.015` y_{90} == -0.004`$,
$0.985` y_{89} - 1.9999` y_{90} + 1.015` y_{91} == -0.004`$,

```
  0.985` y90 - 1.9999` y91 + 1.015` y92 == -0.004`,
  0.985` y91 - 1.9999` y92 + 1.015` y93 == -0.004`,
  0.985` y92 - 1.9999` y93 + 1.015` y94 == -0.004`,
  0.985` y93 - 1.9999` y94 + 1.015` y95 == -0.004`,
  0.985` y94 - 1.9999` y95 + 1.015` y96 == -0.004`,
  0.985` y95 - 1.9999` y96 + 1.015` y97 == -0.004`,
  0.985` y96 - 1.9999` y97 + 1.015` y98 == -0.004`,
  0.985` y97 - 1.9999` y98 + 1.015` y99 == -0.004`,
  0.985` y98 - 1.9999` y99 + 1.015` y100 == -0.004`,
  0.985` y99 - 1.9999` y100 + 1.015 * 20 == -0.004`},
 {y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12,
  y13, y14, y15, y16, y17, y18, y19, y20, y21, y22, y23,
  y24, y25, y26, y27, y28, y29, y30, y31, y32, y33, y34,
  y35, y36, y37, y38, y39, y40, y41, y42, y43, y44, y45,
  y46, y47, y48, y49, y50, y51, y52, y53, y54, y55, y56,
  y57, y58, y59, y60, y61, y62, y63, y64, y65, y66, y67,
  y68, y69, y70, y71, y72, y73, y74, y75, y76, y77, y78,
  y79, y80, y81, y82, y83, y84, y85, y86, y87, y88, y89,
  y90, y91, y92, y93, y94, y95, y96, y97, y98, y99, y100}]
```

Out[▫]=

$y_1 == 3.94917$ && $y_2 == 4.86595$ &&
$y_3 == 5.75122$ && $y_4 == 6.60581$ && $y_5 == 7.43055$ &&
$y_6 == 8.22625$ && $y_7 == 8.99367$ && $y_8 == 9.73358$ &&
$y_9 == 10.4467$ && $y_{10} == 11.1338$ && $y_{11} == 11.7956$ &&
$y_{12} == 12.4327$ && $y_{13} == 13.0458$ &&
$y_{14} == 13.6355$ && $y_{15} == 14.2025$ && $y_{16} == 14.7475$ &&

$y_{17} == 15.2709$ && $y_{18} == 15.7734$ && $y_{19} == 16.2556$ &&
$y_{20} == 16.7179$ && $y_{21} == 17.1611$ && $y_{22} == 17.5855$ &&
$y_{23} == 17.9916$ && $y_{24} == 18.3801$ && $y_{25} == 18.7513$ &&
$y_{26} == 19.1058$ && $y_{27} == 19.4439$ && $y_{28} == 19.7662$ &&
$y_{29} == 20.0731$ && $y_{30} == 20.365$ && $y_{31} == 20.6424$ &&
$y_{32} == 20.9055$ && $y_{33} == 21.1549$ && $y_{34} == 21.3909$ &&
$y_{35} == 21.6138$ && $y_{36} == 21.8242$ && $y_{37} == 22.0221$ &&
$y_{38} == 22.2082$ && $y_{39} == 22.3826$ && $y_{40} == 22.5457$ &&
$y_{41} == 22.6978$ && $y_{42} == 22.8392$ && $y_{43} == 22.9703$ &&
$y_{44} == 23.0913$ && $y_{45} == 23.2025$ && $y_{46} == 23.3042$ &&
$y_{47} == 23.3967$ && $y_{48} == 23.4801$ && $y_{49} == 23.5549$ &&
$y_{50} == 23.6212$ && $y_{51} == 23.6792$ && $y_{52} == 23.7293$ &&
$y_{53} == 23.7716$ && $y_{54} == 23.8063$ && $y_{55} == 23.8338$ &&
$y_{56} == 23.8542$ && $y_{57} == 23.8676$ && $y_{58} == 23.8744$ &&
$y_{59} == 23.8747$ && $y_{60} == 23.8687$ && $y_{61} == 23.8565$ &&
$y_{62} == 23.8384$ && $y_{63} == 23.8146$ && $y_{64} == 23.7852$ &&
$y_{65} == 23.7504$ && $y_{66} == 23.7103$ && $y_{67} == 23.6651$ &&
$y_{68} == 23.615$ && $y_{69} == 23.5602$ && $y_{70} == 23.5006$ &&
$y_{71} == 23.4366$ && $y_{72} == 23.3682$ && $y_{73} == 23.2956$ &&
$y_{74} == 23.2189$ && $y_{75} == 23.1382$ && $y_{76} == 23.0538$ &&
$y_{77} == 22.9655$ && $y_{78} == 22.8737$ && $y_{79} == 22.7785$ &&
$y_{80} == 22.6798$ && $y_{81} == 22.5779$ && $y_{82} == 22.4728$ &&
$y_{83} == 22.3647$ && $y_{84} == 22.2536$ && $y_{85} == 22.1397$ &&
$y_{86} == 22.023$ && $y_{87} == 21.9037$ && $y_{88} == 21.7818$ &&
$y_{89} == 21.6574$ && $y_{90} == 21.5306$ && $y_{91} == 21.4015$ &&

$y_{92} == 21.2702$ && $y_{93} == 21.1367$ && $y_{94} == 21.0011$ &&
$y_{95} == 20.8636$ && $y_{96} == 20.7241$ && $y_{97} == 20.5827$ &&
$y_{98} == 20.4395$ && $y_{99} == 20.2947$ && $y_{100} == 20.1481$

In[ ]:=

```
ListLinePlot[
 {3.949167722286919, 4.865951258917841,
  5.7512174544405665, 6.6058106375385375,
  7.430553203339273, 8.226246180672662,
  8.99366978466805, 9.733583955069026,
  10.446728880634987, 11.133825509989085,
  11.795576049262767, 12.432664446878185,
  13.045756865800842, 13.635502143586299,
  14.202532240536362, 14.747462676272084,
  15.27089295502288, 15.773406979923402,
  16.25557345660224, 16.717946286339174,
  17.161064949060602, 17.58545487643568,
  17.991627815329082, 18.380082181859592,
  18.751303406307244, 19.105764269105574,
  19.443925228149364, 19.766234737642286,
  20.073129558703133, 20.365035061943587,
  20.642365522225017, 20.905524405796427,
  21.15490465001048, 21.390888935809336,
  21.613849953167257, 21.824150659671925,
  22.022144532421805, 22.208175813412335,
  22.38257974857916, 22.54568282066237,
```

22.697802976051427, 22.839249845766318,
22.970324960726508, 23.091321961455286,
23.202526802363362, 23.304217950751756,
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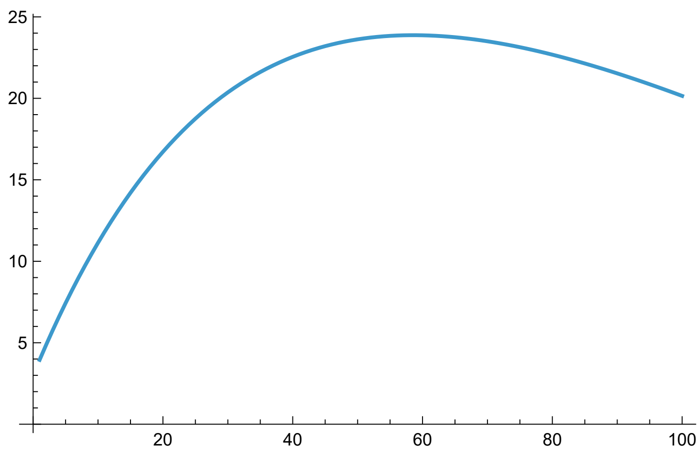
23.554885643246504, 23.621163633092884,
23.67921457263512, 23.72927590661723,
23.77157884886522, 23.806348543672495,
23.833804223013182, 23.854159359691284,
23.867621816530654, 23.87439399170812,
23.874672960329434, 23.868650612345157,
23.856513786901072, 23.838444403215245,
23.81461958807154, 23.785211800017,
23.750388950348302, 23.710314520970268,
23.665147679207255, 23.615043389646182,
23.560152523087933, 23.50062196268184,
23.436594707316058, 23.368209972334746,
23.295603287651176, 23.218906593324,
23.13824833266232, 23.05375354292339,
22.965543943665118, 22.873738022814116,
22.778451120508187, 22.67979551077086,
22.57788048107396, 22.472812409842874,
22.364694841957547, 22.253628562301152,
22.139711667406782, 22.023039635251415,
21.903705393244948, 21.781799384461014,
21.65740963215498, 21.530621802613442,

```
21.40151926637829, 21.27018315788739,
21.13669243357278, 21.001123928456277,
20.863552411281297, 20.724050638218745,
20.58268940518384, 20.439537598799692,
20.294662246042783, 20.1481285626042}]
```

Out[∘]=



In[∘]:=

```
data = {3.949167722286919, 4.865951258917841,
   5.751217454440566 5, 6.6058106375385375,
   7.430553203339273, 8.226246180672662,
   8.99366978466805, 9.733583955069026,
   10.446728880634987, 11.13382509989085,
   11.795576049262767, 12.43266444687818 5,
   13.045756865800842, 13.635502143586299,
   14.202532240536362, 14.747462676272084,
   15.2708929550228 8, 15.773406979923402,
   16.2555734566022 4, 16.717946286339174,
   17.161064949060602, 17.5854548764356 8,
   17.991627815329082, 18.380082181859592,
```

18.751303406307244, 19.105764269105574,
19.443925228149364, 19.766234737642286,
20.073129558703133, 20.365035061943587,
20.642365522225017, 20.905524405796427,
21.15490465001048, 21.390888935809336,
21.613849953167257, 21.824150659671925,
22.022144532421805, 22.208175813412335,
22.38257974857916, 22.5456828206237,
22.697802976051427, 22.839249845766318,
22.970324960726508, 23.091321961455286,
23.202526802363362, 23.304217950751756,
23.396666580670466, 23.480136761765895,
23.554885643246504, 23.621163633092884,
23.67921457263512, 23.7292759066l723,
23.77157884886522, 23.806348543672495,
23.833804223013182, 23.854159359691284,
23.867621816530654, 23.87439399170812,
23.874672960329434, 23.868650612345157,
23.856513786901072, 23.838444403215245,
23.81461958807154, 23.785211800017,
23.750388950348302, 23.710314520970268,
23.665147679207255, 23.615043389646182,
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23.436594707316058, 23.368209972334746,
23.295603287651176, 23.218906593324,

```
23.13824833266232, 23.05375354292339,
22.965543943665118, 22.873738022814116,
22.778451120508187, 22.67979551077086,
22.57788048107396, 22.47281240984287 4,
22.364694841957547, 22.253628562301152,
22.139711667406782, 22.023039635251415,
21.903705393244948, 21.781799384461014,
21.65740963215498, 21.530621802613442,
21.40151926637829, 21.27018315788739,
21.13669243357278, 21.001123928456277,
20.863552411281297, 20.724050638218745,
20.58268940518384, 20.439537598799692,
20.294662246042783, 20.1481285626042};
```

In[∘]:=
```
quadraticFit = Fit[data, {1, x, x^2}, x]
cubicFit = Fit[data, {1, x, x^2, x^3}, x]
```

Out[∘]=

$$5.83615 + 0.590288\, x - 0.00464446\, x^2$$

Out[∘]=

$$3.52534 + 0.858199\, x - 0.011243\, x^2 + 0.0000435548\, x^3$$

In[ ]:=

```
Plot[{3.0629320049862114 + 1.0697424491320113 * x - 0.012664482298342118 * x^2}, {x, 0, 50}]
```

Out[ ]=