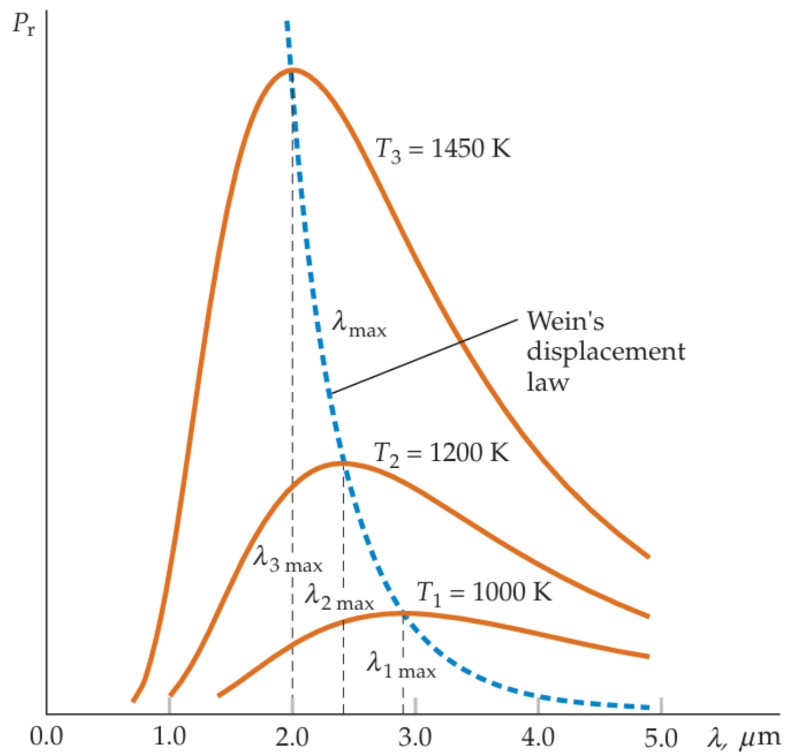
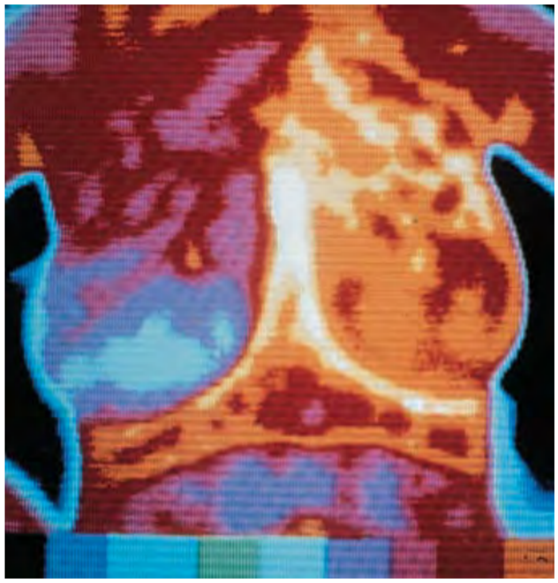
- **BLACKBODY RADIATION**
- Radiation
  - All bodies emit and absorb electromagnetic radiation. When a body and its surrounding have the same temperature, the amount of radiation emitted is equal to the amount of radiation absorbed. The power of body's radiation is proportional to its surface area and to the fourth power of its absolute temperature. Josef Stefan observed this result in the year 1879 but it was Ludwig Boltzmann who derived this finding theoretically after five years. This result is now called **Stefan-Boltzmann law:**
    - $P_r = e\sigma AT^4$ where
    - P_r = power radiated
    - A = surface area
    - σ = universal constant = $5.6703 \times 10^{-8}\ \mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{K}^4)$
    - e = emissivity of the radiating surface. It is unitless constant between 0 and 1 that depends on surface properties of the body
    - Part of electromagnetic radiation on a body is reflected and the other part is absorbed but dark body absorbs most of the radiation.
    - If the body has temperature T_0 it absorbs radiation at a rate given by
    - $P_a = e\sigma AT_o^4$

- If the body at temperature T and the objects at its surrounding have temperature T_o the net power of radiation coming the body is
- $P_{\text{net}} = e\sigma A\left(T^4 - T_o^4\right)$
- If the body emits more radiant energy it becomes cooler but if the surrounding emits more radiant energy the body absorbs radiation and becomes warmer.
- If the body and the surrounding have the same temperature then the body emits and absorbs radiation equally.
- A **blackbody** (or an ideal radiator) is a body that absorbs all radiation on it with emissivity equal to 1.
- It is possible to calculate theoretically the characteristics of the radiation.
- Between 600 °C and 700°C the radiant energy of the object becomes visible as shown in its lightly reddening surface.
- As the temperature increases the surface of the object becomes brightly red, orange , or even yellow at higher temperature.
- THe figure below shows the blackbody power as a function of wavelengths of three different temperatures.



- The wavelength is inversely proportional to the Kelvin temperature at which the power is maximum. This law is also know as Wein's displacement law:
- $\lambda_{max} = \frac{2.898\ \text{mm}\cdot\text{K}}{T}$
- The surface temperature of the stars can be determined using this law. The temperatures in the different regions of the surface of the object can be mapped out using Wien's displacement law.
- A thermograph is a device that continuously records temperature changes over time.
- In medical thermography a thermograph is used to detect heat patterns, specially the cancerous skin, in the human body.



A thermograph was used to detect this cancerous tumor. *(Science Photo Library/ Photo Researchers, Inc.)*

- Classical thermodynamics calculated theoretically the blackbody spectral distributions but Max Planck quantized the energy which resolved the discrepancy with the Rayleigh-Jeans law that has an ultraviolet catastrophe.

- **Sample problem** The maximum power of radiation emitted by the Sun surface has approximately a wavelength of 500 nm. (a) What is the surface temperature of the Sun considering that it is a blackbody emitter? (b) Find the wavelength λ_{max} of a blackbody at 100°C. (c) Compare both wavelengths. Compare also their temperatures.

- SOLUTION

  - (a) We use the Wien's law
    $\lambda_{max} = \frac{(2.898\,\text{mm}\cdot\text{K})}{T} \to T = \frac{2.989\,\text{mm}\cdot\text{K}}{500\,\text{nm}} \approx 5800\,\text{K}$

- (b) $\lambda_{max} = \frac{2.898\ \text{mm}\cdot\text{K}}{(100+273.15)\ \text{K}} = 0.000007766$
  $\approx 7.77 \times 10^{-6} = 7.77\ \mu\text{m}$.
- (c) 0.000007766/500 nm ≈ 16. The wavelength at 100°C is almost 16 times greater than the wavelength near the surface of the Sun.
- Also their temperatures comparison is 5800 K/273.15 K ≈ 16. The temperature at the sun surface is 16 times greater than the temperature of the blackbody at 100°C.
- The Sun maximum wavelength is visible in the spectrum so the Sun's radiation spectrum is fair.

- The infrared maximum wavelength is longer than the wavelength visible to the eyes.
- The human skins although they are not black to our eyes can be considered as blackbodies for absorption and emission so the emissivity of the skin 1.0.
- RADIATION FROM HUMAN BODY
- **Problem**: Suppose a naked person as a blackbody has a total surface area of 1.3 m^2 inside a room where the temperature is 22°C. If the temperature of the person is 34°C, find the rate of heat loss that radiates from the person.
- **SOLUTION**
  - Substitue $T_0 = (22 + 273.15)\ \text{K} = 295.15\ \text{K}$,
    $T = (34 + 273.15)\ K = 307.15\ K$,
    Stefan constant $\sigma = 5.6703 \times 10^{-8}\ \text{W}/(\text{m}^2 \cdot \text{K}^4)$,
    and $e = 1$ in

- $P_{\text{net}} = e\sigma A(T^4 - T_0^4) = 96.67 \approx 97$ W.
- By wearing clothing with low thermal conductivity we protect our selves from this loss of heat.

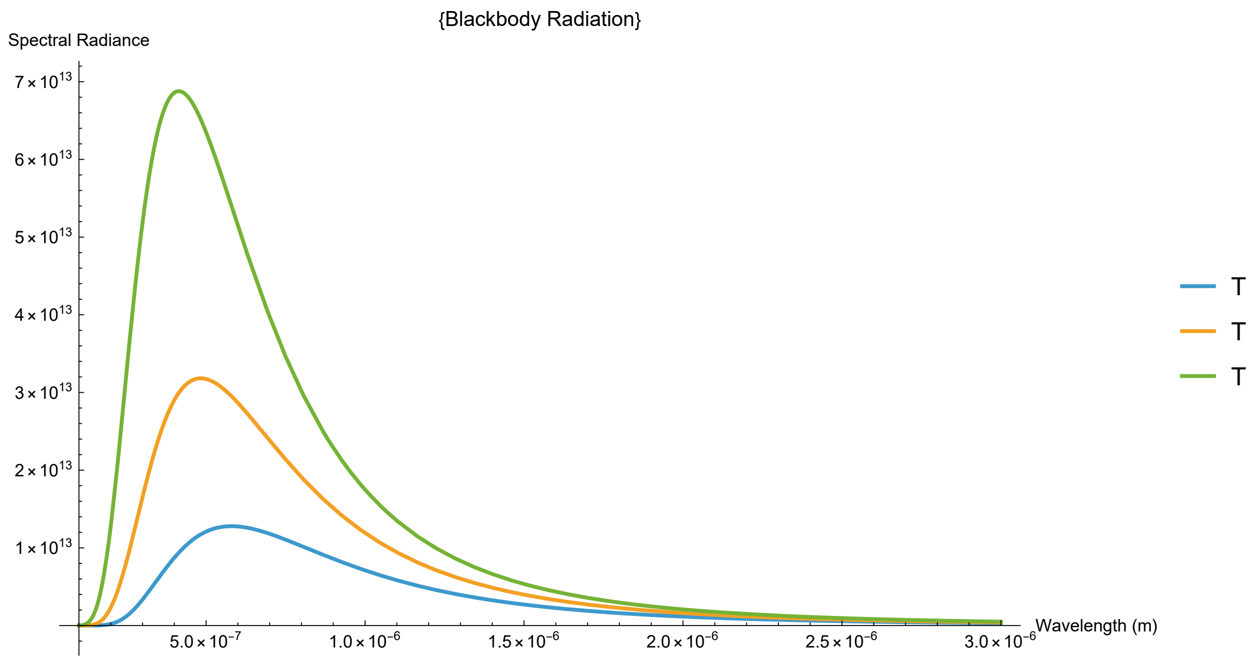
- If the temperature T of the object is approximate equal to the temperature T_0 of the surroundings then
  - $P_{\text{net}} = e\sigma A(T^4 - T_0^4) = e\sigma A\,(T^2 + T_0^2)\,(T^2 - T_0^2)$. By replacing T with T_0 we have
  - $e\sigma A\,(T_0^2 + T_0^2)\,(T_0 + T_0)\,(T - T_0) = 4\,e\sigma A T_0^3 \Delta T$ or we have
  - $\Delta P_r \approx \frac{dP_r}{dT}\,|_{T=T_0}\,(T - T_0) = 4\,e\sigma A T_0^3 \Delta T$

- ---

In[58]:=

```
h = 6.626 * 10^-34;
c = 300 000 000;
k = 1.381 * 10^-23;
Plot[{(2 * h * c^2)/(λ^5 * (Exp[(h*c)/(λ*k*5000)] - 1)), (2 * h * c^2)/(λ^5 * (Exp[(h*c)/(λ*k*6000)] - 1)), (2 * h * c^2)/(λ^5 * (Exp[(h*c)/(λ*k*7000)] - 1))},
 {λ, 100 * 10^-9, 3000 * 10^-9}, PlotRange → All,
 ImageSize → {600}, PlotLabel → {"Blackbody Radiation"},
 PlotLegends → {"T = 5000 K", "T = 6000 K", "T = 7000 K"},
 AxesLabel → {"Wavelength (m)", "Spectral Radiance"}]
```

Out[61]=



■ PLANCK'S LAW

Planck's Law describes the spectral radiance of a blackbody:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

where:

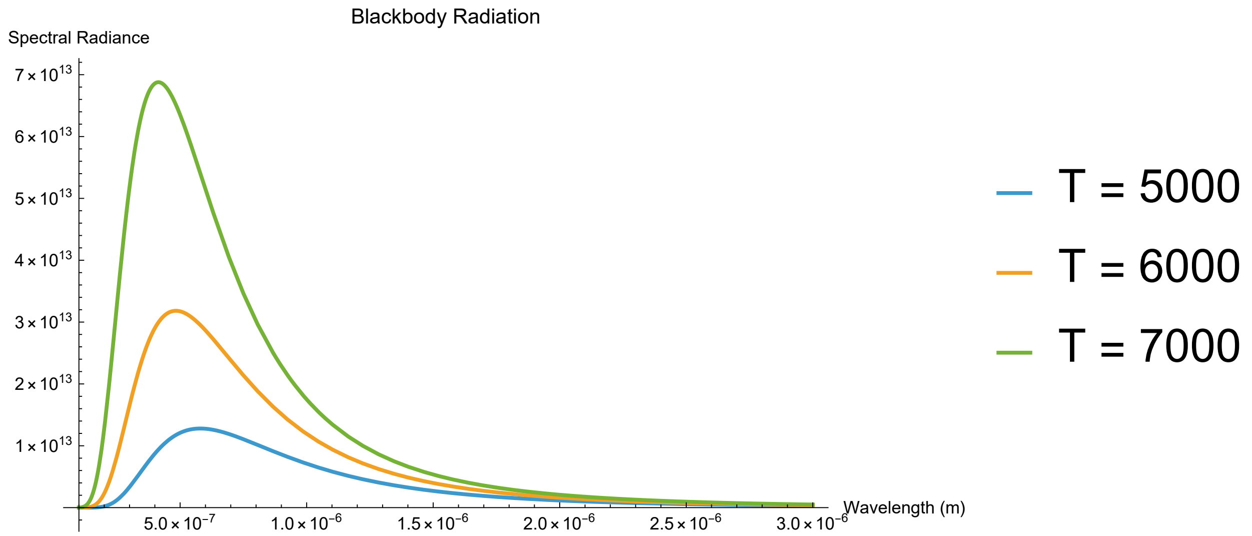
- $B(\lambda, T)$ is the spectral radiance (power per unit area per unit wavelength per unit solid angle)
- λ is the wavelength
- T is the temperature in Kelvin
- h is Planck's constant ($6.626 \times 10^{-34} Js$)
- c is the speed of light ($3 \times 10^8 m/s$)
- k_B is Boltzmann's constant ($1.381 \times 10^{-23} J/K$)

In[62]:=
```
h = 6.626 * 10^-34;
c = 3 * 10^8;
kB = 1.381 * 10^-23;

planck[λ_, T_] :=
  (2 h c^2) / (λ^5 (Exp[(h c) / (λ kB T)] - 1));

Plot[{planck[λ, 5000], planck[λ, 6000],
  planck[λ, 7000]}, {λ, 100 * 10^-9, 3000 * 10^-9},
 PlotRange → All, AxesLabel → {"Wavelength (m)",
   "Spectral Radiance"}, PlotLegends →
  {"T = 5000 K", "T = 6000 K", "T = 7000 K"},
 PlotLabel → "Blackbody Radiation",
 ImageSize → {500}]
```

Out[66]=



- Rayleigh-Jeans law

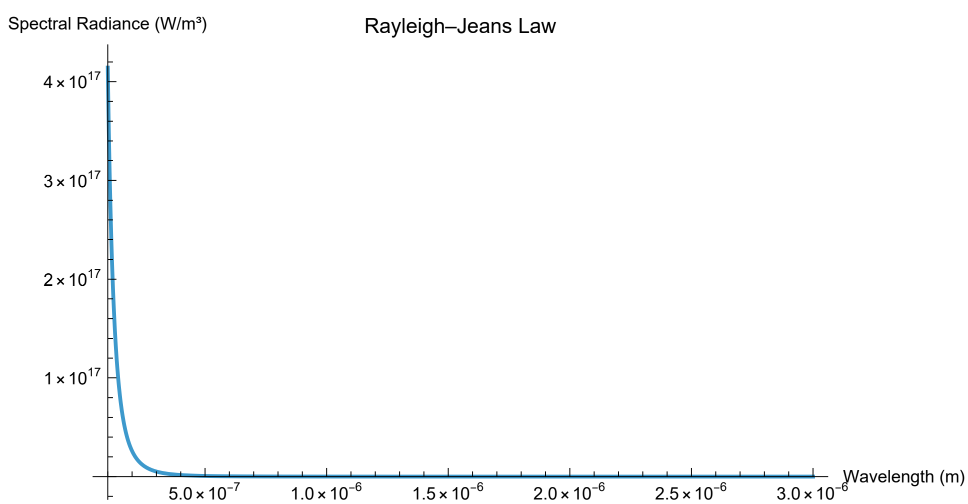
In[67]:=

```
c = 300 000 000;
k = 1.381 * 10^-23;
```

In[69]:=

```
Plot[(2 * c * k * 5000)/λ^4, {λ, 100 * 10^-9, 3000 * 10^-9},
 PlotRange → All, PlotLabel → "Rayleigh-Jeans Law",
 AxesLabel → {"Wavelength (m)", "Spectral Radiance (W/m³)"}, ImageSize → {500}]
```

Out[69]=



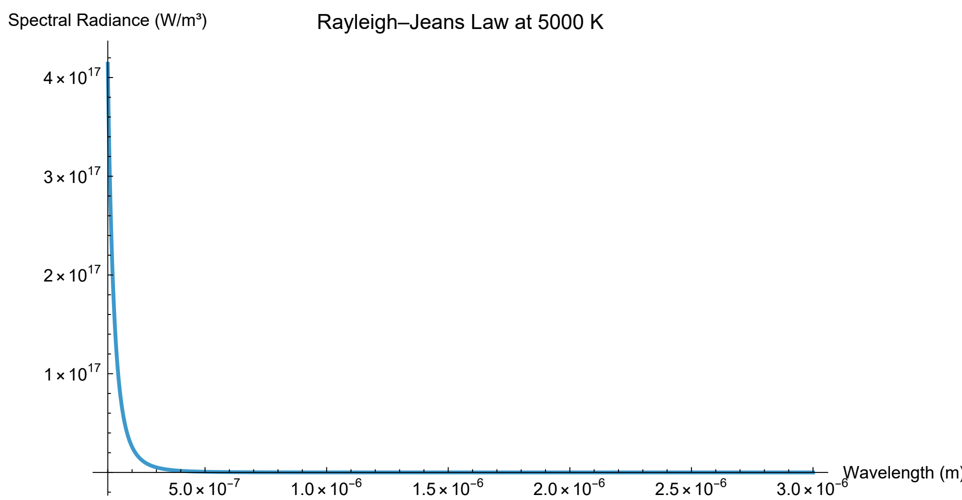
In[70]:=
```
(*Define physical constants in SI units*)
c = Quantity["SpeedOfLight"] // UnitConvert // QuantityMagnitude;
kB = Quantity["BoltzmannConstant"] // UnitConvert // QuantityMagnitude;

(*Define the Rayleigh-Jeans law function for spectral radiance*)
rayleighJeans[λ_, T_] := (2 * c * kB * T) / λ^4;

(*Choose a temperature,e.g.,5000 Kelvin*)
T0 = 5000;

(*Plot the Rayleigh-Jeans spectral radiance for wavelengths from 100 nm to 3000 nm*)
Plot[rayleighJeans[λ, T0], {λ, 100 * 10^-9, 3000 * 10^-9},
 AxesLabel → {"Wavelength (m)", "Spectral Radiance (W/m³)"},
 PlotLabel → "Rayleigh-Jeans Law at " <> ToString[T0] <> " K",
 PlotRange → All, ImageSize → {500}]
```

Out[74]=



- c = Quantity[“SpeedOfLight”] // UnitConvert // QuantityMagnitude // N
  kB = Quantity[“BoltzmannConstant”] // UnitConvert // QuantityMagnitude // N

In[75]:=
```
c = Quantity["SpeedOfLight"] // UnitConvert // QuantityMagnitude // N
kB = Quantity["BoltzmannConstant"] // UnitConvert // QuantityMagnitude // N
```

Out[75]=

2.99792×10^{8}

Out[76]=

1.38065×10^{-23}

- DIFFERENCE BETWEEN RAYLEIGH-JEANS LAW AND PLANCK’S LAW
  - The **Rayleigh-Jeans Law** and Planck’s Law both describe the spectral radiance of blackbody radiation, but they differ in their assumptions and accuracy at different wavelengths.
  - Rayleigh-Jeans Law is a classical approximation for blackbody radiation, given by:

    - $B_\lambda(\lambda, T) = \frac{2 c k_B T}{\lambda^4}$ where
    - B_λ is the spectral radiance,
    - c is the speed of light,
    - k_B is Biltzmann' s constant,
    - T is the temperature in Kelvin,
    - λ is the wavelength
    - Valid at long wavelength (low frequencies).
    - Fails at short wavelengths (high frequencies), leading to the **ultraviolet catastrophe,** where the predicted energy diverges to infinity.
- **Planck's law**
    - Planck's law correctly describes blackbody radiation by incorporating quantum mechanics
    - $B_\lambda(\lambda, T) = \frac{2 h c^2}{\lambda^5\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)}$ where
    - h is the Planck' s constant equal to 6.626×10^{-34} J·s,
    - Accurate at all wavelengths, solving the ultraviolet catastrophe
    - Reduces to the Rayleigh-Jeans law at long wavelengths (when is large, $e^{\mathrm{hc}/(\lambda \mathrm{k}_B T)} \approx 1 + \mathrm{hc}/(\lambda \mathrm{k}_B T)$, leading to the classical result).

    - Reduces to Wien'sLaw at short wavelengths (where only high-energy photons contribute).
- Key Difference

In[57]:=
```
Grid[{{"Feature", "Rayleigh-Jeans Law", "Planck's Law"},
  {"Based on", "Classical physics", "Quantum Mechanics"},
  {"Validity Range", "Long wavelengths (low frequency)", "All wavelengths"},
  {"Ultraviolet Catastrophe", "Yes (diverges at short wavelengths)",
   "No (finite energy at all wavelengths)"},
  {"Derived from", "Equipartition Theorem", "Photon energy quantization"}},
 Frame → All, Background → {{1 → LightGray}, None}]
```

(* **Key Differences** *)

Out[57]=

Feature	Rayleigh–Jeans Law	Planck’s Law
Based on	Classical physics	Quantum Mechanics
Validity Range	Long wavelengths (low frequency)	All wavelengths
Ultraviolet Catastrophe	Yes (diverges at short wavelengths)	No (finite energy at all wavelengths)
Derived from	Equipartition Theorem	Photon energy quantization

- Planck’s law corrected the flaws of the Rayleigh-Jeans law and led to the development of quantum mechanics.