- Our problem here is to solve the nonhomogeneous differential equation:

$\frac{d^2 y}{dx^2} - 5\, y =$ $= 1.$

x^2 subject to the boundary conditions $y(0) = 2$ and $y(10)$

- By Mathematica we have

In[∘]:=
```
solution = NDSolve[{y''[x] - 5 y[x] == x^2,
   y[0] == 2, y[10] == 1}, y, {x, 0, 10}]
```

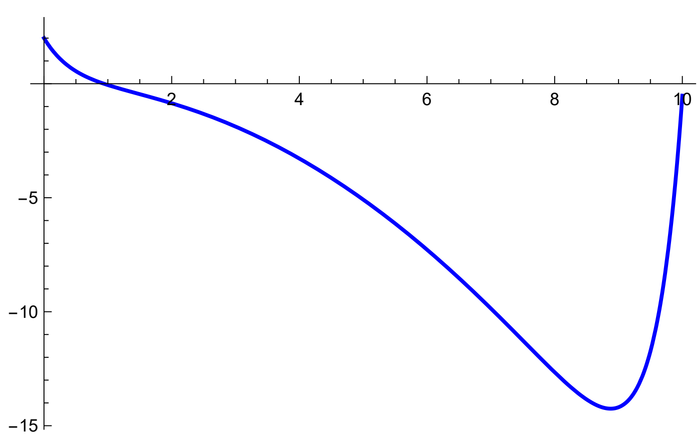
NDSolve: The equations derived from the boundary conditions are numerically ill−conditioned. The boundary conditions may not be sufficient to uniquely define a solution. If a solution is computed, it may match the boundary conditions poorly.

NDSolve: The scaled boundary value residual error of 265.117078727927` indicates that the boundary values are not satisfied to specified tolerances. Returning the best solution found.

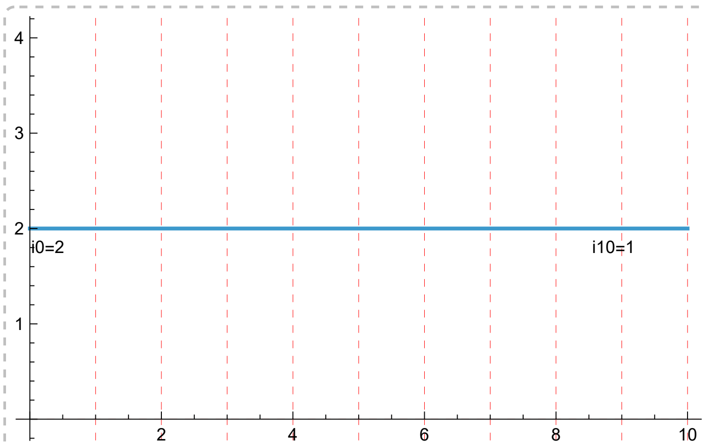
Out[∘]=
```
{{y →
   InterpolatingFunction[ Domain: {{0., 10.}} Output: scalar ]}}
```

In[∘]:=
```
Plot[Evaluate[y[x] /. solution],
 {x, 0, 10}, PlotStyle → Blue]
```

Out[∘]=



```
Plot[2, {x, 0, 10},
 GridLines → {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {0}},
 GridLinesStyle → Directive[Red, Dashed]]
(* Step size = (9-1)/(9-1) =1. i0=2 (boundary condition),
i1=3,i2=4,i3=5,i4=6,i5=7,i6=8,i7=9,
i8=10,i9=11,i10=1(boundary condition) *)
```



- Second derivative approximation: $y''(x) \approx \dfrac{y(x+h) - 2y(x) + y(x-h)}{h^2}$
- $\frac{y_{i+1} - 2\,y_i + y_{i-1}}{h^2}$ - 5 $y_i = x_i^2$. For h = 1, then $y_{i-1} - 7\,y_i + y_{i+1} = x_i^2$
- For i=1: $y_0 - 7\,y_1 + y_2 = x_1^2$. For y0=2 then $2 - 7\,y_1 + y_2 = 1^2 = 1$ >> $-7\,y_1 + y_2 = -1$
- For i =2: $y_1 - 7\,y_2 + y_3 = x_2^2 = 4$. Our last equation is for $y_{10} = 1$,
- $y_8 - 7\,y_9 + y_{10} = x_9^2$ >> $y_8 - 7\,y_9 + 1 = 81$>> $y_8 - 7\,y_9 = 80$.
- We have 9 variables y1, y2, y3, y4, y5, y6, y7, y8, and y9.

In[ ]:=
```
eqs = Table[Subscript[y, i - 1] - 7 Subscript[y, i] +
    Subscript[y, i + 1] == i^2, {i, 2, 8}];

eqs
```

Out[ ]=

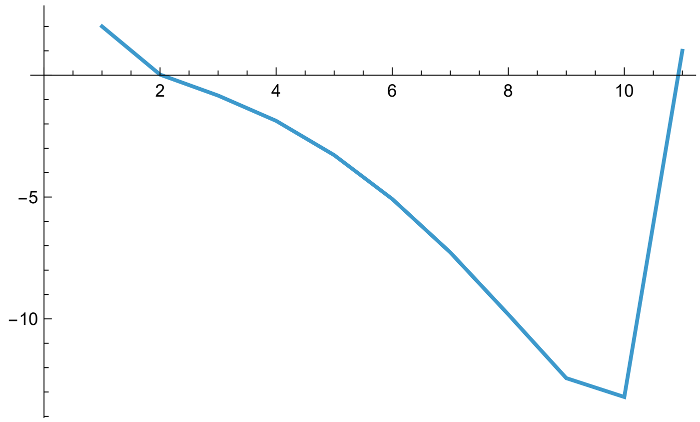
$$\{y_1 - 7 y_2 + y_3 == 4,\ y_2 - 7 y_3 + y_4 == 9,\ y_3 - 7 y_4 + y_5 == 16,\ y_4 - 7 y_5 + y_6 == 25,\ y_5 - 7 y_6 + y_7 == 36,\ y_6 - 7 y_7 + y_8 == 49,\ y_7 - 7 y_8 + y_9 == 64\}$$

In[ ]:=
```
NSolve[{2 - 7 * y1 + y2 == 1,
   y1 - 7 y2 + y3 == 4, y2 - 7 y3 + y4 == 9,
   y3 - 7 y4 + y5 == 16, y4 - 7 y5 + y6 == 25,
   y5 - 7 y6 + y7 == 36, y6 - 7 y7 + y8 == 49,
   y7 - 7 y8 + y9 == 64, y8 - 7 y9 + 1 == 81},
  {y1, y2, y3, y4, y5, y6, y7, y8, y9}]
```

Out[ ]=

$$\{\{y_1 \to 0.0234685,\ y_2 \to -0.83572,\ y_3 \to -1.87351,\ y_4 \to -3.27885,\ y_5 \to -5.07847,\ y_6 \to -7.27043,\ y_7 \to -9.81453,\ y_8 \to -12.4313,\ y_9 \to -13.2045\}\}$$

In[ ]:=
```
ListLinePlot[{2, 0.023468528176149982, -0.8357203027669501,
  -1.8735106475448002, -3.2788542300466537, -5.0784689627817805, -7.270428509425812,
  -9.814530603198902, -12.431285712966506, -13.204469387566643, 1}]
```
Out[ ]=



- Our solution becomes more accurate if the step size is for example $\frac{10-0}{101-1}$ =0.1 where y_0 =2 and y_{101} = 1.
- In our example $y_{i+1} - \left(2 + 5\,h^2\right) y_i + y_{i-1} = h^2\, x_i^2$. For $h = 0.1$ then
- $y_{i-1} - 2.05\, y_i + y_{i+1} = 0.01 x_i^2$.
- The number of equations is 100, that is, from equation $2 - 2.05\, y_1 + y_2 = 0.01 x_1^2$ where x_1= 1 and $y_0 = 2$ to equation $y_{99} - 2.05\, y_{100} + 1 = (0.1)^2\, x_{100}^2$= $(0.1)^2\, (100)^2$

In[•]:=
```
eqs = Table[Subscript[y, i - 1] -
    2.05 Subscript[y, i] + Subscript[y, i + 1] ==
   0.01 * i^2, {i, 1, 100}];

eqs
```

Out[•]=

$\{y_0 - 2.05\,y_1 + y_2 == 0.01,\ y_1 - 2.05\,y_2 + y_3 == 0.04,\ y_2 - 2.05\,y_3 + y_4 == 0.09,$
$y_3 - 2.05\,y_4 + y_5 == 0.16,\ y_4 - 2.05\,y_5 + y_6 == 0.25,\ y_5 - 2.05\,y_6 + y_7 == 0.36,\ y_6 - 2.05\,y_7 + y_8 == 0.49,$
$y_7 - 2.05\,y_8 + y_9 == 0.64,\ y_8 - 2.05\,y_9 + y_{10} == 0.81,\ y_9 - 2.05\,y_{10} + y_{11} == 1.,$
$y_{10} - 2.05\,y_{11} + y_{12} == 1.21,\ y_{11} - 2.05\,y_{12} + y_{13} == 1.44,\ y_{12} - 2.05\,y_{13} + y_{14} == 1.69,$
$y_{13} - 2.05\,y_{14} + y_{15} == 1.96,\ y_{14} - 2.05\,y_{15} + y_{16} == 2.25,\ y_{15} - 2.05\,y_{16} + y_{17} == 2.56,$
$y_{16} - 2.05\,y_{17} + y_{18} == 2.89,\ y_{17} - 2.05\,y_{18} + y_{19} == 3.24,\ y_{18} - 2.05\,y_{19} + y_{20} == 3.61,$
$y_{19} - 2.05\,y_{20} + y_{21} == 4.,\ y_{20} - 2.05\,y_{21} + y_{22} == 4.41,\ y_{21} - 2.05\,y_{22} + y_{23} == 4.84,$
$y_{22} - 2.05\,y_{23} + y_{24} == 5.29,\ y_{23} - 2.05\,y_{24} + y_{25} == 5.76,\ y_{24} - 2.05\,y_{25} + y_{26} == 6.25,$
$y_{25} - 2.05\,y_{26} + y_{27} == 6.76,\ y_{26} - 2.05\,y_{27} + y_{28} == 7.29,\ y_{27} - 2.05\,y_{28} + y_{29} == 7.84,$
$y_{28} - 2.05\,y_{29} + y_{30} == 8.41,\ y_{29} - 2.05\,y_{30} + y_{31} == 9.,\ y_{30} - 2.05\,y_{31} + y_{32} == 9.61,$
$y_{31} - 2.05\,y_{32} + y_{33} == 10.24,\ y_{32} - 2.05\,y_{33} + y_{34} == 10.89,\ y_{33} - 2.05\,y_{34} + y_{35} == 11.56,$
$y_{34} - 2.05\,y_{35} + y_{36} == 12.25,\ y_{35} - 2.05\,y_{36} + y_{37} == 12.96,\ y_{36} - 2.05\,y_{37} + y_{38} == 13.69,$
$y_{37} - 2.05\,y_{38} + y_{39} == 14.44,\ y_{38} - 2.05\,y_{39} + y_{40} == 15.21,\ y_{39} - 2.05\,y_{40} + y_{41} == 16.,$
$y_{40} - 2.05\,y_{41} + y_{42} == 16.81,\ y_{41} - 2.05\,y_{42} + y_{43} == 17.64,\ y_{42} - 2.05\,y_{43} + y_{44} == 18.49,$
$y_{43} - 2.05\,y_{44} + y_{45} == 19.36,\ y_{44} - 2.05\,y_{45} + y_{46} == 20.25,\ y_{45} - 2.05\,y_{46} + y_{47} == 21.16,$
$y_{46} - 2.05\,y_{47} + y_{48} == 22.09,\ y_{47} - 2.05\,y_{48} + y_{49} == 23.04,\ y_{48} - 2.05\,y_{49} + y_{50} == 24.01,$
$y_{49} - 2.05\,y_{50} + y_{51} == 25.,\ y_{50} - 2.05\,y_{51} + y_{52} == 26.01,\ y_{51} - 2.05\,y_{52} + y_{53} == 27.04,$
$y_{52} - 2.05\,y_{53} + y_{54} == 28.09,\ y_{53} - 2.05\,y_{54} + y_{55} == 29.16,\ y_{54} - 2.05\,y_{55} + y_{56} == 30.25,$
$y_{55} - 2.05\,y_{56} + y_{57} == 31.36,\ y_{56} - 2.05\,y_{57} + y_{58} == 32.49,\ y_{57} - 2.05\,y_{58} + y_{59} == 33.64,$
$y_{58} - 2.05\,y_{59} + y_{60} == 34.81,\ y_{59} - 2.05\,y_{60} + y_{61} == 36.,\ y_{60} - 2.05\,y_{61} + y_{62} == 37.21,$
$y_{61} - 2.05\,y_{62} + y_{63} == 38.44,\ y_{62} - 2.05\,y_{63} + y_{64} == 39.69,\ y_{63} - 2.05\,y_{64} + y_{65} == 40.96,$
$y_{64} - 2.05\,y_{65} + y_{66} == 42.25,\ y_{65} - 2.05\,y_{66} + y_{67} == 43.56,\ y_{66} - 2.05\,y_{67} + y_{68} == 44.89,$
$y_{67} - 2.05\,y_{68} + y_{69} == 46.24,\ y_{68} - 2.05\,y_{69} + y_{70} == 47.61,\ y_{69} - 2.05\,y_{70} + y_{71} == 49.,$
$y_{70} - 2.05\,y_{71} + y_{72} == 50.41,\ y_{71} - 2.05\,y_{72} + y_{73} == 51.84,\ y_{72} - 2.05\,y_{73} + y_{74} == 53.29,$
$y_{73} - 2.05\,y_{74} + y_{75} == 54.76,\ y_{74} - 2.05\,y_{75} + y_{76} == 56.25,\ y_{75} - 2.05\,y_{76} + y_{77} == 57.76,$
$y_{76} - 2.05\,y_{77} + y_{78} == 59.29,\ y_{77} - 2.05\,y_{78} + y_{79} == 60.84,\ y_{78} - 2.05\,y_{79} + y_{80} == 62.41,$
$y_{79} - 2.05\,y_{80} + y_{81} == 64.,\ y_{80} - 2.05\,y_{81} + y_{82} == 65.61,\ y_{81} - 2.05\,y_{82} + y_{83} == 67.24,$
$y_{82} - 2.05\,y_{83} + y_{84} == 68.89,\ y_{83} - 2.05\,y_{84} + y_{85} == 70.56,\ y_{84} - 2.05\,y_{85} + y_{86} == 72.25,$
$y_{85} - 2.05\,y_{86} + y_{87} == 73.96,\ y_{86} - 2.05\,y_{87} + y_{88} == 75.69,\ y_{87} - 2.05\,y_{88} + y_{89} == 77.44,$
$y_{88} - 2.05\,y_{89} + y_{90} == 79.21,\ y_{89} - 2.05\,y_{90} + y_{91} == 81.,\ y_{90} - 2.05\,y_{91} + y_{92} == 82.81,$
$y_{91} - 2.05\,y_{92} + y_{93} == 84.64,\ y_{92} - 2.05\,y_{93} + y_{94} == 86.49,\ y_{93} - 2.05\,y_{94} + y_{95} == 88.36,$
$y_{94} - 2.05\,y_{95} + y_{96} == 90.25,\ y_{95} - 2.05\,y_{96} + y_{97} == 92.16,\ y_{96} - 2.05\,y_{97} + y_{98} == 94.09,$
$y_{97} - 2.05\,y_{98} + y_{99} == 96.04,\ y_{98} - 2.05\,y_{99} + y_{100} == 98.01,\ y_{99} - 2.05\,y_{100} + y_{101} == 100.\}$

In[ ]:=

```
NSolve[{2 - 2.05` y1 + y2 == 0.01`, y1 - 2.05` y2 + y3 == 0.04`,
  y2 - 2.05` y3 + y4 == 0.09`, y3 - 2.05` y4 + y5 == 0.16`, y4 - 2.05` y5 + y6 == 0.25`,
  y5 - 2.05` y6 + y7 == 0.36`, y6 - 2.05` y7 + y8 == 0.49`, y7 - 2.05` y8 + y9 == 0.64`,
  y8 - 2.05` y9 + y10 == 0.81`, y9 - 2.05` y10 + y11 == 1.`, y10 - 2.05` y11 + y12 == 1.21`,
  y11 - 2.05` y12 + y13 == 1.44`, y12 - 2.05` y13 + y14 == 1.69`, y13 - 2.05` y14 + y15 == 1.96`,
  y14 - 2.05` y15 + y16 == 2.25`, y15 - 2.05` y16 + y17 == 2.56`, y16 - 2.05` y17 + y18 == 2.89`,
  y17 - 2.05` y18 + y19 == 3.24`, y18 - 2.05` y19 + y20 == 3.61`, y19 - 2.05` y20 + y21 == 4.`,
  y20 - 2.05` y21 + y22 == 4.41`, y21 - 2.05` y22 + y23 == 4.84`, y22 - 2.05` y23 + y24 == 5.29`,
  y23 - 2.05` y24 + y25 == 5.76`, y24 - 2.05` y25 + y26 == 6.25`, y25 - 2.05` y26 + y27 == 6.76`,
  y26 - 2.05` y27 + y28 == 7.29`, y27 - 2.05` y28 + y29 == 7.84`, y28 - 2.05` y29 + y30 == 8.41`,
  y29 - 2.05` y30 + y31 == 9.`, y30 - 2.05` y31 + y32 == 9.61`, y31 - 2.05` y32 + y33 == 10.24`,
  y32 - 2.05` y33 + y34 == 10.89`, y33 - 2.05` y34 + y35 == 11.56`, y34 - 2.05` y35 + y36 == 12.25`,
  y35 - 2.05` y36 + y37 == 12.96`, y36 - 2.05` y37 + y38 == 13.69`, y37 - 2.05` y38 + y39 == 14.44`,
  y38 - 2.05` y39 + y40 == 15.21`, y39 - 2.05` y40 + y41 == 16.`, y40 - 2.05` y41 + y42 == 16.81`,
  y41 - 2.05` y42 + y43 == 17.64`, y42 - 2.05` y43 + y44 == 18.490000000000002`,
  y43 - 2.05` y44 + y45 == 19.36`, y44 - 2.05` y45 + y46 == 20.25`, y45 - 2.05` y46 + y47 == 21.16`,
  y46 - 2.05` y47 + y48 == 22.09`, y47 - 2.05` y48 + y49 == 23.04`, y48 - 2.05` y49 + y50 == 24.01`,
  y49 - 2.05` y50 + y51 == 25.`, y50 - 2.05` y51 + y52 == 26.01`, y51 - 2.05` y52 + y53 == 27.04`,
  y52 - 2.05` y53 + y54 == 28.09`, y53 - 2.05` y54 + y55 == 29.16`, y54 - 2.05` y55 + y56 == 30.25`,
  y55 - 2.05` y56 + y57 == 31.36`, y56 - 2.05` y57 + y58 == 32.49`, y57 - 2.05` y58 + y59 == 33.64`,
  y58 - 2.05` y59 + y60 == 34.81`, y59 - 2.05` y60 + y61 == 36.`, y60 - 2.05` y61 + y62 == 37.21`,
  y61 - 2.05` y62 + y63 == 38.44`, y62 - 2.05` y63 + y64 == 39.69`, y63 - 2.05` y64 + y65 == 40.96`,
  y64 - 2.05` y65 + y66 == 42.25`, y65 - 2.05` y66 + y67 == 43.56`, y66 - 2.05` y67 + y68 == 44.89`,
  y67 - 2.05` y68 + y69 == 46.24`, y68 - 2.05` y69 + y70 == 47.61`, y69 - 2.05` y70 + y71 == 49.`,
  y70 - 2.05` y71 + y72 == 50.410000000000004`, y71 - 2.05` y72 + y73 == 51.84`,
  y72 - 2.05` y73 + y74 == 53.29`, y73 - 2.05` y74 + y75 == 54.76`,
  y74 - 2.05` y75 + y76 == 56.25`, y75 - 2.05` y76 + y77 == 57.76`, y76 - 2.05` y77 + y78 == 59.29`,
  y77 - 2.05` y78 + y79 == 60.84`, y78 - 2.05` y79 + y80 == 62.410000000000004`,
  y79 - 2.05` y80 + y81 == 64.`, y80 - 2.05` y81 + y82 == 65.61`, y81 - 2.05` y82 + y83 == 67.24`,
  y82 - 2.05` y83 + y84 == 68.89`, y83 - 2.05` y84 + y85 == 70.56`, y84 - 2.05` y85 + y86 == 72.25`,
  y85 - 2.05` y86 + y87 == 73.96000000000001`, y86 - 2.05` y87 + y88 == 75.69`,
  y87 - 2.05` y88 + y89 == 77.44`, y88 - 2.05` y89 + y90 == 79.21000000000001`,
  y89 - 2.05` y90 + y91 == 81.`, y90 - 2.05` y91 + y92 == 82.81`, y91 - 2.05` y92 + y93 == 84.64`,
  y92 - 2.05` y93 + y94 == 86.49`, y93 - 2.05` y94 + y95 == 88.36`,
  y94 - 2.05` y95 + y96 == 90.25`, y95 - 2.05` y96 + y97 == 92.16`, y96 - 2.05` y97 + y98 == 94.09`,
  y97 - 2.05` y98 + y99 == 96.04`, y98 - 2.05` y99 + y100 == 98.01`, y99 - 2.05` y100 + 1 == 100},
 {y1, y2, y3, y4, y5, y6, y7, y8, y9, y10, y11, y12, y13, y14, y15, y16, y17, y18, y19, y20, y21,
  y22, y23, y24, y25, y26, y27, y28, y29, y30, y31, y32, y33, y34, y35, y36, y37, y38, y39, y40, y41,
  y42, y43, y44, y45, y46, y47, y48, y49, y50, y51, y52, y53, y54, y55, y56, y57, y58, y59, y60, y61,
  y62, y63, y64, y65, y66, y67, y68, y69, y70, y71, y72, y73, y74, y75, y76, y77, y78, y79, y80, y81,
  y82, y83, y84, y85, y86, y87, y88, y89, y90, y91, y92, y93, y94, y95, y96, y97, y98, y99, y100}]
```

Out[ ]=

$\{\{y_1 \to -0.2, y_2 \to -2.4, y_3 \to -4.68, y_4 \to -7.104, y_5 \to -9.7232, y_6 \to -12.5786, y_7 \to -15.7028,$
$y_8 \to -19.1223, y_9 \to -22.8578, y_{10} \to -26.9263, y_{11} \to -31.341, y_{12} \to -36.1128, y_{13} \to -41.2502,$
$y_{14} \to -46.7602, y_{15} \to -52.6481, y_{16} \to -58.9185, y_{17} \to -65.5748, y_{18} \to -72.6198,$
$y_{19} \to -80.0559, y_{20} \to -87.8847, y_{21} \to -96.1077, y_{22} \to -104.726, y_{23} \to -113.741,$
$y_{24} \to -123.153, y_{25} \to -132.962, y_{26} \to -143.17, y_{27} \to -153.776, y_{28} \to -164.78, y_{29} \to -176.184,$
$y_{30} \to -187.987, y_{31} \to -200.19, y_{32} \to -212.792, y_{33} \to -225.793, y_{34} \to -239.194, y_{35} \to -252.995,$
$y_{36} \to -267.196, y_{37} \to -281.796, y_{38} \to -296.796, y_{39} \to -312.196, y_{40} \to -327.996,$
$y_{41} \to -344.196, y_{42} \to -360.795, y_{43} \to -377.794, y_{44} \to -395.193, y_{45} \to -412.992, y_{46} \to -431.19,$
$y_{47} \to -449.788, y_{48} \to -468.785, y_{49} \to -488.181, y_{50} \to -507.976, y_{51} \to -528.171,$
$y_{52} \to -548.763, y_{53} \to -569.754, y_{54} \to -591.143, y_{55} \to -612.929, y_{56} \to -635.111,$
$y_{57} \to -657.688, y_{58} \to -680.661, y_{59} \to -704.026, y_{60} \to -727.782, y_{61} \to -751.928, y_{62} \to -776.46,$
$y_{63} \to -801.374, y_{64} \to -826.668, y_{65} \to -852.335, y_{66} \to -878.369, y_{67} \to -904.761,$
$y_{68} \to -931.501, y_{69} \to -958.576, y_{70} \to -985.971, y_{71} \to -1013.66, y_{72} \to -1041.63,$
$y_{73} \to -1069.84, y_{74} \to -1098.25, y_{75} \to -1126.81, y_{76} \to -1155.46, y_{77} \to -1184.12,$
$y_{78} \to -1212.7, y_{79} \to -1241.08, y_{80} \to -1269.1, y_{81} \to -1296.57, y_{82} \to -1323.27, y_{83} \to -1348.88,$
$y_{84} \to -1373.06, y_{85} \to -1395.32, y_{86} \to -1415.1, y_{87} \to -1431.68, y_{88} \to -1444.14, y_{89} \to -1451.38,$
$y_{90} \to -1451.98, y_{91} \to -1444.17, y_{92} \to -1425.76, y_{93} \to -1394., y_{94} \to -1345.45, y_{95} \to -1275.81,$
$y_{96} \to -1179.72, y_{97} \to -1050.45, y_{98} \to -879.61, y_{99} \to -656.712, y_{100} \to -368.64\}\}$

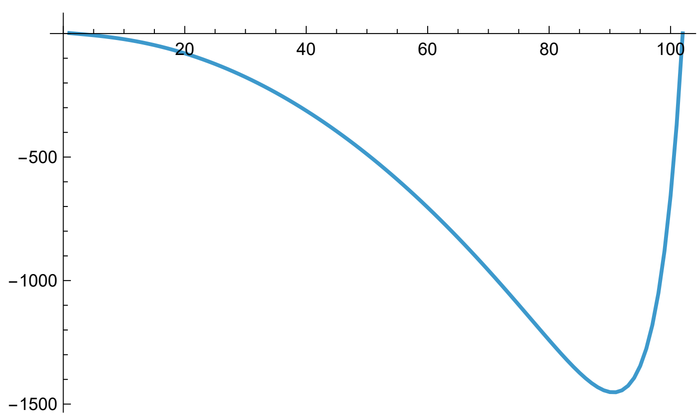
In[ ]:=

```
Clear[d]
```

In[∘]:=

```
d = ListLinePlot[
  {2, -0.1999998497254204, -2.399999691937112, -4.6799995187456584, -7.103999321491488,
   -9.72319909031189, -12.578558813647884, -15.702846477666272, -19.122276465567975,
   -22.857820276748072, -26.926255101765566, -31.341002681871338, -36.11280039607067,
   -41.25023813007352, -46.760187770580046, -52.64814679961556, -58.91851316863184,
   -65.5748051960797, -72.61983748333151, -80.05586164474988, -87.88467888840574,
   -96.10773007648187, -104.72616776838208, -113.74091384870137, -123.15270562145568,
   -132.96213267528276, -143.16966636287395, -153.77568336860884, -164.7804845427741,
   -176.1843099440781, -187.98735084258595, -200.18975928322308, -212.79165568802136,
   -225.7931348772207, -239.19427081028104, -252.99512028385539, -267.19572577162245,
   -281.79611754797065, -296.79631520171733, -312.1963286155499, -327.99615846015985,
   -344.1957962277778, -360.7952238067845, -377.79441257613036, -395.1933219742827,
   -412.9918974711492, -431.190067841573, -449.78774160407534, -468.7848024467813,
   -488.1811034118262, -507.9764595474624, -528.1706386604717, -548.7633497065044,
   -569.7542282378623, -591.1428181811131, -612.9285490334194, -635.1107073373964,
   -657.6884010082431, -680.6605147295019, -704.0256541872358, -727.7820763543311,
   -751.9276023391429, -776.4595084409118, -801.3743899647262, -826.6679909867768,
   -852.334991558166, -878.3687417074634, -904.7609289421337, -931.5011626239102,
   -958.5764544368822, -985.9705689716982, -1013.6632119550989, -1041.6290155362542,
   -1069.8362698942221, -1098.245337746901, -1126.8066724869245, -1155.4583408512942,
   -1184.1229262582285, -1212.7036579780738, -1241.0795725968226, -1269.099465845412,
   -1296.5743323862716, -1323.267915546445, -1348.8848944839403, -1373.0561181456326,
   -1395.3201477146063, -1415.10018466931, -1431.6752308574792, -1444.1440385885219,
   -1451.3800482489905, -1451.9750603219084, -1444.1688254109215, -1425.7610317704805,
   -1394.0012897185634`, -1345.4516121525742, -1275.8145151942135, -1179.7181439955632,
   -1050.447679996691, -879.609599997653, -656.7119999984977, -368.6399999992672, 1}]
```

Out[∘]=



In[ ]:=

```
d = {2, -0.1999998497254204, -2.399999691937112, -4.6799995187456584, -7.103999321491488,
   -9.72319909031189, -12.578558813647884, -15.702846477666272, -19.122276465567975,
   -22.857820276748072, -26.926255101765566, -31.341002681871338, -36.11280039607067,
   -41.25023813007352, -46.760187770580046, -52.64814679961556, -58.91851316863184,
   -65.5748051960797, -72.61983748333151, -80.05586164474988, -87.88467888840574,
   -96.10773007648187, -104.72616776838208, -113.74091384870137, -123.15270562145568,
   -132.96213267528276, -143.16966636287395, -153.77568336860884, -164.7804845427741,
   -176.1843099440781, -187.98735084258595, -200.18975928322308, -212.79165568802136,
   -225.7931348772207, -239.19427081028104, -252.99512028385539, -267.19572577162245,
   -281.79611754797065, -296.79631520171733, -312.1963286155499, -327.99615846015985,
   -344.1957962277778, -360.7952238067845, -377.79441257613036, -395.1933219742827,
   -412.9918974711492, -431.190067841573, -449.78774160407534, -468.7848024467813,
   -488.1811034118262, -507.9764595474624, -528.1706386604717, -548.7633497065044,
   -569.7542282378623, -591.1428181811131, -612.9285490334194, -635.1107073373964,
   -657.6884010082431, -680.6605147295019, -704.0256541872358, -727.7820763543311,
   -751.9276023391429, -776.4595084409118, -801.3743899647262, -826.6679909867768,
   -852.334991558166, -878.3687417074634, -904.7609289421337, -931.5011626239102,
   -958.5764544368822, -985.9705689716982, -1013.6632119550989, -1041.6290155362542,
   -1069.8362698942221, -1098.245337746901, -1126.8066724869245, -1155.4583408512942,
   -1184.1229262582285, -1212.7036579780738, -1241.0795725968226, -1269.099465845412,
   -1296.5743323862716, -1323.267915546445, -1348.8848944839403, -1373.0561181456326,
   -1395.3201477146063, -1415.10018466931, -1431.6752308574792, -1444.1440385885219,
   -1451.3800482489905, -1451.9750603219084, -1444.1688254109215, -1425.7610317704805,
   -1394.0012897185634`, -1345.4516121525742, -1275.8145151942135, -1179.7181439955632,
   -1050.44767999691, -879.609599997653, -656.7119999984977, -368.6399999992672, 1};
```

In[ ]:= **cubicFit = Fit[d, {1, x, x^3}, x]**

Out[ ]=

$$248.114 - 18.0642\, x + 0.000338964\, x^3$$

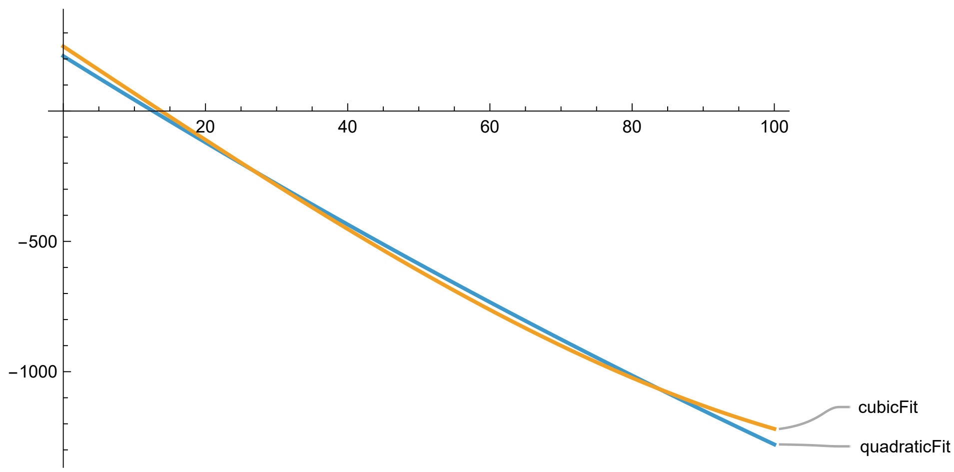
In[ ]:= **quadraticFit = Fit$\left[d, \left\{1, x, x^2\right\}, x\right]$**

Out[ ]=

$$210.982 - 16.9904\, x + 0.0208944\, x^2$$

In[ ]:=
```
Plot[{quadraticFit, cubicFit}, {x, 0, 100},
 PlotLabels → {"quadraticFit", "cubicFit"},
 ImageSize → {500}]
 (* The approximation is not good. *)
```

Out[ ]=



- Convert your data to csv file extension then to Microsoft Excel.

In[∘]:=
```
e = {{1, 2}, {2, -0.1999998497254204}, {3, -2.399999691937112}, {4, -4.6799995187456584},
  {5, -7.103999321491488}, {6, -9.72319909031189}, {7, -12.578558813647884},
  {8, -15.702846477666272}, {9, -19.122276465567975}, {10, -22.857820276748072},
  {11, -26.926255101765566}, {12, -31.341002681871338}, {13, -36.11280039607067},
  {14, -41.25023813007352}, {15, -46.760187770580046}, {16, -52.64814679961556},
  {17, -58.91851316863184}, {18, -65.5748051960797}, {19, -72.61983748333151},
  {20, -80.05586164474988}, {21, -87.88467888840574}, {22, -96.10773007648187},
  {23, -104.72616776838208}, {24, -113.74091384870137}, {25, -123.15270562145568},
  {26, -132.96213267528276}, {27, -143.16966636287395}, {28, -153.77568336860884},
  {29, -164.7804845427741}, {30, -176.1843099440781}, {31, -187.98735084258595},
  {32, -200.18975928322308}, {33, -212.79165568802136}, {34, -225.7931348772207},
  {35, -239.19427081028104}, {36, -252.99512028385539}, {37, -267.19572577162245},
  {38, -281.79611754797065}, {39, -296.79631520171733}, {40, -312.1963286155499},
  {41, -327.99615846015985}, {42, -344.1957962277778}, {43, -360.7952238067845},
  {44, -377.79441257613036}, {45, -395.1933219742827}, {46, -412.9918974711492},
  {47, -431.190067841573}, {48, -449.78774160407534}, {49, -468.7848024467813},
  {50, -488.1811034118262}, {51, -507.9764595474624}, {52, -528.1706386604717},
  {53, -548.7633497065044}, {54, -569.7542282378623}, {55, -591.1428181811131},
  {56, -612.9285490334194}, {57, -635.1107073373964}, {58, -657.6884010082431},
  {59, -680.6605147295019}, {60, -704.0256541872358}, {61, -727.7820763543311},
  {62, -751.9276023391429}, {63, -776.4595084409118}, {64, -801.3743899647262},
  {65, -826.6679909867768}, {66, -852.334991558166}, {67, -878.3687417074634},
  {68, -904.7609289421337}, {69, -931.5011626239102}, {70, -958.5764544368822},
  {71, -985.9705689716982}, {72, -1013.6632119550989}, {73, -1041.6290155362542},
  {74, -1069.8362698942221}, {75, -1098.245337746901}, {76, -1126.8066724869245},
  {77, -1155.4583408512942}, {78, -1184.1229262582285}, {79, -1212.7036579780738},
  {80, -1241.0795725968226}, {81, -1269.099465845412}, {82, -1296.5743323862716},
  {83, -1323.267915546445}, {84, -1348.8848944839403}, {85, -1373.0561181456326},
  {86, -1395.3201477146063}, {87, -1415.10018466931}, {88, -1431.6752308574792},
  {89, -1444.1440385885219}, {90, -1451.3800482489905}, {91, -1451.9750603219084},
  {92, -1444.1688254109215}, {93, -1425.7610317704805}, {94, -1394.0012897185634},
  {95, -1345.4516121525742}, {96, -1275.8145151942135}, {97, -1179.7181439955632},
  {98, -1050.447679996691}, {99, -879.609599997653}, {100, -656.7119999984977}, {101, 1}};
```

In[∘]:=

```
Export["data.csv", e, "csv"]
```

In[∘]:=
```
"data.csv"
Directory[]
```

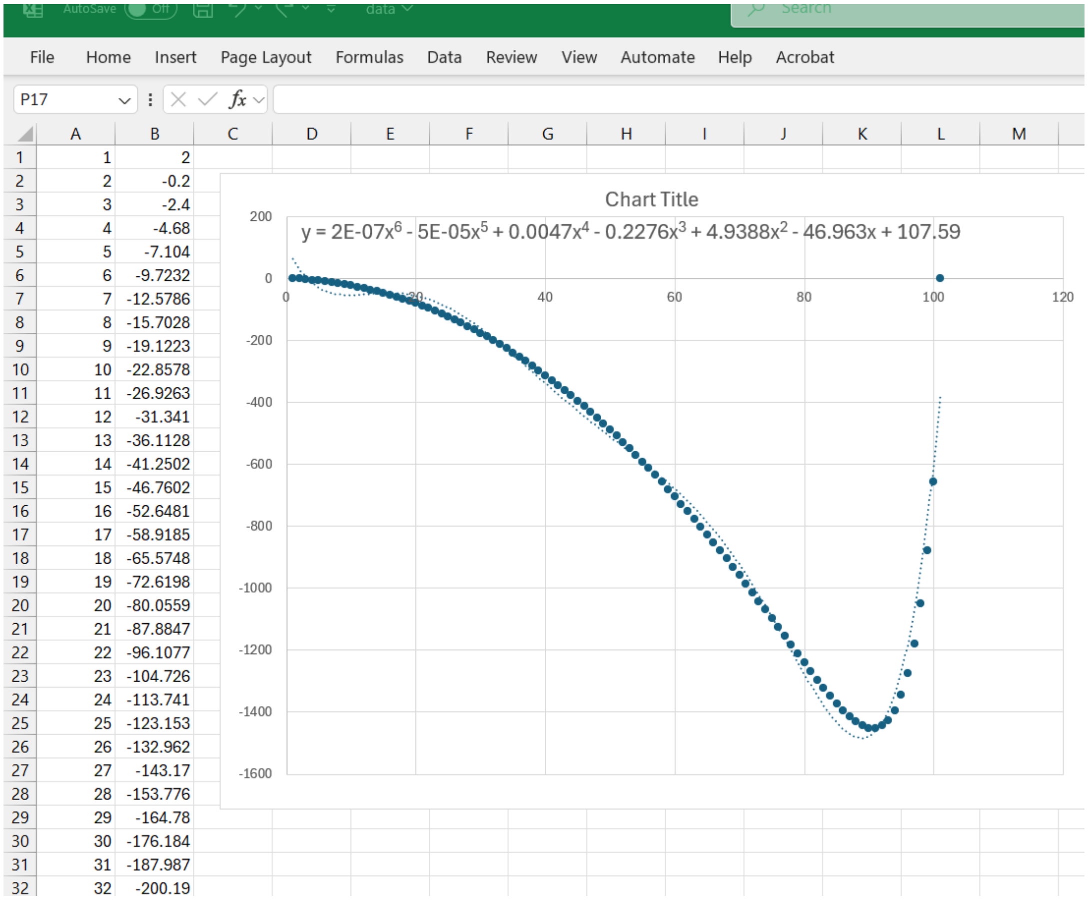
Out[∘]=
```
data.csv
```

Out[∘]=
```
C:\Users\Loreto Juelar
```

- The approximate solution is given by

$$y = 2E\text{-}07x^6 - 5E\text{-}05x^5 + 0.0047x^4 - 0.2276x^3 + 4.9388x^2 - 46.963x + 107.59$$ by

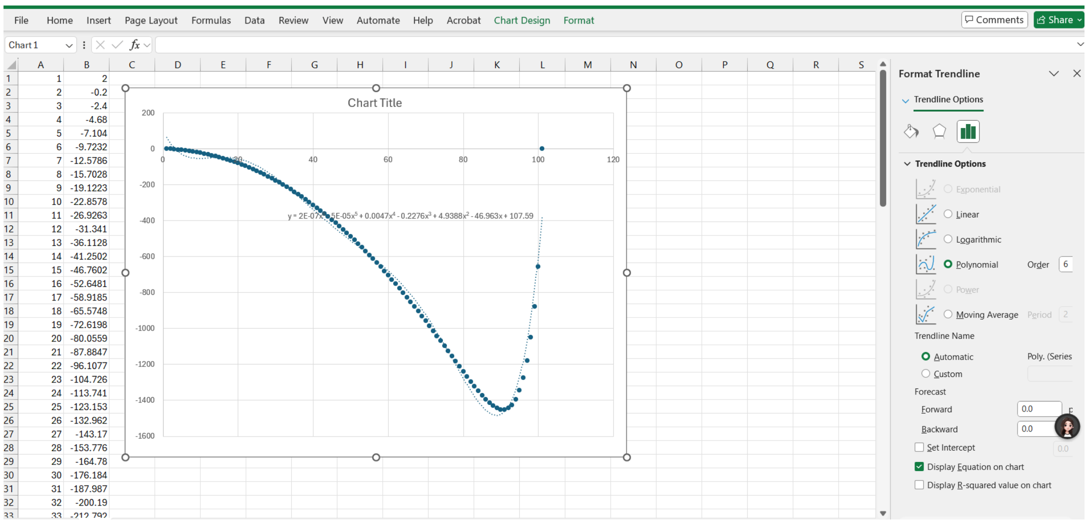
polynomial of order 6 in Microsoft excel. But we can consider the values of x and y in our plot as out final solution.



33	33	-212.792
34	34	-225.793
35	35	-239.194
36	36	-252.995
37	37	-267.196
38	38	-281.796
39	39	-296.796
40	40	-312.196
41	41	-327.996
42	42	-344.196
43	43	-360.795
44	44	-377.794
45	45	-395.193
46	46	-412.992
47	47	-431.19
48	48	-449.788
49	49	-468.785
50	50	-488.181
51	51	-507.976
52	52	-528.171
53	53	-548.763
54	54	-569.754
55	55	-591.143
56	56	-612.929
57	57	-635.111
58	58	-657.688
59	59	-680.661
60	60	-704.026
61	61	-727.782
62	62	-751.928
63	63	-776.46
64	64	-801.374
65	65	-826.668
66	66	-852.335
67	67	-878.369
68	68	-904.761
69	69	-931.501
70	70	-958.576
71	71	-985.971
72	72	-1013.66
73	73	-1041.63
74	74	-1069.84
75	75	-1098.25
76	76	-1126.81
77	77	-1155.46
78	78	-1184.12
79	79	-1212.7
80	80	-1241.08
81	81	-1269.1
82	82	-1296.57
83	83	-1323.27
84	84	-1348.88
85	85	-1373.06
86	86	-1395.32
87	87	-1415.1
88	88	-1431.68
89	89	-1444.14
90	90	-1451.38
91	91	-1451.98
92	92	-1444.17

93	93	-1425.76
94	94	-1394
95	95	-1345.45
96	96	-1275.81
97	97	-1179.72
98	98	-1050.45
99	99	-879.61
100	100	-656.712
101	101	1

- On the upper right side of the plot click the plus sign + then click the Trendline Options > Polynomial of Order 6 then click the Display Equation chart.



- Approximation of the solution:

$y = 2E\text{-}07x^6 - 5E\text{-}05x^5 + 0.0047x^4 - 0.2276x^3 + 4.9388x^2 - 46.963x + 107.59$ Bu

t the data points in our solution is our final solution.

In[ ]:=

```
Plot[2 * 10^-7 * x^6 - 5 * 10^-5 * x^5 +
  0.0047 * x^4 - 0.2276 * x^3 + 4.9388 * x^2 -
  46.963 * x + 107.59, {x, 0, 120}]
```

Out[ ]=