- TAYLOR'S SERIES GEOMETRICAL INTERPRETATION
  - Plot Exp[x] and its Maclaurin series

```
Series[Exp[x], {x, 0, 1}](* 2 terms *)
```

Out[1]= $1 + x + O[x]^2$

```
Series[Exp[x], {x, 0, 2}](* 3 terms *)
```

Out[2]= $1 + x + \frac{x^2}{2} + O[x]^3$

```
Series[Exp[x], {x, 0, 3}](* 4 terms *)
```

Out[3]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + O[x]^4$

```
Series[Exp[x], {x, 0, 4}](* 5 terms *)
```

Out[4]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + O[x]^5$

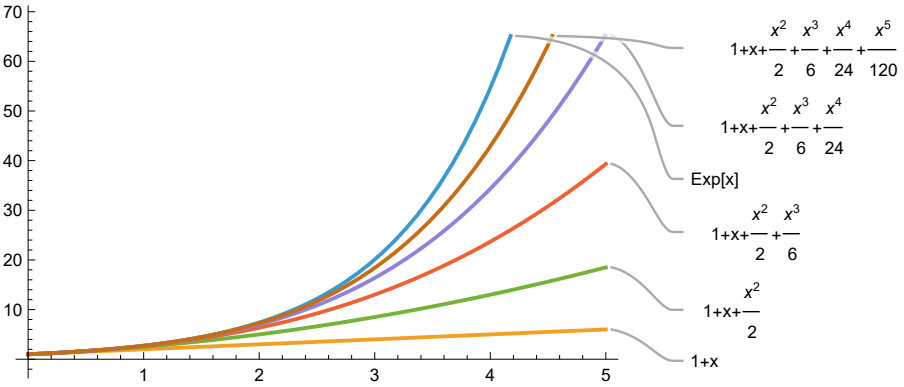
```
Series[Exp[x], {x, 0, 5}](* 6 terms *)
```

Out[5]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O[x]^6$

- Plot Exp[x] and all the Taylor's series terms of Exp[x]:

```
Plot[{Exp[x], 1 + x, 1 + x + x^2/2, 1 + x + x^2/2 + x^3/6, 1 + x + x^2/2 + x^3/6 + x^4/24, 1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120},
 {x, 0, 5}, PlotLabels → {"Exp[x]", "1+x", "1+x+x^2/2", "1+x+x^2/2+x^3/6",
  "1+x+x^2/2+x^3/6+x^4/24", "1+x+x^2/2+x^3/6+x^4/24+x^5/120"}, ImageSize → {500}]
(* All the plots have the same slope at x= 0. The Taylor's series polynimials
 become closer and closer to Exp[x] *)
```

Out[8]=



■ ==========

■ Let us plot the Taylor's series of Exp[x] at x=5 and compare their plots to the plot of Exp[x].

In[9]:=
```
Series[Exp[x], {x, 5, 1}]
```
Out[9]= $e^5 + e^5 (x-5) + O[x-5]^2$

In[10]:=
```
Series[Exp[x], {x, 5, 2}]
```
Out[10]=
$$e^5 + e^5 (x-5) + \frac{1}{2} e^5 (x-5)^2 + O[x-5]^3$$

In[11]:=
```
Series[Exp[x], {x, 5, 3}]
```
Out[11]=
$$e^5 + e^5 (x-5) + \frac{1}{2} e^5 (x-5)^2 + \frac{1}{6} e^5 (x-5)^3 + O[x-5]^4$$

In[12]:=
```
Series[Exp[x], {x, 5, 4}]
```
Out[12]=
$$e^5 + e^5 (x-5) + \frac{1}{2} e^5 (x-5)^2 + \frac{1}{6} e^5 (x-5)^3 + \frac{1}{24} e^5 (x-5)^4 + O[x-5]^5$$

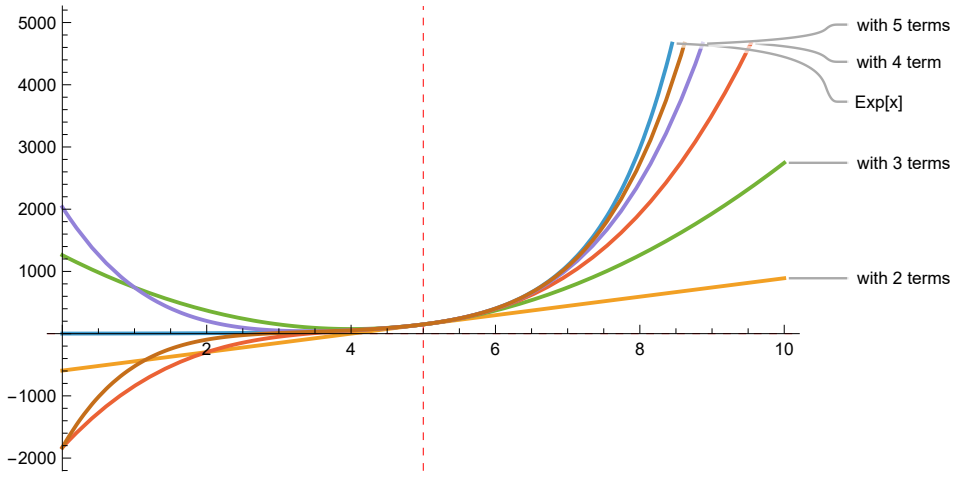
In[13]:=
```
Series[Exp[x], {x, 5, 5}]
```

Out[13]=

$$e^5 + e^5 (x-5) + \frac{1}{2} e^5 (x-5)^2 + \frac{1}{6} e^5 (x-5)^3 + \frac{1}{24} e^5 (x-5)^4 + \frac{1}{120} e^5 (x-5)^5 + O[x-5]^6$$

```
Plot[{Exp[x], e^5 + e^5 (x - 5),
  e^5 + e^5 (x - 5) + 1/2 e^5 (x - 5)^2, e^5 + e^5 (x - 5) + 1/2 e^5 (x - 5)^2 + 1/6 e^5 (x - 5)^3,
  e^5 + e^5 (x - 5) + 1/2 e^5 (x - 5)^2 + 1/6 e^5 (x - 5)^3 + 1/24 e^5 (x - 5)^4,
  e^5 + e^5 (x - 5) + 1/2 e^5 (x - 5)^2 + 1/6 e^5 (x - 5)^3 + 1/24 e^5 (x - 5)^4 + 1/120 e^5 (x - 5)^5}, {x, 0, 10},
 PlotLabels → {"Exp[x]", "with 2 terms", "with 3 terms", "with 4 term", "with 5 terms"},
 GridLines → {{5}, {0}}, GridLinesStyle → Directive[Red, Dashed], ImageSize → {500}]
(* All the plots have the same slope at x=5 *)
```

Out[77]=



■ Plot the function sin(x) and its Taylor's expansion at x=5.

In[51]:=
```
Series[Sin[x], {x, 5, 1}]
```

Out[51]=

$$\text{Sin}[5] + \text{Cos}[5] (x-5) + O[x-5]^2$$

In[52]:=
```
Series[Sin[x], {x, 5, 2}]
```

Out[52]=

$$\text{Sin}[5] + \text{Cos}[5] (x-5) - \frac{1}{2} \text{Sin}[5] (x-5)^2 + O[x-5]^3$$

In[53]:=
```
Series[Sin[x], {x, 5, 3}]
```

Out[53]=

$$\text{Sin}[5] + \text{Cos}[5] (x-5) - \frac{1}{2} \text{Sin}[5] (x-5)^2 - \frac{1}{6} \text{Cos}[5] (x-5)^3 + O[x-5]^4$$

In[54]:=
```
Series[Sin[x], {x, 5, 4}]
```

Out[54]=

$$\text{Sin}[5] + \text{Cos}[5]\ (x-5) - \frac{1}{2}\text{Sin}[5]\ (x-5)^2 - \frac{1}{6}\text{Cos}[5]\ (x-5)^3 + \frac{1}{24}\text{Sin}[5]\ (x-5)^4 + O[x-5]^5$$

In[55]:=
```
Series[Sin[x], {x, 5, 5}]
```

Out[55]=

$$\text{Sin}[5] + \text{Cos}[5]\ (x-5) - \frac{1}{2}\text{Sin}[5]\ (x-5)^2 - \frac{1}{6}\text{Cos}[5]\ (x-5)^3 + \frac{1}{24}\text{Sin}[5]\ (x-5)^4 + \frac{1}{120}\text{Cos}[5]\ (x-5)^5 + O[x-5]^6$$

```
Plot[{Sin[x], Sin[5] + Cos[5] (x - 5),
  Sin[5] + Cos[5] (x - 5) - 1/2 Sin[5] (x - 5)^2, Sin[5] + Cos[5] (x - 5) - 1/2 Sin[5] (x - 5)^2,
  Sin[5] + Cos[5] (x - 5) - 1/2 Sin[5] (x - 5)^2 - 1/6 Cos[5] (x - 5)^3 + 1/24 Sin[5] (x - 5)^4,
  Sin[5] + Cos[5] (x - 5) - 1/2 Sin[5] (x - 5)^2 - 1/6 Cos[5] (x - 5)^3 +
   1/24 Sin[5] (x - 5)^4 + 1/120 Cos[5] (x - 5)^5}, {x, 2, 8},
 GridLines → {{5}, {0}}, GridLinesStyle → Directive[Red, Dashed],
 PlotLabels → {"sin(x)", "with 2 terms", "with 3 terms", "with 4 terms", "with 5 terms"}]
(* All the plots have the same slope at x=
5 and become closer and
 closer to the plot of sin(x) *)
```

Out[67]=