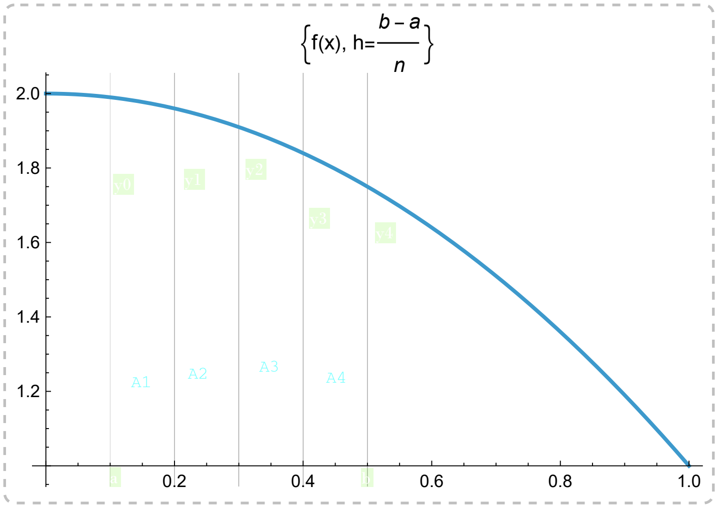
- TRAPEZOIDAL RULE

In[ ]:=

```
Plot[-x^2 + 2, {x, 0, 1},
 GridLines → {{0.1, 0.2, 0.3, 0.4, 0.5}, {0}}, PlotLabel → {"f(x)", "h=(b - a)/n"}]
```



- $A_1 = h\left(\frac{y_0+y_1}{2}\right)$
- $A_T = A_1 + A_2 + A_3 + A_4$
- $A_2 = h\left(\frac{y_1+y_2}{2}\right)$, $A_3 = h\left(\frac{y_2+y_3}{2}\right)$, and $A_4 = h\left(\frac{y_3+y_4}{2}\right)$. The total area now is
  $A_T = \frac{h}{2}\,(y_0 + 2\,y_1 + 2\,y_2 + 2\,y_3 + y_4)$
- **Example:**

  Approximate the integral

  $$\int_0^2 x^2\,dx$$

  using the **Trapezoidal Rule** with **n = 10** subintervals.

  Step 1: Understand the Trapezoidal Rule Formula

  The Trapezoidal Rule for approximating

$$\int_a^b f(x)\,dx$$

with **n** subintervals is:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

where:

- $h = \frac{b-a}{n}$ is the width of each subinterval,
- $x_0 = a$, $x_1 = a + h$, $x_2 = a + 2h$, ..., $x_n = b$.

Step 2: Apply the Values

We are given:

- $a = 0$
- $b = 2$
- $n = 10$
- $f(x) = x^2$

So:

$$h = \frac{2-0}{10} = 0.2$$

Now compute the points:

$$x_k = 0 + k \cdot h = 0.2k \quad \text{for } k = 0, 1, 2, \ldots, 10$$

So the points are:

$$x_0 = 0.0,\ x_1 = 0.2,\ x_2 = 0.4,\ x_3 = 0.6,\ x_4 = 0.8,\ x_5 = 1.0,$$
$$x_6 = 1.2,\ x_7 = 1.4,\ x_8 = 1.6,\ x_9 = 1.8,\ x_{10} = 2.0$$

Now compute $f(x_k) = x_k^2$:

K	X_K	$F(X_K) = X_K^2$
0	0.0	0.00
1	0.2	0.04
2	0.4	0.16
3	0.6	0.36
4	0.8	0.64
5	1.0	1.00
6	1.2	1.44
7	1.4	1.96
8	1.6	2.56
9	1.8	3.24
10	2.0	4.00

Step 3: Apply the Trapezoidal Rule Formula

$$\int_0^2 x^2\,dx \approx \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + \cdots + f(x_9)) + f(x_{10})\right]$$

Plug in the values:

$$\approx \frac{0.2}{2}\left[0.00 + 2(0.04 + 0.16 + 0.36 + 0.64 + 1.00 + 1.44 + 1.96 + 2.56 + 3.24) + 4.00\right]$$

First, sum the interior terms:

$$0.04 + 0.16 = 0.20$$
$$0.20 + 0.36 = 0.56$$
$$0.56 + 0.64 = 1.20$$
$$1.20 + 1.00 = 2.20$$
$$2.20 + 1.44 = 3.64$$
$$3.64 + 1.96 = 5.60$$
$$5.60 + 2.56 = 8.16$$
$$8.16 + 3.24 = 11.40$$

Now multiply by 2:

$$2 \times 11.40 = 22.80$$

Now add the first and last terms:

$$0.00 + 22.80 + 4.00 = 26.80$$

- Now multiply by $\frac{0.2}{2} = 0.1$:

  $$0.1 \times 26.80 = 2.68$$

  Final Answer:

  $$\int_0^2 x^2 \, dx \approx 2.68$$

  Compare with Exact Value:

  The exact value is:

- $$\int_0^2 x^2 \, dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3} \approx 2.6667$$

  So our approximation is:

  $$2.68 \quad \text{vs} \quad 2.6667$$

  Error ≈ $2.68 - 2.6667 = 0.0133$, which is quite small!

- ✅ **Conclusion:**
  Using the Trapezoidal Rule with $n = 10$, we approximated $\int_0^2 x^2 \, dx \approx 2.68$, close to the exact value $\frac{8}{3} \approx 2.6667$

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- SIMULATION BY MATHEMATICA

```
In[13]:= (*Define the function*)f[x_] := x^2

(*Integration limits*)
a = 0;
b = 2;
n = 10;

(*Step size*)
h = (b - a) / n;

(*Generate the x-values*)
xValues = Table[a + i * h, {i, 0, n}];

(*Evaluate f(x) at each point*)
fValues = f /@ xValues;

(*Apply Trapezoidal Rule formula*)
approxIntegral = (h / 2) * (fValues[[1]] + 2 * Sum[fValues[[i]], {i, 2, n}] + fValues[[n + 1]])

(*Optional:Compare with exact value*)
exactIntegral = Integrate[x^2, {x, 0, 2}]
error = approxIntegral - exactIntegral
```

Out[20]= $\frac{67}{25}$

Out[21]= $\frac{8}{3}$

Out[22]= $\frac{1}{75}$

In[12]:= $\frac{67.0}{25}$

Out[12]= 2.68

Explanation of the Code:

- `f[x_] := x^2` — defines the function.
- `a = 0; b = 2; n = 10;` — sets up interval and number of subintervals.
- `h = (b - a)/n` — computes step size.
- `Table[a + i*h, {i, 0, n}]` — creates the 11 points from 0 to 2 in steps of 0.2.
- `f /@ xValues` — applies the function to all x-values.

- • The trapezoidal sum uses the standard formula:

  $$\frac{h}{2}\left[f(x_0)+2\sum_{i=1}^{n-1} f(x_i)+f(x_n)\right]$$

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