- Our problem here is to compute for absolute error which is the difference between the actual value and the approximate value given by the Taylor's polynomial.
- Approximate the function y= $\sqrt[5]{x}$ by a Taylor's polynomial of degree 3.
- Approximate the function $f(x) = \sqrt[5]{x}$ by a Taylor polynomial of degree 3 at $a = 5$.
  - SOLUTION

```
f = x^(1/5) (* Given function *)
f' = D[f, x] (* First derivative *)
f'' = D[f, {x, 2}] (* Second derivative *)
f'''[x_] = D[f, {x, 3}] (* Third derivative *)
f^iv[x_] = D[f, {x, 4}] (* Fourth derivative *)
```

Out[∘]=

$$x^{1/5}$$

In[∘]:=

(* **Given function:** $x^{1/5}$ *)

- First derivative: $\frac{1}{5\,x^{4/5}}$

Out[∘]=

$$-\frac{4}{25\,x^{9/5}}$$

- Second derivative: $-\frac{4}{25\,x^{9/5}}$
- 3rd derivative: $\frac{36}{125\,x^{14/5}}$
- 4th derivative: $-\frac{504}{625\,x^{19/5}}$

Out[ ]=

$$\frac{36}{125\,x^{14/5}}$$

Set: Tag Power in $x^{iv/5}$[x_] is Protected.

Out[ ]=

$$-\frac{504}{625\,x^{19/5}}$$

$x^{1/5}$ (* given function. *)

Out[ ]=

$$\frac{1}{5\,x^{4/5}}$$

Out[ ]=

$$-\frac{4}{25\,x^{9/5}}$$

Out[ ]=

$$\frac{36}{125\,x^{14/5}}$$

Set: Tag Power in $x^{iv/5}$[x_] is Protected.

Out[ ]=

$$-\frac{504}{625\,x^{19/5}}$$

- At x=5
- $f = 5^{1/5}$
- $f' = \frac{1}{5\,(5)^{4/5}}$
- $f'' = -\frac{4}{25\,(5^{9/5})}$
- $f''' = \frac{36}{125\,(5^{14/5})}$
- $f^{iv} = -\frac{504}{4!*625\,(5)^{19/5}}$

- The Taylor's series is $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$.

  By substitution we have the approximation for degree 3:

  - $f\,(x) = T_3(x) = \sqrt[5]{x} =$

    $$5^{1/5} + \frac{1}{5\,(5)^{4/5}}\,(x-5) - \frac{4}{(2)\,25\,(5^{4/5})}\,(x-a)^2 + \frac{36}{(6)\,125\,(5^{14/5})}\,(x-3)^3$$

In[ ]:=

```
Series[x^(1/5), {x, 5, 4}]
(* The output below verifies the answer. *)
```

Out[ ]=

$$5^{1/5} + \frac{x-5}{5\times 5^{4/5}} - \frac{2\,(x-5)^2}{125\times 5^{4/5}} + \frac{6\,(x-5)^3}{3125\times 5^{4/5}} - \frac{21\,(x-5)^4}{78\,125\times 5^{4/5}} + O[x-5]^5$$

- ==

- What is the absolute error when x=4 for degree 3.

  - SOLUTION: Error = |Exact value - approximate value|

    - $= \left|\, x^{1/5} - \left(5^{1/5} + \frac{x-5}{5\times 5^{4/5}} - \frac{2\,(x-5)^2}{125\times 5^{4/5}} + \frac{6\,(x-5)^3}{3125\times 5^{4/5}}\right)\right|$

In[ ]:=

```
h[x_] := Abs[x^(1/5) - (5^(1/5) + (x - 5)/(5 × 5^(4/5)) - (2 (x - 5)^2)/(125 × 5^(4/5)) + (6 (x - 5)^3)/(3125 × 5^(4/5)))];
(* Absolute error at x=4 *)
h[4] // N
```

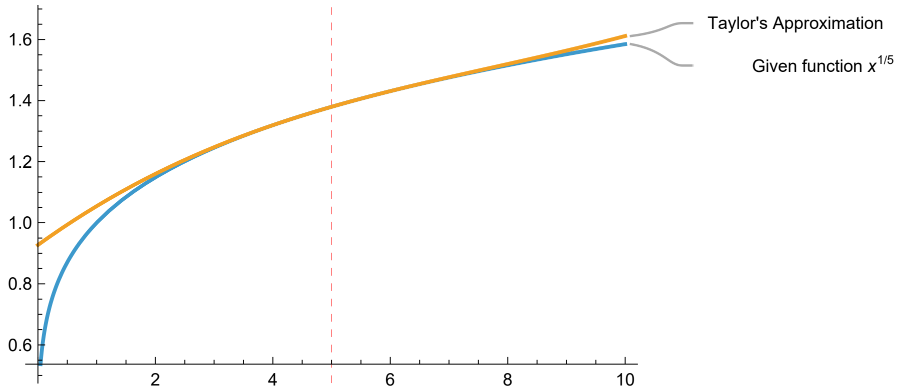
Out[ ]=

0.0000876131

In[∘]:=

$$\text{Plot}\left[\left\{x^{1/5}, 5^{1/5}+\frac{x-5}{5\times 5^{4/5}}-\frac{2\,(x-5)^2}{125\times 5^{4/5}}+\frac{6\,(x-5)^3}{3125\times 5^{4/5}}\right\}, \{x, 0, 10\},\right.$$
$$\text{GridLines}\to\{\{5\},\{0\}\}, \text{GridLinesStyle}\to\text{Directive}[\text{Red}, \text{Dashed}],$$
$$\left.\text{PlotLabels}\to\left\{\text{"Given function } x^{1/5}\text{"}, \text{"Taylor's Approximation"}\right\}, \text{ImageSize}\to\{500\}\right]$$

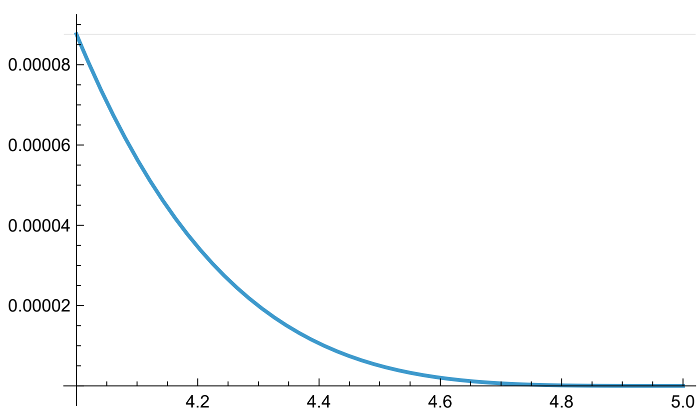
Out[∘]=



In[∘]:=

$$\text{Plot}\left[\text{Abs}\left[x^{1/5}-\left(5^{1/5}+\frac{x-5}{5\times 5^{4/5}}-\frac{2\,(x-5)^2}{125\times 5^{4/5}}+\frac{6\,(x-5)^3}{3125\times 5^{4/5}}\right)\right],\right.$$
$$\{x, 4, 5\}, \text{GridLines}\to\{\{0\}, \{0.00008761312319505166`\}\},$$
$$\left.\text{GridLinesStyle}\to\text{Directive}[\text{Red}, \text{Thick}]\}\right]$$

Out[∘]=




```
h[5] (* Absolute error is at x=5 *)
```

Out[∘]=

0