▪ APPLICATION OF TAYLOR'S SERIES IN THE SPECIAL THEORY OF RELATIVITY

In Einstein's theory of special relativity the mass of an object moving with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

- where m_0 is the mass of the object when at rest and c is the speed of light. The kinetic energy of the object is the difference between its total energy and its energy at rest:

$$K = mc^2 - m_0c^2$$

- Show that when velocity v is much much less than c (speed of light) the kinetic energy K becomes in Newtonian physics
  - $K= \frac{1}{2} m_0 v^2$
  - SOLUTION
    - $K = m c^2 - m_0 c^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2 = m_0 c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right]$
    - Let $x = \frac{-v^2}{c^2}$ then $(1 + x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + O[x]^4$

In[∘]:=

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Series[(1 + x)^(-1/2), {x, 0, 3}]
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Out[∘]=

$$1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + O[x]^4$$

- So by substitution
  - $K = m_0 c^2\left(\frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \frac{5v^6}{16c^6} + \ldots\right)$
  - If $v << c$ then
  - $K = m_0 c^2\left(\frac{v^2}{2c^2}\right) = \frac{1}{2} m_0 v^2$
- Find the error when $v \leq 200$ m/s.

- SOLUTION
  - $f(x) = m_0 c^2[(1+x)^{-1/2} - 1]$

In[ ]:=

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D[m0 * c^2 * (1 + x)^(-1/2) - 1, {x, 2}]
(* the second derivative
 of f(x) with respect to x. *)
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Out[ ]=

$$\frac{67\,500\,000\,000\,000\,000\ \text{m0}}{(1+x)^{5/2}}$$

- $|f''(x)| = \frac{3\,c^2\,\text{m0}}{4\left(1-\frac{v^2}{c^2}\right)^{5/2}} \leq \frac{3\,c^2\,\text{m0}}{4\left(1-\frac{200^2}{c^2}\right)^{5/2}} = M =$

In[ ]:=

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c = 3 * 10^8
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Out[ ]=

300 000 000

$$\frac{(3*(3*10^8)*\text{m0})}{4*\left(1-\frac{200^2}{(3*10^8)^2}\right)^{5/2}} \text{ // N (* } f''(200) = \frac{200^4}{c^4} \text{ *)}$$

Out[ ]=

2.25×10^8 m0

$$\frac{1}{2!}\left(\frac{(3*(3*10^8)*\text{m0})}{4*\left(1-\frac{200^2}{(3*10^8)^2}\right)^{5/2}}\right)\left(\frac{200^4}{c^4}\right) \text{ // N (* } f''(200) = \frac{200^4}{c^4} \text{ *)}$$

Out[ ]=

2.22222×10^{-17} m0

- This is the error
  - $|R_1(x)| = \frac{M}{2!} f''(x) = \frac{1}{2!}\left(\frac{(3*(3*10^8)*\text{m0})}{4*\left(1-\frac{200^2}{(3*10^8)^2}\right)^{5/2}}\right)\left(\frac{200^4}{c^4}\right) = 2.22222 \times 10^{-17}\ \text{m0}$