- **Plotting the gamma function**
- The gamma function is defined by $\Gamma(n)=\int_0^\infty x^{n-1}\, e^{-x}\, dx$
- For n=1 the integral has a value of 1.
- A recursion formula of gamma is given by Γ(n+1) = n!. For example: Γ(3+1) = 3!= 6. Γ(5+1) = 5!= 120.
- $\frac{\Gamma(9)}{\Gamma(5)} = \frac{8!}{4!} = 1680$
- In Mathematica Gamma[x] = value of gamma function at x. For example Gamma[1.5]= 0.886227
  . For n =1 the gamma function at x=1 is 1, that is, Gamma[1]=1.

In[4]:=

```
Gamma[1.5]
```

Out[4]= 0.886227

In[ ]:=

```
Gamma[1]
```

Out[ ]=

1

- Let us use the definition of gamma function now:

In[5]:=

```
g[n_] := ∫_0^Infinity x^(n-1) E^(-x) dx;
```

$$g[n\_] := \int_0^{\text{Infinity}} x^{n-1}\, E^{-x}\, dx;$$

In[6]:=

```
g[1]
```

Out[6]= 1

In[ ]:=

```
g[1.1]
```

Out[ ]=

```
0.951351
```

In[ ]:=

```
g[1.2]
```

Out[ ]=

```
0.918169
```

In[ ]:=

```
g[1.3]
```

Out[ ]=

```
0.897471
```

In[ ]:=

```
g[1.4]
```

Out[ ]=

```
0.887264
```

In[ ]:=

```
g[1.5]
```

Out[ ]=

```
0.886227
```

In[ ]:=

```
g[1.6]
```

Out[ ]=

```
0.893515
```

In[ ]:=

```
g[1.7]
```

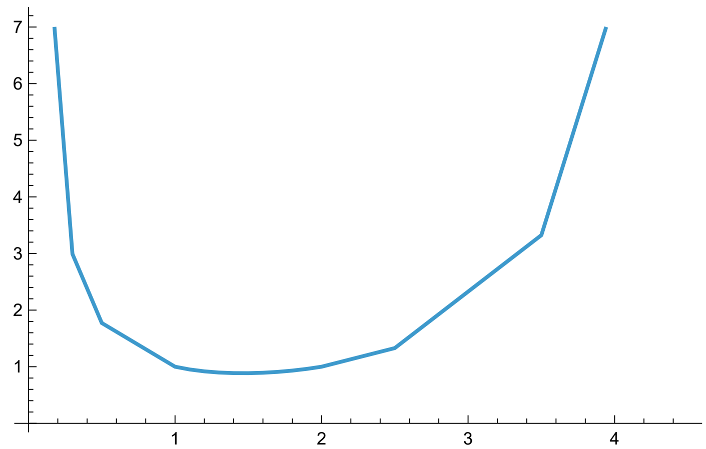
Out[ ]=

```
0.908639
```

In[ ]:=
```
g[1.8]
```
Out[ ]=
```
0.931384
```
In[ ]:=
```
g[1.9]
```
Out[ ]=
```
0.961766
```
In[ ]:=
```
g[2]
```
Out[ ]=
```
1
```
In[ ]:=
```
g[2.5]
```
Out[ ]=
```
1.32934
```
In[ ]:=
```
g[3.5]
```
Out[ ]=
```
3.32335
```
In[ ]:=
```
g[4.5]
```
Out[ ]=
```
11.6317
```

```
c = {{0.1, g[0.1]}, {0.3, g[0.3]},
   {0.5, g[0.5]}, {1, g[1]}, {1.1, g[1.1]},
   {1.2, g[1.2]}, {1.3, g[1.3]}, {1.4, g[1.4]},
   {1.5, g[1.5]}, {1.6, g[1.6]}, {1.7, g[1.7]},
   {1.8, g[1.8]}, {1.9, g[1.9]}, {2, g[2]},
   {2.5, g[2.5]}, {3.5, g[3.5]}, {4.5, g[4.5]}};
ListLinePlot[c] (* From n= 0.1 to n=4.5 *)
```

Out[∘]=



- For m < 0 the gamma function to be used is

In[∘]:=

$$h[m\_] := \frac{1}{m} * \int_0^{Infinity} x^m * E^{-x} \, dx;$$

- But $n\Gamma(n) = \Gamma(n+1)$ or $\Gamma(n) = \frac{\Gamma(n+1)}{n}$ by recurrence relation. Let n = -1 to 0 but let's use the -0.9 to - 0.1 since the gamma function is undefined at x = -1 and at x=0.

In[ ]:= (* Γ(0.9) *)h1 =
$$\frac{h[-0.9+1]}{-0.9}$$

Out[ ]=
-10.5706

(* Γ(0.8) *)

In[ ]:= $$h2 = \frac{h[-0.8+1]}{-0.8}$$

Out[ ]=
-5.73855

(* Γ(0.7) *)

In[ ]:= $$h3 = \frac{h[-0.7+1]}{-0.7}$$

Out[ ]=
-4.27367

(* Γ(0.6) *)

In[ ]:= $$h4 = \frac{h[-0.6+1]}{-0.6}$$

Out[ ]=
-3.69693

(* Γ(0.5) *)

In[ ]:= $$h5 = \frac{h[-0.5+1]}{-0.5}$$

Out[ ]=
-3.54491

(* Γ(0.4) *)

In[•]:= $h6 = \frac{h[-0.4 + 1]}{-0.4}$

Out[•]=

-3.72298

```
(* Γ(0.3) *
```

In[•]:= $h7 = \frac{h[-0.3 + 1]}{-0.3}$

Out[•]=

-4.32685

```
(* Γ(0.2) *
```

In[•]:= $h8 = \frac{h[-0.2 + 1]}{-0.2}$

Out[•]=

-5.82115

```
(* Γ(0.1) *
```

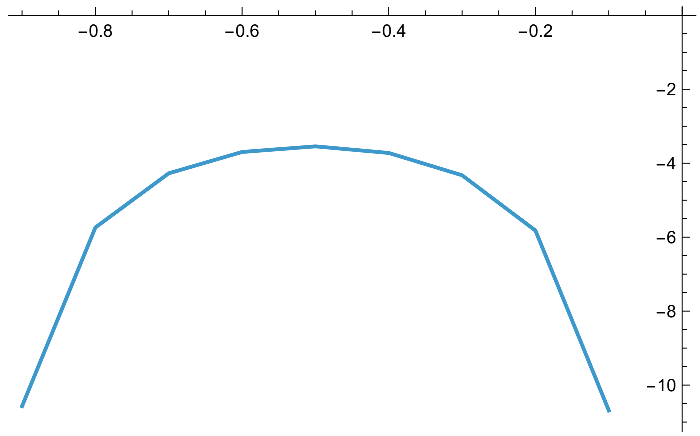
In[•]:= $h9 = \frac{h[-0.1 + 1]}{-0.1}$

Out[•]=

-10.6863

In[ ]:=
```
data = {{-0.9, h1}, {-0.8, h2}, {-0.7, h3},
   {-0.6, h4}, {-0.5, h5}, {-0.4, h6},
   {-0.3, h7}, {-0.2, h8}, {-0.1, h9}};
ListLinePlot[data]
```

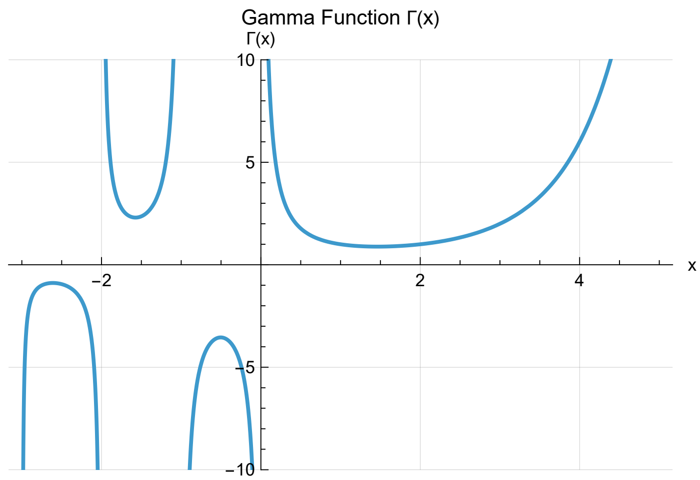
Out[ ]=



In[ ]:=
```
ListLinePlot[{c, data}, PlotRange → {-10, 10}]
```

In[ ]:=
```
Plot[Gamma[x], {x, -3, 5},
 PlotRange → {-10, 10}, AxesLabel → {"x", "Γ(x)"},
 PlotLabel → "Gamma Function Γ(x)",
 GridLines → Automatic]
```

Out[ ]=



In[ ]:=
```
Gamma[-.9]
```

Out[ ]=

−10.5706