- Let us find the Laplace Transform of t^n, where $n \geq 0$ is a real number (most commonly a non-negative integer).
- The Laplace Transform of a function $f(t)$ is defined by:
  - $L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$ ---------------- [1]
    - For $f(t) = t^n$, we get --------------- [2]
    - $L\{t^n\} = \int_0^\infty e^{-st} f(t)\, dt$ (Laplace integral)---------------[3]
  - Use the substitution to match the Gamma integral:
    - $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$ ---------------------[4]
    - Let $u = st \Rightarrow t = \frac{u}{s}$, $dt = \frac{du}{s}$ ------------[5]
    - Substitute equations [5] into the Laplace integral [3]:
    - $L\{t^n\} = \int_0^\infty e^{-st} t^n\, dt = \int_0^\infty e^{-u} \left(\frac{u}{s}\right)^n \frac{du}{s}$ ---------------[6] or
    - $= \frac{1}{s^{n+1}} \int_0^\infty u^n e^{-u}\, du = \frac{1}{s^{n+1}} \Gamma(n+1)$ -------------------[7] (from equation [4])
  - Use the identity $\Gamma(n+1) = n!$ ---------------[8] for integers. So for integer n:
    - $L\{t^n\} = \frac{n!}{s^{n+1}}$ (Final answer)
- By using Mathematica:

In[3]:=
```
LaplaceTransform[t^n, t, s]
```

Out[3]= s^{-1-n} Gamma[1 + n]