- Find the Riemann sum of $f(x) = x(x - 10)$ from $x = -1$ to $x = 10$ using the left – end points ..
- By using the most accurate solution we have

In[1]:=
```
∫_{-1}^{10} x * (x - 10) dx // N
```
Out[1]= -161.333

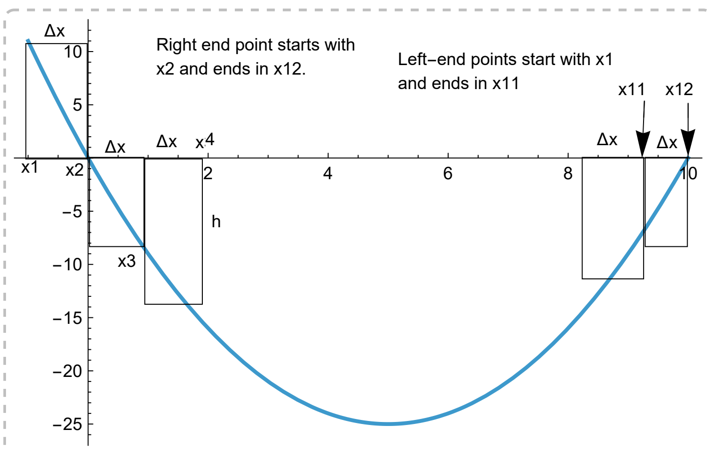
In[2]:=
```
∫_{0}^{10} x * (x - 10) dx // N
```
Out[2]= -166.667

In[3]:=
```
∫_{-1}^{0} x * (x - 10) dx // N
```
Out[3]= 5.33333

In[4]:=
```
∫_{-1}^{0} x * (x - 10) dx + Abs[∫_{0}^{10} x * (x - 10) dx // N]
```
Out[4]= 172.

In[5]:=
```
166.665 + 5.333333
```
Out[5]= 171.998

In[ ]:=
```
Plot[x * (x - 10), {x, -1, 10}]
```



- Let $\Delta x = \frac{b-a}{n} = \frac{10-(-1)}{11} = \frac{11}{11} = 1.0$
  - Using $f(x) = x(x-10)$ we have $f(x_1{=}{-1}) = 11$, $f(x_2 = 0) = 0$, $f(x_3 = 1) = |-9| = 9$, $f(x_4 = 2) = |-16| = 16$, $f(x_5 = 3) = |-21| = 21$, $f(x_6 = 4) = |-24| = 24$, $f(x_7 = 5) = |-25| = 25$, $f(x_8 = 6) = |-24| = 24$, $f(x_9 = 7) = |-21| = 21$, $f(x_{10} = 8) = |-16| = 16$, $f(x_{11} = 11) = 11$.
  - Area A_L using the left end points is given by: $\Delta x(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7) + f(x_8) + f(x_9) + f(x_{10}) + f(x_{11}) = 11$
    - $(1.0)(11 + 0 + 9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 11) = 176$
    - The accepted value is 172 so the percentage error is

In[ ]:=

```
Print[Abs[172 - 176]/172 * 100 // N, "%"]
```

2.32558%

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- Using the right-end points
  - Using $f(x) = x(x-10)$ we have $f(x_2 = 0) = 0$, $f(x_3 = 1) = |9|$, $f(x_4 = 2) = |16|$, $f(x_5 = 3) = |21|$, $f(x_6 = 4) = |24|$, $f(x_7 = 5) = |25|$, $f(x_8 = 6) =$ 24, $f(x_9 = 7) = |\ 21|$, $f(x_{10} = 8) = |16|$, $f(x_{11} = 9) = |9|$, $f(x_{12} = 10) = 0$.
  - So the area A_R using the right-points is
    - (1)(0 + 9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9 + 0) = 165
  - The average are is $\frac{176+165}{2}$ = 170.5
    - The percentage error of the average value with respect to the accepted value is $\frac{|172-170|}{172}\times 100 = 1.16\%$

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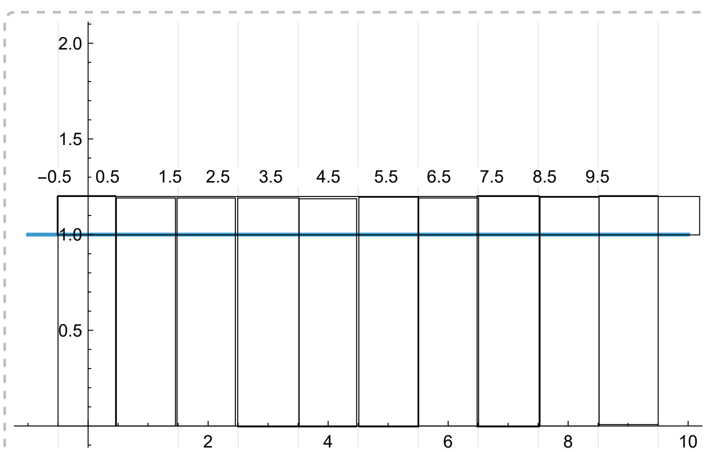
- Using the mid-points
  - Using $f(x) = x(x-10)$ we have $f(x_1 = -0.5) = 5.25$, $f(x_2 = 0.5) = |\text{-}4.75| = 4.75$, $f(x_3 = 1.5) = |\text{-}12.75| = 12.75$, $f(x_4 = 2.5) = |\text{-}18.75| = 18.75$, $f(x_5 = 3.5) = |\text{-}22.75| = 22.75$, $f(x_6 = 4.5) = |\text{-}24.75| = 24.75$, $f(x_7 = 5.5) = |\text{-}24.75| = 24.75$, $f(x_8 = 6.5) = |\ \text{-}22.75| = 22.75$, $f(x_9 = 7.5) = |\text{-}18.75| = 18.75$, $f(x_{10} = 8.5) = |\text{-}12.75| = 12.75$, $f(x_{11} = 9.5) = |\text{-}4.75| = 4.75$.
    - The area A_{midpoint} = (1)(5.25 + 4.75 + 12.75 + 18.75 + 22.75 + 24.75 + 24.75 + 22.75 + + 18.75 + 12.75 + 4.75 = 172.75
    - The percentage error with respect to the accepted value is $\left|\frac{172-172.75}{172}\right|\times 100 = 0.26\%$

In[ ]:=

```
Plot[1, {x, -1, 10}, GridLines → {{-0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5}, {0}}]
```



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- Evaluate $\int_{-1}^{10} x(x-10)\,dx$ using Riemann sum.
- Solution: For area above the positive x-axis:
  - $\Delta x = \frac{b-a}{n} = \frac{0-(-1)}{n} = \frac{1}{n}$
  - $x_i = a + (\Delta x)\,i = -1 + (\Delta x)i.$
  - So $f(x_i) = [-1 + (\Delta x)\,i]^2 - 10[-1 + (\Delta x)i] = 1 - 2(\Delta x)i + (\Delta x)^2\,i^2 + 10 - 10\,(\Delta x)i = 11 - 12(\frac{1}{n})\,i + \left(\frac{1}{n}\right)^2 i^2$
  - $[11 - 12(\frac{1}{n})\,i + \left(\frac{1}{n}\right)^2 i^2](\frac{1}{n}) = \frac{11}{n} - 12(\frac{1}{n})^2 i + \left(\frac{1}{n}\right)^3 i^2$
  - $\sum_{i=1}^{n} i = \frac{n^2+n}{2}, \sum_{i=1}^{n} 1 = n$, and $\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6}$. So by substitution
  - $\frac{11}{n}(\text{n}) - 12(\frac{1}{n})^2 \left(\frac{n^2+n}{2}\right) + \left(\frac{1}{n}\right)^3 (\frac{2n^3+3n^2+n}{6}) = 11 - 6(1 + \frac{1}{n}) + \left(\frac{2+3/n+1/n^2}{6}\right)$
  - As n→∞, then summation becomes (11 -6 +1/3) = 5.33
  - For the area below the x-axis we have
    - $\Delta x = \frac{b-a}{n} = \frac{10-0}{n} = \frac{10}{n}$
    - $x_i = 0 + (\Delta x)\,i$
  - So $x^2 - 10\,x = (\Delta x)^2\,i^2 - 10(\Delta x)i = (\frac{10}{n})^2\,i^2 - 10(\frac{10}{n})i = (\frac{10}{n})^2(\frac{2n^3+3n^2+n}{6}) - 10\left(\frac{n^2+n}{2}\right)$
  - $[(\frac{10}{n})^2(\frac{2n^3+3n^2+n}{6}) - 10(\frac{10}{n})\left(\frac{n^2+n}{2}\right)]\left(\frac{10}{n}\right) = (\frac{10}{n})^3(\frac{2n^3+3n^2+n}{6}) - 10\,\frac{(n^2+n)}{2}\,\left(\frac{10}{n}\right)^2 = \frac{10^3}{6}(2 + \frac{3}{n} + \frac{1}{n^2}) - 500(1 + \frac{1}{n})$
  - As n→∞, the absolute value of the sum becomes $|\frac{1000}{3} - 500| = 166.667$
  - The sum of the area above and below the x-xis is 5.33 + 166.667 ≈ 172 consistent with our previous answer.