Example 1:

Use the Newton Method to find a root of x = 2sin x.

Let f(x) = x − 2sin x. Then f'(x)=1 − 2cos x , and the Newton iteration is

$$x_{n+1} = Xn - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n - 2\sin x_n}{1 - 2\cos x_n}$$

Let x0 =1.1, f'(1.1)=0.092807757 so we can use it. The next six estimates are:

X1	8.4529922
X2	5.2564136
X3	203.3841837
X4	118.0193304
X5	-87.4706733
X6	-203.664234

The result get worse. X1 quite far from X0, and X2 is far from X1. the chaotic continues. The trouble was caused by the choice of X0.

Let X0 = π/3 , f'(π/3)=0 . we can not use it because x1 does not exist.

Let X0 = 1.5 f'(1.5)=0.858525 so we can use it. The next six estimates are:

X1	2.076558201
X2	1.910506616
X3	1.895622003
X4	1.895494276
X5	1.895494267
X6	1.895494267

The X5 agrees with X6 so the root is 1. 895494267.

Example 2:

Newton's equation $y^3 - 2y - 5 = 0$ has a root near y = 2. Starting with y0 = 2, compute y1, y2, and y3, the next three Newton-Raphson estimates for the root.

Let f(x)= $y^3 - 2y - 5$. Then f'(x)= $3y^2 - 2$, and the Newton iteration is

$$x_{n+1} = Xn - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{y^3 - 2y - 5}{3y^2 - 2}$$

Y0=2 then the Y1,Y2 and Y3 is

Y1	2.1
Y2	2.094568121
Y3	2.094551482

Example 3:

Use Newton's Method to find the only real root of the equation cos x = 2x correct to 9 decimal places

Let f(x) = cos x − 2x ,then f'(x)= − sin xn – 2 , and the Newton iteration is

$$x_{n+1}=Xn-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{\cos x_n-2x_n}{-\sin x_n-2}$$

Let X0=0.5 f'(0.5)=-2.4794255 and f(0.5)$\approx$ 0

X1	0.45063
X2	0.45018
X3	0.45018

with no further changes in the digits, to five decimal places. Therefore, to this degree of accuracy, the root is x = 0.45018

Example 4:

Use Newton's Method to find the only real root of the equation x = tan x correct to 9 decimal places

Let f(x) = x − tan x ,then f'(x)= 1 − $\sec^2 x$, and the Newton iteration is

$$x_{n+1}=Xn-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{x_n-\tan x_n}{1-\sec^2 x_n}$$

Let try X0=4 then

X1	6.12016
X2	238.40428
X3	1957.26490

Clearly these numbers are not converging. We need to try a new initial guess.

Let's try x0 = 4.6

X1	4.54573

X2	4.50615
X3	4.49417
X4	4.49341
X5	4.49341

with no further changes in the digits, to five decimal places. Therefore, to this degree of accuracy, the root is x = 4.49341