Mini Lesson #2

Computer Number Systems & Recursive Functions

Octal & Hexadecimal Number System

Octal (Base 8)

The octal numeral system, characterized by a base 8 representation, uses the digits 0 through 7. In this system, a total of 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7) are used to construct different numbers. You call a number system with **base 8** an **octal number system**.

1	8	64	512	4096	32768	262144	2.1×10^6	1.7×10^7	1.3×10^8	1.1×10^9
8^0	8^1	8^2	8^3	8^4	8^5	8^6	8^7	8^8	8^9	8^{10}

↑ Useful for when we convert from octal to decimal.

Counting in Octal

Counting in octal is like counting in decimal except you never use 8 and 9.

0	11	22	33	44	55	66	77
1	12	23	34	45	56	67	?
2	13	24	35	46	57	70	
3	14	25	36	47	60	71	
4	15	26	37	50	61	72	
5	16	27	40	51	62	73	
6	17	30	41	52	63	74	
7	20	31	42	53	64	75	
10	21	32	43	54	65	76	

What do you think might come next?

Answer: **100!**

When you want to multiply octals with each other, you have to convert them to decimal first and then multiply, and then convert it back to octal.

If you had 6 × 5. In decimal that is still 6 × 5. So that is 30 in decimal, and then you have to convert back to octal to get 36.

Converting from Octal to Decimal

Let's you are given the number **145_8** (Note: The '8' in the 145_8 is base 8, suggesting that this number is in octal).

1) Look at the rightmost digit first (5).
    a) 5 is in the 8^0 place, so its value would be $5 \times 8^0 = 5$.
2) Look at the next digit after 5 (4).
    a) 4 is in the 8^1 place, so its value would be $4 \times 8^1 = 32$.
3) Look at the leftmost digit (1).
    a) 1 is in the 8^2 place, so its value would be $1 \times 8^2 = 64$.
4) Now add up all three numbers: $5 + 32 + 64 = 101$.

Answer: 101_{10}

Converting from Decimal to Octal

Let's you are given the number $\mathbf{53_{10}}$.

1) Divide the decimal number 53 by 8.
    a) 53 ÷ 8 = 6 r **5**.
    b) Another way to get remainder: 53 % 8 = 5
2) Divide the quotient (excluding the remainder) by 8 again.
    a) 6 ÷ 8 = 0 r **6**.
    b) Alternative: 6 % 8 = 6
3) Take the last remainder you found and make it the leftmost digit.
    a) 6_
4) Then include the other digit(s) in that order.
    a) 65

Answer: $\mathbf{65_8}$

What is 1005_{10} in octal? What is 237_8 in decimal?

1005_{10} (DECIMAL to OCTAL)

$1005 \div 8 = 125$ r **5**

$125 \div 8 = 15$ r **5**

$15 \div 8 = 1$ r **7**

$1 \div 8 = 0$ r **1**

reverse order

$1 \rightarrow 7 \rightarrow 5 \rightarrow 5$

$1005_{10} \rightarrow 1755_8$

Answer: 1755_8

237_8 (OCTAL to DECIMAL)

$7 \times 8^0 = 7 \times 1 = 7$

$3 \times 8^1 = 3 \times 8 = 24$

$2 \times 8^2 = 2 \times 64 = 128$

$128 + 24 + 7 = 159$

$237_8 \rightarrow 159_{10}$

Answer: 159_{10}

What is 16_8 in binary? (Octal to Binary)

0 → 0

1 → 1

10 → 2

11 → 3

100 → 4

101 → 5

110 → 6

111 → 7

1000 → 10

16 = 1 and 6

1 (octal) = 1 (binary)

6 (octal) = 110 (binary)

Answer: 1110

Hexadecimal (Base 16)

The hexadecimal system, also referred to as hex, is a numerical system with a base value of 16. It uses 16 symbols, namely 0-9 and A-F, where each symbol represents a decimal value.

For example D in hex is equivalent to 13 in decimal.

Counting in Hex

0	B	16	21	2D	38	44	50	5B	66	71	7C	87	93	9F	AA	B7	C2
1	C	17	22	2E	39	45	51	5C	67	72	7D	88	94	A0	AB	B8	C3
2	D	18	23	2F	3A	46	52	5D	68	73	7E	89	95	A1	AC	B9	C4
3	E	19	24	30	3B	47	53	5E	69	74	7F	8A	96	A2	AD	BA	C5
4	F	1A	25	31	3C	48	54	5F	6A	75	80	8B	97	A3	AE	BB	C6
5	10	1B	26	32	3D	49	55	60	6B	76	81	8C	98	A4	AF	BC	C7
6	11	1C	27	33	3E	4A	56	61	6C	77	82	8D	99	A5	B0	BD	C8
7	12	1D	28	34	3F	4B	57	62	6D	78	83	8E	9A	A6	B1	BE	C9
8	13	1E	29	35	40	4C	58	63	6E	79	84	8F	9B	A7	B2	BF	CA
9	14	1F	2A	36	41	4D	59	64	6F	7A	85	90	9C	A8	B3	C0	CB
A	15	20	2B	37	42	4E	5A	65	70	7B	86	91	9D	A9	B4	C1	CC
			2C		43	4F						92	9E		B5		
															B6		

Converting from Hex to Decimal (Multiplication)

Given 7C5.

1) What is 7 equal to in decimal?
   a) 7 = 7
2) What is C equal to in decimal?
   a) C = 12
3) What is 5 equal to in decimal?
   a) 5 = 5
4) $5 \times 16^0 = 5$
   $12 \times 16^1 = 192$
   $7 \times 16^2 = 1792$
   $1792 + 192 + 5 = 1989$

Answer: 1989

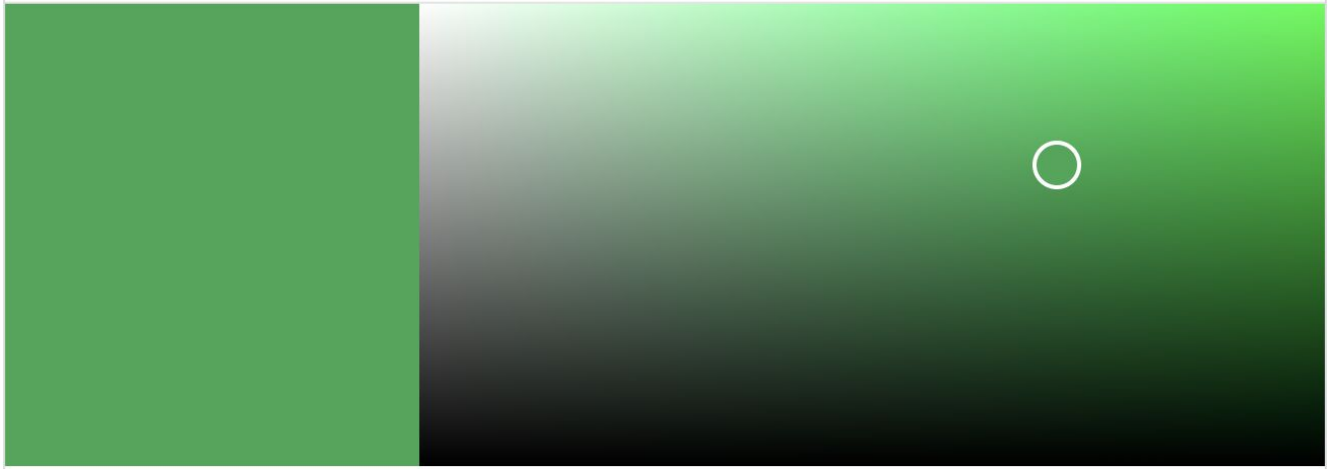
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
A	10
B	11
C	12
D	13
E	14
F	15

Converting from Decimal to Hex (Division)

Given 960_{10}.

1) Divide decimal number by 16.
    a) 960 ÷ 16 = 60 r **0**
2) Divide quotient by 16 till you get 0 as the quotient (excluding the remainder).
    a) 60 ÷ 16 = 3 r **12**
    b) 3 ÷ 16 = 0 r **3**
3) Convert the remainders to their corresponding hex value.
    a) 0 = 0
    b) 12 = C
    c) 3 = 3
4) Put the last remainder you got as the first digit and continue: 3_ **Answer: 3C0**

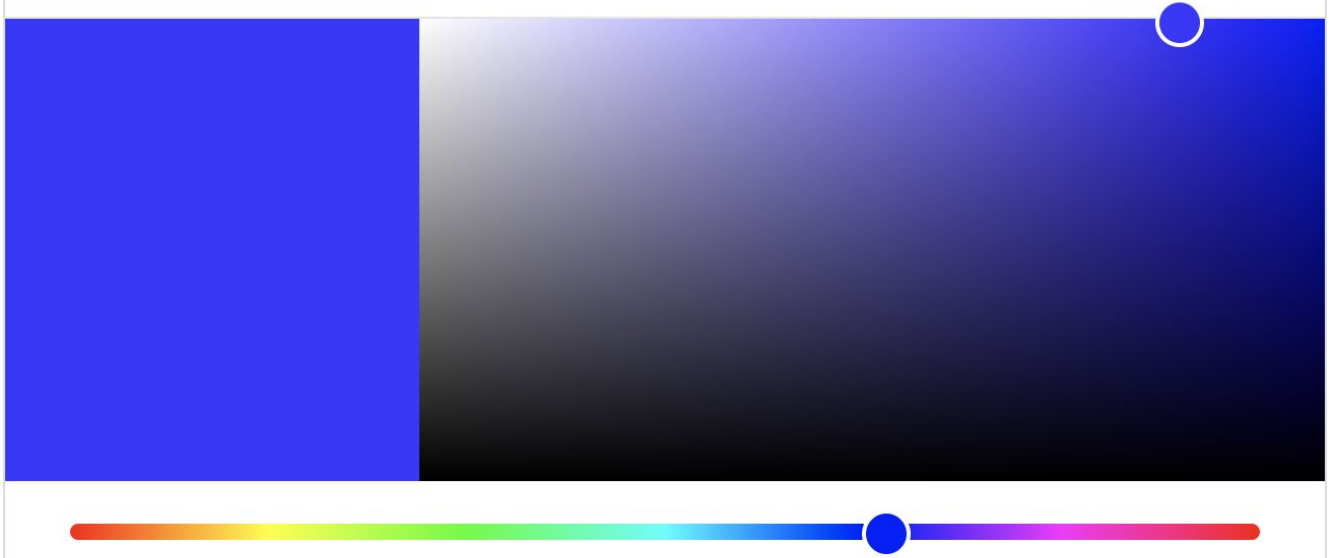
Color picker



HEX
#32a852

RGB	CMYK	HSV	HSL
50, 168, 82	70%, 0%, 51%, 34%	136°, 70%, 66%	136°, 54%, 43%

Color picker



HEX
#402aff

RGB	CMYK	HSV	HSL
64, 42, 255	75%, 84%, 0%, 0%	246°, 84%, 100%	246°, 100%, 58%

Review!

Question 1: Convert $3A9B_{16}$ to Octal

$3A9B_{16}$

1) Convert to decimal.
   a) 3 = 3
      A = 10
      9 = 9
      B = 11
   b) $11 \times 16^0 = 11$
      $9 \times 16^1 = 144$
      $10 \times 16^2 = 2560$
      $3 \times 16^3 = 12288$
      15003

2) Convert from decimal to octal.
   a) 15003 ÷ 8 = 1875 r 3
   b) 1875 ÷ 8 = 234 r 3
   c) 234 ÷ 8 = 29 r 2
   d) 29 ÷ 8 = 3 r 5
   e) 3 ÷ 8 = 0 r 3
   f) 35233

Answer: 35233_8

Question 2: Evaluate and express your answer in hex:

$$32_8 + 1011_2 + 352_{10} + AF_{16}$$

32_8	1011_2	352_{10}	AF_{16}
To decimal:	To decimal:	Already in decimal:	To decimal:
$2 \times 8^0 = 2$	$2^0 + 2^1 + 2^3$		$F = 15 \times 16^0 = 15$
$3 \times 8^1 = 24$	$1 + 2 + 8 = 11$		$A = 10 \times 16^1 = 160$
26 +	11 +	352 +	175

$26 + 11 + 352 + 175 = 564_{10}$

Answer: 234_{16}

To hexadecimal:

$564 \div 16 = 35$ r 4

$35 \div 16 = 2$ r 3

$2 \div 16 = 0$ r 2

Upcoming: Prefix/Infix/Postfix Notation