GEE.

$i = 1 \ldots k.$ $\quad$ k : sample size for our case.

$Y_i = (y_{i1} \ldots, y_{in_i})$. $\quad \underline{n_i \equiv 1}$ ← single observation for each person.

$E(y_{it}) = \mu_{it} \; (= p_{it}) \qquad g(\mu_{it}) = x_{it}^T \beta.$

$g(s) = \log \frac{s}{1-s}$. ∵ we use the logistic model

$Var(y_{it}) =. \; a_{it} = a(\mu_{it}) = a(p_{it}) = p_{it}(1-p_{it}) \cdot \phi = 1.$

∵ Bernoulli distribution

$$\frac{\partial \mu_i}{\partial \beta} = \frac{\partial}{\partial \beta}\left(\frac{\exp(x_i^T\beta)}{1+\exp(x_i^T\beta)}\right) = \frac{\exp(x_i^T\beta)}{(1+\exp(x_i^T\beta))^2} x_i^T$$

∵ the logistic model

$$= p_i(1-p_i)\, x_i^T$$

$R_i(\alpha) \equiv 1$

↑ 1×1 scalar for our case ∵ $n_i \equiv 1$

$$\therefore V_i = \phi A_i^{\frac{1}{2}} R_i(\alpha) A_i^{\frac{1}{2}} = A_i = p_i(1-p_i)$$

$$\Sigma = \lim_{k \to \infty} k \cdot \Sigma_0^{-1} \Sigma_1 \Sigma_0^{-1}. \approx k \cdot \Sigma_0^{-1} \Sigma_1 \Sigma_0^{-1}$$

$$\Sigma_0 = \sum_{i=1}^{k} \left(\frac{\partial \mu_i}{\partial \beta}\right) V_i^{-1} \left(\frac{\partial \mu_i}{\partial \beta}\right)^T = \sum_{i=1}^{k} p_i(1-p_i)\, x_i x_i^T$$

$$\Sigma_1 = \sum_{i=1}^{k} \left(\frac{\partial \mu_i}{\partial \beta}\right) V_i^{-1} Cov(Y_i)\, V_i^{-1} \left(\frac{\partial \mu_i}{\partial \beta}\right)^T$$

$$= \sum_{i=1}^{k} (y_i - p_i)\, x_i x_i^T$$

Generalized Estimation Equation

$i = 1, 2, \cdots, k$ (sample size : k)

$Y = (Y_1, \cdots, Y_k)^T$

$E(Y_i) = \mu_i$, $\text{logit}(\mu_i) = x_i^T \beta$

$\therefore \mu_i = \dfrac{\exp(x_i^T \beta)}{1 + \exp(x_i^T \beta)}$ $\qquad V_i = \mu_i \cdot (1 - \mu_i)$

$$X = \overset{2}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}\, n \qquad X_i = \overset{2}{\begin{pmatrix} & \end{pmatrix}}'$$

$$\frac{\partial}{\partial \beta} \mu_i = \frac{\exp(x_i^T \beta)}{(1 + \exp(x_i^T \beta))^2} = \mu_i \cdot (1 - \mu_i) \cdot x_i^T$$

$$k^{1/2} (\hat{\beta} - \beta) \xrightarrow{d} N(0, \Sigma)$$

$$\Sigma = k * \Sigma_0^{-1} \cdot \Sigma_1 \cdot \Sigma_0^{-1}$$

$$\Sigma_0 = \sum_{i=1}^{k} \mu_i \cdot (1 - \mu_i) \cdot \underbrace{\overset{2\times 1}{x_i} \cdot \overset{1\times 2}{x_i^T}}_{2\times 2}, \quad \Sigma_1 = \sum_{i=1}^{k} (Y_i - \mu_i)^2 \cdot \underbrace{\overset{2\times 1}{x_i} \cdot \overset{1\times 2}{x_i^T}}_{2\times 2}$$

$\hat{\beta}$'s Sandwitch variance $= \widehat{\Sigma_0^{-1} \cdot \Sigma_1 \cdot \Sigma_0^{-1}}$

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weighted glm

$$\mathrm{Var}(Y_i) = a_i \rightarrow \mathrm{Var}(\underset{\text{"}Y_i^*\text{"}}{w_i \cdot Y_i}) = w_i^2 \cdot a_i = w_i^2 \cdot \mu_i \cdot (1 - \mu_i)$$

$$\underset{\mu_i^*}{w_i \cdot \mu_i} = w_i * \frac{\exp(x_i^T \beta)}{1 + \exp(x_i^T \beta)}$$

$$\underset{\beta_0, \beta_1}{\text{argmin}} \sum w_i * (y_i - \hat{\mu}_i)^2 \quad \cdots ?$$

정리) $\mu_i^* = w_i * \dfrac{\exp(x_i^T\beta)}{1+\exp(x_i^T\beta)}$

$$Var(Y_i^*) = w_i^2 \cdot a_i$$
$$= w_i^2 \cdot \mu_i \cdot (1-\mu_i)$$

$$\frac{\partial}{\partial\beta}\mu_i^* = \frac{\exp(x_i^T\cdot\beta)}{(1+\exp(x_i^T\cdot\beta))^2} * w_i * x_i^T = \mu_i\cdot(1-\mu_i)\, w_i\, x_i^T$$

$$\Sigma_0 = \sum_{i=1}^{k}\left(\frac{\partial}{\partial\beta}\mu_i^*\right)^T \cdot V_i^{-1}\cdot\left(\frac{\partial}{\partial\beta}\mu_i^*\right)$$

$$= \sum_{i=1}^{k} \mu_i\cdot\cancel{(1-\mu_i)}\cdot w_i x_i\left[w_i^2\cdot\mu_i\cancel{(1-\mu_i)}\right]^{-1}\cdot\mu_i\cdot(1-\mu_i)w_i\cdot x_i^T$$