

Projekt Robot 2D

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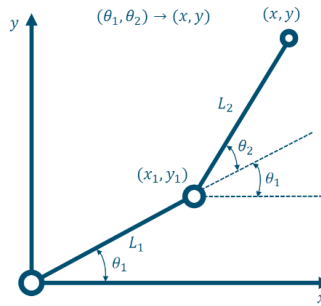
31 maja 2022

1 Cel ćwiczenia

Celem ćwiczenia było stworzenie symulacji działania robota 2D.

2 Obliczenia

2.1 Kinematyka prosta



Rysunek 1: Schemat kinematyki prostej

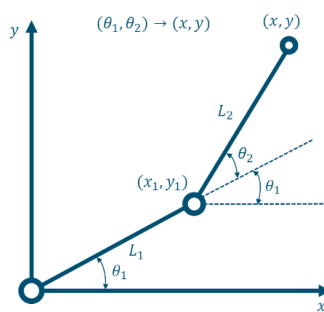
| | a_i | α_i | d_i | ϕ |
|---|-------------|------------|-------|--------|
| 1 | $a_1 = L_1$ | 0 | 0 | d_1 |
| 2 | $a_2 = L_2$ | 0 | 0 | d_2 |

Tablica 1: Notacja D-H

Wzory potrzebne przy wyznaczaniu pozycji manipulatora:

$$\begin{aligned}x_1 &= L_1 \cos \phi_1 \\y_1 &= L_1 \sin \phi_1 \\x &= L_1 \cos \phi_1 + L_2 \cos \phi_1 + \phi_2 \\y &= L_1 \sin \phi_1 + L_2 \sin \phi_1 + \phi_2\end{aligned}$$

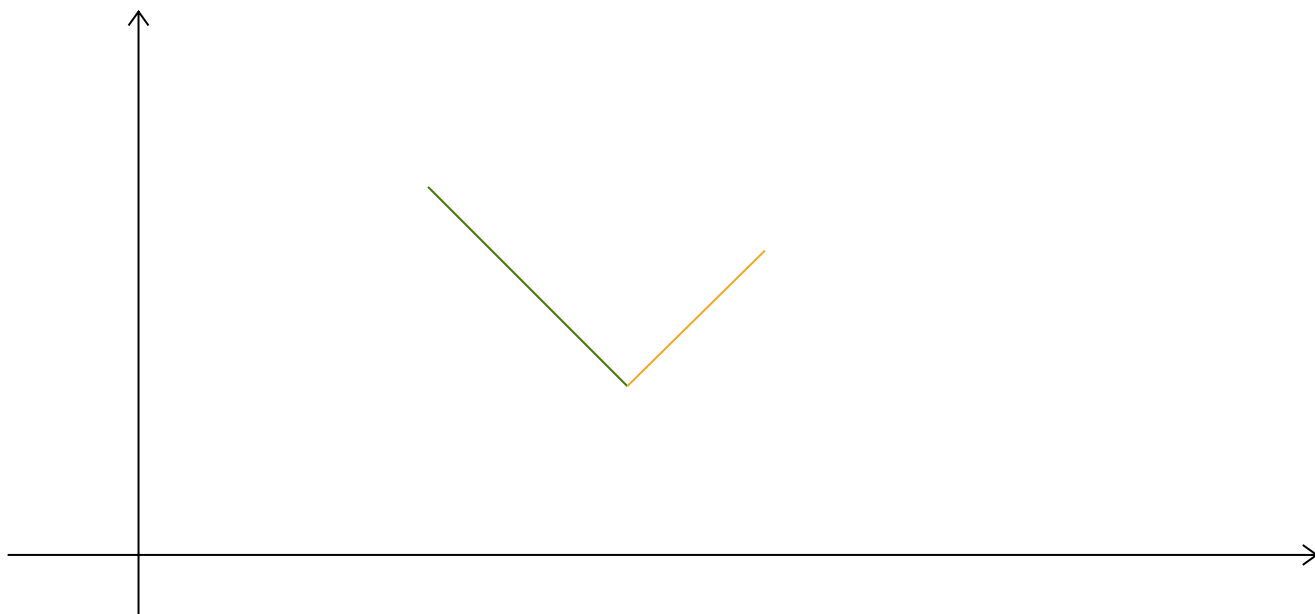
2.2 Kinematyka odwrotna



Rysunek 2: Schemat kinematyki odwrotnej

Potrzebne wzory

$$\phi_1 = \arctan \frac{y}{x} - \arccos \frac{L_1^2 + L^2 - L_2^2}{2L_1 L_2} \quad \phi_2 = \pi - \arccos \frac{L_1^2 + L^2 - L_2^2}{2L_1 L_2}$$



2.3 Python

```
1 from numpy import sin, cos
2 import numpy as np
3 import matplotlib.pyplot as plt
4 import scipy.integrate as integrate
5 import matplotlib.animation as animation
6
7 class robot_model:
8     def __init__(self,
9                 init_state = [120, 0, 120, 0],
10                L1=1.0, # dlugosc ramienia 1
11                L2=1.0, # dlugosc ramienia 2
12                M1=1.0, # masa ramienia 1 [kg]
13                M2=1.0, # masa ramienia 2 [kg]
14                G=9.8, # grawitacja m/s^2
15                origin=(0, 0)):
16         self.init_state = np.asarray(init_state, dtype='float')
17         self.params = (L1, L2, M1, M2, G)
18         self.origin = origin
19         self.time_elapsed = 0
20         self.dt = 0.05
21         self.state = self.init_state * np.pi / 180.
22         self.trajectory = [[], [], []]
23
24         self.__theta_target = []
25
26         self.p = np.zeros(2)
27         self.__p_target = None
28         self.__p_start = None
29
30         # Parametry macierzy DH
31         theta_0 = [0.0, 0.0]
32         a = [L1, L2]
33         d = [0.0, 0.0]
34         alpha = [0.0, 0.0]
35         self.dh_params = {"theta": theta_0, "a": a, "d": d, "alpha": alpha}
36
37         # Dlugosc ramienia [m]
38         self.L = [L1, L2]
39         # Polowa dlugosci ramienia [m]
40         self.lg = [L1/2, L2/2]
41         # Masa [kg]
42         self.m = [M1, M2]
43         # Moment inercyjny [kg.m^2]
44         self.I = [(1/3)*(M1)*(L1**2), (1/3)*(M2)*(L2**2)]
45         # Przyspieszenie grawitacyjne [m/s^2]
46         self.g = G
47         # Czas animacji
48         self.t = np.arange(0.0, 20, self.dt)
49
50         self.fig = plt.figure()
51         self.ax = self.fig.add_subplot(111, autoscale_on = False, xlim = (-2, 2), ylim
= (-2, 2))
52         self.ax.grid()
53         line, = self.ax.plot([], [], 'o-', lw = 2)
54         __line1, = self.ax.plot([], [], 'o-', lw = 2)
55         __line2, = self.ax.plot([], [], 'o-', lw = 2)
56         __line3, = self.ax.plot([], [], 'o-', lw = 2)
57         __line4, = self.ax.plot([], [], 'o-', lw = 2)
58
59         self.__line = [__line1, __line2, __line3, __line4]
60         self.__animation_dMat = np.zeros((1, 4), dtype=np.float64)
61
62     def position(self):
63         (L1, L2, M1, M2, G) = self.params
64
65         x = np.cumsum([self.origin[0],
66                       L1 * sin(self.state[0]),
67                       L2 * sin(self.state[2])])
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```

69         y = np.cumsum([self.origin[1],
70                        -L1 * cos(self.state[0]),
71                        -L2 * cos(self.state[2])])
72         return x.tolist(), y.tolist()
73
74     def dstate_dt(self, input_p, t):
75         theta_1 = input_p[0]; theta_2 = input_p[2]
76         dtheta_1 = input_p[1]; dtheta_2 = input_p[3]
77
78         M_Mat = np.matrix([
79             [self.I[0] + self.I[1] + self.m[0] * (self.lg[0]**2) + self.m[1] * ((self.
80             L[0]**2) + (self.lg[1]**2) + 2 * self.L[0] * self.lg[1] * np.cos(theta_2)), self.I
81             [1] + self.m[1] * ((self.lg[1]**2) + self.L[0] * self.lg[1] * np.cos(theta_2))],
82             [self.I[1] + self.m[1] * ((self.lg[1]**2) + self.L[0] * self.lg[1] * np.
83             cos(theta_2)), self.I[1] + self.m[1] * (self.lg[1]**2)]
84         ])
85
86         b_Mat = np.matrix([
87             [(-1) * self.m[1] * self.L[0] * self.lg[1] * dtheta_2 * (2 * dtheta_1 +
88             dtheta_2) * np.sin(theta_2)],
89             [self.m[1] * self.L[0] * self.lg[1] * (dtheta_1**2) * np.sin(theta_2)]
90         ])
91
92         g_Mat = np.matrix([
93             [self.m[0] * self.g * self.lg[0] * np.cos(theta_1) + self.m[1] * self.g *
94             (self.L[0] * np.cos(theta_1) + self.lg[1] * np.cos(theta_1 + theta_2))],
95             [self.m[1] * self.g * self.lg[1] * np.cos(theta_1 + theta_2)]
96         ])
97
98         # Ordinary Differential Equations (ODE)
99         ode_r = np.linalg.inv(M_Mat).dot(-b_Mat - g_Mat)
100
101         return [dtheta_1, ode_r[0][0], dtheta_2, ode_r[1][0]]
102
103     def forward_kin(self):
104
105         self.p[0] = round(self.dh_params["a"][0]*np.cos(self.dh_params["theta"][0]) +
106         self.dh_params["a"][1]*np.cos(self.dh_params["theta"][0] + self.dh_params["theta"
107         ][1]), 5)
108
109         self.p[1] = round(self.dh_params["a"][0]*np.sin(self.dh_params["theta"][0]) +
110         self.dh_params["a"][1]*np.sin(self.dh_params["theta"][0] + self.dh_params["theta"
111         ][1]), 5)
112
113     def forward_kinematics(self, theta):
114         self.__theta_target = np.zeros(2)
115         self.__theta_target[0] = theta[0]
116         self.__theta_target[1] = theta[1]
117
118         self.dh_params["theta"] = self.__theta_target
119
120         self.forward_kin()
121
122     def inverse_kinematics(self, p):
123
124         theta_aux = np.zeros(2)
125         self.__p_target = np.zeros(2)
126         self.__p_target[0] = p[0]
127         self.__p_target[1] = p[1]
128
129         cosT_beta_numerator = ((self.dh_params["a"][0]**2) + (self.__p_target[0]**2
130         + self.__p_target[1]**2) - (self.dh_params["a"][1]**2))
131         cosT_beta_denominator = (2*self.dh_params["a"][0]*np.sqrt(self.__p_target
132         [0]**2 + self.__p_target[1]**2))
133
134         # THETA 1
135         if cosT_beta_numerator/cosT_beta_denominator > 1:
136             theta_aux[0] = np.arctan2(self.__p_target[1], self.__p_target[0])
137             print('[INFO] Theta 1 Error: ', self.__p_target[0], self.__p_target[1])
138         elif cosT_beta_numerator/cosT_beta_denominator < -1:
139             theta_aux[0] = np.arctan2(self.__p_target[1], self.__p_target[0]) - np.pi

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129         print('[INFO] Theta 1 Error: ', self.__p_target[0], self.__p_target[1])
130     else:
131         theta_aux[0] = np.arctan2(self.__p_target[1], self.__p_target[0]) + np.
arccos(cosT_beta_numerator/cosT_beta_denominator)
132
133         cosT_alpha_numerator = (self.dh_params["a"][0]**2) + (self.dh_params["a"
] [1]**2) - (self.__p_target[0]**2 + self.__p_target[1]**2)
134         cosT_alpha_denominator = (2*(self.dh_params["a"][0]*self.dh_params["a"][1]))
135
136         # THETA2
137         if cosT_alpha_numerator/cosT_alpha_denominator > 1:
138             theta_aux[1] = np.pi
139             print('[INFO] Theta 2 Error: ', self.__p_target[0], self.__p_target[1])
140         elif cosT_alpha_numerator/cosT_alpha_denominator < -1:
141             theta_aux[1] = 0.0
142             print('[INFO] Theta 2 Error: ', self.__p_target[0], self.__p_target[1])
143         else:
144             theta_aux[1] = np.arccos(cosT_alpha_numerator/cosT_alpha_denominator) - np
.pi
145
146         self.theta = theta_aux
147         self.forward_kinematics(self.theta)
148
149     def generate_trajectory(self, trajectory_point):
150
151         self.__p_target = trajectory_point[0], trajectory_point[1]
152
153         start_theta = self.dh_params["theta"]
154
155         self.inverse_kinematics(self.__p_target)
156         self.__theta_target = self.theta
157         x = []
158         y = []
159
160         self.inverse_kinematics(trajectory_point)
161
162         # self.step()
163
164         self.inverse_kinematics(trajectory_point)
165         target_theta = self.__theta_target
166
167         start_theta_dt = np.linspace(start_theta[0], target_theta[0], 100)
168         target_theta_dt = np.linspace(start_theta[1], target_theta[1], 100)
169
170         # start_theta_dt = self.state[0]
171         # target_theta_dt = self.state[1]
172
173         for i in range(len(start_theta_dt)):
174             self.forward_kinematics([start_theta_dt[i], target_theta_dt[i]])
175             x.append(self.p[0])
176             y.append(self.p[1])
177
178         # x, y = self.position()
179         self.trajectory[0] = x
180         self.trajectory[1] = y
181
182     def step(self):
183         self.state = integrate.odeint(self.dstate_dt, self.state, np.linspace(0, 128,
128))
184         self.time_elapsed += self.dt
185
186     def __animation_data_generation(self):
187         self.__animation_dMat = np.zeros((len(self.trajectory[0]), 4), dtype=np.
float64)
188
189         for i in range(len(self.trajectory[0])):
190             self.inverse_kinematics([self.trajectory[0][i], self.trajectory[1][i]])
191             self.__animation_dMat[i][0] = self.dh_params["a"][0]*np.cos(self.dh_params
["theta"][0])
192             self.__animation_dMat[i][1] = self.dh_params["a"][0]*np.sin(self.dh_params
["theta"][0])

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193         self.__animation_dMat[i][2] = self.p[0]
194         self.__animation_dMat[i][3] = self.p[1]
195
196     def init_animation(self):
197         self.__animation_data_generation()
198
199         self.__line[0].set_data([0.0, self.__animation_dMat[0][0]], [0.0, self.
200         __animation_dMat[0][1]])
201         self.__line[1].set_data([self.__animation_dMat[0][0], self.__animation_dMat
202         [0][2]], [self.__animation_dMat[0][1], self.__animation_dMat[0][3]])
203         self.__line[2].set_data(self.__animation_dMat[0][0], self.__animation_dMat
204         [0][1])
205         self.__line[3].set_data(self.__animation_dMat[0][2], self.__animation_dMat
206         [0][3])
207
208         return [self.__line[0], self.__line[1], self.__line[2], self.__line[3]]
209
210     def start_animation(self, i):
211
212         self.__line[0].set_data([0.0, self.__animation_dMat[i][0]], [0.0, self.
213         __animation_dMat[i][1]])
214         self.__line[1].set_data([self.__animation_dMat[i][0], self.__animation_dMat[i
215         ] [2]], [self.__animation_dMat[i][1], self.__animation_dMat[i][3]])
216         self.__line[2].set_data(self.__animation_dMat[i][0], self.__animation_dMat[i
217         ] [1])
218         self.__line[3].set_data(self.__animation_dMat[i][2], self.__animation_dMat[i
219         ] [3])
220
221         return [self.__line[0], self.__line[1], self.__line[2], self.__line[3]]
222
223 def animate_animation():
224     p1, p2 = input("WPISZ PORZADANY PUNKT X (OD 0 DO 2): "), input("WPISZ PORZADANY
225     PUNKT Y (OD 0 DO 2): ")
226
227     robot2D = robot_model([240.0, 0.0, 240.0, 0.0])
228     dt = 1./30 # 30 fps
229
230     robot2D.generate_trajectory([p1, p2]) #([-0.5, 0.5])
231
232     line, = robot2D.ax.plot([], [], 'o-', lw=2, animated=True)
233     time_text = robot2D.ax.text(0.02, 0.95, '', transform=robot2D.ax.transAxes)
234     energy_text = robot2D.ax.text(0.02, 0.90, '', transform=robot2D.ax.transAxes)
235
236     animator = animation.FuncAnimation(robot2D.fig, robot2D.start_animation, init_func
237     =robot2D.init_animation, frames=len(robot2D.trajectory[0]), interval=2)
238
239     plt.show()
240
241 animate_animation()

```

3 Podsumowanie

Zadanie zostało zrealizowane zgodnie z [wymaganiami](#). Symulacja robota 2d odbywa się w czasie ciągłym.

Literatura

[1] Dr inż. Krzysztof Armiński, *Tematy*