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On the sharpest upper bound on the MP-ratio: approaching the problem by geometric insights

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joint work with Bojan Bašić

Department of Mathematics and Informatics, University of Novi Sad, Serbia

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- Σ^* the set of finite words, Σ^{∞} the set of finite or infinite words:

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- A word u is called a:
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 - subword of $w \in \Sigma^*$ if and only if there exist words $x_1, x_2, \ldots, x_n, x_{n+1} \in \Sigma^*$ and $y_1, y_2, \ldots, y_n \in \Sigma^*$ such that $u = y_1 y_2 \ldots y_n$ and $w = x_1 y_1 x_2 y_2 \ldots x_n y_n x_{n+1}$;

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- w[i,j] the factor of w that begins at the ith position in w and ends at the jth position in w;
- $|u|_{v} = |\{i : 1 \le i \le |u| |v| + 1, u[i, i + |v| 1] = v\}|.$

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- The role of palindromic words in both pure and applied mathematics, as well as in completely different areas, attracts more and more attention in recent times.
- For that reason, a number of ways to measure how palindromic a given word is have been defined and researched in the literature.

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Definition (Brlek, Hamel, Nivat, Reutenauer; 2004) *Palindromic defect:*

$$D(w) = |w| + 1 - |\operatorname{Pal}(w)|, w \in \Sigma^*;$$

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Definition (Frid, Puzynina, Zamboni; 2013)

Palindromic length of $w \in \Sigma^*$ – the least number of palindromes whose concatenation is the given word w.

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- r, s ∈ {0,1}*, (r,s) is an MP-extension of w ↔ rws is minimal-palindromic;
- (r, s) is an SMP-extension

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- (r, s) is an SMP-extension $\leftrightarrow |r| + |s|$ is the least possible;
- MP-ratio of the word w: $\frac{|rws|}{|w|}$, (r, s) SMP-extension of w.

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Theorem (Holub, Saari; 2009)

Every binary minimal-palindromic word is abelian unbordered.

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Theorem (Holub, Saari; 2009)

Every binary minimal-palindromic word is abelian unbordered.

Theorem (Holub, Saari; 2009)

Every binary word is characterized, up to reversal, by the set of its subpalindromes.

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Theorem (Holub, Saari; 2009)

The MP-ratio of any binary word is at most 4.

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The MP-ratio of any binary word is at most 4.

Proof (sketch).

$$0^{|w|+|w|_1}w1^{|w|+|w|_0}$$

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- w n-ary word on $\{0, 1, ..., n 1\}$;
- w contains a subpalindrome of length $\left| \frac{|w|}{n} \right|$;
- w is minimal-palindromic \leftrightarrow does not contain a subpalindrome longer than $\left\lceil \frac{|w|}{n} \right\rceil$;
- r, s ∈ {0, 1, ..., n − 1}*, (r, s) is an MP-extension of w
 rws is minimal-palindromic;
- (r, s) is an SMP-extension $\leftrightarrow |r| + |s|$ is the least possible;
- MP-ratio of the word w: $\frac{|rws|}{|w|}$, (r, s) SMP-extension of w.

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- $r, s \in \{0, 1, ..., n-1\}^*$, (r, s) is an MP-extension of $w \leftrightarrow rws$ is minimal-palindromic; Does it always exist???
- (r, s) is an SMP-extension $\leftrightarrow |r| + |s|$ is the least possible;
- MP-ratio of the word w: $\frac{|rws|}{|w|}$, (r, s) SMP-extension of w.

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Theorem (A., Bašić; 2021)

Each n-ary word has an MP-extension.

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Proof (sketch).

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Theorem (A., Bašić; 2021)

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Proof (sketch).

Let $w \in \{0, 1, \dots, n-1\}^*$, $n \geqslant 4$.

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Theorem (A., Bašić; 2021)

Each n-ary word has an MP-extension.

Proof (sketch).

Let $w \in \{0, 1, \dots, n-1\}^*$, $n \geqslant 4$.

Let $M = 2^{\lceil \frac{n}{2} \rceil} + 2^{\lfloor \frac{n}{2} \rfloor - 1} - 3$; exceptionally, if n = 4 or n = 5, we define M = 4, respectively M = 8 instead.

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Theorem (A., Bašić; 2021)

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Proof (sketch).

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Let $M = 2^{\lceil \frac{n}{2} \rceil} + 2^{\lfloor \frac{n}{2} \rfloor - 1} - 3$; exceptionally, if n = 4 or n = 5, we define M = 4, respectively M = 8 instead. Let:

$$r = 0^{l_0} 1^{l_1} \dots (n-2)^{l_{n-2}}, \ \ s = 1^{r_1} 2^{r_2} \dots (n-1)^{r_{n-1}},$$

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Well-definedness for $n \ge 4$

Theorem (A., Bašić; 2021)

Each n-ary word has an MP-extension.

Proof (sketch).

Let $w \in \{0, 1, ..., n-1\}^*, n \geqslant 4$.

Let $M = 2^{\lceil \frac{n}{2} \rceil} + 2^{\lfloor \frac{n}{2} \rfloor - 1} - 3$; exceptionally, if n = 4 or n = 5, we define M = 4, respectively M = 8 instead. Let:

$$r = 0^{l_0} 1^{l_1} \dots (n-2)^{l_{n-2}}, \ \ s = 1^{r_1} 2^{r_2} \dots (n-1)^{r_{n-1}},$$

where $I_0 = M|w| - |w|_0$ and

$$I_i = \left\{ \begin{array}{ll} (M-2^i+1)|w|, & \text{for } i = 1, \dots, \lceil \frac{n}{2} \rceil - 1; \\ (2^{n-1-i}-1)|w| - |w|_i, & \text{for } i = \lceil \frac{n}{2} \rceil, \dots, n-2, \end{array} \right.$$

while $r_{n-1} = M|w| - |w|_{n-1}$ and

$$r_i = \left\{ \begin{array}{ll} (2^i - 1)|w| - |w|_i, & \text{for } i = 1, \dots, \lceil \frac{n}{2} \rceil - 1; \\ (M - 2^{n - 1 - i} + 1)|w|, & \text{for } i = \lceil \frac{n}{2} \rceil, \dots, n - 2. \end{array} \right.$$

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Theorem (A., Bašić; 2021)

Each n-ary word has an MP-extension.

Proof (sketch).

Let $w \in \{0, 1, \dots, n-1\}^*, n \geqslant 4$.

Let $M = 2^{\lceil \frac{n}{2} \rceil} + 2^{\lfloor \frac{n}{2} \rfloor - 1} - 3$; exceptionally, if n = 4 or n = 5, we define M = 4, respectively M = 8 instead. Let:

$$r = 0^{l_0} 1^{l_1} \dots (n-2)^{l_{n-2}}, \ \ s = 1^{r_1} 2^{r_2} \dots (n-1)^{r_{n-1}},$$

where $I_0 = M|w| - |w|_0$ and

$$I_{i} = \begin{cases} (M - 2^{i} + 1)|w|, & \text{for } i = 1, \dots, \lceil \frac{n}{2} \rceil - 1; \\ (2^{n-1-i} - 1)|w| - |w|_{i}, & \text{for } i = \lceil \frac{n}{2} \rceil, \dots, n-2, \end{cases}$$

while $r_{n-1} = M|w| - |w|_{n-1}$ and

$$r_i = \begin{cases} (2^i - 1)|w| - |w|_i, & \text{for } i = 1, \dots, \lceil \frac{n}{2} \rceil - 1; \\ (M - 2^{n-1-i} + 1)|w|, & \text{for } i = \lceil \frac{n}{2} \rceil, \dots, n - 2. \end{cases}$$

Then (r, s) is an MP-extension of w.



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Theorem (A., Bašić; 2021)

Let w be an n-ary word for $n \ge 4$, and let

$$M = \begin{cases} 4, & \text{if } n = 4; \\ 8, & \text{if } n = 5; \\ 2^{\lceil \frac{n}{2} \rceil} + 2^{\lfloor \frac{n}{2} \rfloor - 1} - 3, & \text{if } n \geqslant 6. \end{cases}$$

Then the MP-ratio of w is not greater than nM.

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Theorem (A., Bašić; 2021)

Let w be an n-ary word for $n \ge 4$, and let

$$M = \left\{ \begin{array}{ll} 4, & \text{if } n = 4; \\ 8, & \text{if } n = 5; \\ 2^{\left \lceil \frac{n}{2} \right \rceil} + 2^{\left \lfloor \frac{n}{2} \right \rfloor - 1} - 3, & \text{if } n \geqslant 6. \end{array} \right.$$

Then the MP-ratio of w is not greater than nM.

Theorem (A., Bašić; 2021)

The optimal upper bound on the MP-ratio is not less than 2n.

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Now we know that the optimal upper bound on the MP-ratio for $n \ge 4$ is somewhere between

$$2n$$
 and $\sim 2^{\frac{n}{2}}n$,

and it remains an open problem to narrow (or even better, close) this gap.

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Theorem (A., Bašić; 2021)

The MP-ratio is well-defined in the ternary case, it is bounded from above by 6 and this upper bound is the best possible.

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Theorem (A., Bašić; 2021)

The MP-ratio is well-defined in the ternary case, it is bounded from above by 6 and this upper bound is the best possible.

$$f(w) = 0^{2|w|-|w|_0} 2^{2|w|-|w|_2-g'(w,0,2)} w 2^{g'(w,0,2)} 1^{2|w|-|w|_1}$$

$$f'(w) = 1^{2|w|-|w|_1} 2^{g'(\widetilde{w},0,2)} w 2^{2|w|-|w|_2-g'(\widetilde{w},0,2)} 0^{2|w|-|w|_0}$$

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$$f(w) = 0^{2|w|-|w|_0} 2^{2|w|-|w|_2-g'(w,0,2)} w 2^{g'(w,0,2)} 1^{2|w|-|w|_1}$$

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$$g'(w, a, b) = \max (\{2|w[i, |w|]|_2 - |w[i, |w|]|_b : i = 1, 2, \dots, j(a, w)\} \cup \{0\}),$$

where j(a, w) denotes the position of the last occurrence of a in w, and j(a, w) = 0 if a does not occur in w.

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$$g'(w, a, b) = \max (\{2 |w[i, |w|]|_a - |w[i, |w|]|_b : i = 1, 2, \dots, j(a, w)\} \cup \{0\}),$$

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Theorem (A., Bašić; 2022)

Let $u \in \{1,2\}^*$, $t,v \in 2^*$ and let p and q be subpalindromes of tu and uv, respectively. If |p| + |q| > 2|u|,

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$$g'(w, a, b) = \max \big(\big\{ 2 \big| w[i, |w|] \big|_{a} - \big| w[i, |w|] \big|_{b} : i = 1, 2, \dots, j(a, w) \big\} \cup \{0\} \big),$$

where j(a, w) denotes the position of the last occurrence of a in w, and j(a, w) = 0 if a does not occur in w.

Theorem (A., Bašić; 2022)

Let $u \in \{1,2\}^*$, $t,v \in 2^*$ and let p and q be subpalindromes of tu and uv, respectively. If |p| + |q| > 2|u|, then

$$|u|_1\leqslant \frac{|tv|-1}{|tv|}|tuv|_2.$$

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Theorem (A., Bašić; 2025+)

The MP-ratio of any 4-ary word is at most 8.

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Theorem (A., Bašić; 2025+)

The MP-ratio of any 4-ary word is at most 8.

Proof (sketch).

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Theorem (A., Bašić; 2025+)

The MP-ratio of any 4-ary word is at most 8.

Proof (sketch).

We define two extensions of w, and show that at least one of them is an MP-extension.

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We define two extensions of w, and show that at least one of them is an MP-extension.

$$0^{2|w|-|w|_0}3^{2|w|-|w|_3-g(w,0,3)}2^{g(\widetilde{w},1,2)}\ \ w\ \ 3^{g(w,0,3)}2^{2|w|-|w|_2-g(\widetilde{w},1,2)}1^{2|w|-|w|_1};$$

$$1^{2|w|-|w|_1}2^{2|w|-|w|_2-g(w,1,2)}3^{g(\widetilde{w},0,3)} w 2^{g(w,1,2)}3^{2|w|-|w|_3-g(\widetilde{w},0,3)}0^{2|w|-|w|_0}.$$

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Theorem (A., Bašić; 2025+)

The MP-ratio of any 4-ary word is at most 8.

Proof (sketch).

We define two extensions of w, and show that at least one of them is an MP-extension.

Let us call them $f_1(w)$ and $f_2(w)$, respectively.

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The MP-ratio of any 4-ary word is at most 8.

Proof (sketch).

We define two extensions of w, and show that at least one of them is an MP-extension.

Let us call them $f_1(w)$ and $f_2(w)$, respectively. We have $|f_1(w)| = |f_2(w)| = 8|w|$, and we prove that at least one of the words $f_1(w)$ and $f_2(w)$ does not have a subpalindrome whose length exceeds 2|w|.

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$$0^{2|w|-|w|_0} 3^{2|w|-|w|_3-g(w,0,3)} 2^{g(\widetilde{w},1,2)} \quad w \quad 3^{g(w,0,3)} 2^{2|w|-|w|_2-g(\widetilde{w},1,2)} 1^{2|w|-|w|_1};$$

$$1^{2|w|-|w|_1} 2^{2|w|-|w|_2-g(w,1,2)} 3^{g(\widetilde{w},0,3)} \quad w \quad 2^{g(w,1,2)} 3^{2|w|-|w|_3-g(\widetilde{w},0,3)} 0^{2|w|-|w|_0}.$$

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$$0^{2|w|-|w|_0}3^{2|w|-|w|_3-g(w,0,3)}2^{g(\widetilde{w},1,2)} \quad w \quad 3^{g(w,0,3)}2^{2|w|-|w|_2-g(\widetilde{w},1,2)}1^{2|w|-|w|_1};$$

$$1^{2|w|-|w|_1}2^{2|w|-|w|_2-g(w,1,2)}3^{g(\widetilde{w},0,3)} \quad w \quad 2^{g(w,1,2)}3^{2|w|-|w|_3-g(\widetilde{w},0,3)}0^{2|w|-|w|_0}$$

Proposition

The length of an arbitrary subpalindrome of the form 0p0 in each of the words $f_1(w)$ and $f_2(w)$ is less than or equal to 2|w|.

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$$0^{2|w|-|w|_0}3^{2|w|-|w|_3-g(w,0,3)}2^{g(\widetilde{w},1,2)} \ \ w \ \ 3^{g(w,0,3)}2^{2|w|-|w|_2-g(\widetilde{w},1,2)}1^{2|w|-|w|_1};$$

$$1^{2|w|-|w|_1}2^{2|w|-|w|_2-g(w,1,2)}3^{g(\widetilde{w},0,3)} \ \ w \ \ 2^{g(w,1,2)}3^{2|w|-|w|_3-g(\widetilde{w},0,3)}0^{2|w|-|w|_0}$$

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The length of an arbitrary subpalindrome of the form 0p0 in each of the words $f_1(w)$ and $f_2(w)$ is less than or equal to 2|w|.

Proposition

The length of an arbitrary subpalindrome of the form 1 p1 in each of the words $f_1(w)$ and $f_2(w)$ is less than or equal to 2|w|.

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Proposition 3

Proposition

At least one among the words $f_1(w)$ and $f_2(w)$ does not contain a subpalindrome of the form 2p2 longer than 2|w|.

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Proposition 3

Proposition

At least one among the words $f_1(w)$ and $f_2(w)$ does not contain a subpalindrome of the form 2p2 longer than 2|w|.

 In proving this result, we used a geometric approach in one part of the proof. We defined certain mappings on sequences of letters in a word, inspired by Euclidean isometries—especially reflections and translations—and used these mappings in our analysis.

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Proposition

At least one among the words $f_1(w)$ and $f_2(w)$ does not contain a subpalindrome of the form 3p3 longer than 2|w|.

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Propositions 4-5

Proposition

At least one among the words $f_1(w)$ and $f_2(w)$ does not contain a subpalindrome of the form 3p3 longer than 2|w|.

Proposition

One of the following is true: the word $f_1(w)$ does not contain a subpalindrome of the form 2p2 longer than 2|w|, or the word $f_2(w)$ does not contain a subpalindrome of the form 3p3 longer than 2|w|. Analogously, the word $f_1(w)$ does not contain a subpalindrome of the form 3p3 longer than 2|w|, or the word $f_2(w)$ does not contain a subpalindrome of the form 2p2 longer than 2|w|.

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$f_1(w)$	$f_2(w)$
	$f_1(w)$

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Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1		
2 <i>p</i> 2		
3 <i>p</i> 3		

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	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2		
3 <i>p</i> 3		

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Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	
3 <i>p</i> 3		

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Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	
3 <i>p</i> 3	×	

The main theorem

Putting everything together

Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	
3 <i>p</i> 3	×	
	✓	

The main theorem

Putting everything together

Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	
3 <i>p</i> 3	\in	

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Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	
3 <i>p</i> 3	\in	×

The main theorem

Putting everything together

Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	×
3 <i>p</i> 3	\in	×

The main theorem

Putting everything together

Putting everything together

	$f_1(w)$	$f_2(w)$
0 <i>p</i> 0	×	×
1 <i>p</i> 1	×	×
2 <i>p</i> 2	×	×
3 <i>p</i> 3	\in	×
		\checkmark