Exercise 5

FY8503 Advanced theoretical physics
Transport modelling with Stochastic differential equations

October 9, 2023

Problem 1

In this exercise, we will consider the concentration (density) of diffusing particles. The diffusion equation (for constant diffusivity) is given by

$$\frac{\partial p}{\partial t} = K \frac{\partial^2 p}{\partial x^2}. \tag{1}$$

Task a

Find the SDE corresponding to Eq. (1).

Task b

Implement a solver for the SDE found above, with a reflecting boundary at $x = 0$. Solve the SDE for a large number of particles, all starting out at some position $x_0 > 0$. Choose suitable values for K, x_0 and the integration time, such that you can clearly see the effect of the reflecting boundary.

Compare the density of particle positions to the analytical solution of Eq. (1), which in this case is a sum of two Gaussians, both with variance $\sigma^2 = 2Kt$, and with means $\mu_+ = x_0$ and $\mu_- = -x_0$. Note that using two Gaussians, symmetrically placed about the reflecting boundary in this way is the same idea as the use of "image charges" in electrostatics.

Task c

Repeat the problem above, but now with an absorbing boundary at $x = 0$. In this case, the analytical solution is a difference of two Gaussians, instead of the sum.