Exercise 4

FY8503 Advanced theoretical physics
Transport modelling with Stochastic differential equations

October 3, 2023

Problem 1

Here we will again look at a version of the Duffing-van der Pol oscillator, and try some calculations as described on pages 222–225 in Kloeden, Platen & Schurz. The oscillator is described by two coupled SDEs:

$$\mathrm{d}X = V\,\mathrm{d}t \tag{1}$$

$$\mathrm{d}V = \big(X(\alpha - X^2) - V\big)\mathrm{d}t + \sigma X\,\mathrm{d}W_t, \tag{2}$$

where α and σ are constants. We can equivalently write this as a vector-valued SDE with scalar noise:

$$\begin{pmatrix} \mathrm{d}X \\ \mathrm{d}V \end{pmatrix} = \begin{pmatrix} V \\ \big(X(\alpha - X^2) - V\big) \end{pmatrix} \mathrm{d}t + \begin{pmatrix} 0 \\ \sigma X \end{pmatrix} \mathrm{d}W_t. \tag{3}$$

Task a

Repeat Problem 2, Task B from Exercise 3, but solving Eq. (3) with the order 1.5 strong Itô-Taylor scheme for scalar noise. See Eq. 4.5 on page 353 of Kloeden & Platen, and see page 339 for the definitions of L^0 and L^1. You can choose if you want to try to implement a fully general version of the scheme, or if you prefer to implement the scheme specifically for Eq. (3).

References

Kloeden, Platen & Schurz, 1997, *Numerical Solution of SDE Through Computer Experiments*, Springer-Verlag, Berlin, Heidelberg, corrected second printing. `https://link.springer.com/book/10.1007/978-3-642-57913-4`

Kloeden & Platen, 1992, *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag, Berlin, Heidelberg. `https://link.springer.com/book/10.1007/978-3-662-12616-5`