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函数的微分

一、填空题.

1. $\frac{1}{1+4x^2}dx=\frac{1}{2}\frac{1}{1+(2x)^2}d\underline{2x}=\frac{1}{2}d\underline{\arctan(2x)}=d\underline{\frac{1}{2}\arctan(2x)}$;

2. $\frac{1}{\sqrt{1+2x}}dx=\frac{1}{2}\frac{1}{\sqrt{1+2x}}d\underline{2x}=d\underline{\sqrt{1+2x}}$;

3. $\frac{f'(\arctan x)}{1+x^2}dx=f'(\arctan x)d\underline{\arctan x}=d\underline{f(\arctan x)}$;

4. $d2^{\arctan^3 x}=2^{\arctan^3 x}\ln 2d\underline{\arctan^3 x}$

二、计算微分.

1. $d(x^2\ln x+\arcsin 2x)$;

$=\ln x\,dx^2+x^2 d\ln x+d\arcsin 2x$

$=\left[2x\ln x+x+\frac{2}{\sqrt{1-4x^2}}\right]dx$

2. $d(\arctan e^{\sqrt{x}})$;

$=\frac{1}{1+e^{2\sqrt{x}}}de^{\sqrt{x}}$

$=\frac{e^{\sqrt{x}}}{1+e^{2\sqrt{x}}}d\sqrt{x}$

$=\frac{e^{\sqrt{x}}}{2(1+e^{2\sqrt{x}})\sqrt{x}}dx$

3. $d\left(\frac{2^x}{x^2+1}\right)$;

$=\frac{d2^x\cdot(x^2+1)-d(x^2+1)\cdot 2^x}{(x^2+1)^2}$

$=\frac{(x^2+1)2^x\ln 2-2x2^x}{(x^2+1)^2}dx$

4. $u=u(x),v=v(x)$ 为可导函数,求 $y=\arctan\frac{u}{v}$ 的微分.

$d(\arctan\frac{u}{v})=\frac{1}{1+\frac{u^2}{v^2}}d(\frac{u}{v})$

$=\frac{1}{1+\frac{u^2}{v^2}}\frac{vdu-udv}{v^2}$

$=\frac{vdu-udv}{u^2+v^2}$

$=\frac{v(x)u'(x)-u(x)v'(x)}{u^2(x)+v^2(x)}dx$

三、求隐函数或参数方程决定函数的导数.

1. $y=y(x)$ 由方程 $x^2y+e^y=\ln x$ 决定,求 $\dfrac{dy}{dx}$;

$$2xy+x^2\frac{dy}{dx}+e^y\frac{dy}{dx}=\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\frac{1}{x}-2xy}{x^2+e^y}=\frac{1-2x^2y}{x^3+xe^y}$$

2. $y=y(x)$ 由参数方程 $\begin{cases}x=e^{2t}-2e^t+3,\\ y=3e^{4t}-4e^{3t}+7\end{cases}$ 确定,求 $\dfrac{dy}{dx},\dfrac{d^2y}{dx^2}$.

$$\frac{dy}{dx}=\frac{12e^{4t}-12e^{3t}}{2e^{2t}-2e^t}=\frac{6e^{2t}(e^t-1)}{e^t-1}=6e^{2t}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dt}(6e^{2t})\frac{dt}{dx}=12e^{2t}\frac{1}{2e^{2t}-2e^t}=\frac{6e^t}{e^t-1}$$

四、求 arctan1.05 的近似值.

$$\arctan(1.05)\approx\arctan\left(1+\frac{1}{20}\right)$$

$$\approx\arctan 1+\frac{1}{1+1}\cdot\frac{1}{20}$$

$$\approx\frac{\pi}{4}+\frac{1}{40}$$

五、利用微分的近似公式证明:$(1+x)^\alpha\approx1+\alpha x$,当 $|x|$ 充分小时. 并由此求 $\sqrt[3]{8012}$ 的近似值.

$$\left.\frac{d}{dx}(1+x)^\alpha\right|_{x=0}=\left.\alpha(1+x)^{\alpha-1}\right|_{x=0}=\alpha$$

于是 $(1+x)^\alpha\approx1+\alpha x$

$$\sqrt[3]{8012}=\sqrt[3]{8000+12}=20\sqrt[3]{1+\frac{3}{2000}}\approx20\left(1+\frac{1}{3}\times\frac{3}{2000}\right)=20.01$$