学院＿＿＿＿　姓名＿＿＿＿　学号＿＿＿＿　教师＿＿＿＿

不定积分的概念与性质

一、选择题.

1. 下列函数中,不是 $f(x)=4\sin x\cos x$ 的原函数的是(A).

A. $\sin 2x$　　B. $-\cos 2x$　　C. $2\sin^2 x$　　D. $-2\cos^2 x$

2. 设 $f(x)$ 为可导函数,$F'(x)=f(x)$,且 $f(0)=1$,又 $F(x)=xf(x)+x^2$,则 $f(x)=$(C).

A. $-2x-1$　　B. $-x^2+1$　　C. $-2x+1$　　D. $-x^2-1$

3. 下列各式中(C)是 $f(x)=\sin|x|$ 的原函数.

A. $y=-\cos|x|$

B. $y=-|\cos x|$

C. $y=\begin{cases}-\cos x, & x\geqslant 0,\\ \cos x-2, & x<0\end{cases}$

D. $y=\begin{cases}-\cos x+C_1, & x\geqslant 0,\\ \cos x+C_2, & x<0,\end{cases}$ C_1,C_2 为任意常数

二、计算下列不定积分.

1. $\displaystyle\int\frac{(1-x)^2}{\sqrt{x}}\mathrm{d}x$;

$=\int\frac{1-2x+x^2}{\sqrt{x}}dx$

$=\int x^{-\frac{1}{2}}dx-2\int x^{\frac{1}{2}}dx+\int x^{\frac{3}{2}}dx$

$=2x^{\frac{1}{2}}-\frac{4}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}+C$

2. $\displaystyle\int\frac{x^4}{1+x^2}\mathrm{d}x$; [多项式的带余除法]

$=\int\frac{x^4-1+1}{1+x^2}dx$

$=\int\frac{(x^2-1)(x^2+1)}{1+x^2}dx+\int\frac{1}{1+x^2}dx$

$=\int(x^2-1)dx+\arctan x+C$

$=\frac{1}{3}x^3-x+\arctan x+C$

3. $\displaystyle\int\frac{1}{x^2(1+x^2)}\mathrm{d}x$;

$=\int\left(\frac{1}{x^2}-\frac{1}{1+x^2}\right)dx$

$=\int\frac{1}{x^2}dx-\int\frac{1}{1+x^2}dx$

$=-\frac{1}{x}-\arctan x+C$

4. $\displaystyle\int(\mathrm{e}^x-1)^2 2^x\mathrm{d}x$;

展开

$=\int(e^{2x}-e^x+1)2^x dx$

$=\int(e^{2x}-e^x+1)e^{x\ln 2}dx$

$=\int e^{x(2+\ln 2)}dx-\int e^{x(1+\ln 2)}dx+\int e^{x\ln 2}dx$

$=\frac{e^{x(2+\ln 2)}}{2+\ln 2}-\frac{e^{x(1+\ln 2)}}{1+\ln 2}+\frac{e^{x\ln 2}}{\ln 2}+C$

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5. $\int \frac{1}{\sin^2 x\cos^2 x}dx$;

$= \int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}dx$

$= \int \frac{1}{\cos^2 x}dx + \int \frac{1}{\sin^2 x}dx$

$= \tan x - \cot x + C$

6. $\int \frac{2+\sin^2 x}{\cos^2 x}dx$;

$\sin^2 x = 1-\cos^2 x$

$= \int \frac{3-\cos^2 x}{\cos^2 x}dx$

$= 3\int \frac{1}{\cos^2 x}dx - x + C$

$= 3\tan x - x + C$

$\int \frac{1-\sin x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx + \int \frac{d\cos x}{\cos^2 x} = \tan x - \sec x + C$

$\left(\frac{g(x)}{f(x)}\right)' = \frac{g'(x)f(x)-f'(x)g(x)}{f^2(x)}$

7. $\int \frac{1}{1+\sin x}dx$;

$= \int \frac{1}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$= \int \frac{1}{(\sin\frac{x}{2}+\cos\frac{x}{2})^2}dx = \int \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{(\sin\frac{x}{2}+\cos\frac{x}{2})^2}dx = \frac{\sin\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}} + C$

$= \int \frac{1}{2\sin^2(\frac{x}{2}+\frac{\pi}{4})}dx = \int \frac{d(\frac{x}{2}+\frac{\pi}{4})}{\sin^2(\frac{x}{2}+\frac{\pi}{4})} = -\cot(\frac{x}{2}+\frac{\pi}{4}) + C$

8. $\int d\int f'(x)dx$;

$= \int dF(x)$ $[F(x)+C \triangleq \int f'(x)dx]$

$= \int f'(x)dx = f(x)+C$

9. $\int d[f(x)+2]$;

$= \int f'(x)dx$

$= f(x)+C$

10. $\int \cot^2 x dx$.

$= \int \frac{\cos^2 x}{\sin^2 x}dx$

$= \int \frac{1-\sin^2 x}{\sin^2 x}dx$

$= \int \frac{1}{\sin^2 x}dx - x + C$

$= -\cot x - x + C$