Apple delivery

Let $N(r)$ be the number of integer points within distance r from the origin. One thing we will definitely need in order to solve the problem is a fast way of computing $N(r)$. A slow way is to iterate through all the $(2r+1)^2$ points in the square $[-r, r] \times [-r, r]$ and checking if $x^2 + y^2 \leq r^2$. This takes $O(r^2)$ time. A faster way is to instead iterate through all x-coordinates in $[-r, r]$ and counting the number of points in the circle having that particular x-coordinate (which is $2\lfloor\sqrt{r^2 - x^2}\rfloor + 1$). This instead takes $O(r)$ time.

Subtask 1
For the first subtask we can now find $N(r)$ for every radius (with the slow or the fast method), and then try all subsets.

Subtask 2
For the second subtask we will again find every $N(r)$ (only the fast method will work now). Then we need a slightly better way to find the answer, for example by running a knapsack-like DP with $8 \cdot n$ states. Another way that will be more useful later is to partition the radii into groups with respect to the remainder of $N(r_i)$ modulo 8. Then we can note that it only makes sense to remove the smallest 7 radii from each group, so we can just check all the 8^8 possibilities. Actually the only possible remainders modulo 8 are 1 and 5, so there are only 8^2 possibilities.

Subtask 3
The third subtask is pretty much the same as the second except that you need to memoize the $N(r)$ to make it $O(R^2)$ rather than $O(n \cdot R)$, where R is the maximum r_i.

Subtask 4
To get full score we cannot possibly compute all the $N(r)$ even with the fast method. But as we saw in subtask 2, we actually only need to know 16 of the values, namely the smallest 8 of the ones with remainders 1 and 5. Since $N(r)$ is an increasing function, we can sort the radii themselves in order to find these 16 interesting radii. Now only one thing remains, a fast way to compute the remainder of $N(r)$ modulo 8. This can be done with a symmetry argument. Let's pick all the points within radius r of the origin. Now we remove all points (x, y) where $x = 0$ or $y = 0$ or $|x| = |y|$. The number of points left is now a multiple of eight (this is can be seen by drawing the region or by partitioning the remaining points into equivalence classes $(x, y) = (x, -y) = (-x, y) = (-x, -y) = (y, x) = (y, -x) = (-y, x) = (-y, -x)$). So to find $N(r)$ modulo 8 we just need to find the number of removed points, which turns out to be $1 + 4r + 4\lfloor\frac{r}{\sqrt{2}}\rfloor$. So now we know all $N(r)$ modulo 8, which allows us to solve the problem as in subtask 2. This takes $O(R + n)$ time.