Robotics Assignment 2

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Sheet 6

6.1)

(a) Update rule:

$\boldsymbol{l}(m_{i,j})$ **= inv_sensor_model(mi, xt, zt) +** $\boldsymbol{l_{t-1,i}} - \boldsymbol{l_0}$

$p(m_{i,j}) = 0.2$ => *prior probability*

$l_0 = l_{prior}(m_{i,j}) = log(\frac{p(mi,j)}{1-p(mi,j)}) = ln(\frac{0.2}{1-0.2}) = -1.3863$

$p(m = occ|z = d) = 0.8$

$l_{inv,z=d} = l(m = occ|z = d) = ln(\frac{0.8}{1-0.8}) = 1.3863$

$l_{inv,z>d} = p(m = occ|z > d) = 0.2$

$l(m = occ|z > d) = ln(\frac{0.2}{1-0.2}) = -1.3863$

- $z = d$:

  $l(m_{i,j}) = \boldsymbol{l_{t-1,i}} + l(m = occ|z = d) - l_0$

  $l(m_{i,j}) = \boldsymbol{l_{t-1,i}} + 1.3863 - (-1.3863)$
  $= \boldsymbol{l_{t-1,i}} + 2.7726$

- $z > d$:

  $l(m_{i,j}) = \boldsymbol{l_{t-1,i}} + l(m = occ|z > d) - l_0$

  $l(m_{i,j}) = \boldsymbol{l_{t-1,i}} + -1.3863 - (-1.3863)$
  $= \boldsymbol{l_{t-1,i}} + 0$

(b)

$l(x) = l_{100} = 60*2.7726+40*0 -(-1.3863) = 167.742$

$p(x) = 1 - \frac{1}{1+exp(l(x))} = 1 - \frac{1}{1+exp(167.742)} \simeq 1$

(c)

hits = 60 , misses = 40

reflection probability of the cell

$$Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y)+\text{misses}(x,y)}$$

Bel $(m^{[x,y]}) = \frac{60}{60+40}$ **= 0.6**

(d)

What are the benefits of the reflection map representation, and where are the problems?

- **Benefits:**
    1. Reflection probability maps are alternative representation of occupancy grid maps
    2. They store in each cell the probability that a beam is reflected by this cell
    3. Given the described sensor model, counting yields the maximum likelihood model
    4. The counting model determines how often a cell reflects a beam

- **Problems :**

  Although a cell might be occupied by an object, the reflection probability of this object might be very small

6.2)

b0 = 0.25, b1 = 1/3 , b2 = 0.5 , b3 = 1.Given the three measurements
zt0 = 0, zt2 = 3, zt3 = 1,
compute the value of the measurement zt1.

	c0	c1	c2	c3
zt0	**Hit**	**Not subject to update**	**Not subject to update**	**Not subject to update**
zt1	**?**	**?**	**?**	**?**
zt2	**miss**	**miss**	**miss**	**hit**
zt3	**miss**	**hit**	**Not subject to update**	**Not subject to update**

Hit/Miss scenario for the three given measurements

	c0	c1	c2	c3
hits	**1**	**1**	**0**	**1**
misses	**2**	**1**	**1**	**0**

- b0 = 0.25 when we look at the above table b0 = $\frac{1}{1+2} = 1/3$ we need another miss for b0 to be 0.25 so zt1 > 0
- b1=1/3 when we look at the above table b1 = $\frac{1}{1+1} = 1/2$ we need another miss for b1 to be 1/3 so zt1 >1
- b2 = 0.5 when we look at the above table b2 = $\frac{0}{0+1} = 0$ we need a hit for b2 to be 0.5 so zt1 = 2
- b3 = 1 when we look at the above table b3 = $\frac{1}{1+0} = 0$ we need to make it not subject to update

So in the result zt1 = 2

Sheet7

7.2)

Introduction to the problem

There are Range and Bearing SLAM methods that demonstrate good performance in real-time applications, even in large three-dimensional environments. However, these methods rely on sensors that have certain drawbacks. These sensors are often fragile, large in size, and expensive. Alternatively, we can consider utilizing vision-based solutions. A low-cost, compact, and reliable camera can provide a significant amount of spatial information. However, one trade-off of using such a camera is the loss of the distance dimension to the observed objects. This approach is known as Bearing-Only SLAM.

Initializing landmarks in Bearing-Only EKF-SLAM is a challenging task. EKF requires Gaussian representations for all the random variables involved in the map, including the robot's position and the positions of all landmarks. Additionally, the variances of these variables need to be small to approximate nonlinear functions with their linearized forms effectively.
It is not possible to determine the position of a landmark based on a single bearing measurement.

- ➔ Estimating the landmark position requires multiple measurements from different viewpoints, once a sufficient baseline has been established.
- ➔ These methods rely on waiting for the necessary baseline to be available.

These approaches have two drawbacks: they require a criterion to determine if the baseline is adequate, and they introduce a delay in landmark initialization until this criterion is satisfied.

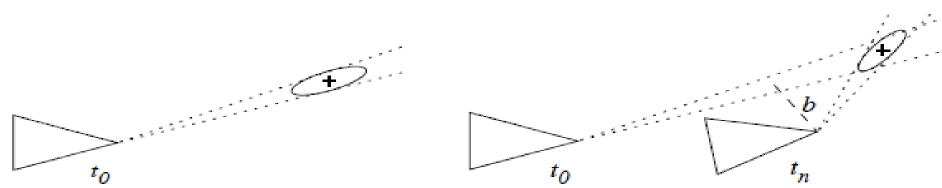

Fig. 1. Landmark initializations. *Left*: Range and Bearing SLAM. *Right*: Acquiring baseline b in Bearing-Only SLAM.

Solution

an undelayed method. It defines a set of hypothesis for the position of the landmark, and includes them all inside the map from the beginning. On successive observations

employing multi-hypothesis reasoning suggests the creation of a collection of weighted maps, with each hypothesis having its own map (Fig. 2 center). However, this approach leads to computationally challenging algorithms like **the Gaussian Sum Filter (GSF)**, where the computational load increases exponentially. In contrast, the method we propose is an approximation of the GSF that enables immediate initialization while only adding to the problem size in an additive manner like **Federated Information Sharing** (FIS) (Fig. 2 right).

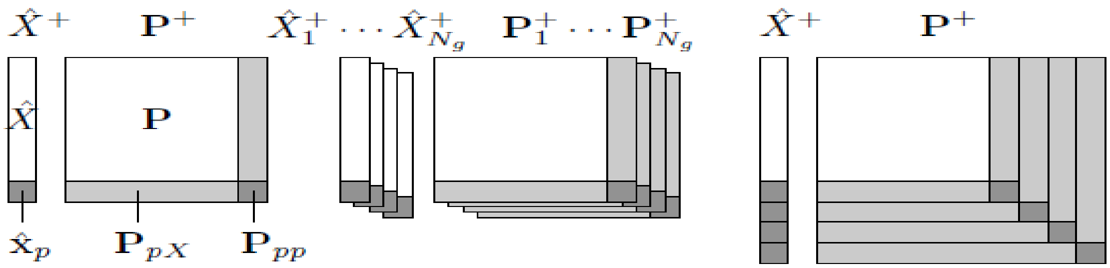

Fig. 2. Landmark initializations in EKF-based systems. Mean and covariances matrices are shown for *Left*: EKF-SLAM; *Center*: GSF-SLAM; *Right*: FIS-SLAM.

Solution algorithms

In the case of Bearing-Only measurements, the available data lacks range information, making the initialization procedure more complex.

To address this, we can separate the range (s) from the bearing (b_p), with all variables except range (s) being considered Gaussian.

Initially, the range (s) values cover the interval $s \in (0, \infty)$, but based on application-specific knowledge, this range can be narrowed down to $s \in [s_{min}, s_{max}]$.

However, this interval defines a uniform probability density function (PDF) p(s), which may not be sufficiently small. As a result, the linear approximation of function g is no longer valid, and the landmark initialization procedure of EKF-SLAM cannot be applied.

To overcome this challenge, **we need to define a non-Gaussian characterization of p(s) and explore an alternative approach to EKF for managing it. In this case, we propose the**

1-the Gaussian Sum Filter (GSF)

$$p(s) \approx \sum_{j=1}^{N_g} c_j \cdot \Gamma(s - s_j; \sigma_j^2)$$

where $\Gamma(s - s_j; \sigma_j^2) = \exp((s - s_j)^2/2(\sigma_j)^2)/\sqrt{2\pi}\sigma_j$.

For each new landmark, we will generate N_g maps, and if we initialize m new landmarks, there will be N_g^m maps. While managing these maps, the standard GSF will be employed. However, the substantial increase in the size of the problem due to multiplication makes this solution impractical to handle.so we need to find a way to handle this which is

2-Federated Information Sharing (FIS)

We must identify a computationally efficient alternative to GSF. Following the same multi-hypothesis reasoning, we can associate each hypothesis with a distinct landmark. Consequently, we can initialize all these landmarks within a single Gaussian map using the conventional EKF SLAM procedure.

To mitigate the risks of divergence and inconsistency arising from having all hypotheses correlated in a single map, the proposed FIS technique relies on likelihood evaluations of the hypotheses to weigh the influence of subsequent corrections. Aggregated likelihoods are also employed to progressively eliminate incorrect hypotheses.

The Ray: a geometric series of Gaussians

A concise representation for the Gaussian sum defining the PDF of the range. This approach aims to minimize the number of hypotheses.

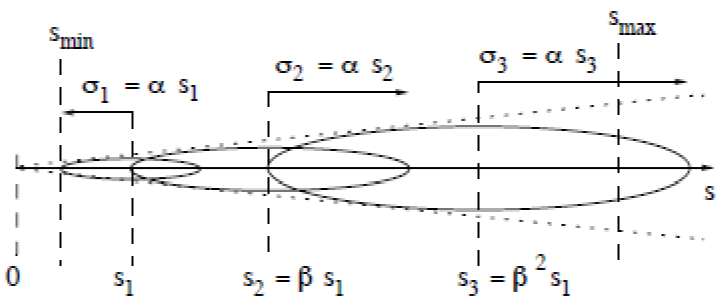

Fig. 3. The conic Ray: a geometric series of Gaussian distributions.

Map management

The aim of the initialization procedure is twofold: we want to choose the Gaussian in the ray that best represents the real landmark, while using at the same time the angular information this ray provides.

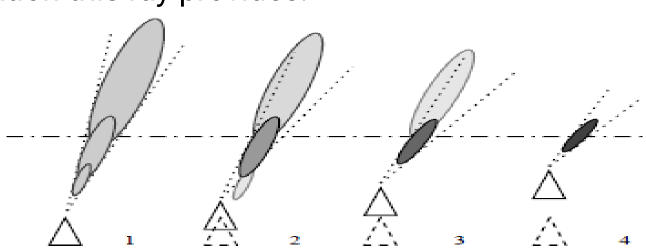

Fig. 5. Ray updates on 4 consecutive poses. Grey level indicates Aggregated Likelihood that is used to discard bad hypothesis. Dash and dot line is the true distance to the landmark

It consists of three main operations:

1. Iterated Ray Initialization

We include all landmark hypothesis that conform the ray in a single Gaussian map. All ray members are stacked in the same random state vector as if they were different landmarks:

$$X^+ = \begin{bmatrix} X^\top & \mathbf{x}_p^{1\,\top} & \dots & \mathbf{x}_p^{N_g\,\top} \end{bmatrix}^\top$$

An iterated method is used to construct its mean and covariances matrix (Fig. 6). Landmark hypothesis are stacked one by one by iteratively . The result looks like this:

$$\hat{X}^+ = \begin{bmatrix} \hat{X} \\ \hat{\mathbf{x}}_p^1 \\ \vdots \\ \hat{\mathbf{x}}_p^{N_g} \end{bmatrix} \qquad \mathbf{P}^+ = \begin{bmatrix} \mathbf{P} & \mathbf{P}_{pX}^{1\ \top} & \cdots & \mathbf{P}_{pX}^{N_g\,\top} \\ \mathbf{P}_{pX}^1 & \mathbf{P}_{pp}^1 & & \\ \vdots & & \ddots & \\ \mathbf{P}_{pX}^{N_g} & & & \mathbf{P}_{pp}^{N_g} \end{bmatrix}$$

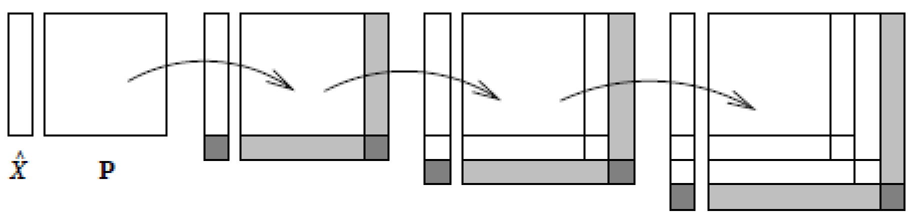

Fig. 6. Iterated Ray Initialization for $N_g = 3$. Each arrow states for an EKF-SLAM-based landmark initialization.

2. Map updates via Federated Information Sharing

This is the most delicate stage.We have a fully correlated map with all hypothesis in it, so a correction step on one hypothesis has an effect over the whole map.

If the hypothesis is wrong, this effect will cause the map to diverge.

FF applies the Principle of Measurement Reproduction to overcome inconsistency. This principle can be resumed as follows: The correction of the estimate of a random variable by a set of measurement tuples {y;Rj} is equivalent to the unique correction by {y;R}

This is what is done by FIS. The idea (Fig. 7) is to share the information given by the observation tuple {yp;R} among all hypothesis.

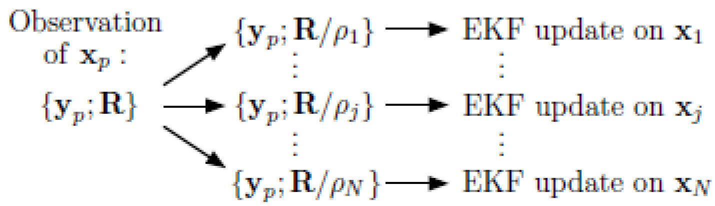

Fig. 7. Update via Federated Information Sharing

3. Ray member pruning

The divergence risk also calls for a criteria for pruning those members with very low likelihood. This will in turn allow the ray to collapse to a single Gaussian.

Paper Link :
https://www.researchgate.net/publication/224623382_Undelayed_Initialization_in_Bearing_Only_SLAM