Cairo University
Faculty of Engineering

Department of Computer Engineering

Control Assignment
Team : 14

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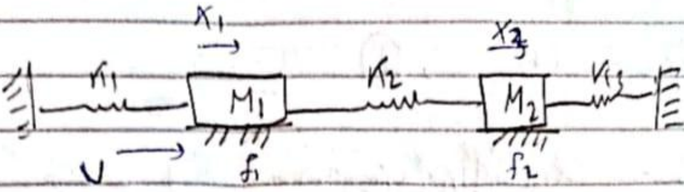
Req 1

(4)



find $\frac{X_1(s)}{U(s)}$, $\frac{X_2(s)}{U(s)}$, if $U = 1N$ find the steady state values ~~for~~ of x_1 and x_2

(M_1) (M_2)

$\xrightarrow{U}$

$\xleftarrow{k_1 x_1}$

$\xleftarrow{k_2 x_1}$ $\quad\quad$ $\xrightarrow{k_2 x_1}$

$\xrightarrow{k_2 x_2}$ $\quad\quad$ $\xleftarrow{k_2 x_2}$

$\xleftarrow{k_3 x_2}$

$\xleftarrow{f_1 \dot{x}_1}$

$\xleftarrow{f_2 \dot{x}_2}$

(for M_1)

$$U(t) - k_1 x_1(t) - k_2 x_1(t) + k_2 x_2(t) - f_1 \dot{x}_1(t) = M_1 \ddot{x}_1(t)$$

$$U(s) - k_1 X_1(s) - k_2 X_1(s) + k_2 X_2(s) - f_1 s X_1(s) = M_1 s^2 X_1(s)$$

$$U(s) = X_1(s)\left[k_1 + k_2 + f_1 s + M_1 s^2\right] - k_2 X_2(s) \rightarrow (1)$$

(for M_2)

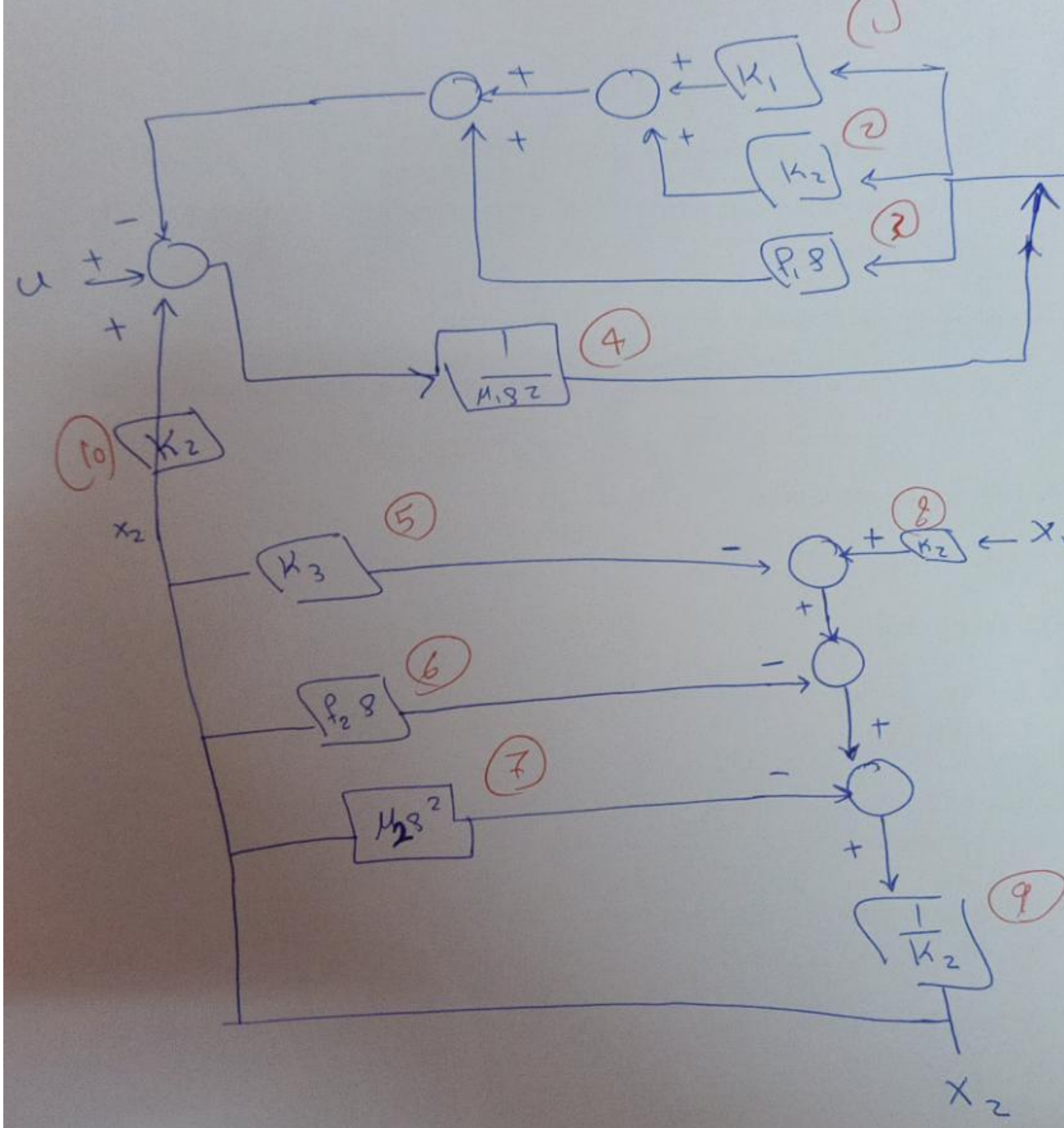
$$k_2 X_1(t) - k_2 x_2(t) - k_3 x_2(t) - f_2 \dot{x}_2(t) = M_2 \ddot{x}_2(t)$$

$$k_2 X_1(s) - k_2 X_2(s) - k_3 X_2(s) - f_2 s X_2(s) = M_2 s^2 X_2(s)$$

$$k_2 X_1(s) = X_2(s)\left[k_2 + k_3 + f_2 s + M_2 s^2\right] \rightarrow (2)$$

$$M_1 s^2 X_1 = -K_1 X_1 - F_1 s X_1 - K_2 X_1 + K_2 X_2 + U$$

$$M_2 s^2 X_2 = K_2 X_1 - K_2 X_2 - F_2 s X_2 - K_3 X_2$$



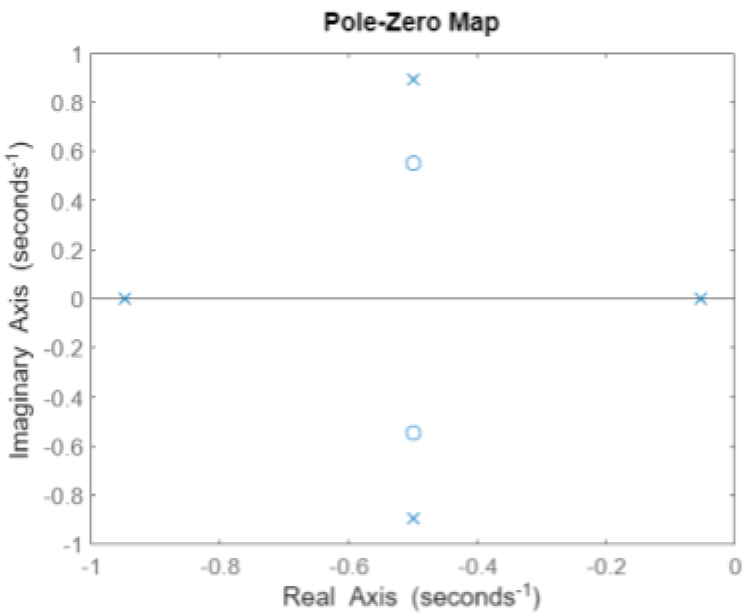
u
+
-
+
+
+
+
+
K_1
K_2
$f_1 s$
$\frac{1}{M_1 s^2}$
x_1
K_2
x_2
K_3
$f_2 s$
$M_2 s^2$
-
-
-
+
+
+
+
K_2
x_1
$\frac{1}{K_2}$
x_2
1
2
3
4
5
6
7
8
9
10

Req 2

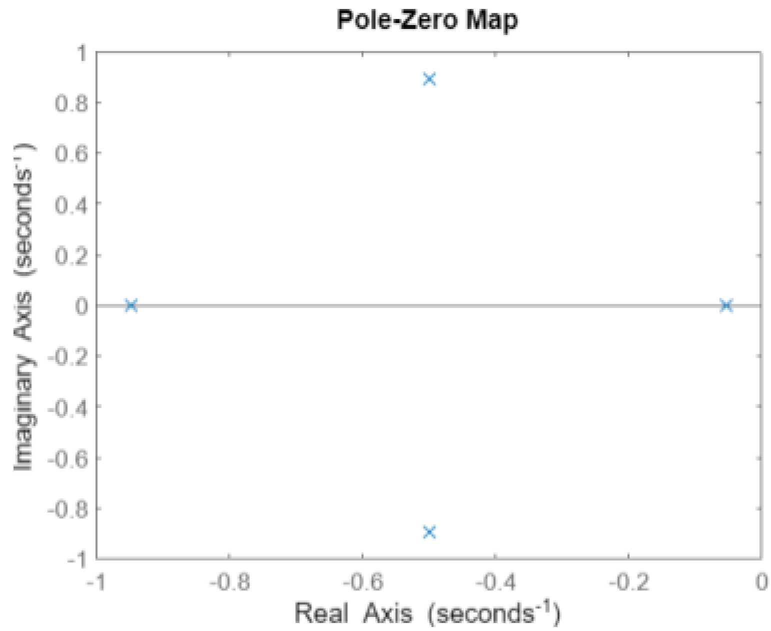
$$tf1 = \frac{0.01\,s^2 + 0.01\,s + 0.0055}{s^4 + 2\,s^3 + 2.1\,s^2 + 1.1\,s + 0.0525}$$

$$tf2 = \frac{0.005}{s^4 + 2\,s^3 + 2.1\,s^2 + 1.1\,s + 0.0525}$$

Req 3



Transfer function x1/U plot



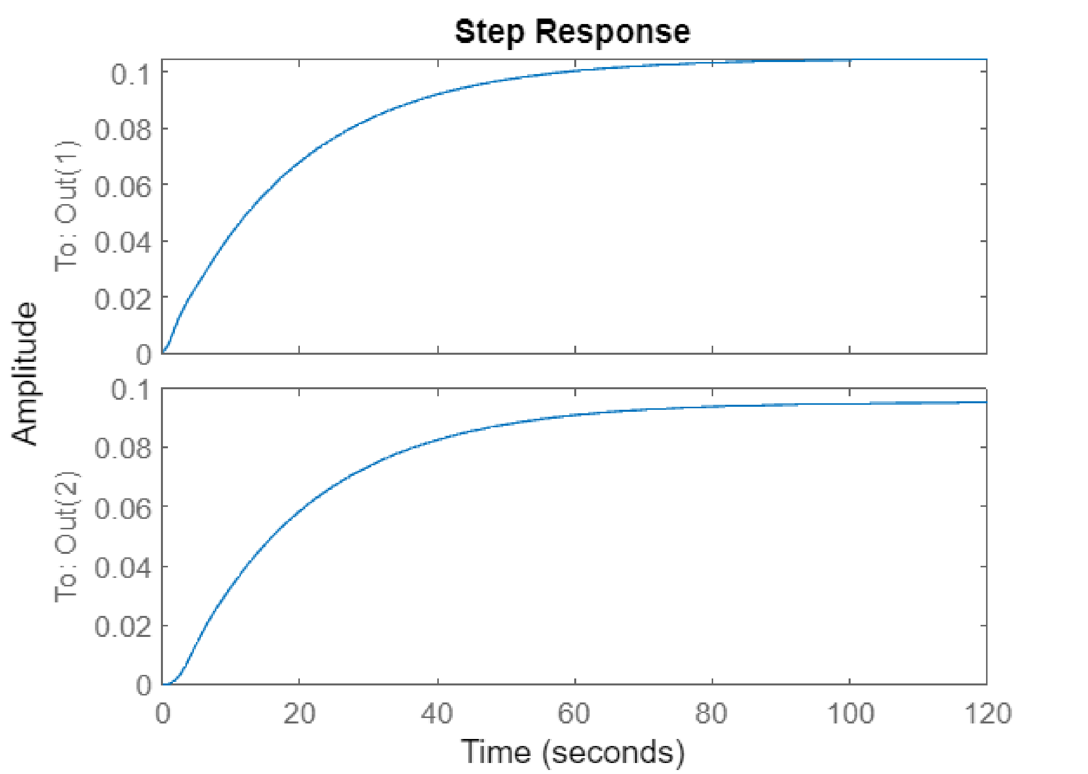
Transfer function X2/U plot

To study the stability of the system with the transfer function $\frac{X1}{U}$ and $\frac{X2}{U}$, we analyzed the poles and zeros of the transfer functions. The poles of a transfer function are the values of s that make the transfer function infinite or undefined, while the zeros are the values of s that make the transfer function zero.

Since all the poles in both plots lie in the left half of the S-plane and no zeros in both plots lie in the right half of the S-plane , then the system is stable.

Overall, the stability of the system with the transfer functions: $\frac{X1}{U}$ and $\frac{X2}{U}$ depends on the location of its poles and zeros in the S-plane. If all the poles lie in the left half of the plane and there are no zeros in the right half of the plane, then the system is stable. Otherwise, the system is unstable.

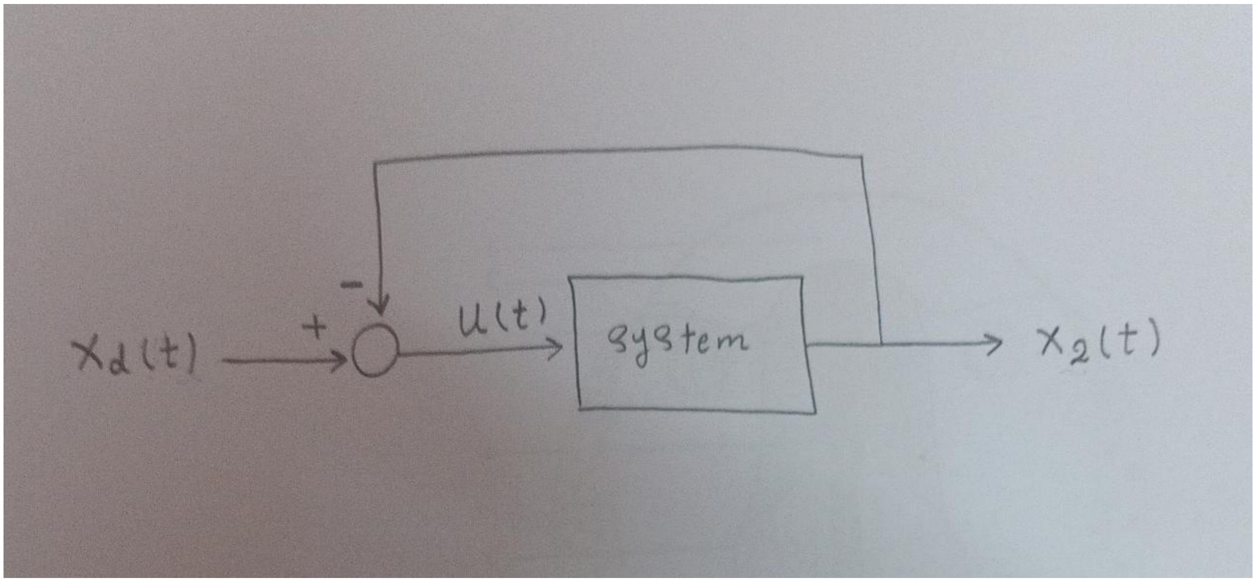
Req 4



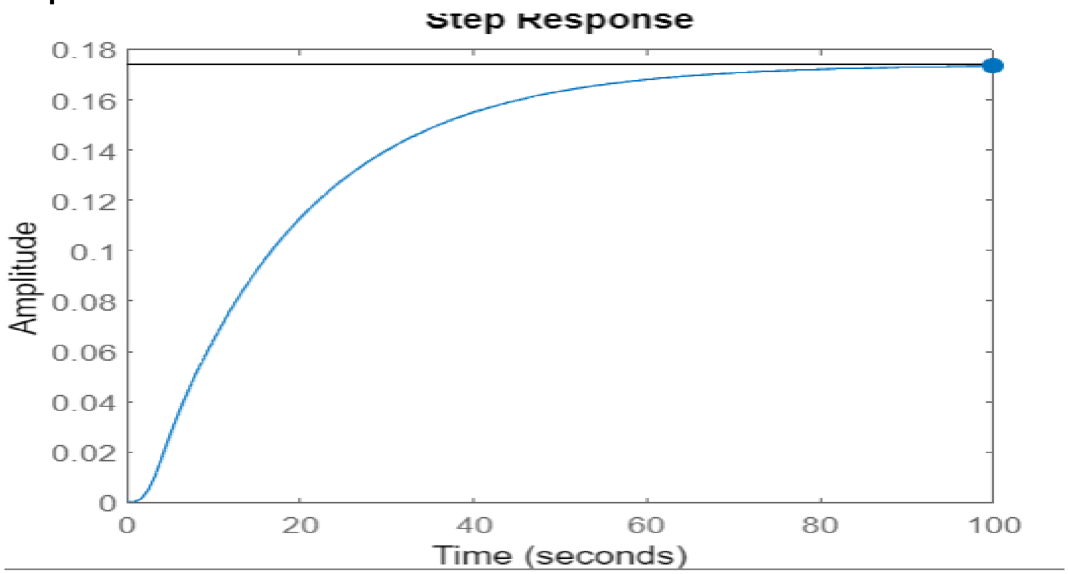
X1
rise time= 41.700422956062113
peak time= 1.387141379497528e+02
max peak= 0.104691936202951
Settling time= 74.318449376120768
X2ss= 0.104691936202951
ess= 0.8956

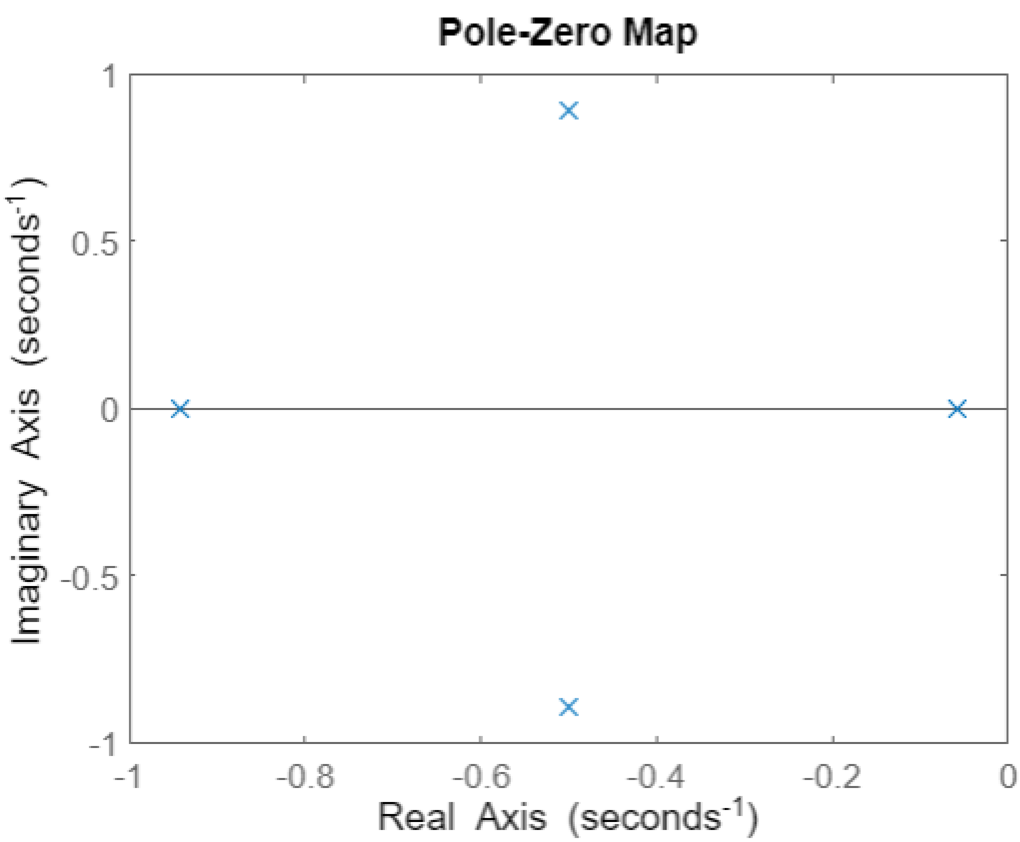
X2
rise time= 41.507635562886918
peak time= 1.387141379497528e+02
max peak= 0.095168126679141
Settling time= 76.124819172111145
X2ss= 0.095168126679141
ess= 0.9051

Req 5



Req 6

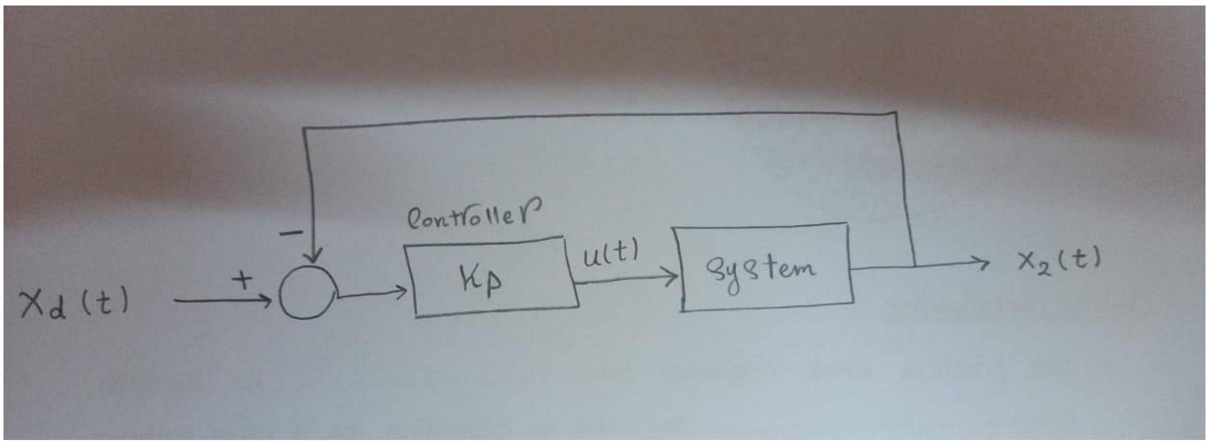

Step Response
Amplitude
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0
20
40
60
80
100
Time (seconds)


Pole-Zero Map
Imaginary Axis (seconds⁻¹)
1
0.5
0
-0.5
-1
-1
-0.8
-0.6
-0.4
-0.2
0
Real Axis (seconds⁻¹)

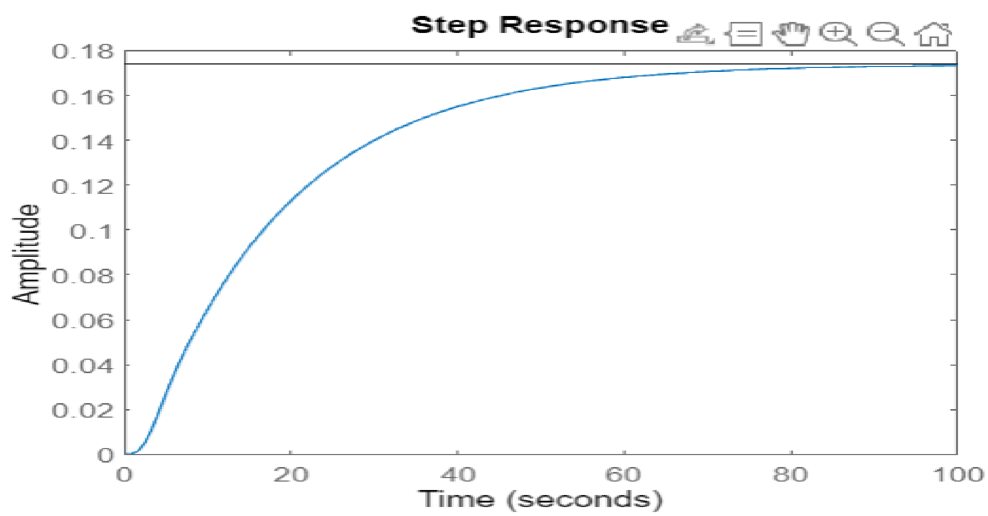
Req 7

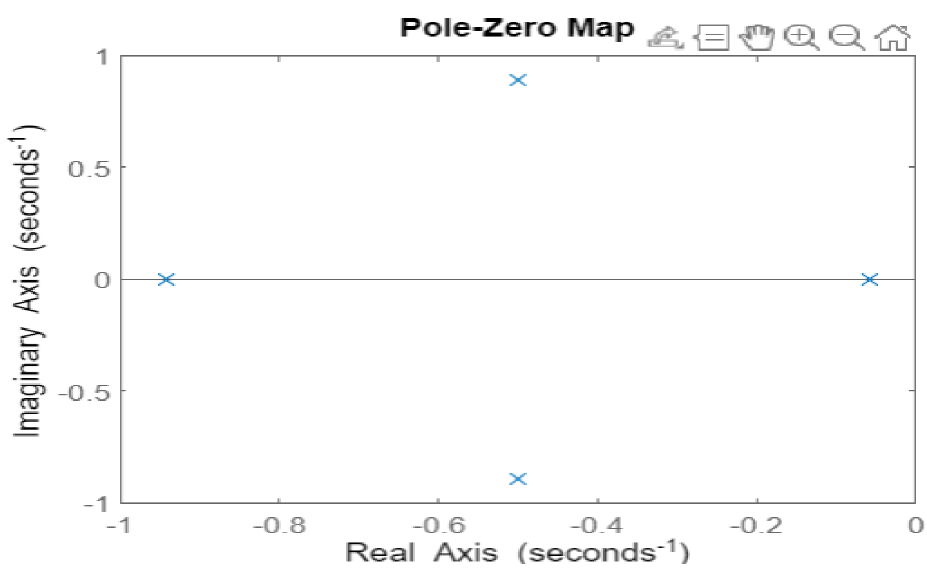
rise time= 37.467621515312409
peak time= 1.252934896436744e+02
max peak=0.173
Settling time= 68.966829376855586
X2ss= 0.174
ess= 1.826678046342734

Req 8

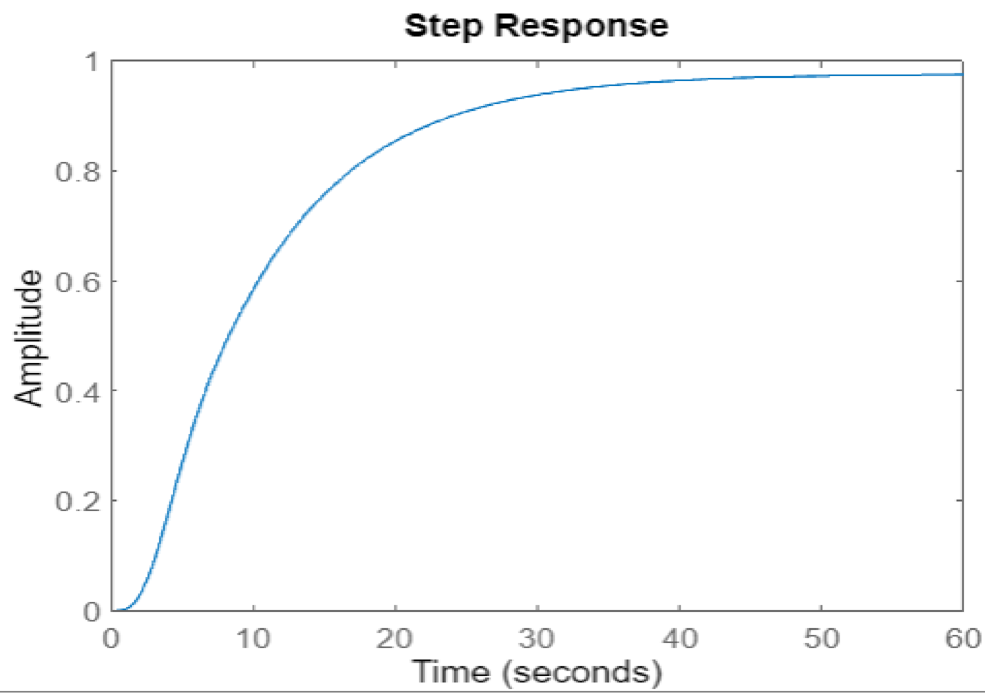


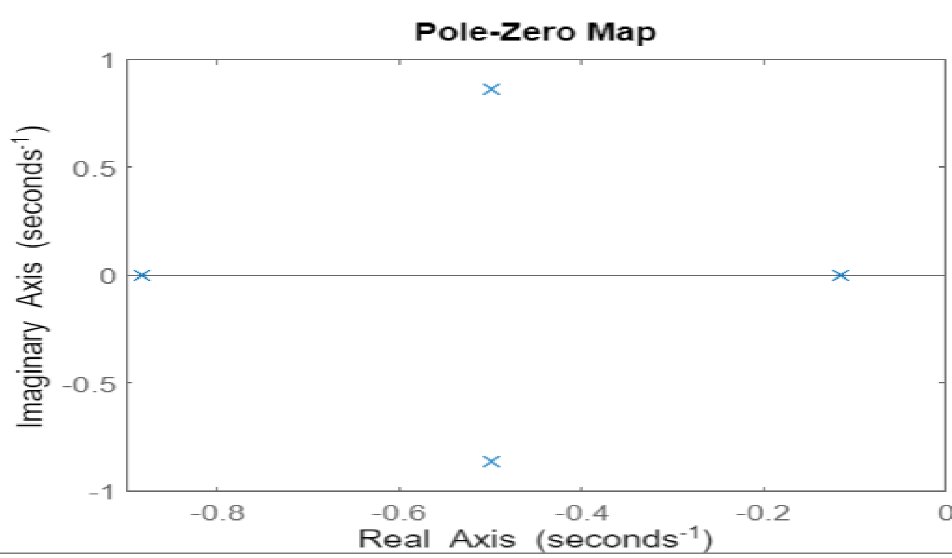
kp=1
rise time= 37.467621515312409
peak time= 1.252934896436744e+02
max peak=0.173
Settling time= 68.966829376855586
X2ss= 0.174
ess= 1.8267



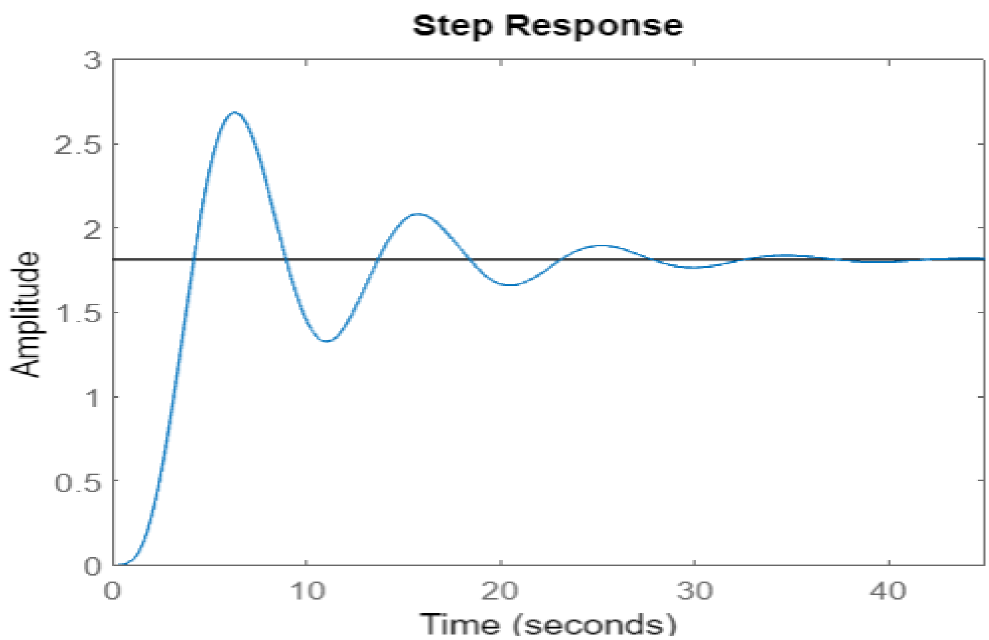


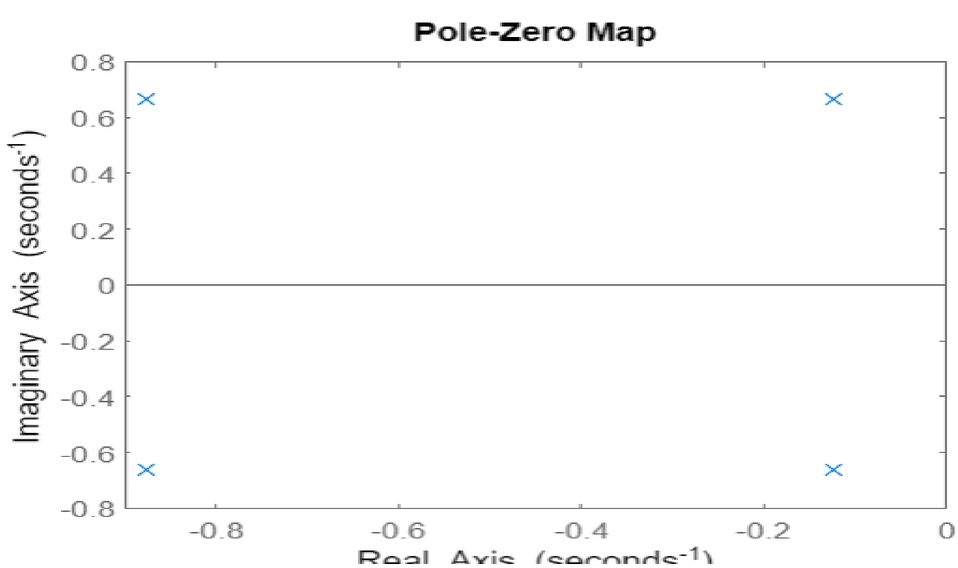
kp=10
rise time= 18.846485591043255
peak time= 61.389467388124636
max peak= 0.974
Settling time= 35.781502479245354
X2ss= 0.976
ess= 1.0271



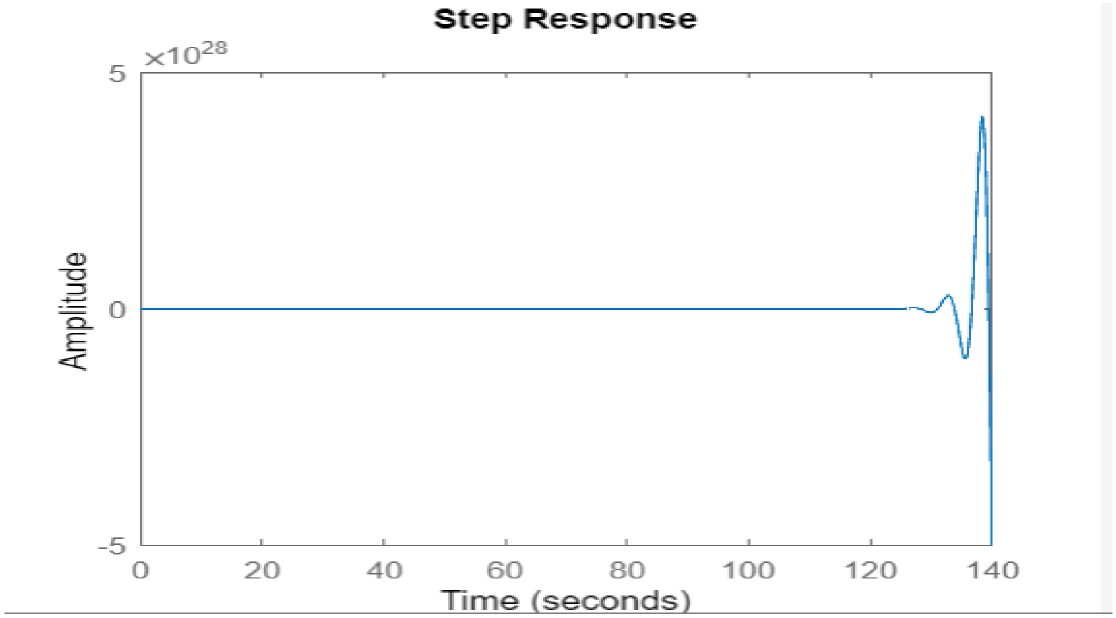


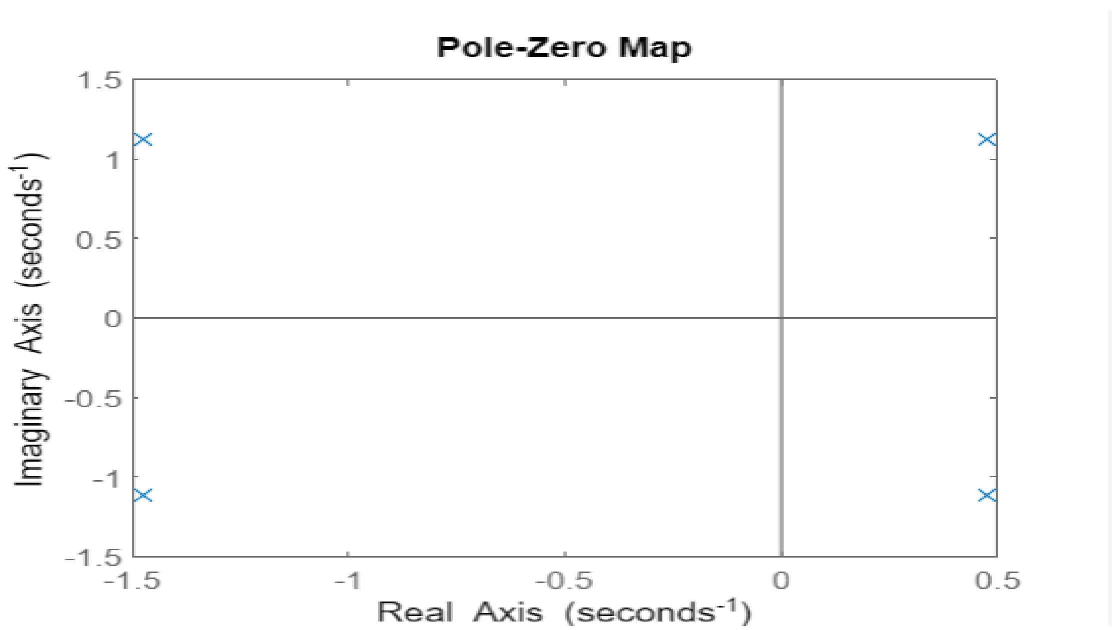
kp=100
rise time= 2.2180
peak time= 6.306809980973551
max peak= 2.68
Settling time= 31.014091607682847
X2ss= 1.81
ess= 0.1830





kp=1000
rise time= NaN
peak time= Inf
max peak= Inf
Settling time= NaN
X2ss= inf
ess= 1.8001e+12





Comment on Requirement 8 results:

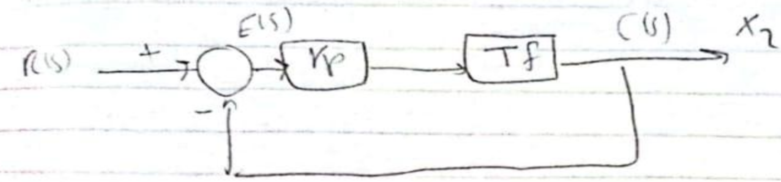
As Kp increases, we observed the following trends:

1. **Steady-state error (ess):** The steady-state error decreases as Kp increases, which means that the system becomes more accurate in reaching the desired output value, but after a certain point (Kp = 1000) it starts to increase again.
2. **Rise time:** The rise time decreases as Kp increases, indicating that the system reacts faster to changes in the input signal, but after a certain point (Kp = 1000) it becomes NaN.
3. **Settling time:** The settling time decreases initially as Kp increases, but after a certain point (Kp = 1000) it becomes NaN. This is because high gains can introduce oscillations and instability.
4. **Peak time and overshoot:** As Kp increases, the peak time decreases, and the overshoot increases. A high Kp value causes the system to oscillate more and have more overshoot, which may result in instability, but after a certain point (Kp = 1000) peak time starts to increase again.

From the results, we conclude that increasing the proportional controller gain Kp can improve the system's steady-state error and speed up its response. However, there is a tradeoff between accuracy, speed, and stability. High gains lead to increased overshoot and oscillations.

Req 9

Subject موضوع الدرس Date التاريخ



$$Tf = \frac{0.005}{s^4 + 2s^3 + 2.1s^2 + 1.1s + 0.0525}$$

$$E(s) = R(s) - C(s)\,H(s)$$

$$C(s) = G(s)\,G_c(s)\,E(s)$$

$$R(s) = \frac{4}{s}$$

$$E(s) = R(s) - G(s)\,G_c(s)\,E(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)\,G_c(s)} = \frac{\frac{4}{s}}{1 + K_p \dfrac{0.005}{s^4 + 2s^3 + 2.1s^2 + 1.1s + 0.0525}}$$

$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$e_{ss} = \lim_{s \to 0} \frac{4(s^4 + 2s^3 + 2.1s^2 + 1.1s + 0.0525)}{s^4 + 2s^3 + 2.1s^2 + 1.1s + 0.0525 + 0.005K_p}$$

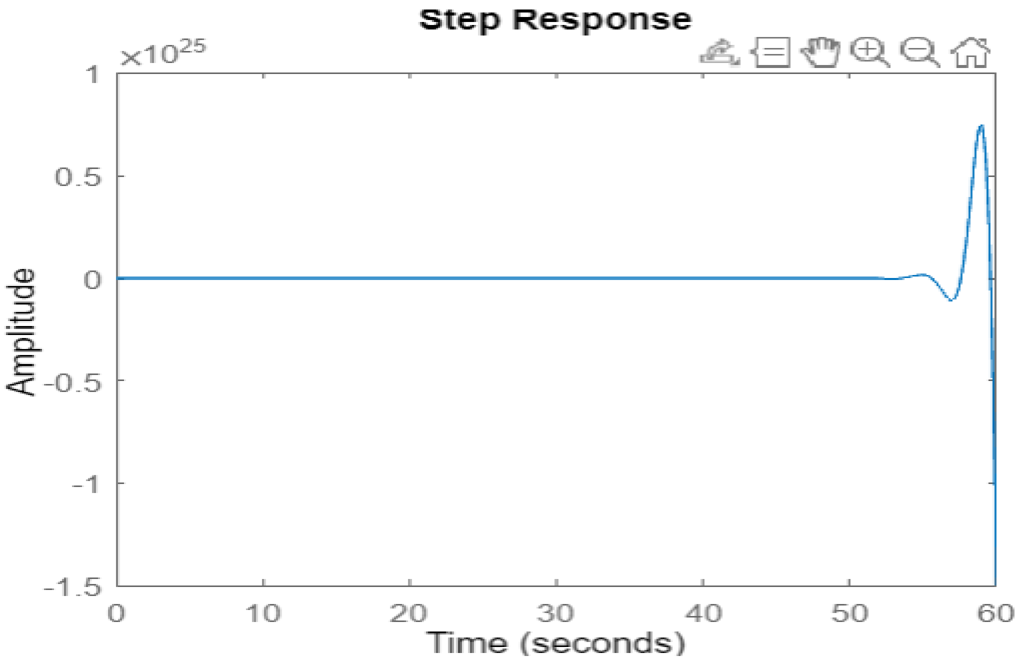
$$e_{ss} = \frac{4(0.0525)}{0.0525 + 0.005K_p} = \frac{0.21}{0.0525 + 0.005K_p}$$

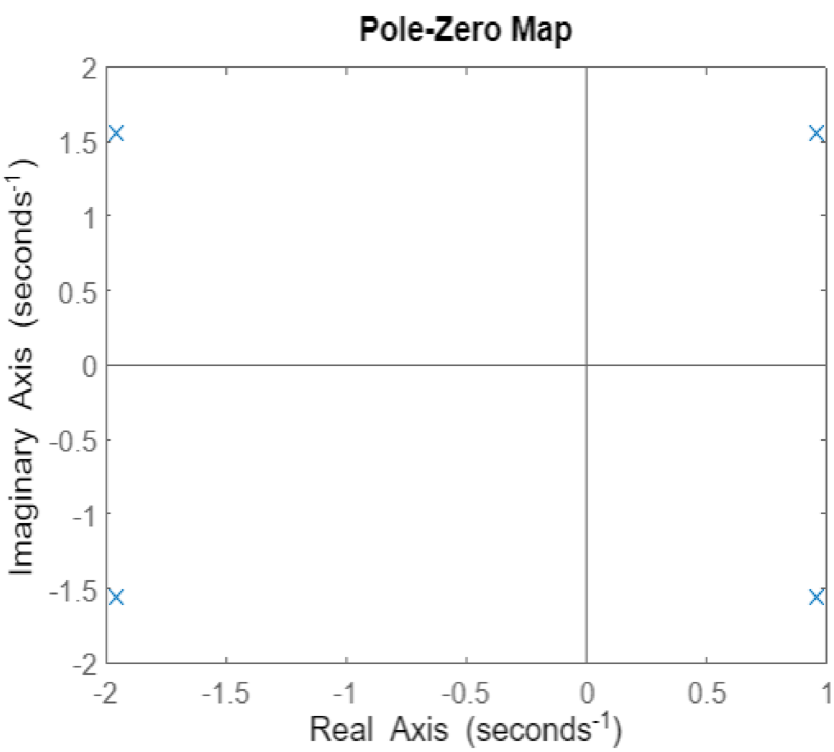
$$e_{ss} \le 0.01$$

$$\frac{0.21}{0.0525 + 0.005K_p} \le 0.01$$

$$K_p \geq 4189.5$$

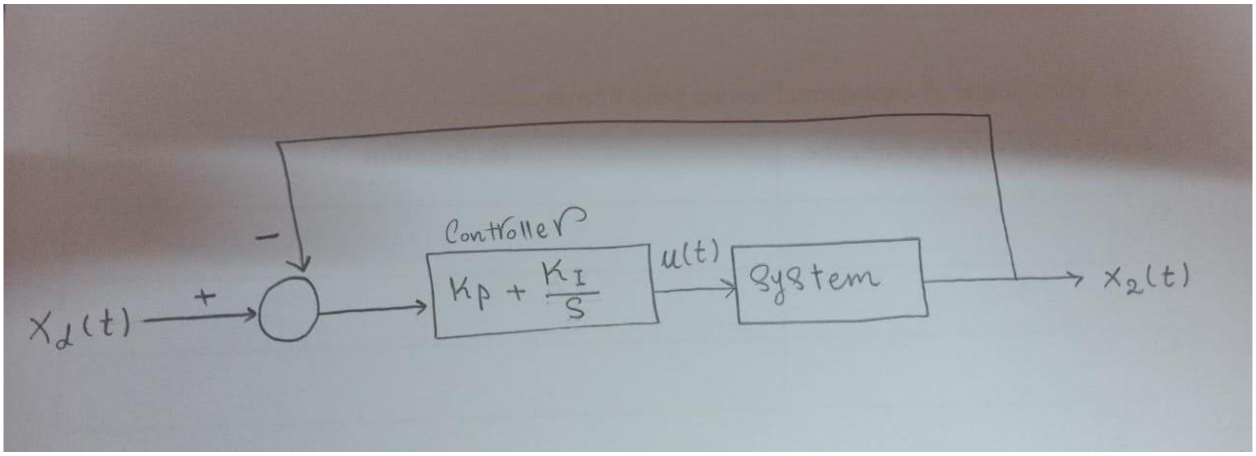
At kp = 4189.5





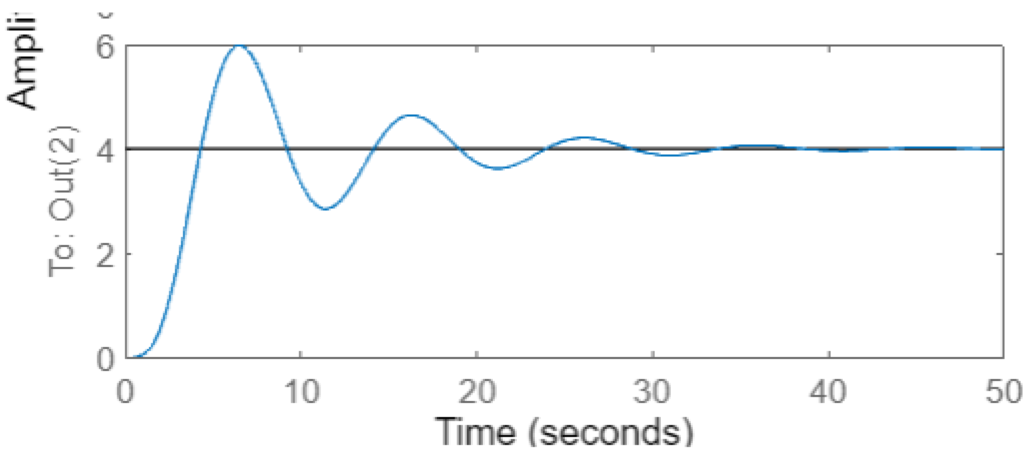
The system will not be stable so we can't use only a proportional compensator to achieve the required ess

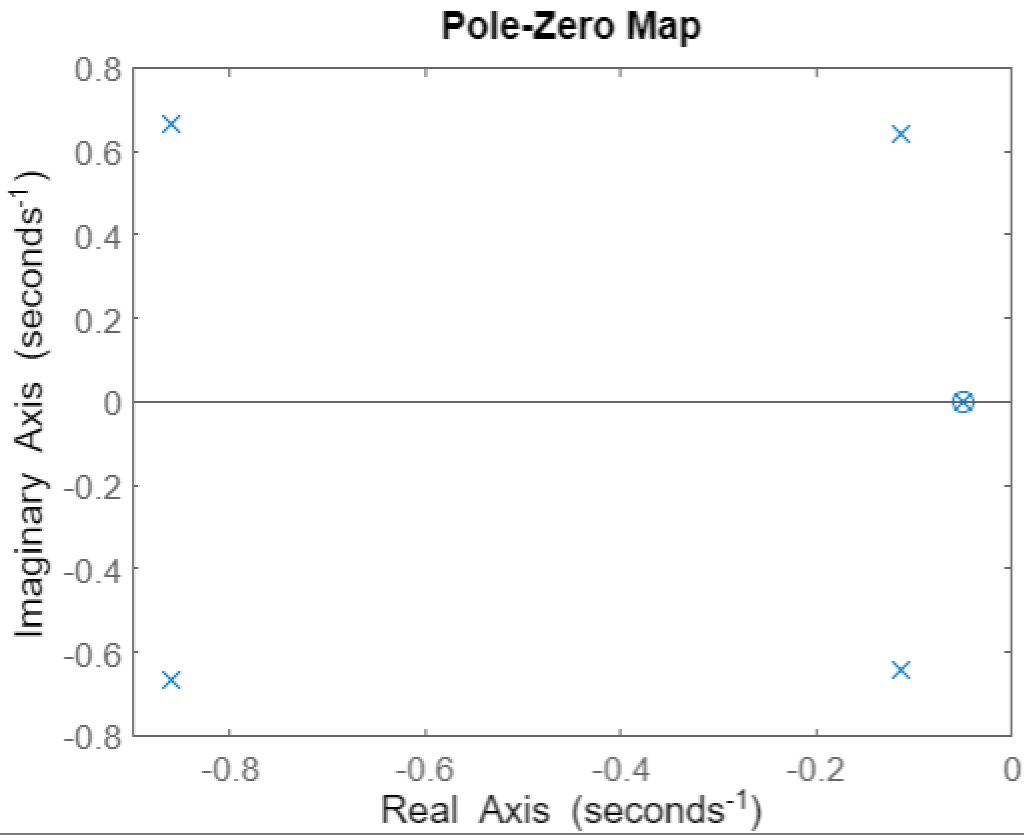
Req10



We choose PI controller (Kp + Ki/S) with Kp = 100 & KI = 5

rise time= 2.267588161154840
peak time= 6.524670874962880
max peak= 5.98
Settling time= 32.475805724765706
X2ss= 4
ess= 0.0086 less than 0.01




Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.8
-0.6
-0.4
-0.2
0