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Problem 1.6

ⓐ the Probability of drawing 10 green marbles in a row is $(1-N)^{10}$

since each draw is independent

for

* $N = 0.05$

$P(\text{not red}) = (1-0.05)^{10} = 0.59874$

* $N = 0.5$

$P(\text{not red}) = (1-0.5)^{10} = 9.76563 \times 10^{-4}$

* $N = 0.8$

$P(\text{not red}) = (1-0.8)^{10} = 1.024 \times 10^{-7}$

(b)

P(at least one of samples has $\nu = 0$) = 1 - P($\nu = 0$)

= 1 - P(no red)

for 1000 independent sample

$$P(\text{at least one has } \nu = 0) = 1 - (1 - \underbrace{P(\nu = 0)}_{\text{P for 1 sample}})^{1000}$$

for:

* $\mu = 0.05$

$$P(\text{at least one has } \nu = 0) = 1 - (1 - 0.59874)^{1000} = 1$$

* $\mu = 0.5$

$$P = 1 - (1 - 9.76563 \times 10^{-4})^{1000} = 0.6236$$

* $\mu = 0.8$

$$P = 1 - (1 - 1.024 \times 10^{-7})^{1000} = 1.0239 \times 10^{-4}$$

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For 1000000 independent samples

$$P(\text{at least one has } V=0) = 1 - (1 - P(V=0))^{1000000}$$

P for 1 sample

For:

* $N = 0.05$

~~[illegible]~~

$$P = 1 - (1 - 0.59874)^{1000000} = 1$$

* $N = 0.5$

$$P = 1 - (1 - 9.76563 \times 10^{-4})^{1000000} = 1$$

* $N = 0.8$

$$P = 1 - (1 - 1.024 \times 10^{-7})^{1000000} = 0.09733$$

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Problem 2.5

base case

for $N=1$

- If $D=0$

$$\sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} = 1$$

$$N^D + 1 = 1^0 + 1 = 2$$

$$\therefore \sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1 \quad \text{for } (D=0)$$

- If $D=1$

$$\sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} + \binom{1}{1} = 2$$

$$N^D + 1 = 1^1 + 1 = 2$$

$$\therefore \sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1 \quad \text{for } D=1$$

$\therefore$ the base case holds

lets check for the general case

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general case

Assume that $\sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1$ ~~Theorem~~

is valid for N or less let's check if this inequality holds for N+1

$$\sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \binom{N}{i} + \binom{N}{i-1}$$

$$= \sum_{i=0}^{D} \binom{N}{i} + \sum_{r=0}^{D-1} \binom{N}{r}$$

$$\leq N^D + 1 + N^{D-1} + 1 = (N^D + N^{D-1} + 1) + 1$$

$$\leq (N+1)^D + 1$$

This Term is larger than $(N^D + N^{D-1} + 1)$ so the inequality still valid after replacement

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the last inequality is obvious when $D > 1$
and we expand $(N+1)^D$ into terms of N

$$(N+1)^D = \sum_{k=0}^{D} \binom{D}{k} N^k$$

- when $D = 0$

$$\sum_{i=0}^{D} \binom{N}{i} = \binom{N}{0} \leq N^0 + 1 = 1$$

- when $D = 1$

$$\sum_{i=0}^{D} \binom{N}{i} + \binom{N}{1} = 1 + N \leq N^1 + 1$$

from the above results

$$\therefore \sum_{i=0}^{D} \binom{N}{i} \leq N^D + 1 \rightarrow (1)$$

from the lecture we have

$$m_H(N) \leq \sum_{i=0}^{d_{vc}} \binom{N}{i} \rightarrow (2)$$

from 1 , 2

Hence

$$\boxed{m_H(N) \le N^{d_{vc}} + 1}$$