

1 Lie Groups

A Lie Group is a group whose elements are organized continuously and smoothly, making it a smooth manifold.

Special Orthogonal group $\mathbf{SO}(3)$

B Group of 3D rotation matrix:

$$\mathbf{SO}(3) = \{ \mathbf{C} \in \mathbf{GL}(3, \mathbb{R}) \mid \det(\mathbf{C}) = 1, \mathbf{C}^T \mathbf{C} = \mathbf{I} \}$$

Special Euclidian group $\mathbf{SE}(3)$

Group of 3D transformation matrix:

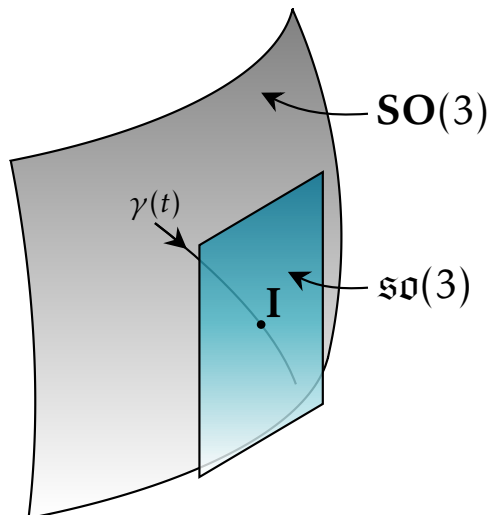
$$\mathbf{SE}(3) = \left\{ \mathbf{T} = \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbf{GL}(4, \mathbb{R}) \mid \mathbf{C} \in \mathbf{SO}(3), \mathbf{r} \in \mathbb{R}^3 \right\}$$

2 Lie algebra

A Lie algebra associated to a Lie Group is the tangent space of the group at the identity element.

Special Orthogonal Group $\mathfrak{so}(3)$

Special Euclidian Group $\mathfrak{se}(3)$



4 Interpolation

4.1 In $\mathbf{SO}(3)$

$$\mathbf{C} = (\mathbf{C}_2 \mathbf{C}_1^T)^\alpha \mathbf{C}_1 \quad \text{with } \alpha \in [0, 1]$$

4.2 In $\mathbf{SE}(3)$

$$\mathbf{T} = (\mathbf{T}_2 \mathbf{T}_1^{-1})^\alpha \mathbf{T}_1 \quad \text{with } \alpha \in [0, 1]$$

3 Adjoint of $\mathbf{SE}(3)$