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Lecture Notes of Spring 2013

Algorithms I

University of Mannheim
2013

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TODO: 1st lecture missing

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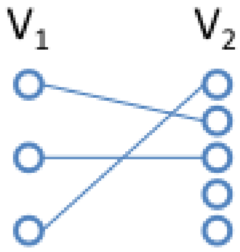
Definition 1.4
Let $G = (V, E)$ be a graph without loops. If there exists $V_1, V_2 \leq V$ and $V_1 \cup V_2 = V$ such that $V_1 \cap V_2 = \emptyset$ and every edge e has one endpoint in V_1 and the other in V_2, then we call G a bipartite.

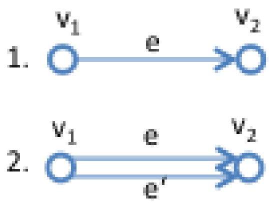
Beispiel



Definition 1.5
A directed graph is a pair $G = (V, E)$ where V is a set of nodes (vertices) and E is a set of edges together with a function $i : E -> VxV$. If $i(e) = (v_1, v_2)$ then v_1 is called start point, v_2 is called end point.

Graphically:

If $i(e) = (v_1, v_2)$ we draw 1.
If $i(e') = (v_1, v_2)$ then this indices a second edge (2.).
If $i(e_1) = i(e_2)$ we call e_1, e_2 parallel.
If $i(e) = (v, v)$ then e is called a directed loop.



$g_{out}(v)$ is the number of edges that have starting point v.
$g_{in}(v)$ is the number of edges with endpoint v.

Lemma 1.2
$\sum_{v \in V} g_{in}(v) = \sum_{v \in V} g_{out}(v)$

Proof: We start with a graph without edges. Then we insert one after the other edges in E. Each edge contributes 1 to both sides of the equation.

Definition 1.6
A directed path is a sequence of edges $e_1, e_2 ...$ such that the end point of e_i is the start point of $e_1 + 1, i > 1$([NW] +1 seems strange to me, correct?).

A directed path $e_1 ... e_k$ is called a (directed) cycle, if the start point of e_1 and the end point of e_k coincide.
A simple (directed) path is a path where every node occurs at most once.
A directed cycle is called simple if every node except for the start and end node occurs at most once.

Definition 1.7
A graph directed or undirected is called simple, if it does not contain parallel edges.

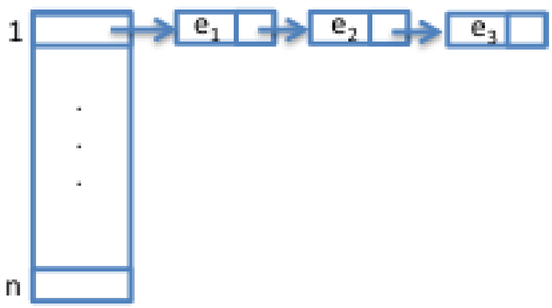
Definition 1.8
A directed graph is called strongly connected if for any pair of nodes (u, v) there is a directd path from u to v.

Let G be a directed graph $G = (V, E)$. $x, y \in V x\ y$ ([NW] does mean are "connected"?) if there is a directed path from x to y and vice versa.

The equivalence classes of this relation $cV x V$ are called strongly connected components. (Analogously: Define connected components for undirected graphs)

> We should know how the following terms are defined: reflexivity, symetry, transitivity.

Implementation:
1. Adjacency Lists $V = 1...n, E = e_1 ... e_t$



2. Dynamically changing graphs:
e.g. multi user databses: Nodes ≡ transactions of user; Edges ≡ waiting situations

1	1	2
2	1	3
3	1	2

Graph is used to detect dead locks. Waiting arises when data are locked by a user that modifies these data.
U_1 $write(d)$, $read(d')$
U_2 $read(d)$, $write(d')$

Definition 1.9
An undirected graph is called a tree if it is connected and does not have simple cycles.
Let G be a directed graph, $G = (V, E)$. A node r is called root if every other node can be reached from r via a directed path.
A directed graph is called a tree if it has a root and the underlying undirected graph is a tree.
Let G be a directed graph. A node is called source if $g_{in}(v) = 0$. v is called sink if $g_{out}(v) = 0$

Lemma 1.3
NW: what was lemma 1.3? the next one was 1.4 in my notes

Lemma 1.4
If $G = (V, E)$ is a directed graph without directed cycles, then tere is always a source and sink.

We use this theorem to detect cycles

Proof: Source (sink analogously): Select an arbitrary node v_1. If v_1 is a source we are done. If it is not, then there must be an edge e_1 leading to it $v_2 \xrightarrow{e_1} v_1$.
If v_2 is a source we are done. If not, there must be an edge e_2 leading to it $v_3 \xrightarrow{e_2} v_2 \xrightarrow{e_1} v_1$. We continue this process. It must stop because there are only finitely many nodes and if a node would appear once more on such a path, there would be a directed cycle.

2

Euler Graphs and Hamilton Graphs

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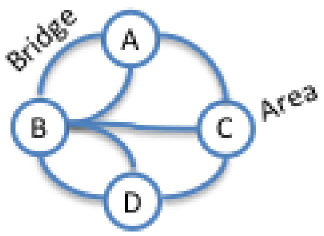
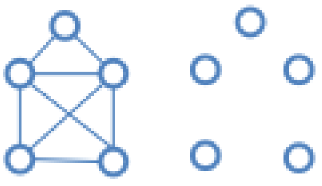
2.1 Euler Graphs

2.1.1 Euler 1736: Königsberger Brückenproblem

Is it possible to do a round walk crossing every bridge exactly once?



Beispiel 2.1



Definition 2.1
Let G be a finite undirected graph. A path $e_1..e_t$ is called a euler path if every edge in E occurs exactly once in the list.
A graph is a euler graph if it has a euler path.

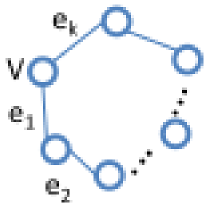
Theorem 2.1
A finite connected graph is a euler graph if and only if:

i) It ha eiter exactly two nodes of odd degree. or

ii) All nodes have even degree.

In the last case the path is a cycle. In the first case no euler path is a cycle. Check is possible in linear time.

Proof: " $>$ " Let $G = (V, E)$ be a graph that has a euler path that is not a cycle. Let $|E| = k$
$\circ \xrightarrow{e_1} \circ \xrightarrow{e_2} \ldots \circ \xrightarrow{e_k}$ In this path v_1 and v_{k+1} have od degreee and all other nodes have even degree. Now consider the case teht G has a euler cycle.



Hence every node has even degree.

" $<$ " Let G be a graph with exactly two nodes with odd degree, let this be a and b. We contradict a euler path as follows:
Start at node a and folow an edge ?inktt? on a. $a - - v$ If the node that we currently deal with is neither a nor b then passing this node will use up two edges. If we look at the residual degree at v, it is an even number. If at some point we reach a node which we can enter but not leave it must be b.

3
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