Simulation and Prediction Evidence On the Usefulness of Seasonal Unit Root Models: Additional Results

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1 Accompanying Material for Seasonality Paper - Swanson and Urbach

The following contains some material from the paper as well as some additional tables. For complete details, including final revised table footnotes, etc., see the paper.

2 Linear Seasonal Model

We have the following time series data from Jan 1959 to Dec 2005 (except last two)

1) CPI for all urban consumers: all items ($CPI1$)

2) CPI for all urban consumers: energy ($CPI2$)

3) M1 ($M1$)

4) M2 ($M2$)

5) Housing Start (H_start)

6) Industrial Production Index: Total index; 2002=100 (IP)

7) Total nonfarm: Total Employment (thousands) ($NonF\_Emp$)

8) Industrial Production Index: Automotive products (IP_Auto)

9) Industrial Production Index: Durable consumer goods (IP_Dur)

10) Industrial Production Index: Nondurable consumer goods (IP_NDur)

11) Durable Goods Shipment New Orders and Unfulfilled Orders (D_Ship)

12) Total Inventories, Manufacturing ($Invent$)

13) Retail Sales (1967:1 to 2000:12) (Ret_sales)

14) Motor Vehicle Unit Retail Sales (thousands) (1967:1 to 2005:12) (Veh_sales)

We are using first log differences for all of the series, further seasonal unit root tests are carried out for log levels.

3 Empirical Evidence of Seasonality

Results based on estimation of the following model

$$\phi(L)y_t = \Sigma_{s=1}^{S}\theta_s d_{s,t} + \varepsilon_t$$

In Table 1 we report the estimated values of the seasonal dummies. Entries in brackets are the t-statistics, starred entries indicate significance at 5% level. The last column reports the likelihood ratio statistics for testing the null hypothesis, $\theta_s = \theta, s = 1 \cdots 12$. Starred entries indicate rejection of the null at 5% level.

Table 1 Estimated Seasonal Dummies $^{(*)}$

Series	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	LR
CPI1	0.003	0.004	0.004	0.004	0.003	0.004	0.003	0.003	0.004	0.004	0.002	0.001	6.60*
	(6.70*)	(8.39*)	(8.21*)	(7.97*)	(6.6*)	(8.48*)	(6.68*)	(6.5*)	(8.04*)	(7.18*)	(3.36*)	(2.73*)	
CPI2	0.006	0.003	0.004	0.008	0.010	0.015	0.001	0.004	0.007	-0.007	-0.005	-0.002	6.39*
	(2.4*)	(1.13)	(1.6)	(3.09*)	(4.01*)	(5.80*)	(0.54)	(1.44)	(2.89*)	(-2.65*)	(-2.1*)	(-0.60)	
M1	-0.007	-0.022	0.008	0.02	-0.016	0.013	0.006	-0.002	0.007	0.006	0.012	0.023	142.02*
	(-5.77*)	(-19.7*)	(6.82*)	(17.89*)	(-13.95*)	(11.61*)	(5.63*)	(-1.72)	(5.93*)	(5.39*)	(10.17*)	(20.21*)	
M2	0.004	-0.002	0.009	0.011	-0.004	0.009	0.007	0.003	0.006	0.006	0.007	0.01	85.27*
	(7.19*)	(-2.55*)	(14.36*)	(17.96*)	(-6.53*)	(15.46*)	(11.37*)	(5.06*)	(9.65*)	(10.34*)	(11.14*)	(16.79*)	
H_Start	-0.072	0.038	0.307	0.139	0.037	-0.008	-0.043	-0.011	-0.059	0.042	-0.187	-0.177	105.04*
	(-5.30*)	(2.87*)	(22.92*)	(10.35*)	(2.73*)	(-0.62)	(-3.21*)	(-0.79)	(-4.42*)	(3.13*)	(-14*)	(-13.2*)	
IP	0.005	0.017	0.006	-0.008	0.004	0.025	-0.047	0.034	0.017	0.001	-0.012	-0.01	138.70*
	(2.85*)	(9.68*)	(3.74*)	(-4.75*)	(2.26*)	(14.28*)	(-27.12*)	(19.86*)	(9.66*)	(0.63)	(-6.83*)	(-5.53*)	
NonF_Emp	-0.022	0.003	0.006	0.009	0.007	0.008	-0.009	0.002	0.007	0.004	0.002	0.002	383.04*
	(-48.02*)	(5.96*)	(13.69*)	(18.32*)	(15.53*)	(17.23*)	(-18.49*)	(4.33*)	(15.44*)	(9.2*)	(4.92*)	(5.09*)	
IP_Auto	0.056	0.062	0.023	-0.002	0.003	0.024	-0.264	0.012	0.173	0.08	-0.06	-0.071	36.43*
	(3.08*)	(3.43*)	(1.28)	(-0.09)	(0.15)	(1.3)	(-14.51*)	(0.68)	(9.52*)	(4.37*)	(-3.3*)	(-3.9*)	
IP_Dur	0.012	0.048	0.017	-0.004	0.001	0.023	-0.15	0.056	0.085	0.042	-0.039	-0.053	82.75*
	(1.78)	(7.15*)	(2.58*)	(-0.54)	(0.13)	(3.41*)	(-22.54*)	(8.38*)	(12.8*)	(6.3*)	(-5.86*)	(-8.02*)	
IP_NDur	0.025	-0.001	-0.011	-0.021	-0.002	0.038	-0.011	0.043	-0.008	-0.015	-0.015	0.002	61.04*
	(9.09*)	(-0.5)	(-4.18*)	(-7.86*)	(-0.66)	(14.19*)	(-4.26*)	(16.09*)	(-2.94*)	(-5.65*)	(-5.6*)	(0.84)	
D_Ship	-0.013	0.041	0.029	-0.02	-0.001	0.027	-0.066	0.029	0.036	-0.002	-0.013	0.01	112.77*
	(-4.62*)	(14.57*)	(10.4*)	(-6.96*)	(-0.2)	(9.65*)	(-23.52*)	(10.26*)	(12.74*)	(-0.84)	(-4.7*)	(3.46*)	
Invent	0.018	0.010	0.000	0.007	0.005	-0.005	0.004	0.006	-0.002	0.007	0.003	-0.007	20.40*
	(12.49*)	(7.13*)	(-0.07)	(5.01*)	(3.55*)	(-3.36*)	(2.46*)	(3.85*)	(-1.44)	(4.88*)	(2.24*)	(-4.48*)	
Ret_Sales	-0.298	-0.015	0.136	0.005	0.054	-0.004	-0.01	0.023	-0.046	0.038	0.014	0.175	813.16*
	(-68.19*)	(-3.39*)	(31.25*)	(1.19)	(12.35*)	(-1.01)	(-2.32*)	(5.37*)	(-10.61*)	(8.74*)	(3.14*)	(40.20*)	
Veh_Sales	-0.06	0.11	0.17	-0.047	0.073	-0.011	-0.104	-0.035	-0.013	0.064	-0.13	-0.025	34.93*
	(-3.38*)	(6.19*)	(9.58*)	(-2.62*)	(4.1*)	(-0.63)	(-5.84*)	(-1.97*)	(-0.76)	(3.6*)	(-7.3*)	(-1.43)	

$^{(*)}$ In this table we report the estimated values of the seasonal dummies. Entries in brackets are the t-statistics, starred entries indicate significance at 5% level. The last column reports the likelihood ratio statistics for testing the null hypothesis, $\theta_s = \theta, s = 1 \cdots 12$. Starred entries indicate rejection of the null at 5% level

For the stationary data we consider models of the form:

I. $y_t = \theta_0 + \varepsilon_t$ – Random Walk (RW, No change) type Models

II. $\phi(L)y_t = \Sigma_{s=1}^{S}\theta_s d_{s,t} + \varepsilon_t$ – Difference Stationary (DS) type Models With Deterministic Seasonal Components

III. $\phi(L)\triangle_S y_t = (1+\theta L^s)\varepsilon_t$ – Seasonal Unit Root (SUR) type Models

IV. $y_t = \theta_s + \Sigma_{i=1}^{p}\theta_{i,s}y_{t-i} + \varepsilon_t$ – Periodic Autoregression (PAR) type Models

In these models: y_t is the "target" variable; $\triangle_S$ denotes the S^{th} difference operator, where S is the number of seasons presumed to be in the data (i.e. $\triangle_S$ is the seasonal difference operator); $\phi(L) = (1-\phi_1 L - ... - \phi_p L)$ is a standard lag polynomial of order p, expressed using the lag operator, L; the θ_s denote seasonal intercepts, with associated and conformably defined dummy variables, $d_{s,t}$. Notice that the PAR model is a generalization of the DS model, as it allows the intercept as well as the slope coefficients to vary according to the season. Notice also that both of these model generalize the RW model, which is used as a "strawman" model in our analysis. Under fairly general conditions, the parameters of the PAR model can be readily estimated via least squares (LS), for example. In particular, consider the PAR model with $p = 1$ (i.e. the PAR(1) model) and $p_1 = ... = p_k = 0$. This model can be estimated via least squares using the equation: $y_t = \Sigma_{s=1}^{S}\theta_s d_{s,t} + \Sigma_{s=1}^{S}\theta_{1,s}d_{s,t}y_{t-1} + \varepsilon_t$. Here, under error normality and given fixed stating values, the LS estimator is the MLE, and standard asymptotics pertain (see Franses and Paap (1999) for further details).

Notice that the PAR(1) model has a unit root when $\theta_{1,1}\theta_{1,2}\theta_{1,3}\cdots\theta_{1,s} = 1$. Clearly, the PAR(1) model nest the simple random walk model where $\triangle_1 y_t = (1-\theta L)y_t = \theta_0 + \varepsilon_t$, with $\theta = 1$. In this case, the characteristic equation is $(1-\theta^S z) = 0$, so that when $\theta = 1$, y_t has a single nonseasonal unit root, corresponding to the simple random walk model. Also, when $\theta = -1$, y_t has a seasonal unit root. Thus, as mentioned above, both seasonal and non-seasonal unit root processes such as those given in our RW, DS, and SUR models, are nested within PAR models (see Hylleberg, Engle, Granger and Yoo (HEGY: 1990) for further details). Nonlinear variants of the above models are discussed in Franses and van Dijk (2001), for example. Boswijk and Franses (1996: BF) and Boswijk, Franses and Haldrup (1997) outline tests for $\theta_{1,s}\theta_{2,s}\theta_{3,s}\theta_{4,s} = 1$ and for $\theta = 1$ and $\theta = -1$ (see Franses and Paap (1999) for a summary).

The 14 variables listed above will be used in the simulation below, as well as in the prediction experiments carried out subsequently. Lags are selected using the Schwarz Information Criterion (SIC), and estimation is carried out using least squares. Additionally, Dickey Fuller, HEGY, and BF type unit root tests are run (on log levels). HEGY_PW, refers to HEGY test with pre whitening, which involves estimating

$\triangle_S y_t = \sum_{i=1}^{p} \phi_i \triangle_S y_{t-i}$, HEGY is then applied to prewhitened data $y_t' = \left(1 - \sum_{i=1}^{p} \phi_i L^i\right) y_t$ as suggested by Psaradakis (1997). The estimated models are summarized as follows:

Table 2: Summary Statistics (see paper for final table) $^{(*)}$

Series	Mean	Std. Dev.	Skewness	Kurtosis	Jarque Bera	ADF	BF	HEGY	HEGY_PW
CPI1	0.0041	0.0032	0.9168	4.2301	83.488	-1.31*	15.20	-7.45	-6.99
CPI2	0.0042	0.0165	0.4440	5.5488	124.76	-0.22*	6.84*	-8.39	-7.41
M1	0.0045	0.0158	-0.4169	2.8838	12.134	-1.16*	5.79*	-3.99	-4.21
M2	0.0058	0.0061	-0.3555	3.5043	13.014	-2.08*	29.97	-4.53	-4.67
H_Start	0.0023	0.1610	0.5117	3.9207	32.452	-4.29	1.66*	-7.49	-7.18
IP	0.0023	0.0222	-0.5048	3.5371	22.393	-1.31*	51.84	-5.63	-5.59
NonF_Emp	0.0017	0.0086	-1.5923	4.9595	239.43	-1.54*	158.69	-3.64	-3.77
IP_Auto	0.0026	0.1366	-0.1845	6.6850	234.87	-0.76*	3.40*	-2.50*	-2.65*
IP_Dur	0.0027	0.0694	-0.4980	4.7093	67.019	-0.86*	0.21*	-2.59*	-2.66*
IP_NDur	0.0016	0.0251	0.2133	2.5401	6.7389	-3.18	187.14	-4.68	-4.35
D_Ship	0.0045	0.0344	0.7707	4.0412	59.249	-1.94*	6.36*	-4.26	-5.37
Invent	0.0044	0.0120	3.5928	48.027	35604	-1.91*	83.15	-5.27	-4.99
Veh_Sales	-0.0002	0.1373	0.0748	3.0104	0.3470	-1.63*	3.27*	-5.74	-6.26
Ret_Sales	0.0062	0.1126	-1.2410	5.3646	201.24	-2.37*	18.89	-4.27	-4.58

$^{(*)}$ Mean, Std. Dev. Skewness Kurtosis and Jarque-Bera statistics are calculated for level series. Unit root tests are carried out for log levels except for the interest rates. For the unit root tests the starred entries represent the cases where we fail to reject the null of unit root at 5% nominal level. HEGY test statistics are reported for seasonal frequency π, for test at all the other seasonal frequencies and non seasonal unit root see table 3 and 4.

Table 3 HEGY Test Details [(*)]

	CPI1	CPI2	M1	M2	H_Start	R10	IP	NonF_Emp	IP_Auto	IP_Dur	IP_NDur	D_Ship	Invent	Veh_Sales	Ret_Sales
π_1	-1.20*	-0.35*	-1.22*	-1.93*	-3.93	-1.84*	-1.99*	-2.57*	-0.72*	-1.24*	-3.47	-2.14*	-2.14*	-2.47*	-3.02*
π_2	-7.45	-8.39	-3.99	-4.53	-7.49	-8.07	-5.63	-3.64	-2.50*	-2.59*	-4.68	-4.26	-5.27	-5.74	-4.27
π_3	-4.55	-5.34	-3.82	-0.56*	-6.57	-3.00	-4.49	-3.32	-5.28	-5.19	-2.68	-7.88	-4.60	-6.07	-4.51
π_4	-8.48	-9.45	-4.99	-5.82	-3.35	-9.01	-3.97	-3.03*	-1.39*	-2.13*	-2.84*	-3.89	-6.78	-0.67*	-2.53*
π_5	-7.26	-6.33	-3.53	-4.17	-7.35	-8.21	-6.20	-4.07	-4.08	-4.28	-6.00	-3.78	-4.04	-6.50	-5.99
π_6	5.62*	9.03*	0.39*	1.69*	2.17*	7.32*	3.88*	0.88*	0.20*	1.08*	2.89*	-0.67*	2.36*	-4.45	0.19*
π_7	-2.40	-4.79	-3.02	-0.03*	-5.20	-5.13	-1.16*	-2.07*	-3.18	-3.24	-1.42*	-5.01	-1.98*	-6.70	-5.23
π_8	-7.00	-7.18	-6.59	-7.09	-4.18	-9.36	-4.41	-3.93	-2.44*	-2.98*	-2.13*	-7.52	-7.90	-2.46*	-3.24*
π_9	-8.99	-8.28	-3.57	-5.00	-9.81	-8.68	-6.13	-6.00	-3.97	-4.16	-5.01	-5.04	-8.08	-6.92	-6.69
π_{10}	1.14*	5.09*	3.51*	5.77*	1.38*	6.97*	1.48*	1.19*	-0.21*	0.17*	1.42*	0.68*	2.68*	0.29*	1.40*
π_{11}	-1.22*	-1.52*	-0.33*	0.95*	-1.72*	-3.36	0.81*	0.38*	-5.09	-3.33	-2.22*	-0.85*	2.31	-5.42	-6.88
π_{12}	-8.31	-7.60	-6.77	-7.28	-7.29	-9.23	-7.77	-5.25	-3.88	-5.25	-2.80*	-8.85	-8.22	-4.65	-4.29
$F_{3,4}$	46.2	58.65	20.51	17.09	28.21	45.10	18.00	10.15	14.91	15.76	7.75	40.74	33.61	18.62	13.39
$F_{5,6}$	42.97	66.91	6.31*	10.11	29.41	63.17	27.01	8.63	8.35	9.71	22.18	7.38	10.94	32.95	17.92
$F_{7,8}$	27.07	37.27	26.32	25.11	22.18	56.77	10.33	9.86	8.10	9.79	3.26*	40.98	32.67	26.64	19.71
$F_{9,10}$	40.66	52.16	13.01	31.27	48.10	73.78	19.93	18.52	7.92	8.66	13.40	12.82	36.53	23.93	23.21
$F_{11,12}$	34.96	29.77	22.94	27.89	27.41	49.15	31.15	13.92	21.85	19.94	6.42*	39.18	39.08	28.82	38.37

[(*)] Starred entries represent the cases where we fail to reject the null of unit root at 10% nominal level. Data is for the period Jan. 1959 till Dec 2004. In order to test the hypothesis of unit root at various seasonal frequencies the equation to be estimated is $\theta(L)\, y_{13t} = \sum_{k=1}^{12} \pi_k y_{k,t-1} + \epsilon_t$, where $\theta(L)$ is a polynomial in lag operator L and $y_{13t} = \left(1 - B^{12}\right) y_t$.For monthly data the frequencies for seasonal unit root are π, $\pm\frac{\pi}{2}$, $\pm\frac{2\pi}{3}$, $\pm\frac{\pi}{3}$, $\pm\frac{5\pi}{6}$, and $\pm\frac{\pi}{6}$, thus testing for the presence of seasonal unit root boils down to testing the null $\pi_k = 0$, for $k > 1$. Testing the null that $\pi_1 = 0$, corresponds to the test for null of non seasonal unit root. To test the presence of unit root at any seasonal frequency first we need to test that $\pi_k = 0$, for $k = 2$ against an alternative that $\pi_k < 0$. For other roots we test $\pi_k = 0$ for even $k\,(> 2)$, against a two sided alternative, finally if we fail to reject the null for even k we test $\pi_{k-1} = 0$ against the alternative $\pi_{k-1} < 0$. As we estimate the equation by OLS the tests can be carried out as a sequence of t-tets, or a set of f-tests for the even odd pairs $\pi_k = \pi_{k-1} = 0$. The estimation details and the critical values are reported in Miron and Beaulieu (1993). These results further confirm the finding of Miron and Beaulieu (1993) that the data rejects the null for seasonal unit roots at most of the frequencies.

Table 4 HEGY_PW Test Details (*)

	CPI1	CPI2	M1	M2	H_Start	R10	IP	NonF_Emp	IP_Auto	IP_Dur	IP_NDur	D_Ship	Invent	Veh_Sales	Ret_Sales
π_1	-1.82*	-0.77*	-1.66*	-2.27*	-4.94	-2.28*	-1.08*	-1.28*	-0.73*	-0.93*	-1.85*	-1.57*	-1.57*	-1.78*	-2.50*
π_2	-6.99	-7.41	-4.21	-4.67	-7.18	-8.04	-5.59	-3.77	-2.65*	-2.66*	-4.35	-5.37	-4.99	-6.26	-4.58
π_3	-8.84	-10.18	-6.34	-5.73	-8.12	-9.41	-5.90	-4.61	-5.15	-5.32	-3.64	-8.44	-7.05	-5.56	-5.30
π_4	-0.37*	1.03*	0.13*	-1.30*	0.45*	-1.89*	1.17*	0.19*	1.77*	1.65*	-0.93*	1.79*	-1.24*	-1.99*	0.13*
π_5	-8.65	-10.24	-3.02	-3.98	-7.23	-10.71	-6.72	-4.50	-4.62	-4.84	-6.34	-3.17	-4.60	-6.05	-5.60
π_6	0.39*	-1.31*	-1.76*	-2.07*	-0.14*	-2.93*	0.76*	-0.31*	-1.22*	-0.73*	0.42*	-1.80*	0.16*	-3.84	-1.82*
π_7	-6.95	-7.41	-7.10	-7.15	-7.62	-10.38	-4.52	-6.56	-4.44	-4.72	-2.31	-8.78	-7.01	-5.86	-6.18
π_8	1.42*	2.25*	0.32*	-0.03*	0.96*	1.31*	0.69*	0.37*	0.94*	1.37*	-0.24*	-1.22*	0.89*	-0.35*	0.68*
π_9	-8.76	-8.69	-4.90	-7.35	-9.72	-11.33	-5.94	-7.20	-4.48	-4.56	-4.82	-5.91	-8.28	-7.97	-7.11
π_{10}	-1.37*	-0.56*	2.10*	3.03*	0.07*	1.25*	0.63*	0.05*	-1.30*	-0.89*	0.36*	-0.27*	-0.13*	-1.31*	0.07*
π_{11}	-9.08	-7.44	-6.53	-7.54	-8.01	-10.20	-8.78	-5.13	-6.82	-6.53	-3.67	-8.08	-8.07	-6.39	-8.72
π_{12}	2.03*	1.08*	-0.95*	0.22*	-2.30*	1.47*	0.91*	1.30*	1.39*	1.16*	1.14*	-2.24*	1.29*	1.88*	0.40*
$F_{3,4}$	39.16	52.62	20.13	17.37	33.11	46.69	18.14	10.66	15.00	15.64	7.05	37.68	25.81	17.72	14.03
$F_{5,6}$	37.55	53.53	6.14*	10.19	26.14	63.65	22.89	10.17	11.42	11.98	20.18	6.70	10.58	27.16	17.55
$F_{7,8}$	25.60	30.83	25.24	25.57	29.62	55.11	10.47	21.57	10.34	12.15	2.70*	39.44	25.01	17.23	19.37
$F_{9,10}$	39.65	37.85	14.69	32.57	47.22	65.45	17.91	26.00	10.98	10.85	11.68	17.47	34.32	32.91	25.29
$F_{11,12}$	43.95	28.39	21.82	28.46	35.35	53.63	38.81	14.12	24.36	22.09	7.39	35.74	33.58	22.69	38.17

(*) See notes to Table 3.

4 Small Monte Carlo Study

In this section results of two basic experiments are reported, in both of them the DGP is Seasonal Unit Root (SUR) type Models

$$\phi(L)\triangle_S y_t = (1 + \theta L^s)\varepsilon_t$$

where $\phi(L) = 1 - \phi L$, and $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$. Parameter values are chosen to mimic the estimates from the 15 series, they are $\theta \in (-0.6, -0.7, -0.8, -0.9)$, $\phi \in (-0.5, 0.0, 0.5)$, and $\sigma_\varepsilon \in (0.005, 0.01, 0.1)$, further to mimic the unit root in actual data the DGP was $\phi(L)\triangle_S (1 - L)\, y_t = (1 + \theta L^s)\varepsilon_t$, where $s = 12$. (see paper for complete details).

4.1 Seasonal Unit Root, Parameter Estimation Error and Out of Sample Forecasting

In order to judge the contribution of parameter estimation error for seasonal unit root models first the data was generated using the above models. Next SUR and RW models were fit in a recursive estimation scheme and out of sample forecasts (1, 3, 12, and 60 step ahead) were generated. Following are the results of MSFE comparison and DM test for predictive accuracy. Table 5 on next page reports the results. The first column reports the parameter values used in data generation. The sample size was 500. Results are based on 500 Monte Carlo simulations. Numbers in the tables report the proportions of times SUR beat RW in terms of MSFE.

Table 5: MSFE comparison, DGP SUR $^{(*)}$

$(\theta, \sigma_\varepsilon, \phi)$	1-Step	3-Step	12-Step	60-Step
$(-0.6, 0.005, 0.0)$	1.00	1.00	0.87	0.67
$(-0.7, 0.005, 0.0)$	1.00	1.00	0.85	0.55
$(-0.8, 0.005, 0.0)$	1.00	1.00	0.67	0.47
$(-0.9, 0.005, 0.0)$	1.00	1.00	0.70	0.20
$(-0.6, 0.01, 0.0)$	1.00	1.00	0.90	0.80
$(-0.7, 0.01, 0.0)$	1.00	1.00	0.85	0.65
$(-0.8, 0.01, 0.0)$	1.00	1.00	0.75	0.55
$(-0.9, 0.01, 0.0)$	1.00	0.97	0.70	0.30
$(-0.6, 0.1, 0.0)$	1.00	1.00	0.90	0.87
$(-0.7, 0.1, 0.0)$	1.00	1.00	0.89	0.69
$(-0.8, 0.1, 0.0)$	1.00	1.00	0.85	0.45
$(-0.9, 0.1, 0.0)$	1.00	0.95	0.75	0.25
$(-0.6, 0.005, 0.5)$	1.00	1.00	0.86	0.76
$(-0.7, 0.005, 0.5)$	1.00	1.00	0.87	0.72
$(-0.8, 0.005, 0.5)$	1.00	0.99	0.70	0.54
$(-0.9, 0.005, 0.5)$	1.00	0.97	0.57	0.27
$(-0.6, 0.01, 0.5)$	1.00	1.00	0.82	0.71
$(-0.7, 0.01, 0.5)$	1.00	1.00	0.86	0.74
$(-0.8, 0.01, 0.5)$	1.00	1.00	0.77	0.43
$(-0.9, 0.01, 0.5)$	1.00	0.98	0.62	0.22
$(-0.6, 0.1, 0.5)$	1.00	1.00	0.90	0.84
$(-0.7, 0.1, 0.5)$	1.00	1.00	0.84	0.81
$(-0.8, 0.1, 0.5)$	1.00	1.00	0.72	0.52
$(-0.9, 0.1, 0.5)$	1.00	0.99	0.50	0.21
$(-0.6, 0.005, -0.5)$	1.00	1.00	0.86	0.76
$(-0.7, 0.005, -0.5)$	1.00	1.00	0.88	0.78
$(-0.8, 0.005, -0.5)$	1.00	0.99	0.71	0.51
$(-0.9, 0.005, -0.5)$	1.00	0.96	0.66	0.36
$(-0.6, 0.01, -0.5)$	1.00	1.00	0.94	0.74
$(-0.7, 0.01, -0.5)$	1.00	1.00	0.88	0.68
$(-0.8, 0.01, -0.5)$	1.00	1.00	0.79	0.42
$(-0.9, 0.01, -0.5)$	1.00	0.97	0.67	0.19
$(-0.6, 0.1, -0.5)$	1.00	1.00	0.84	0.68
$(-0.7, 0.1, -0.5)$	1.00	1.00	0.86	0.70
$(-0.8, 0.1, -0.5)$	1.00	0.98	0.68	0.42
$(-0.9, 0.1, -0.5)$	1.00	0.96	0.49	0.17

$^{(*)}$ The first column reports the parameter values used in data generation. Numbers in other columns report the proportions of times SUR beat RW in terms of MSFE. The sample size was 500. Forecasts were based on recursive estimation scheme, where the intitial sample size was 120. Results are based on 500 Monte Carlo simulations.

Table 6: Forecasting Performance DM Test, DGP SUR [(*)]

DGP	1-Step		3-Step		12-Step		60-Step	
$(\theta, \sigma_\varepsilon, \phi)$	SUR	RW	SUR	RW	SUR	RW	SUR	RW
$(-0.6, 0.005, 0.0)$	1.00	0.00	1.00	0.00	0.30	0.00	0.07	0.05
$(-0.7, 0.005, 0.0)$	1.00	0.00	1.00	0.00	0.21	0.00	0.05	0.04
$(-0.8, 0.005, 0.0)$	1.00	0.00	1.00	0.00	0.17	0.00	0.07	0.13
$(-0.9, 0.005, 0.0)$	1.00	0.00	0.97	0.00	0.11	0.05	0.02	0.27
$(-0.6, 0.01, 0.0)$	1.00	0.00	1.00	0.00	0.32	0.00	0.10	0.05
$(-0.7, 0.01, 0.0)$	1.00	0.00	1.00	0.00	0.26	0.00	0.15	0.05
$(-0.8, 0.01, 0.0)$	1.00	0.00	1.00	0.00	0.21	0.03	0.05	0.06
$(-0.9, 0.01, 0.0)$	0.97	0.00	0.95	0.00	0.14	0.11	0.04	0.19
$(-0.6, 0.1, 0.0)$	1.00	0.00	1.00	0.00	0.35	0.01	0.17	0.04
$(-0.7, 0.1, 0.0)$	1.00	0.00	1.00	0.00	0.30	0.01	0.09	0.05
$(-0.8, 0.1, 0.0)$	1.00	0.00	1.00	0.00	0.22	0.03	0.05	0.13
$(-0.9, 0.1, 0.0)$	1.00	0.00	0.94	0.00	0.15	0.07	0.03	0.26
$(-0.6, 0.005, 0.5)$	1.00	0.00	1.00	0.00	0.26	0.01	0.17	0.07
$(-0.7, 0.005, 0.5)$	1.00	0.00	1.00	0.00	0.27	0.02	0.12	0.05
$(-0.8, 0.005, 0.5)$	1.00	0.00	0.98	0.00	0.20	0.05	0.04	0.12
$(-0.9, 0.005, 0.5)$	1.00	0.00	0.96	0.00	0.17	0.12	0.02	0.32
$(-0.6, 0.01, 0.5)$	1.00	0.00	1.00	0.00	0.31	0.02	0.11	0.10
$(-0.7, 0.01, 0.5)$	1.00	0.00	1.00	0.00	0.36	0.00	0.10	0.03
$(-0.8, 0.01, 0.5)$	1.00	0.00	1.00	0.00	0.27	0.10	0.03	0.15
$(-0.9, 0.01, 0.5)$	1.00	0.00	0.98	0.00	0.22	0.12	0.02	0.26
$(-0.6, 0.1, 0.5)$	1.00	0.00	1.00	0.00	0.28	0.00	0.14	0.02
$(-0.7, 0.1, 0.5)$	1.00	0.00	1.00	0.00	0.24	0.03	0.09	0.05
$(-0.8, 0.1, 0.5)$	1.00	0.00	1.00	0.00	0.22	0.08	0.04	0.14
$(-0.9, 0.1, 0.5)$	1.00	0.00	0.97	0.00	0.10	0.18	0.02	0.25
$(-0.6, 0.005, -0.5)$	0.95	0.00	1.00	0.00	0.24	0.01	0.16	0.06
$(-0.7, 0.005, -0.5)$	1.00	0.00	1.00	0.00	0.28	0.01	0.08	0.05
$(-0.8, 0.005, -0.5)$	1.00	0.00	0.99	0.00	0.21	0.04	0.11	0.11
$(-0.9, 0.005, -0.5)$	1.00	0.00	0.95	0.00	0.16	0.12	0.06	0.31
$(-0.6, 0.01, -0.5)$	1.00	0.00	1.00	0.00	0.34	0.01	0.14	0.03
$(-0.7, 0.01, -0.5)$	1.00	0.00	1.00	0.00	0.28	0.04	0.11	0.08
$(-0.8, 0.01, -0.5)$	1.00	0.00	1.00	0.00	0.19	0.06	0.07	0.15
$(-0.9, 0.01, -0.5)$	1.00	0.00	0.96	0.00	0.11	0.12	0.02	0.32
$(-0.6, 0.1, -0.5)$	1.00	0.00	1.00	0.00	0.24	0.01	0.18	0.04
$(-0.7, 0.1, -0.5)$	1.00	0.00	1.00	0.00	0.26	0.06	0.10	0.02
$(-0.8, 0.1, -0.5)$	1.00	0.00	0.98	0.00	0.18	0.08	0.06	0.07
$(-0.9, 0.1, -0.5)$	1.00	0.00	0.95	0.01	0.09	0.11	0.07	0.24

[(*)] Diebold and Mariano (DM) test statistics are based on MSFE loss, and application of the test assumes that parameter estimation error vanishes and that the standard normal limiting distribution is asymptotically valid. Results are based on 1, 3, 12 and 60 step ahead forecasting based on recursive estimation scheme. The numbers indicates the proportions of times DM test prefers the respective model (SUR and RW) at a 5% nominal significance level, based on MSFE loss. Results are based on 500 Monte Carlo simulations.

The next table reports the performance of Seasonal unit root tests in the presence of a large MA term (θ), this table illustrates the poor performance of these tests as is very well reported in the literature. HEGY and HEGY_PW test results are reported for seasonal frequency π_2,

Table 7: Monte Carlo Results: rejection frequency of null of seasonal unit root at 5% level.

DGP		Test	
$(\theta, \sigma_\varepsilon, \phi)$	BF	HEGY	HEGY_PW
$(-0.6, 0.005, 0.0)$	0.64	0.46	0.45
$(-0.7, 0.005, 0.0)$	0.58	0.71	0.68
$(-0.8, 0.005, 0.0)$	0.46	0.95	0.95
$(-0.9, 0.005, 0.0)$	0.34	1.00	1.00
$(-0.6, 0.01, 0.0)$	0.61	0.38	0.36
$(-0.7, 0.01, 0.0)$	0.55	0.66	0.63
$(-0.8, 0.01, 0.0)$	0.43	0.92	0.90
$(-0.9, 0.01, 0.0)$	0.34	1.00	1.00
$(-0.6, 0.1, 0.0)$	0.54	0.41	0.39
$(-0.7, 0.1, 0.0)$	0.51	0.71	0.71
$(-0.8, 0.1, 0.0)$	0.41	0.98	0.98
$(-0.9, 0.1, 0.0)$	0.30	1.00	1.00
$(-0.6, 0.005, 0.5)$	0.58	0.34	0.33
$(-0.7, 0.005, 0.5)$	0.56	0.68	0.72
$(-0.8, 0.005, 0.5)$	0.41	0.98	0.95
$(-0.9, 0.005, 0.5)$	0.35	1.00	1.00
$(-0.6, 0.01, 0.5)$	0.62	0.38	0.36
$(-0.7, 0.01, 0.5)$	0.60	0.72	0.68
$(-0.8, 0.01, 0.5)$	0.42	0.97	0.97
$(-0.9, 0.01, 0.5)$	0.34	1.00	1.00
$(-0.6, 0.1, 0.5)$	0.58	0.42	0.43
$(-0.7, 0.1, 0.5)$	0.53	0.77	0.74
$(-0.8, 0.1, 0.5)$	0.47	0.96	0.95
$(-0.9, 0.1, 0.5)$	0.33	1.00	1.00
$(-0.6, 0.005, -0.5)$	0.62	0.32	0.35
$(-0.7, 0.005, -0.5)$	0.66	0.58	0.65
$(-0.8, 0.005, -0.5)$	0.48	0.85	0.95
$(-0.9, 0.005, -0.5)$	0.41	1.00	1.00
$(-0.6, 0.01, -0.5)$	0.60	0.37	0.39
$(-0.7, 0.01, -0.5)$	0.53	0.54	0.71
$(-0.8, 0.01, -0.5)$	0.50	0.82	0.94
$(-0.9, 0.01, -0.5)$	0.33	1.00	1.00
$(-0.6, 0.1, -0.5)$	0.57	0.34	0.37
$(-0.7, 0.1, -0.5)$	0.52	0.57	0.64
$(-0.8, 0.1, -0.5)$	0.44	0.86	0.94
$(-0.9, 0.1, -0.5)$	0.30	1.00	1.00

4.2 Simulation

In order to evaluate the performance of the different models from the perspective of simulation, each parameterization was used to simulate B observations. The starting value for the simulations are fixed to the last observation of the historical sample used in estimation. In addition, the models used for simulation are the models outlined above, including the error term from the original estimated model, where the error is assumed to be *iid* normal, and where the variance of the error term is estimated by using the residuals of the of fitted model. Thus, simulation models take forms such as $y_t = \theta + \varepsilon_t$, for example, where $\varepsilon_t \sim iidN(0, \sigma_\varepsilon^2)$, and σ_ε^2 is calibrated from the data. Finally, all results are based on the analysis of 100 simulation paths.

Table 8: Distributional Accuracy Tests Based on the Comparison of Historical and Simulated Data, Benchmark Model: RW$^{(*)}$

S,l	Z	$Crit.Val.(Z^{*})$		$Crit.Val.(Z^{**})$		CS Distributional Loss			
		10%	5%	10%	5%	RW	DS	SUR	PAR
				$\ln(CPI1_t) - \ln(CPI1_{t-1})$					
5T,4	-0.0031	2.893	2.9863	2.8997	2.9954	1.5634	4.7753	2.5993	1.5665
5T,12	-0.0031	2.4984	2.5606	2.5006	2.5465	1.5634	4.7753	2.5993	1.5665
10T,4	-0.0048	3.1109	3.1499	3.1134	3.1397	1.5631	4.9371	2.6047	1.5679
10T,12	-0.0048	2.6184	2.8337	2.6068	2.8256	1.5631	4.9371	2.6047	1.5679
				$\ln(CPI2_t) - \ln(CPI2_{t-1})$					
5T,4	-0.0067	0.2552	0.2652	0.2518	0.2673	1.1283	1.1412	1.3952	1.1351
5T,12	-0.0067	0.268	0.2728	0.2659	0.2763	1.1283	1.1412	1.3952	1.1351
10T,4	-0.0022	0.261	0.2703	0.2605	0.2713	1.1285	1.1446	1.3959	1.1307
10T,12	-0.0022	0.2602	0.2676	0.2613	0.2710	1.1285	1.1446	1.3959	1.1307
				$\ln(M1_t) - \ln(M1_{t-1})$					
5T,4	0.0174	0.6724	0.704	0.6696	0.6990	2.2174	2.2000	2.9743	2.2013
5T,12	0.0174	0.6813	0.6965	0.6879	0.6995	2.2174	2.2000	2.9743	2.2013
10T,4	0.0167	0.6786	0.7034	0.6788	0.7063	2.2167	2.2000	2.9743	2.2005
10T,12	0.0167	0.6774	0.7037	0.6729	0.7040	2.2167	2.2000	2.9743	2.2005
				$\ln(M2_t) - \ln(M2_{t-1})$					
5T,4	0.0046	0.8092	0.9512	0.8069	0.9393	1.8239	2.4718	3.1176	1.8193
5T,12	0.0046	0.8386	1.0808	0.8413	1.0817	1.8239	2.4718	3.1176	1.8193
10T,4	0.0042	0.9672	1.0918	0.9688	1.0909	1.8234	2.4716	3.157	1.8192
10T,12	0.0042	0.9044	1.0065	0.9025	1.0039	1.8234	2.4716	3.157	1.8192
				$\ln(H\_start_t) - \ln(H\_start_{t-1})$					
5T,4	0.0077	0.0627	0.0796	0.0658	0.0814	1.9628	1.9551	2.0481	1.9570
5T,12	0.0077	0.0760	0.0855	0.0810	0.0857	1.9628	1.9551	2.0481	1.9570
10T,4	0.0104	0.0666	0.0779	0.0673	0.0783	1.9650	1.9546	2.0514	1.9575
10T,12	0.0104	0.0784	0.0859	0.0789	0.0877	1.9650	1.9546	2.0514	1.9575
				$\ln(IP_t) - \ln(IP_{t-1})$					
5T,4	0.0151	0.3824	0.4107	0.3845	0.4119	2.0792	2.0677	2.5311	2.0641
5T,12	0.0151	0.4156	0.4262	0.4138	0.4275	2.0792	2.0677	2.5311	2.0641
10T,4	0.0167	0.4073	0.4177	0.407	0.4179	2.0808	2.0667	2.5327	2.0641
10T,12	0.0167	0.4067	0.4172	0.4059	0.4183	2.0808	2.0667	2.5327	2.0641
				$\ln(NonF\_Emp_t) - \ln(NonF\_Emp_{t-1})$					
5T,4	0.1401	0.6085	0.6974	0.5943	0.6931	2.0522	1.9366	2.8813	1.9121
5T,12	0.1401	0.6427	0.7317	0.6410	0.7238	2.0522	1.9366	2.8813	1.9121
10T,4	0.1237	0.7064	0.7334	0.7070	0.7405	2.0363	1.9397	2.8811	1.9126
10T,12	0.1237	0.6524	0.7127	0.6629	0.7195	2.0363	1.9397	2.8811	1.9126

Table 8 (Contd.): Distributional Accuracy Tests Based on the Comparison of Historical and Simulated Data, Benchmark Model: RW$^{(*)}$

S,l	Z	$Crit.Val.(Z^{*})$ 10%	$Crit.Val.(Z^{*})$ 5%	$Crit.Val.(Z^{**})$ 10%	$Crit.Val.(Z^{**})$ 5%	CS Distributional Loss RW	DS	SUR	PAR
$\ln(IP\_Auto_t) - \ln(IP\_Auto_{t-1})$									
5T,4	0.0231	0.1317	0.1372	0.1317	0.1358	1.3325	1.3291	1.4475	1.3094
5T,12	0.0231	0.1442	0.1498	0.1454	0.1532	1.3325	1.3291	1.4475	1.3094
10T,4	0.0198	0.1068	0.1166	0.1069	0.1185	1.3288	1.3299	1.4159	1.309
10T,12	0.0198	0.11	0.1167	0.1098	0.1153	1.3288	1.3299	1.4159	1.309
$\ln(IP\_Dur_t) - \ln(IP\_Dur_{t-1})$									
5T,4	0.0217	0.2924	0.3102	0.2938	0.3075	1.8412	1.8301	2.1449	1.8195
5T,12	0.0217	0.2974	0.3114	0.3021	0.309	1.8412	1.8301	2.1449	1.8195
10T,4	0.0219	0.2976	0.311	0.297	0.3119	1.8421	1.8307	2.1624	1.8202
10T,12	0.0219	0.3205	0.3381	0.3201	0.3388	1.8421	1.8307	2.1624	1.8202
$\ln(IP\_NDur_t) - \ln(IP\_NDur_{t-1})$									
5T,4	0.0015	0.3974	0.4088	0.4014	0.407	2.2186	2.2929	2.6528	2.2171
5T,12	0.0015	0.4023	0.4061	0.3996	0.4075	2.2186	2.2929	2.6528	2.2171
10T,4	0.0018	0.3751	0.3949	0.3768	0.3939	2.2195	2.2971	2.6557	2.2177
10T,12	0.0018	0.3969	0.4051	0.3967	0.4062	2.2195	2.2971	2.6557	2.2177
$\ln(D\_Ship_t) - \ln(D\_Ship_{t-1})$									
5T,4	0.0058	0.2615	0.2807	0.2605	0.2777	1.8453	1.8419	2.1582	1.8395
5T,12	0.0058	0.2628	0.2808	0.2628	0.2821	1.8453	1.8419	2.1582	1.8395
10T,4	0.0082	0.2669	0.2801	0.2694	0.2802	1.8466	1.8408	2.1629	1.8384
10T,12	0.0082	0.2638	0.2783	0.2668	0.2782	1.8466	1.8408	2.1629	1.8384
$\ln(Invent_t) - \ln(Invent_{t-1})$									
5T,4	-0.0178	0.5438	0.5506	0.5426	0.5497	0.7184	0.8054	1.2663	0.7362
5T,12	-0.0178	0.5239	0.5372	0.5247	0.5386	0.7184	0.8054	1.2663	0.7362
10T,4	-0.0186	0.5272	0.5393	0.5282	0.538	0.7183	0.7976	1.2663	0.7369
10T,12	-0.0186	0.5291	0.5354	0.5292	0.5354	0.7183	0.7976	1.2663	0.7369
$\ln(Ret\_sales_t) - \ln(Ret\_sales_{t-1})$									
5T,4	0.1019	0.3401	0.3461	0.3411	0.353	1.9304	2.0016	2.2441	1.8285
5T,12	0.1019	0.3357	0.3448	0.3362	0.3457	1.9304	2.0016	2.2441	1.8285
10T,4	0.1004	0.3431	0.357	0.3421	0.3532	1.9291	2.0002	2.2407	1.8286
10T,12	0.1004	0.3368	0.3507	0.3366	0.3502	1.9291	2.0002	2.2407	1.8286
$\ln(Veh\_sales_t) - \ln(Veh\_sales_{t-1})$									
5T,4	0.0011	0.0556	0.0627	0.0538	0.0623	1.7596	1.7616	1.8359	1.7585
5T,12	0.0011	0.0585	0.0599	0.0577	0.0600	1.7596	1.7616	1.8359	1.7585
10T,4	0.0006	0.0484	0.0555	0.0512	0.0542	1.7590	1.7606	1.8259	1.7584
10T,12	0.0006	0.0495	0.0553	0.0502	0.0553	1.7590	1.7606	1.8259	1.7584

$^{(*)}$ Distributions of historical and simulated series are compared based on the statistics discussed above. Table reports the results where the benchmark model is RW. The null hypothesis corresponds to the case where no alternative model outperforms the benchmark. Second column reports the test statistics. Next four columns report the 5% and 10% bootstrap critical values based on bootstrap statistics constructed allowing for parameter estimation error ($Z**$) and assuming that parameter estimation error vanishes asymptotically ($Z*$). Additionally T denotes the historical sample size, S the length of simulated sample size, and l is the bootstrap block length. The historical sample length in this table is also the period used to estimate the models. All statistics are based on a grid of 20x20 values for u distributed uniformly across the historical data rang. Bootstrap empirical distributions are constructed using 100 bootstrap replications.

4.3 Prediction

Table 9: Predictive Accuracy Test Results for Various Macroeconomic Variables $^{(*)}$

	$h=1$			$h=3$			$h=12$			$h=60$		
Series	DS	SUR	PAR	DS	SUR	PAR	DS	SUR	PAR	DS	SUR	PAR
Panel A: MSFE for DS, SUR and PAR, Realtive to the RW Model												
CPI1	1.24	1.17	1.35	0.98	2.41	1.27	0.99	2.35	1.25	0.98	1.74	1.10
CPI2	0.88	1.68	2.37	1.07	3.28	1.04	1.03	3.23	1.03	1.04	2.23	1.02
M1	1.19	0.53	0.57	1.03	3.28	0.45	2.38	3.73	0.45	2.34	2.36	0.45
M2	1.97	0.88	0.85	1.57	5.69	0.75	9.05	5.61	0.78	9.55	2.53	0.76
H_Start	1.57	0.98	0.46	5.49	1.58	0.43	2.00	2.50	0.43	2.05	1.57	0.43
IP	1.26	0.53	0.79	1.27	3.9	0.67	1.91	3.32	0.65	2.00	2.97	0.65
NonF_Emp	9.69	0.14	0.55	1.09	3.77	0.36	1.37	3.53	0.36	1.47	2.07	0.37
IP_Auto	0.98	0.39	2.94	1.05	1.66	3.65	1.06	2.13	3.67	1.10	2.38	3.67
IP_Dur	0.94	0.38	1.55	1.01	2.24	1.42	1.45	2.50	1.45	1.41	2.24	1.45
IP_NDur	2.20	0.56	2.5	1.37	2.19	2.06	1.58	2.58	2.11	1.65	2.15	2.11
D_Ship	1.05	0.52	0.43	1.26	2.95	0.46	1.01	2.99	0.46	1.00	2.12	0.46
Invent	2.38	1.65	1.18	1.2	3.86	0.85	1.28	4.21	0.95	1.43	2.48	0.94
Ret_Sales	7.87	0.08	0.06	2.75	0.77	0.07	2.55	1.58	0.06	2.61	2.64	0.06
Veh_Sales	1.07	1.67	1.55	2.85	2.36	1.20	1.47	2.49	1.23	1.63	2.39	1.23
Panel B: DM Predictive Accuracy Test Statistics - Benchmark is the RW Model												
CPI1	-1.51	-1.36	-2.26*	0.77	-3.81*	-2.62*	0.33	-4.67*	-2.57*	0.85	-3.53*	-1.64
CPI2	2.01*	-2.77*	-3.79*	-1.82	-3.53*	-1.11	-1.55	-3.05*	-0.97	-1.48	-3.16*	-0.62
M1	-4.2*	3.07*	5.29*	-3.85*	-5.23*	4.11*	-8.33*	-4.39*	4.07*	-8.22*	-4.46*	4.08*
M2	-7.51*	0.73	1.64	-6.01*	-5.19*	3.07*	-2.16*	-3.95*	2.68*	-1.89	-4.32*	3.00*
H_Start	-4.15*	0.37	5.8*	-5.26*	-2.81*	6.04*	-5.68*	-5.78*	6.04*	-5.75*	-3.75*	6.04*
IP	-5.12*	3.87*	2.33*	-6.24*	-5.1*	3.42*	-7.95*	-4.82*	3.73*	-8.35*	-5.18*	3.73*
NonF_Emp	-3.96*	7.37*	6.65*	-6.36*	-7.96*	6.88*	-6.16*	-5.14*	6.85*	-6.08*	-7.18*	6.82*
IP_Auto	0.89	4.37*	-5.63*	-4.78*	-3.22*	-5.51*	-5.83*	-6.63*	-5.59*	-5.23*	-4.6*	-5.59*
IP_Dur	2.29*	4.68*	-3.75*	-3.04*	-5.16*	-2.89*	-6.86*	-7.87*	-3.10*	-6.65*	-5.34*	-3.11*
IP_NDur	-9.32*	6.09*	-4.81*	-4.99*	-4.02*	-4.63*	-4.45*	-4.79*	-4.78*	-6.03*	-5.34*	-4.79*
D_Ship	-2.98*	4.45*	6.17*	-5.19*	-5.25*	6.82*	-1.82	-8.5*	6.82*	0.00	-6.03*	6.82*
Invent	-3.48*	-0.89	-1.83	-2.3*	-2.46*	2.48*	-2.09*	-1.98*	1.41	-2.09*	-2.25*	1.55
Ret_Sales	-2.62*	4.52*	4.59*	-4.07*	1.95	4.6*	-8.13*	-5.29*	4.61*	-8.19*	-5.28*	4.61*
Veh_Sales	-2.85*	-2.89*	-2.89*	-4.09*	-2.61*	-2.16*	-4.92*	-2.57*	-2.36*	-5.87*	-3.36*	-2.36*

$^{(*)}$ MSFEs and predictive accuracy test statistics based on MSFE loss are reported in the two panels of this table. All DM tests are pairwise, and compare the benchmark RW model with the model denoted in the column header to the table. Negative values for DM statistics indicate that the point MSFE associated with the benchmark model is lower than that for the other model. Starred DM test statistics indicate rejection of the predictive accuracy null using 5% nominal size critical values (see Section 2 for further details). Prediction models are constructed recursively, starting with $R = 120\text{-}h\text{+}1$ observations, and ending with $R = T - h$ observations, where T is the sample size, and h is the forecast horizon (set equal to 1, 3, 12, and 60 months ahead). For variable definitions, refer to Table 1.

Table: As Table 9, but restricts predictions models to have at least one lag

	1-Step			3-Step			12-Step			60-Step		
Series	DS	SUR	PAR	DS	SUR	PAR	DS	SUR	PAR	DS	SUR	PAR
MSFE relative to RW												
CPI1	1.05	1.18	1.35	3.10	2.46	1.27	7.62	2.38	1.25	9.12	1.76	1.10
CPI2	0.88	1.68	2.37	1.07	3.28	1.04	1.03	3.24	1.03	1.04	2.23	1.02
M1	1.22	0.53	0.57	1.07	3.43	0.45	2.76	3.89	0.45	2.67	2.36	0.45
M2	1.97	0.88	0.85	1.57	5.69	0.75	9.05	5.61	0.78	9.55	2.53	0.76
H_Start	1.57	0.98	0.46	5.49	1.58	0.43	2.00	2.5	0.43	2.05	1.57	0.43
IP	1.26	0.53	0.79	1.27	3.92	0.67	1.91	3.34	0.65	2.00	2.98	0.65
NonF_Emp	9.69	0.14	0.55	1.09	3.77	0.36	1.37	3.53	0.36	1.47	2.08	0.37
IP_Auto	0.98	0.39	2.94	1.05	1.66	3.65	1.06	2.13	3.67	1.10	2.38	3.67
IP_Dur	0.94	0.36	1.55	1.01	1.97	1.42	1.45	2.31	1.45	1.41	2.21	1.45
IP_NDur	2.2	0.52	2.5	1.37	1.97	2.06	1.58	2.47	2.11	1.65	2.11	2.11
D_Ship	1.04	0.53	0.43	1.33	3.08	0.46	1.02	3.12	0.46	1.00	2.13	0.46
Invent	2.38	1.65	1.18	1.2	3.86	0.85	1.28	4.21	0.95	1.43	2.48	0.94
Ret_Sales	7.87	0.08	0.06	2.75	0.77	0.07	2.55	1.58	0.06	2.61	2.64	0.06
Veh_Sales	1.02	1.63	1.55	2.44	2.28	1.20	1.22	2.39	1.23	1.31	2.31	1.23
DM Test, Benchmark Model: *RW*												
CPI1	-0.49	-1.42	-2.26*	-4.37*	-3.67*	-2.62*	-2.00*	-4.63*	-2.57*	-1.20	-3.60*	-1.64
CPI2	2.01*	-2.77*	-3.79*	-1.82	-3.53*	-1.11	-1.55	-3.05*	-0.97	-1.48	-3.16*	-0.62
M1	-5.33*	3.05*	5.29*	-3.47*	-5.22*	4.11*	-8.44*	-4.39*	4.07*	-8.31*	-4.43*	4.08*
M2	-7.51*	0.73	1.64	-6.01*	-5.19*	3.07*	-2.16*	-3.95*	2.68*	-1.89	-4.32*	3.00*
H_Start	-4.15*	0.37	5.80*	-5.26*	-2.81*	6.04*	-5.68*	-5.78*	6.04*	-5.75*	-3.75*	6.04*
IP	-5.12*	4.16*	2.33*	-6.24*	-5.12*	3.42*	-7.95*	-4.81*	3.73*	-8.35*	-5.18*	3.73*
NonF_Emp	-3.96*	7.37*	6.65*	-6.36*	-7.96*	6.88*	-6.16*	-5.14*	6.85*	-6.08*	-7.19*	6.82*
IP_Auto	0.89	4.37*	-5.63*	-4.78*	-3.22*	-5.51*	-5.83*	-6.63*	-5.59*	-5.23*	-4.6*	-5.59*
IP_Dur	2.29*	4.83*	-3.75*	-3.04*	-5.33*	-2.89*	-6.86*	-4.87*	-3.10*	-6.65*	-5.34*	-3.11*
IP_NDur	-9.32*	6.71*	-4.81*	-4.99*	-3.90*	-4.63*	-4.45*	-4.79*	-4.78*	-6.03*	-5.29*	-4.79*
D_Ship	-2.06*	4.42*	6.17*	-5.65*	-5.28*	6.82*	-2.19*	-8.60*	6.82*	-0.46	-5.99*	6.82*
Invent	-3.48*	-0.89	-1.83	-2.30*	-2.46*	2.48*	-2.09*	-1.98*	1.41	-2.09*	-2.25*	1.55
Ret_Sales	-2.62*	4.52*	4.59*	-4.07*	1.95	4.60*	-8.13*	-5.29*	4.61*	-8.19*	-5.28*	4.61*
Veh_Sales	-0.41	-2.90*	-2.89*	-4.37*	-2.78*	-2.16*	-3.31*	-2.78*	-2.36*	-4.49*	-3.29*	-2.36*