Methods for Pastcasting, Nowcasting and Forecasting Using Factor-MIDAS*

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Abstract

We provide a synthesis of methods used for mixed frequency factor-MIDAS, when pastcasting, nowcasting, and forecasting, using real-time data. We also introduce a new real-time Korean GDP dataset Based on a series of prediction experiments, we find that: (i) Factor-MIDAS models outperform various linear benchmark models. Interestingly, 'MSFE-best' MIDAS models contain no AR lag terms when pastcasting and nowcasting, and are only useful for 'true' forecasting. (ii) Models that utilize only 1 or 2 factors are 'MSFE-best' at all forecasting horizons, but not at any pastcasting and nowcasting horizons. (iii) Real-time data are crucial for forecasting Korean GDP, and the use of 'first available' versus 'most recent' data 'strongly' affects model selection and performance. (iv) Recursively estimated models based on autoregressive interpolationare are almost always 'MSFE-best'. (v) Factors estimated using recursive principal component estimation methods have more predictive content than those estimated using other approaches, particularly when estimating factor-MIDAS models.

Keywords: nowcasting, forecasting, factor model, MIDAS.

JEL Classification: C53, G17.

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1 Introduction

In this paper, we begin by introducing a new real-time Korean GDP dataset. We then utilize this dataset, along with a larger monthly dataset including 190 variables, to pastcast, nowcast, and forecast GDP. Our prediction models combine the mixed data sampling (MIDAS) framework of Ghysels et al. (2004), that allows for the incorporation of variables of differing frequencies, with the diffusion index framework of Stock and Watson (2002). Broadly speaking, our primary objective is the synthesis of real-time data methods, mixed frequency modeling methods, and principal components analysis (PCA).

In order to motivate our need for three different prediction models, suppose that the objective is to predict GDP for 2016:Q2, using a simple autoregressive model of order one, say. In a conventional setting where real-time data are not available, it is assumed that information up to 2016:Q1 is available at the time the prediction is made, so that $\widehat{\text{GDP}}_{2016:\text{Q2}} = \hat{\alpha} + \hat{\beta}\text{GDP}_{2016:\text{Q1}}$, where $\hat{\alpha}$ and $\hat{\beta}$ are parameters estimated using maximum likelihood based on recursive or rolling data windows. In a real-time context, however, this prediction is not feasible. Namely, if the prediction is to be made in April or even May of 2016, then $\text{GDP}_{2016:\text{Q1}}$ is not yet available, even in preliminary release. This issue leads to the convention of defining three different types of predictions, including pastcasts (predicting past observations, which are not yet available in real-time), nowcasts (predicting concurrent observations), and forecasts. One advantage of carefully analyzing the data structure used in the formulation of prediction models is that we are able to simulate real-time decision making processes. Girardi et al. (2016) provides an excellent overview of this literature, within the context of nowcasting Euro area GDP in pseudo real-time using dimension reduction techniques.

Various key macroeconomic indicators in many countries, including Korea, are published with considerable delay and at low frequency. One such example is Korean Gross Domestic Product (GDP), which is a component of the so-called system of national accounts (SNA), and has been published quarterly by the Bank of Korea since 1955. These GDP data are 'real-time', in the sense that they are regularly updated and revised. For example, the base year of SNA data is updated every 5 years. Additionally, since the first GDP release in the 1950s, there have been 11 definitional changes affecting the entire historical record. Finally, since 2005, 'first vintage' or first release real GDP has been regularly announced about 28 days after the end of the corresponding calendar quarter. Second vintage data is generally released about 70 days after the end of the quarter (at which time nominal GDP is also released). In approximate conjunction with this second release, the whole prior year of data is also revised and released. Finally, another revision is made approximately 15 months later.

There are several approaches to forecasting lower frequency variables using higher frequency variables. The first approach involves use of the so-called 'bridge' model, which aggregates higher frequency variables with lower frequency variables, such as GDP. This aggregation is called a 'bridge', and this method is commonly used by central banks, since implementation and interpretation is straightforward (see e.g., Rünstler and Sédillot (2003), Golinelli and Parigi (2005) and Zheng and Rossiter (2006)). Indeed, this approach offers a very convenient solution for filtering, or aggregating, variables characterized by different frequencies. However, aggregation may lead to the loss of useful information. This issue has led to the recent development of alternative mixed frequency modeling approaches. One important approach, which is mentioned above, is called MIDAS. This approach involves the use of a regression framework that direct includes variables sampled at different frequencies. Broadly speaking, MIDAS regression offers a parsimonious means by which lags of explanatory variables of differing frequencies can be utilized; and its use for macroeconomic forecasting is succinctly elucidated by Clements and Galvao (2008). Additional recent papers in this area of forecasting include Kuzin et al. (2011), who predict Euro area GDP, Ferrara and Marsilli (2013) who predict French GDP, and Pettenuzzo et al. (2014), who discuss Bayesian implementation of MIDAS. One interesting feature of MIDAS is that the technique readily allows for the inclusion of diffusion indices. For discussion of the combination of factor and MIDAS approaches, see Marcellino and Schumacher (2010), and Section 5 of this paper. For an interesting application to the prediction of German GDP, see Schumacher (2007).

In our forecasting experiments, we implement principal components in order to extract diffusion indices. These diffusion indices are constructed using non real-time monthly data. Hence, in order to retain the real-time feature of our experiments, only suitably lagged 'factors' are used in the construction of forecasting models. For related evidence on the usefulness of factors thus constructed, see Stock and Watson (2012), Boivin and Ng (2005) and Kim and Swanson (2014). A final issue, in the context of real-time prediction, concerns the staggered availability of variables that are published at the same frequency. For example, some of the predictor variables that we use are not available, even in the middle of the current month, while others are. This type of missing data leads to the so-called 'ragged-edge' type problem. In this paper, we tackle this issue following Wallis (1986) and Marcellino and Schumacher (2010), and estimate monthly common factors using PCA coupled with either vertical data realignment, AR data interpolation, EM algorithm based missing value estimation, or a standard state space model. MIDAS prediction models are then implemented, yielding 'factor-MIDAS' predictions that are available at a monthly frequency for our quarterly GDP target variable.

Our findings can be summarized as follows. First and foremost, real-time data makes a difference. The utilization of real-time data in a recursive estimation framework, coupled with MIDAS, leads to the 'MSFE-best' predictions in our experiments. This finding is due in large part to the fact that many important economic indicators, such as CPI and Industrial Production are sampled at monthly or higher frequencies, and are useful for real-time GDP prediction at both monthly and quarterly frequencies. Indeed, when using real-time data, factor-MIDAS prediction models outperform various linear benchmark models. Interestingly, our 'MSFE-best' MIDAS models contain no AR lag terms when pastcasting and nowcasting. AR terms only begin to play a role in 'true' forecasting contexts.

Second, models that utilize only 1 or 2 factors are 'MSFE-best' at all forecasting horizons, but not at any pastcasting and nowcasting horizons. In these latter contexts, much more heavily parameterized models with many factors are preferred. In particular, while 1 or 2 factors are selected around 1/2 of the time in the cases, 5 or 6 factors are also selected around 1/2 of the time. Interestingly, there is little evidence that using an intermediate number of factors is useful. One should either specify very parsimonious 1 or 2 factor models, or one should go with our maximum of 5 or 6 factors. In summary, forecast horizon matters, in the sense that when uncertainty is most prevalent (i.e., longer forecast horizons), then parsimony 'wins' and 'MSFE-best' models utilize only 1 or 2 factors. The reverse holds as the forecast horizon reduces and instead nowcasts and pastcasts are constructed. This finding is quite sensible, given the vast literature indicating that more parsimonious models are usually preferred, particularly when forecasting at longer horizons.

Third, the variable being predicted makes a difference. For Korean GDP, the use of 'first available' versus 'most recent' data 'strongly' affects model selection and performance. One reason for this is that 'first available' data are never revised, and can thus in many cases be viewed as 'noisy' versions of later releases of observations for the same calendar date. This is particularly true if rationality holds (see, e.g. Swanson and van Dijk (2006)). Interestingly, when predictions are constructed using only 'first available' data, and when predictive accuracy is correspondingly carried out with 'first available' data, factor-MIDAS models without AR terms as well as other benchmark models do not work well, regardless of the number of factors specified. In these cases, pure autoregressive models dominate, in terms of MSFE. This suggests that for short forecast horizons, the persistence of Korean GDP growth is strong, and well modeled using linear AR components. Indeed, in many of these cases, our simplest linear AR models are 'MSFE-best'. As the forecast horizon gets longer, simple linear models are no longer 'MSFE-best', and models without AR terms in some cases outperform models with AR terms. This suggests that uncertainty in autoregressive parameters does not carry over to other model parameters, as the horizon increases, and the role for MIDAS thus increases in importance. However, when 'most recent' real-time data are used exclusively in our prediction experiments, MIDAS models dominate at all forecast horizons, as mentioned above, and autoregressive lags of GDP are only useful at longer forecast horizons (of at least 6 months). Given that 'most recent' data are those that are most often used by empirical researchers, we thus have direct empirical evidence of the usefulness of factor-MIDAS coupled with real-time data.

Fourth, recursively estimated models are almost always 'MSFE-best', and models estimated using autoregressive interpolation dominate those estimated using other interpolation methods. In particular, models estimated using rolling data windows are only 'MSFE-best' at 3 forecast horizons, when using 'first available' data, and are never 'MSFE-best' when 'most recent' data are used. Also, when comparing MSFEs, only approximately 10% of models perform best when using vertical alignment or VA interpolation, with 90% favoring autoregressive or AR interpolation.

Fifth, factors constructed using recursive principal component estimation methods have more predictive content than those estimated using a variety of other (more sophisticated) approaches. This result is particularly prevalent for our 'MSFE-best' factor-MIDAS models, across virtually all forecast horizons, estimation schemes, and data vintages that are analyzed.

In summary, this paper introduces a new real-time dataset, offers a first look at the issue of pastcasting, nowcasting, and forecasting real-time Korean GDP, and is meant to add to the burgeoning literature on the usefulness of MIDAS, diffusion indices, and real-time data for prediction. Future research questions include the following: Are robust shrinkage methods such as the lasso and elastic net useful in the context of real-time prediction, and can the methods discussed herein be modified to utilize these sorts of machine learning and shrinkage techniques? Can predictions be improved by utilizing even higher frequency data than those used here, including high frequency financial data? In the context of high frequency data, are measures of risk such as so-called realized volatility useful as predictors? Finally, are alternative "sparse" diffusion index methodologies, such as sparse principal components analysis and independent component analysis useful in real-time prediction (see, e.g. Kim and Swanson (2016))? The rest of the paper is organized as follows. Our real-time Korean GDP dataset is introduced in Section 2. Section 3 briefly describes how to estimate common factors using recursive and non-recursive PCA methods, and discusses approaches to addressing ragged-edge data. The MIDAS framework for pastcasting, nowcasting, and forecasting is discussed in Section 4. Finally, Section 5 presents the results of our forecasting experiments, and Section 6 concludes the paper.

2 Real-Time Data

2.1 Notation

When constructing real-time datasets, both the data vintage (which 'release' of data we are referring to, and when it was released) and the calendar date (the actual calendar date to which the data pertains) must be delineated. Figure 1 depicts this relationship for Korean GDP.

[Insert Figure 1 here]

Moreover, when constructing growth rates (e.g., log differences), data vintage is clearly relevant. It is thus important to carry forward a consistent and sensible notation, when using real-time data in model specification and estimation. Let Z be the level of a variable and z be the log difference thereof. Define:

$$z_t^{(1)} = \ln Z_t^{(1)} - \ln Z_{t-d}^{(1)}, \tag{1}$$

where $Z_t^{(1)}$ denotes the first release of Z_t, for calendar date t, and d denotes the difference taken (i.e., $d = 1$ for quarterly growth rates, and $d = 4$ for annual growth rates, when data are measured at a quarterly frequency). In practice, $z_t^{(1)}$ is not commonly used in empirical analysis, since, at calendar date t, a more recent release than $1st$ may be available for Z_{t-d}. If Z_{t-d} has already been revised once, then use of updated data may be preferred, leading to the following definition:

$$z_t^{(2)} = \ln Z_t^{(1)} - \ln Z_{t-1}^{(2)}. \tag{2}$$

For annual growth rates based on quarterly data, utilizing the latest available revision equates with constructing $z_t^{(3)} = \ln Z_t^{(1)} - \ln Z_{t-4}^{(3)}$. In summary, when we are at calendar date t, the latest observation available for date t is the first release.

In subsequent prediction experiments involving GDP, we update our forecasts at a monthly frequency, even though raw data are accumulated at only a quarterly frequency. It is thus necessary to specify monthly subscripts denoting data vintage. In particular, define:

$$_{t_m}Y_{t_q} = {_{t_m}\mathbf{Y}_{t_q}} - {_{t_m}\mathbf{Y}_{t_q-d}}, \tag{3}$$

where Y and $\mathbf{Y}$ denote the log level and the growth rates of a variable, say GDP, respectively. Here, when $d = 4$, $_{t_m}Y_{t_q}$ are annual growth rates. Suppose that t_m =2016:05 and t_q is 2016:Q1. In practice, we do not know the value of $\mathbf{Y}$ for 2016:Q2, as t_m = May 2016. In light of this, we redefine (3), taking into account the publication lag, k, as follows:

$$_{t_m}Y_{t_q} = {}_{t_m}\mathbf{Y}_{t_q-k} - {}_{t_m}\mathbf{Y}_{t_q-k-d}. \tag{4}$$

Therefore, the annual growth rate of GDP for 2016:Q1 in May 2016 is:

$$_{2014:05}Y_{2016:Q1} = {}_{2014:05}\mathbf{Y}_{2016:Q1} - {}_{2014:05}\mathbf{Y}_{2015:Q1}. \tag{5}$$

Now, let the data release be denoted by adding a superscript to the above expression, as follows:

$$_{2014:05}Y^{(3)}_{2016:Q1} = {}_{2014:05}\mathbf{Y}^{(1)}_{2016:Q1} - {}_{2014:05}\mathbf{Y}^{(3)}_{2015:Q1}, \tag{6}$$

where the superscript 3 corresponds to a third release or vintage of real-time data. Figure 2 depicts the construction of real-time GDP growth rates.

[Insert Figure 2 here]

Putting it all together, our real-time nomenclature for is:

$$_{t_m}Y^{(v)}_{t_q-d}, \tag{7}$$

where the sub- and super-scripts are defined above. Finally, and in order to simplify our notation, we redefine the superscript "v" so that it corresponds directly to the vintage of the growth rate, rather than the vintage of the raw data used in the construction of the growth rate. Namely, let $_{t_m}Y^{(1)}_{t_q-d}$ denote the first vintage growth rate of GDP, instead of (7). Thus, $_{t_m}Y^{(1)}_{t_q-d}$ is simply the first available growth rate of GDP for a particular calendar date, given data reporting agency release lags. Accordingly, $_{t_m+m}Y^{(i)}_{t_q-k}$ is the $i-$th vintage growth rate for calendar date $t_q - d$, at time $t_m + m$, where m is the feasible month for which $i-$ th vintage data are available. In the sequel, when the superscript for the vintage is omitted, we mean first vintage.

Given the above notation, we can specify forecast models using real-time data. Suppose that the objective is to predict h_q steps ahead at time t_m, using an AR(1) model. Then, the prediction model is:

$$_{t_m}Y_{t_q-d+h_q} = \alpha + \beta_{h_q} \cdot {}_{t_m}Y_{t_q-d} + \epsilon_{t_q}, \tag{8}$$

where ϵ_{t_q} is a stochastic noise term, α and β_{h_q} are coefficients estimated using maximum likelihood, and Y is defined as above. Here, vintage notation is omitted for brevity. Note that we forecast h_q periods ahead at time t_m (or t_q), but we do not have real-time information up to t_q. Therefore, the explanatory variable is lagged d quarters. Equation (8) is one of our benchmark forecasting models. Assume that we are at time t_m in the first month of the quarter, t_q. If there is a publication lag equal to 1 (i.e., $d = 1$), we 'pastcast' a value of Y, for time period $t_q - d$, 'nowcast' a value of Y, for time period t_q, and 'forecast' a value of Y, for time period $t_q + h_q$.

2.2 Korean real-time GDP

We have collected real-time Korean GDP beginning with the vintage available in January 2000. The calendar start date of our dataset is 1970:Q1, and data are collected through June 2014. As discussed in the introduction, first release GDP is announced 28 days after the end of the quarter, second GDP release is announced 70 days subsequent to the end of the quarter, and the third release is made available 50 days after a calendar year has passed. Finally, a fourth release is made available a full year later. These release dates have been fixed since 2005. Before then, release dates were relatively irregular, although the first release was usually around 60 days after the end of the quarter, and the second release was around 90 days after the end of the quarter. Even though GDP is finalized after approximately 2 years, there are several definitional changes, as well as regular base-year changes that subsequently affected our dataset. The revision history for Korean GDP is depicted in Figure 3. Panel (a) of the figure shows the growth rate of GDP by vintage. The plot denoted as '1st' is first release GDP, and so on. In Panel (b), revision errors are depicted. Plots denoted as '2nd', '12th' and '24th' all refer to differences relative to the first release. Prior to the 1990's, the differences were relatively large; with notable narrowing of these 'revision errors' more recently. It seems that along with the imposition of stricter release and announcement protocol, early releases have become more accurate. Panel (c) of Figure 3 depicts how GDP for certain calendar dates (i.e., 2001:Q1, 2003:Q1 and 2005:Q1) has evolved across releases. The GDP release dynamics observable in Panels (a), (b) and (c) is indicative of the fact that policy decision-making should carefully account for the real-time nature of GDP data. Panel (d) contains a histogram of first revision errors, which are the difference between first and second releases, over time. Interestingly, the first vintage is biased, as indicated by the asymmetric nature of the histogram. This suggests that the revision error history may be useful for prediction.

[Insert Figure 3 here]

Our monthly predictor dataset is not measured in real-time, as it was infeasible to construct a real-time dataset for the 190 variables utilized in our prediction experiments. The monthly data used are discussed in Kim (2013). These data have been categorized into 12 groups: interest rates, imports/exports, prices, money, exchange rates, orders received, inventories, housing, retail and manufacturing, employment, industrial production, and stocks. We extend this monthly dataset through June 2014 in the current paper. Moreover, all variables are transformed to stationarity, and the final dataset resembles quite closely the well-known Stock and Watson dataset, which has been extensively used to estimate common factors for the U.S. economy. For complete details, see Kim (2013).

3 Estimating Diffusion Indexes

We estimate common latent factors (i.e., diffusion indices) using the 190 monthly macroeconomic and financial variables discussed above. Thereafter, we utilize our estimated factors, along with various additional variables measured at multiple different frequencies, in MIDAS prediction regressions (see Section 5 for complete details). One conventional way to estimate common factors is via the use of PCA. In order to avoid computational burdens associated with matrix inversions, and in order to simulate a 'real-time' environment, we use a variant thereof, called recursive PCA, following Peddaneni et al. (2004). In this section, we discuss PCA and other key details associated with factor estimation, in our context.

3.1 Constructing factors using ragged-edge data

Since we model real-time GDP, it is critical to match monthly data availability with GDP release vintages. In particular, some of our monthly variables are not available at certain calendar dates even though new vintages of GDP have been released by said calendar dates. For example, the consumer price index for the previous month is released early in the current month, whereas the producer price index is released in the middle of the month. In between these releases, new vintages of GDP are often released. This is called a ragged-edge data problem. Denote our N-dimensional monthly dataset as X_{t_m}, where time index t_m denotes the monthly frequency. Assume that the monthly observations have the following factor structure:

$$X_{t_m} = \Lambda F_{t_m} + \xi_{t_m}, \tag{9}$$

where the r-dimensional factor vector is denoted by $F_{t_m} = \left(f'_{1,t_m}, \dots, f'_{r,t_m}\right)$, Λ is an $(N \times r)$ factor loading matrix, and $r << N$. Note that we do not have monthly indicators in real-time, so that there is no prefix subscript in 9. In this formulation, the common components of X_{t_m} consist of the diffusion indices, F_{t_m}. The idiosyncratic components, ξ_{t_m}, are that part of X_{t_m} not explained by the factors. Let data matrix X be a balanced one with dimension $T_m \times N$.

The most widely used methods for estimating F_{t_m} are based on static PCA, as in Stock and Watson (2002); and dynamic PCA, as in Forni et al. (2005). However, PCA is based on an eigenvalue/eigenvector decomposition of the covariance matrix of X_{t_m}, which requires inversion of this matrix. This means that the dataset must be 'completed' (i.e., not ragged). Therefore, we need to resolve the ragged-edge problem in order to obtain PCA estimators of the factors. In this paper, we use *vertical alignment* and *AR interpolation* for missing values. Another convenient way to solve the ragged-edge problem is proposed by Stock and Watson (2002), who use the EM algorithm together with standard PCA. Additionally, one can write the factor model in state-space form in order to handle missing values at the end of each variables' sample, following Doz et al. (2012).[1] Our approaches to the ragged-edge problem are the following:

Vertical alignment (VA) interpolation of missing data:

The simplest way to solve the ragged-edge problem is to directly balance any unbalanced datasets. In particular, assume that variable i is released with a k_i month publication lag. Thus, given a dataset in period T_m, the final observation available for variable i is for period $T_m - k_i$. The realignment proposed by Altissimo et al. (2010) is:

$$\tilde{X}_{i,t_m} = X_{i,t_m - k_i}, \quad \text{for} \quad t_m = k_i + 1, ..., T_m. \tag{10}$$

Applying this procedure for each series, and harmonizing at the beginning of the sample, yields a balanced dataset, $\tilde{X}_{t_m}$, for $t_m = \max\left(\{k_i\}_i = 1^N\right) + 1, ..., T_m$. Given this new dataset, PCA can be immediately implemented. Although easy to use, a disadvantage of this method is that the availability of data determines dynamic cross-correlations between variables. Furthermore, statistical release dates for each variable are not the same over time, for example, due to major revisions.

Autoregressive (AR) interpolation of missing data:

As an alternative to vertical alignment, we use univariate autoregressive models for individual monthly indicators, X_i. Namely, specify and estimate the following models:

$$X_{i,t} = \sum_{s=1}^{p_i} \rho_s X_{i,t-s} + u_{i,t}, \qquad i = 1, \ldots, k, \tag{11}$$

where p_i is the lag length, and is selected using the Schwarz Information Criterion (SIC), coefficients ρ are estimated using maximum likelihood, and $u_{i,t}$ is a white noise error term. This

[1] Doz et al. (2012) use the Kalman filter and smoother for estimation.

AR method depends only on the univariate characteristics of the variable in question, and not on the broader macroeconomic environment from within which the data are generated. However, it is very easy to implement and is an intuitive approach.

EM algorithm for estimating missing data:

The ragged-edge problem essentially concerns estimating missing values. Stock and Watson (2002) propose using the EM algorithm to replace missing values and subsequently carry out PCA. The EM algorithm is initialized with an estimate of the missing data, which is usually set equal to the unconditional mean (this is also the approach that we use). Then, the completed dataset is used to estimate factors using PCA. This algorithm is repeated in two steps, the E-step and the R-step. We briefly explain these steps, and the reader is refereed Schumacher and Breitung (2008) for details. Consider a dataset, X_{t_m}, and pick variable i, say $X_i = (x_{i,1}, ..., x_{i,t_m})'$. Suppose that variable i has missing values due to publication lags. Set $X_i^{obs} = P_i X_i$, where P_i represents the relationship between the full vectors and the ones with missing values. If no missing values are found, then P_i is the identity matrix. As we only observe a subset of X, initialize the EM algorithm by replacing missing values with the unconditional mean of X_i^{obs}, yielding initial estimates of factors and loadings (using PCA), say F^0 and Λ^0. Now iterate this procedure. In the j-th iteration, the E-step updates the estimates of the missing observations using the the expectation of the variable X_i conditional on X_i^{obs}, with factors and loadings from the $j-th$ iteration, F^{j-1} and Λ^{j-1}, as follows:

$$X_i^j = F^{j-1}\Lambda^{j-1} + P_i' \left(P_i' P_i\right)^{-1} \left(X_i^{obs} - P_i F^{j-1}\Lambda_i^{j-1}\right), \tag{12}$$

Run the E-step for all i, in each iteration. The M-step involves re-estimating the factors and loadings using ordinary PCA. Continue until convergence is achieved.

State-space model (Kalman filtering) for estimating missing data:

Another popular approach for estimating factors from large datasets is the state-space approach based on Doz et al. (2012) and Giannone et al. (2008). The factor model represented in state-space form is based on the (9), with factors represented using an autoregressive structure, as follows:

$$\Psi(L_m)F_{t_m} = \mathbf{A}\eta_{t_m}, \tag{13}$$

where $\Psi(L_m)$ is a lag polynomial, given by $\sum_{i=1}^{p} \Psi_i L_m^i$, and η_{t_m} is an orthogonal dynamic shock. The state-space can easily be estimated via maximum likelihood (ML). Doz et al. (2012) propose using quasi-ML for large datasets, when conventional ML is not feasible. In particular, as ML estimation involves initialization of factors based on the use of ordinary

PCA, one needs a completed data matrix. Marcellino and Schumacher (2010) remove missing values from the end of sample to make it balanced, and estimate initial factors using ordinary PCA. In our forecasting experiments, initial factors are extracted from the completed matrix that is completed using VA and AR interpolation. Then, likelihoods are calculated and evaluated using the Kalman filter. More specifically, given an initial set of factors, estimate loadings by regressing X_{t_m} on the factors. Then, obtain the covariance matrix of the idiosyncratic part from (9), $\sum_{\xi}$, where $\xi_{t_m} = X_{t_m} - \Lambda F_{t_m}$. Now, estimate a vector AR(p) on the factors, F_{t_m}, yielding coefficient matrix, $\Psi(L)$, and residual covariance matrix, $\sum_{\varsigma}$ where $\varsigma_{t_m} = \Psi(L_m)F_{t_m}$. Let V be the eigenvectors corresponding to E, where E is a diagonal matrix whose diagonal elements are the eigenvalues in descending order, and zero otherwise. Then, set $P = VE^{-1/2}$. As a final step, the Kalman smoother is used to yield new estimates of the factors.

3.2 Recursive principal component analysis (RPCA)

PCA is widely used to estimate factor or diffusion index models in large data environments (see. e.g. Kim and Swanson (2016) and the references cited therein). Moreover, PCA is quite convenient as it uses standard eigenvalue decompositions of data covariance matrices. However, these matrix operations that may time consuming in certain real-time environments. In light of this, RPCA has been proposed by Peddaneni et al. (2004), and is a natural approach to use in our context, as new data arrive in real-time and need to be incorporated into our prediction models. Also, suppose that F_{t_m} is estimated using PCA. Principal components (factors) in this context are linear combinations of variables that maximize the variance of the data, and there is no guarantee that factor loadings are stationary at each point in time, particularly with large datasets. For example, the factor loadings at times t and $t+1$ may have different signs. Recursive PCA attempts to address these issues, in part by not requiring the calculation the whole covariance matrix of data with the arrival of each new datum. Without loss of generality, consider a standardized random vector at time t, say x_t, with dimension n. Our aim is to find the principal components of x at time t. To begin, define the covariance (or correlation) matrix of x as:

$$\boldsymbol{R}_t = \frac{1}{t}\sum_{i=1}^{t} x_i x_i' = \frac{t-1}{t}\boldsymbol{R}_{t-1} + \frac{1}{t}x_t x_t'. \tag{14}$$

If $\boldsymbol{Q}$ and $\boldsymbol{\Lambda}$ are the orthonormal eigenvector and diagonal eigenvalue matrices of $\boldsymbol{R}$, respectively, then: $\boldsymbol{R}_t = \boldsymbol{Q}_t \boldsymbol{\Lambda_t} \boldsymbol{Q}_t'$ and $\boldsymbol{R}_{t-1} = \boldsymbol{Q}_{t-1} \boldsymbol{\Lambda}_{t-1} \boldsymbol{Q}_{t-1}'$. We can rewrite (14) as:

$$\boldsymbol{Q}_t \left(t \boldsymbol{\Lambda}_t\right) \boldsymbol{Q}_t' = x_t x_t' + (t-1)\, \boldsymbol{Q}_{t-1} \boldsymbol{\Lambda}_{t-1} \boldsymbol{Q}_{t-1}'. \tag{15}$$

If we let $\alpha_t = \boldsymbol{Q}_{t-1}' x_t$, (15) can be written as: $\boldsymbol{Q}_t \left(t \boldsymbol{\Lambda}_t\right) \boldsymbol{Q}_t' = \boldsymbol{Q}_{t-1} \left[(t-1)\, \boldsymbol{\Lambda}_{t-1} + \alpha_t \alpha_t'\right] \boldsymbol{Q}_{t-1}'$. If $\boldsymbol{V}_t$ and $\boldsymbol{D}_t$ are the orthonormal eigenvector and diagonal eigenvalue matrices of $(t-1)\, \boldsymbol{\Lambda}_{t-1} + \alpha_t \alpha_t'$, then:

$$(t-1)\, \boldsymbol{\Lambda}_{t-1} + \alpha_t \alpha_t' = \boldsymbol{V}_t \boldsymbol{D}_t \boldsymbol{V}_t'. \tag{16}$$

Therefore,

$$\boldsymbol{Q}_t \left(t \boldsymbol{\Lambda}_t\right) \boldsymbol{Q}_t' = \boldsymbol{Q}_{t-1} \boldsymbol{V}_t \boldsymbol{D}_t \boldsymbol{V}_t \boldsymbol{Q}_{t-1}'. \tag{17}$$

By comparing both sides of (17), the recursive eigenvector and eigenvalue update rules turn out to be $\boldsymbol{Q}_t = \boldsymbol{Q}_{t-1} \boldsymbol{V}_t$ and $\boldsymbol{\Lambda}_t = \boldsymbol{D}_t / t$. Now,it remains to estimate the eigenvectors and eigenvalues of $(t-1)\, \boldsymbol{\Lambda}_{t-1} + \alpha_t \alpha_t'$, which is equivalent to estimating $\boldsymbol{V}_t$ and $\boldsymbol{D}_t$. It is very difficult to analytically solve for $\boldsymbol{V}_t$ and $\boldsymbol{D}_t$, and so Peddaneni et al. (2004) instead use first order perturbation analysis. Consider the following sample perturbation to the eigenvalue matrix, $(t-1)\, \boldsymbol{\Lambda}_{t-1} + \alpha_t \alpha_t'$. When t is large, this matrix is essentially a diagonal matrix, which means that $\boldsymbol{D}_t$ will be close to $(t-1)\, \boldsymbol{\Lambda}_{t-1}$, and $\boldsymbol{V}_t$ will be close to the identity matrix, $\boldsymbol{I}$. The matrix $\alpha_t \alpha_t'$ is said to perturb the diagonal matrix $(t-1)\, \boldsymbol{\Lambda}_{t-1}$, and as a result, $\boldsymbol{D}_t = (t-1)\, \boldsymbol{\Lambda}_{t-1} + \boldsymbol{P}_\Lambda$ and $\boldsymbol{V}_t = \boldsymbol{I} + \boldsymbol{P}_V$, where $\boldsymbol{P}_\Lambda$ and $\boldsymbol{P}_V$ are small perturbation matrices. Once we find these perturbation matrices, we can solve the problem. Let $\boldsymbol{\Lambda} = (t-1)\, \boldsymbol{\Lambda}_{t-1}$. Then:

$$\begin{aligned}
\boldsymbol{V}_t \boldsymbol{D}_t \boldsymbol{V}_t' &= \left(\boldsymbol{I} + \boldsymbol{P}_V\right) \left(\boldsymbol{\Lambda} + \boldsymbol{P}_\Lambda\right) \left(\boldsymbol{I} + \boldsymbol{P}_V\right)' \\
&= \boldsymbol{\Lambda} + \boldsymbol{\Lambda} \boldsymbol{P}_V' + \boldsymbol{P}_\Lambda + \boldsymbol{P}_\Lambda \boldsymbol{P}_V' + \boldsymbol{P}_V \boldsymbol{\Lambda} + \boldsymbol{P}_V \boldsymbol{\Lambda} \boldsymbol{P}_V' + \boldsymbol{P}_V \boldsymbol{P}_\Lambda + \boldsymbol{P}_V \boldsymbol{P}_\Lambda \boldsymbol{P}_V' \\
&= \boldsymbol{\Lambda} + \boldsymbol{P}_\Lambda + \boldsymbol{D} \boldsymbol{P}_V' + \boldsymbol{P}_V \boldsymbol{D} + \boldsymbol{P}_V \boldsymbol{\Lambda} \boldsymbol{P}_V' + \boldsymbol{P}_V \boldsymbol{P}_\Lambda \boldsymbol{P}_V'
\end{aligned} \tag{18}$$

Substituting this equation into (16), and assuming that $\boldsymbol{P}_V \boldsymbol{\Lambda} \boldsymbol{P}_V'$ and $\boldsymbol{P}_V \boldsymbol{P}_\Lambda \boldsymbol{P}_V'$ are negligible, we have that: $\alpha_t \alpha_t' = \boldsymbol{P}_\Lambda + \boldsymbol{D} \boldsymbol{P}_V' + \boldsymbol{P}_V \boldsymbol{D}$. The fact that $\boldsymbol{V}$ is orthonormal yields an additional characterization of $\boldsymbol{P}_V$. Substituting $\boldsymbol{V} = \boldsymbol{I} + \boldsymbol{P}_V$ into $\boldsymbol{V}\boldsymbol{V}' = \boldsymbol{I}$, and assuming that $\boldsymbol{P}_V \boldsymbol{P}_V' \approx 0$, we have that $\boldsymbol{P}_V = -\boldsymbol{P}_V'$. Thus, combining the fact that the $\boldsymbol{P}_V$ is antisymmetric with the fact that $\boldsymbol{P}_\Lambda$, and $\boldsymbol{D}_t$ are diagonal, yields the following solution to our problem:

$$\alpha_i^2 = (i,i)^{th} \text{ element of } \boldsymbol{P}_\Lambda \tag{19}$$
$$\frac{\alpha_i\alpha_j}{\lambda_j+\alpha_j^2-\lambda_i-\alpha_i^2} = (i,j)^{th} \text{ element of } \boldsymbol{P}_V,\ i\neq j, \text{ and } 0=(i,i)^{th} \text{ element of } \boldsymbol{P}_V.$$

This leads to the following algorithm.

Algorithm: Recursive Principal Component Analysis
At time t, use the covariance matrix, $\mathbf{R}_{k-1}$, which is available for period $t-1$, and collect eigenvalues and eigenvectors into Λ_{t-1} and $\mathbf{Q}_{k-1}$, respectively. The following algorithm is implemented in real-time, as each new observation becomes available.

1. With each a new datum, x_t, calculate $\alpha_t = \mathbf{Q}'_{t-1}x_t$.
2. Use (19), to find the perturbation matrices, $\mathbf{P}_V$ and $\mathbf{P}_\Lambda$.
3. Estimate the eigenvector matrix, $\tilde{\mathbf{Q}}_t = \mathbf{Q}_{t-1}\left(I+\mathbf{P}_\Lambda\right)$.
4. Standardize $\tilde{\mathbf{Q}}_t$, using $\hat{\mathbf{Q}}_t = \tilde{\mathbf{Q}}_t\tilde{\mathbf{S}}_t$, where $\tilde{\mathbf{S}}_t$ is a diagonal matrix containing the inverse of the norms of each column of $\tilde{\mathbf{Q}}_t$.
5. Estimate the eigenvalue, $\hat{\Lambda}_t = \hat{\mathbf{Q}}'_t\mathbf{R}_t\hat{\mathbf{Q}}_t$.

In the sequel, we estimate factors from monthly indicators, and address the ragged-edge problem by introducing VA and AR interpolation, as well as via the use of factor estimation methods including the EM algorithm and the aforementioned state-space model. Additionally, RPCA is used in order to reduce computational issues associated with estimating factors using large and growing datasets. However, standard or ordinary PCA (called OPCA) is also used, for comparison purposes. These methods yield the factors used in our factor-MIDAS prediction models.

4 Pastcasting, Nowcasting, and Forecasting Using MIDAS

4.1 Factor-MIDAS

The MIDAS approach for forecasting with real-time data was developed by Clements and Galvao (2008, 2009). Building on their work, the factor-MIDAS approach utilized in the sequel was developed by Marcellino and Schumacher (2010). Note that factor-MIDAS is essentially conventional MIDAS augmented to include explanatory variables that are common factors extracted from higher frequency variables and datasets. More specifically, suppose that Y_{t_q} is sampled at a quarterly frequency. Let X_{t_m} be sampled at a higher frequency - for example, if it is sampled at a monthly frequency, then $m = 3$. The factor-MIDAS model for forecasting h_q quarters ahead is:

$$Y_{t_q+h_q} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) \hat{F}_{t_m}^{(3)} + \varepsilon_{t_q}, \tag{20}$$

where $B\left(L^{1/m}, \theta\right) = \sum_{j=0}^{j^{\max}} b(j, \theta) L^{j/m}$ is the exponential Almon lag with

$$b(j, \theta) = \frac{\exp\left(\theta_1 j + \theta_2 j^2\right)}{\sum_{j=0}^{j^{max}} \exp\left(\theta_1 j + \theta_2 j^2\right)}, \tag{21}$$

and with $\theta = (\theta_1, \theta_2)$. Here, $\hat{F}_{t_m}$ is a set of monthly factors estimated using one of the various approaches discussed in the previous section, $L^{j/m} X_t^{(m)} = X_{t-j/m}^{(m)}$, and $\hat{F}_{t_m}^{(3)}$ is skip sampled from the monthly factor vector, $\hat{F}_{t_m}$. That is, every third observation starting from the final one is included in the predictor, $\hat{F}_{t_m}^{(3)}$. In this formulation, all monthly factors are in the set of predictors, and are appropriately lagged. If we apply our real-time dataset structure in this framework, the model in (20) is:

$$_{t_m}Y_{t_q-d+h_q} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) {}_{t_m}F_{t_m}^{(3)} + \varepsilon_{t_q}, \tag{22}$$

and assuming that there are r factors, $F_{t_m,1}, F_{t_m,2}, ..., F_{t_m,r}$, we have that:

$$_{t_m}Y_{t_q-d+h_q} = \beta_0 + \sum_{i=1}^{r} \beta_{1,i} B_i\left(L^{1/m}, \theta_i\right) {}_{t_m}F_{t_m,i}^{(3)} + \varepsilon_{t_q-d+h_q}. \tag{23}$$

Since we do not have monthly real-time data and we interpolate missing values at the end of each monthly indicator, F_{t_m} always exists at time t_m. If we are in the first month of the quarter and the dependent variable from previous quarter is not available, we 'pastcast' the previous quarter's value, 'nowcast' the current quarter, and 'forecast' future quarters, as discussed above. For example, the pastcast of Y_{t_q-1} at time t_m, where t_m is the first month of the quarter is:

$$_{t_m}Y_{t_q-1} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) {}_{t_m}F_{t_m-1}^{(3)} + {}_{t_m}\varepsilon_{t_q-1}. \tag{24}$$

Note that $t_q - 1$ denotes the previous quarter and $t_m - 1$ denotes the previous month. The nowcast of Y_{t_q} at time t_m, where t_m is the first month of the quarter is:

$$_{t_m}Y_{t_q} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) {}_{t_m}F_{t_m}^{(3)} + {}_{t_m}\varepsilon_{t_q}, \tag{25}$$

and for the second month of the quarter, the nowcast is:

$$_{t_m+1}Y_{t_q} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) {}_{t_m+1}F_{t_m+1}^{(3)} + {}_{t_m+1}\varepsilon_{t_q}. \tag{26}$$

Now, define the h_q-ahead forecast at time t_m as follows:

$$_{t_m}Y_{t_q+h_q} = \beta_0 + \beta_1 B\left(L^{1/m}, \theta\right) {}_{t_m}F_{t_m}^{(3)} + {}_{t_m}\varepsilon_{t_q+h_q}. \tag{27}$$

Finally, Clements and Galvao (2008) extend MIDAS by adding autoregressive (AR) terms, yielding models of the following variety:

$$_{t_m}Y_{t_q-d+h_q} = \beta_0 + ðY_{t_q-d} + \sum_{i=1}^{r} \beta_{1,i} B_i\left(L^{1/m}, \theta_i\right) {}_{t_m}F_{t_m,i}^{(3)} + \varepsilon_{t_q-d+h_q}. \tag{28}$$

All of the above models are analyzed in our forecasting experiments.

In closing this section, it should be noted that, according to Ghysels et al. (2004) and Andreou et al. (2010), given θ_1 and θ_2, the exponential lag function, $B(L^{1/m}, \theta)$, provides a parsimonious estimate that can proxy for monthly lags of the factors, as long as j is sufficiently large. It remains how to estimate θ and β. Marcellino and Schumacher (2010) suggest using nonlinear least squares (NLS), yielding coefficients, $\hat{\theta}$ and $\hat{\beta}$. In our experiments, all coefficients are estimated using NLS, except in cases where least squares can directly be applied.

4.2 Other MIDAS specifications

Marcellino and Schumacher (2010) utilize two different MIDAS specifications, including smoothed MIDAS, which is a restricted form of the above MIDAS model with different weights on monthly indicators, and unrestricted MIDAS, which relaxes restrictions on the lag polynomial used. These MIDAS models are explained in the context of the models we implement, as given in equations (25).

Smoothed MIDAS

Altissimo et al. (2010) propose a new Eurocoin Index, an indicator of economic activity in real-time. The index is based on a method to obtain a smoothed stationary time series from a large data set. Their index and methodology builds on that discussed in Marcellino and Schumacher (2010), and is used to nowcast and forecast German GDP. In particular, their model can be written as:

$$
\begin{aligned}
{}_{t_m}Y_{t_q-d+h_q} &= \hat{\mu}_Y + \mathbf{G}\hat{F}_{t_m}, \quad \text{and} && (29)\\
\mathbf{G} &= \tilde{\Sigma}_{Y,F}(h_m) \times \hat{\Sigma}_F^{-1}, && (30)
\end{aligned}
$$

where $\hat{\mu}_Y$ is the sample mean of GDP, assuming that the factors are standardized, and $\mathbf{G}$ is a projection coefficient matrix. Here, $\hat{\Sigma}_F$ is the estimated sample covariance of the factors, and $\tilde{\Sigma}_{Y,F}(j)$ is a particular cross-covariance with j monthly lags between GDP and the factors, defined as follows:

$$
\tilde{\sum}_{Y,F}(j) = \frac{1}{t^* - 1} \sum_{m=M+1}^{t_m} {}_mY_{t_q} \hat{F}_{m-j}^{(3)\prime}, \tag{31}
$$

where $t^* = \text{floor}\left[(t_m - (M+1)/3)\right]$ is the number of observations available to compute the cross covariance, for $j = -M, ..., M$; and $M \geq 3h_q = h_m$, under the assumption that both GDP and the factors are demeaned. Note that $h_m = 3 \cdot h_q$. Complete computational details are given in Altissimo et al. (2010) and Marcellino and Schumacher (2010). This so-called 'smoothed MIDAS' is a restricted form of the MIDAS model given in (20), with a different lag structure.

Unrestricted MIDAS

Another alternative version of MIDAS involves using an unrestricted lag polynomial when weighting the explanatory variables (i.e. the factors). Namely, let:

$$
{}_{t_m}Y_{t_q-d+h_q} = \beta_0 + \mathbf{C}(L_m)\hat{F}_{t_m}^{(3)} + \varepsilon_{t_q-k+h_q}, \tag{32}
$$

where $\mathbf{C}(L_m) = \sum_{j=0}^{j^{max}} \mathbf{C}_j L_m^j$ is an unrestricted lag polynomial of order j. Koenig et al. (2003) propose a similar model in the context of forecasting with real-time data, but not with factors. Marcellino and Schumacher (2010) provide a theoretical justification for this model and derive MIDAS as an approximation to a forecast equation from a high-frequency factor model in the presence of mixed sampling frequencies. Here, $\mathbf{C}(L_m)$ and β_0 are estimated using least squares. Lag order specification in our forecasting experiments is done in two different ways. When using a fixed scheme where $j = 0$, automatic lag length selection is carried out using the SIC. Alternatively, if $d = 0$, our model only uses t_m dated factors in forecasting.

5 Empirical Results

5.1 Benchmark models and experimental setup

In addition to the MIDAS models discussed above, we specify and estimate a number of benchmark models, when forecasting real-time GDP. These include:

- *Autoregressive Model:* We pastcast, nowcast and forecast GDP growth rates, ${}_{t_m}\hat{Y}_{t_q+h_q-d}$, h_q-steps ahead, using autoregressions with p lags, where p is selected using the SIC. Note that our AR model does not use monthly indicators; but since lagged GDP, as well as revised GDP, are available at various dates throughout the quarter, we still update our predictions monthly. The model is:

$$ {}_{t_m}\widehat{Y}_{t_q+h_q-d} = \hat{\beta}_0 + \hat{\beta}_1 \cdot {}_{t_m}Y_{t_q-d-1} + \ldots + \hat{\beta}_p \cdot {}_{t_m}Y_{t_q-d-p} \tag{33} $$

- *Random Walk Model:* We implement a standard random walk model, in which the growth rate is assumed to be constant, although this constant value is re-estimated recursively, at each point in time.

- *Combined Bivariate Autoregressive Distributed Lag (CBADL) Model:* We use the so-called bridge equation, since it is widely used to forecast quarterly GDP using monthly data (see, e.g. Baffigi et al. (2004) and Barhoumi et al. (2008)), particularly at central banks. The CBADL model, which is a standard bridge equation, uses monthly indicators as regressors to predict GDP. Forecasts are constructed using a three step procedure, as follows:

  Step 1 - Construct forecasts of all N monthly explanatory variables, where m and q are selected using the SIC. Namely, specify and estimate: $X_{i,t_m} = \rho_1 X_{i,t_m-1} + \cdots \rho_m X_{i,t_m-m} + \zeta_{i,s}$, for all $i = 1, ..., N$.

  Step 2 - Use lagged values of GDP as well as predictions of each individual monthly explanatory variable, order to obtain N alternative quarterly forecasts of GDP. Namely, specify and estimate:

  ${}_{t_m}Y_{i,t_q-d+h_q} = \mu_Y + \gamma_1 Y_{t_q-d-1} + \cdots + \gamma_{q_y} Y_{t_q-d-q_y} + \beta_{i,0}\widehat{X}_{i,t} + \cdots + \beta_{i,q_x}\widehat{X}_{i,t-q_x} + \upsilon_{i,t_q-d+h_q}$.

  Step 3 - Construct a weighted average of the above predictions. Namely:

  ${}_{t_m}\hat{Y}^{CBADL}_{t_q-d+h_q} = \dfrac{1}{N}\sum_{i=1}^{N} {}_{t_m}\hat{Y}_{i,t_q-d+h_q}$.[2]

[2] Stock and Watson (2012) and Kim and Swanson (2016) implement a version of this model.

- *Bridge Equation with Exogenous Variables (BEX) :* This method is identical to the above CBADL model except that the model in Step 2 is replaced with:

$$_{t_m}Y_{i,t_q-d+h_q} = \mu_Y + \beta_{i,0}\widehat{X}_{i,t} + \cdots + \beta_{i,q_x}\widehat{X}_{i,t-q_x} + \upsilon_{i,t_q-d+h_q}. \tag{34}$$

Note that the real-time nature of our experiments is carefully maintained when specifying and estimating these models. Additionally, in all experiments, prediction model estimation is carried out using both recursive and rolling data windows, with the rolling window length set equal to 8 years (i.e., 32 periods of quarterly GDP and 96 monthly observations). All recursive estimations begin with 8 years of data, with windows increasing in length prior to the construction of each new real-time forecast. Out-of-sample forecast performance is evaluated using predictions beginning in 2000:Q1 and ending in 2013:Q4, and for each quarter, three monthly predictions are made. Figure 4 depicts the monthly/quarterly structure of our prediction experiments.

[Insert Figure 4 here]

Table 1 summarizes the forecast models and estimation methods used. In this table, AR, CBADL, and BEX denote the benchmark models, which do not use any factors, and are our alternatives to MIDAS. The two interpolation methods discussed above (i.e., AR and VA interpolation) for addressing the ragged-edge problem are used when estimating factors via implementation of OPCA and RPCA. In addition, the EM algorithm and Kalman Filtering (KF) are used to estimate factors, without interpolation. Once factors are estimated, they are plugged into five different varieties of MIDAS regression model, including: Basic MIDAS w/o AR terms, Basic MIDAS w/ AR terms, Smooth MIDAS, Unrestricted MIDAS w/o AR terms, and Unrestricted MIDAS w/ AR terms. This setup is summarized in Table 1.

[Insert Table 1 here]

In order to assess predictive performance, we construct mean square forecast errors (MSFEs). In conventional datasets that do not contain real-time data, MSFE statistics can be constructed by simply comparing forecasts with actual values of GDP. In the current context, we have two issues. First, we can estimate our forecasting models, in real-time, using only first available data. This is one case considered, and is referred to as our 'first available' case. In this case, when constructing MSFEs, we compare predictions with first available GDP. Second, we can estimate our forecasting models using currently available data, at each point in time. When using currently available data, the most recent observations in any given dataset have undergone the least revision, while the most distant observations have potentially been revised many times. This is the second case considered, and is referred to as our 'most recent' case. In the second case, when constructing MSFEs, we compare predictions with the most recently available (and fully revised or 'final') GDP observations. The second case is closest to that implemented by practitioners that wish to use as much information as possible when constructing forecasts, and in this case, given that Korean GDP is fully revised after 2 years, which corresponds to the 5^{th} vintage, we compare forecasts with actual data defined as ${}_{t_m}Y^{(5)}_{t_q-d}$. In general, the MSFE of the $i-$th model for h_q-step ahead forecasts is defined as follows:

$$MSFE^{(j)}_{i,h_q} = \sum_{t=R-h_q+2}^{T_q-h_q+1} \left({}_{t_m+3(h_q-d)+s}Y^{(j)}_{t_q-d+h_q} - {}_{t_m}\hat{Y}_{i,t_q-d+h_q} \right)^2, \quad j = 1, ..., 5, \tag{35}$$

where $R - h_q + 2$ is the in-sample period, $T_q - h_q + 1$ denotes the total number of observations, ${}_{t_m+3(h_q-d)+s}Y^{(j)}_{t_q-d+h_q}$ is the observed value of the GDP growth rate, for calendar date $t_q - d + h_q$ when it is available, so that s denotes the smallest integer value needed in order to ensure availability of actual GDP growth rate data, $Y^{(j)}_{t_q-d+h_q}$ in real-time, and ${}_{t_m}\hat{Y}_{i,t_q-d+h_q}$ is the predicted value at $t_q - d + h_q$, for the $i-$th model. For example, we forecast the GDP growth rate in 2015:Q1 at 2014:04, called ${}_{2014:04}\hat{Y}_{2015:Q1}$, and the first calendar date at which time we can observe data for 2015:Q1 is May 2015, i.e. ${}_{2015:05}\hat{Y}^{(1)}_{2015:Q1}$. As discussed above, we evaluate model performance using 'first available', and 'most recent' data. In practice, we construct $MSFE^{(first)}_{i,h_q}$ and $MSFE^{(final)}_{i,h_q}$, respectively.

Our strawman model for carrying out statistical inference using MSFEs is the autoregressive model, and said inference is conducted using the Diebold and Mariano (1995) test (hereafter, the DM test). The null hypothesis of the DM test is that two models perform equally, when comparing squared prediction loss. Namely, we test:

$$H_0 : E\left[l\left(\varepsilon^{AR}_{t+h|t}\right)\right] - E\left[l\left(\varepsilon^{i}_{t+h|t}\right)\right] = 0, \tag{36}$$

where $\varepsilon^{AR}_{t+h|t}$ is the prediction error associated with the strawman autoregressive model, $\varepsilon^{i}_{t+h|t}$ is the prediction error of the $i-$th alternative model, and $l(\cdot)$ is the quadratic loss function. If a DM statistic under the null hypothesis is negative and significantly different from zero, then we have evidence that model i outperforms the strawman model. The DM statistic is $DM = \frac{1}{P}\sum_{i=1}^{P}\frac{d_t}{\hat{\sigma}_{\bar{d}}}$, where $d_t = \left(\widehat{\varepsilon^{AR}_{t+h|t}}\right)^2 - \left(\widehat{\varepsilon^{i}_{t+h|t}}\right)^2$, $\bar{d}$ is the mean of d_t, $\hat{\sigma}_{\bar{d}}$ is a heteroskedasticity and autocorrelation robust estimator of the standard deviation of $\bar{d}$, and $\widehat{\varepsilon^{AR}_{t+h|t}}$ and $\widehat{\varepsilon^{i}_{t+h|t}}$ are the estimated prediction errors corresponding to $\varepsilon^{AR}_{t+h|t}$ and $\varepsilon^{i}_{t+h|t}$, respectively.

5.2 Experimental findings

There are a number of methodological as well as empirical conclusions that emerge upon examination of the results from our forecasting experiments. Prior to listing these findings, however, it is useful to recall the structure of our experiments. In particular, recall that we construct pastcasts, nowcasts, and forecasts. Each of these are truly real-time, and they differ only in the timing of the predictions, relative to currently available data. To be specific, recall that in following our above notational setup, we construct three types of MSFEs. Consider construction of MSFEs using 'first available' data as the 'actual data' against which predictions are compared.[3]

The pastcast prediction error of 2009:Q4 at time 2010:01 is defined as,

$$\varepsilon = {}_{2010:02}Y^{(first)}_{2009:Q4} - {}_{2010:01}\hat{Y}_{2009:Q4}, \tag{37}$$

where $\hat{Y}$ denotes the prediction. In this formulation, the first available value for calendar date 2009:Q4 is released in 2010:02, and hence the use of these dates in ${}_{2010:02}Y^{(first)}_{2009:Q4}$. The MSFE, called $MSFE^{first}_{-1}$ is the sum of squared ε, across the out-of-sample prediction period. Note that the subscript "-1" is used to denote pastcasts, when reporting MSFEs. The pastcast involves forecasting the 'past' value of GDP growth. Since there is a release lag in the GDP announcement, we may not know the value of GDP even once the quarter has ended, and hence the need for a pastcast. Note also that these "pastcasts" are made only once every three months (i.e., during the first month of each quarter, prior to the first release of previous quarter GDP growth data).

The nowcast for 2010:01 is the prediction for 2010:Q1 that is made during the first month of Q1 using:

$$\varepsilon = {}_{2010:05}Y^{(first)}_{2010:Q1} - {}_{2010:01}\hat{Y}_{2010:Q1}$$

The MSFE in this case is called $MSFE^{first}_{1}$. In same way, the MSFE for next quarter prediction is denoted by $MSFE^{first}_{4}$, where in this case

$$\varepsilon = {}_{2010:08}Y^{(first)}_{2010:Q2} - {}_{2010:01}\hat{Y}_{2010:Q2}. \tag{38}$$

Note that using predictions from the first month of each quarter, only $MSFE^{first}_{-1}$, $MSFE^{first}_{1}$,

[3] We also use 'most recent' data as our actual data, when constructing MSFEs. This approach is probably the most consistent with actual practice at central banks, for example.

$MSFE_4^{first}$, and $MSFE_7^{first}$ are constructed, where $MSFE_7^{first}$ denotes the MSFE based on two quarter ahead predictions.

During the next month, i.e., 2010:02, we do not construct a pastcast because the first release GDP growth datum for 2009:Q4 has been published by the statistical reporting agency by that time. Therefore, $MSFE_{-1}^{first}$ is not defined during the 2nd month of a quarter, as discussed above. However, we do have a new nowcast; namely ${}_{2010:02}\hat{Y}_{2010:Q1}^{(first)}$, which is the prediction for 2010:Q1 that is made during the second month of Q1. This allows use to form a new nowcast MSFE that is based on predictions that are 'closer' in calendar time to the actual release date of the historical data, using

$$\varepsilon = {}_{2010:05}Y_{2010:Q1}^{(first)} - {}_{2010:02}\hat{Y}_{2010:Q1}, \tag{39}$$

where the MSFE in this case is called $MSFE_2^{first}$. $MSFE_5^{first}$ and $MSFE_8^{first}$ are analogously constructed using

$$\varepsilon = {}_{2010:08}Y_{2010:Q2}^{(first)} - {}_{2010:02}\hat{Y}_{2010:Q2} \tag{40}$$

and

$$\varepsilon = {}_{2010:11}Y_{2010:Q3}^{(first)} - {}_{2010:02}\hat{Y}_{2010:Q3}, \tag{41}$$

respectively.

Finally, we have a third nowcast, made in the third month of the quarter, as well as two true 'forecasts', allowing us to analogously construct $MSFE_3^{first}$, $MSFE_6^{first}$, and $MSFE_9^{first}$, respectively.

Before turning to a discussion of our main prediction experiment results, we summarize three methodological findings that are potentially useful for applied practitioners. First, recall that the ragged-edge data problem can be addressed in a number of ways. One involves use of either AR or VA interpolation of missing data. Another involves directly accounting for this data problem via the use of the EM algorithm or Kalman filtering. Table 2 summarizes the results of a small experiment designed to compare AR and VA interpolation (EM and Kalman filtering methods are discussed later). In this experiment, both AR and VA interpolation are used to construct missing data, and all forecasting models are implemented in order to construct predictions, including MIDAS models, as well as benchmark models. Indeed, the only models not included in this experiment are MIDAS variants based on use of the EM algorithm and Kalman filtering. Entries in the table denote the proportion of forecasting models for which VA interpolation yields lower MSFEs than AR interpolation. Interestingly, proportions are always less than 0.5, regardless of whether pastcasts, nowcasts, or forecasts are compared, and whether 'first available' or 'most recent' data are used. Indeed, in most cases, only approximately 10% of models or less 'prefer' VA interpolation. This is taken as strong evidence in favor of using AR interpolation, and, thus, the remainder of results presented only interpolate data using the AR method. Complete results using both varieties of interpolation are available upon request from the authors.

[Insert Table 2 here]

Second, we compare forecasting performance by estimation type in an experiment for which results are summarized in Table 3. In particular, we are cognizant of the fact that issues relating to structural breaks, model stability, and generic misspecification play an important role on the choice of using either rolling or recursive data windows when constructing real-time forecasting models. In lieu of this fact, we estimated all of our models using both recursive and rolling data windows, and entries in the table report the proportion of models for which the recursive estimation strategy is 'MSFE-best'. In the Korean case it turns out the recursive estimation dominates in all but three horizons, regardless of whether 'first available' or 'most recent' data are used. The fact that the only three instances where rolling windows are 'MSFE-best' are early horizon cases using 'first available' data suggests that only in this case is there sufficient instability to warrant use of said rolling windows. Coupled with the fact that recursively estimated models dominate at all horizons using 'most recent' data, we have evidence that early release Korean data might not condition effectively on all available information. This property can be further investigated via the use of so-called data rationality tests, which is left to future research.

[Insert Table 3 here]

Third, a crucial aspect of forecasting models that utilize diffusion indices is exactly how many factors to specify. Bai and Ng (2002) and many others provide statistics that can be used for selecting the number of factors. However, there is no guarantee that the use of any of the exact tests will yield the 'MSFE-best' forecasting model. In one recent experiment, Kim (2013) uses Bai and Ng (2002), and finds that five to six factors are selected for a large scale Korean dataset. In this paper (see Table 4), we directly examine how many factors are used in 'MSFE-best' forecasting models. In particular, entries in Table 4 denote the proportion of times that models with a given fixed number of factors are MSFE-best among all of our factor-MIDAS models, including those estimated using the EM algorithm, the Kalman filter, AR interpolation (with each of OPCA and RPCA), and those estimated both with and without autoregressive lags. It is very clear from inspection of the results that either 1 or 2 factors, at most, are needed when the prediction horizon in more than 1 quarter ahead. On the other hand, for horizons -1 to 3 (i.e. all pastcasts and nowcasts), the evidence is more mixed. While 1 or 2 factors are selected around 1/2 of the time, 5 or 6 factors are also selected around 1/2 of the time. Interestingly, there is little evidence that using an intermediate number of factors is useful. One should either specify a very parsimonious 1 or 2 factor models, or one should go with our maximum of 5 or 6. It is clear that forecast horizon matters; and this is consistent with the mixed evidence on this issue. Namely, some authors find that very few factors are useful, while others suggest using 5 or more. Both of these results are confirmed in our experiment, with forecast horizon being the critical determining characteristic. The overall conclusion, thus, appears to be that when uncertainty is more prevalent (i.e., longer forecast horizons), then parsimony is the key ingredient to factor selection. This conclusion is not at all surprising, and is in accord with stylized facts concerning model specification when specifying linear models.

[Insert Table 4 here]

We now turn to our forecasting model evaluation. Entries in Tables 5, Panel (a) are MSFEs for all models, relative to the strawman AR(SIC) model. Thus, entries greater than 1 imply that the corresponding model performs worse than the AR(SIC) model. The column headers in the table denote the forecast horizon, ranging from '-1' for pastcasts to 9 for two quarter ahead predictions. In this framework, horizons 1, 2, and 3 are monthly nowcasts for the current quarter, and subsequent horizons pertain to monthly forecasts made during the subsequent two quarters. Notice that the first three rows in the table correspond to our other benchmark models (i.e., the RW, CBADL and BEX models). The rest of the rows in the table report findings for our various MIDAS type models, constructed with 1, 2, and 6 factors. Recall that there are 5 different MIDAS specifications: 'Basic MIDAS with and without AR terms', 'Unrestricted MIDAS with and without AR terms', and 'Smoothed MIDAS'. Estimation is done recursively, the ragged-edge problem is solved by AR interpolation, four different factor estimation methods are reported on, including OPCA, RPCA, EM and KF, and data utilized in these experiments are assumed to be 'first available' data, for the purpose of both estimation and forecast evaluation. Table 5, Panel (b) is the same as Panel (a), except that 'most recent' instead of 'first available' data are used in all experiments reported on. Complete results pertaining to other permutations such as the use of alternative interpolation methods, estimation strategies, and numbers of factors are available upon request.

Digging a bit further into the layout of this table, note that bold entries denote models that are 'MSFE-better' than the AR(SIC) model, entries with superscript 'FB' are 'MSFE-best' for a given forecast horizon and number of factors, and entries with the superscript

'GB' denote models that are 'MSFE-best' across all permutations, for a particular forecast horizon.

[Insert Table 5 here]

When forecast experiments are carried out using 'first available' data (see Panel (a) of Table 5), it turns out that for pastcasting and nowcasting, factor-MIDAS models without AR terms as well as other benchmark models do not work well, regardless of the number of factors specified. In these cases, AR(SIC) models dominate, in terms of MSFE. This suggests that for short forecast horizons, the persistence of GDP growth is strong, and well modeled using linear AR components. As the forecast horizon gets longer, models without AR terms benefit from substantial performance improvement of the other components of the models, such as the MIDAS component. Indeed, in some cases, models without AR terms outperform models with AR terms. This is interesting, as it suggests that uncertainty in autoregressive parameters does not carry over as much to other model parameters, as the horizon increases, and the role for MIDAS thus increases in importance.

Evidently, upon inspection of MSFEs in Panel (a) of Table 5, there is little to choose between OPCA and RPCA estimation methods. Thus, given computing considerations [4], RPCA is preferred when analyzing large datasets. Among the other factor estimation methods, the KF and EM algorithms perform well for longer forecast horizons, but KF outperforms EM for shorter horizons. Turning to the number of factors used in prediction model construction, it is noteworthy that 1 or 2 factors are strongly preferred for longer forecast horizons, in accord with our earlier findings. The exception to this finding is when Smoothed MIDAS is used, in which case 6 factors are always preferred, regardless of horizon. This finding, though, is mitigated somewhat by the finding that Smoothed MIDAS never yields 'MSFE-best' models that are 'globally best' (i.e., GB) for a given forecast horizon. Still, if one must use many factors, then Smoothed MIDAS does yield the 'MSFE-best' model at many horizons.

Panel (b) of Table 5 contains MSFEs that are based on the use of 'most recent' data. In most practical settings, forecasters assess predictive accuracy using this variety of data. Interestingly, in this set of results, we immediately observe that the 'MSFE-best' model is almost never the AR(SIC) model. Moreover, our earlier finding that models without AR terms are not preferred to the AR(SIC) for pastcasting and nowcasting is reversed. Indeed, for these forecast horizons, the 'MSFE-best' models do not contain AR terms, and are factor-MIDAS models. This is interesting, as it suggests that incorporation of the revision process in

[4] Computation when using RPCA is around 10% faster that when using OPCA, based on a run using an Intel i7-3700 processor with 16GB of RAM.

our analysis, the effects of which are captured when 'most' recent' data are utilized, negates the usefulness of autoregressive information, and models specified using MIDAS without AR terms are 'MSFE-best'. This result points to the need to be very careful when specifying models, as the benchmark data used in prediction are crucial to model selection. The rest of the findings in this table, however, mirror those from Panel (a) of the table.

In order to obtain a clearer picture of the rather interesting finding concerning the inclusion (or not) of AR terms, we concisely summarize the findings of Table 5 in Table 6. In particular, in Table 6 the 'GB' models that are 'MSFE-best' across all permutations, for a particular forecast horizon, are given in the rows labels 'All'. The remainder of the table summarizes associated 'MSFE-best' models (and corresponding factor estimation schemes) for a given number of factors, for the cases where both 'first available' and 'most recent' data are used, and for a variety of forecast horizons. The results summarized in our discussion of Table 5 are made even more clear in this summary table. Namely, factor-MIDAS models are almost everywhere 'MSFE-best', with the exception of pastcasts and nowcasts. Additionally, models without AR terms are important when using 'most recent' data at shorter horizons, and when using 'first available' data at longer horizons. Finally, PCA factor estimation methods are almost always preferred, and smoothed MIDAS type models are only useful if including many factors when predicting at the longest horizons. Of course, we do not recommend this, as using many factors for long horizon forecasting has been shown to yield more imprecise predictions than when fewer factors are used.

[Insert Table 6 here]

Figure 5 plots MSFE values that are not relative to the strawman AR(SIC) model, for various prediction models. In the figure, 'Basic' and 'Unrestricted' denote factor-MIDAS models with two factors (refer to above discussion, and to Table 5, for further discussion of this terminology), and AR interpolation with OPCA estimation is used throughout. Panels (a) and (b) correspond to recursively estimated models using 'first available' and 'most recent' data, respectively. Panels (c) and (d) are same, but use rolling estimation. In this figure, $h = 0$ corresponds to pastcasts (called $h = -1$ in the tables), $h = 1, 2, 3$ correspond to nowcasts, and $h = 4, ..., 9$ correspond to forecasts. As discussed above, in conventional forecasting experiments, most forecasters use fully revised data for forecasting evaluation. With these data, factor-MIDAS dominates all other benchmark models, at all horizons, as seen in Panel (b); and RW and CBADL perform poorly at all horizons. Also, among the factor-MIDAS models, 'Basic' factor-MIDAS dominates. If we instead use 'first available' data, factor-MIDAS models as well as BEX models dominate the AR(SIC) model, particularly at long forecast horizons (see Panel (a)). However, as the forecast horizon gets shorter (i.e., we move from forecast→ nowcast→pastcast), AR(SIC) and RW models perform better than other models, as confirmed in our discussion of the results presented in Table 5.

[Insert Figure 5 here]

For the rolling estimation scheme, the forecast performance of factor-MIDAS models and AR(SIC) models are similar for all horizons. However, we know from our earlier discussion that recursively estimated models generally perform better, in our experiments.

In Figure 6, MSFE values are plotted for the same set of models as in Figure 5. However, in this figure, Panels (a)-(d) contain plots based on the use of different factor estimation methods when specifying the models (i.e., OPCA, RPCA, EM and KF), only first available data are used for MSFE construction, all models are specified with one factor, and AR interpolation is implemented. In light of this, Panel (a) in Figures 5 and 6 is the same. A number of conclusions emerge upon inspection of this figure. First, the pattern of increasing MSFE as forecast horizon increases is observed for all factor estimation methods (compare all 4 panels in the figure), as expected. Also, all estimation methods appear to be rather similar, when faced with 'first available' data. However, even though MSFEs are similar across factor estimation methods, the MSFE magnitudes are slightly higher when using EM and KF, than when using OPCA and RPCA are used for estimation. Interestingly, only our top two MIDAS models (that include AR terms) outperform the benchmark AR(SIC) model at all forecast horizons, as can also be seen by inspection of the results in Table 5. Inspection of the plots in Figures 7 and 8, which are the same as Figure 6, except that 2 and 3 factors are specified, respectively, indicate that this finding continues to hold, as the number of factors increases. However, the overall ranking of the entire set of models does become more unclear, particularly with 6 factors. Indeed, in the 6 factor case, MSFE values for some of our non-MIDAS models are so high that the models are completely unreliable. This points to another concern when specifying so many factors, in addition to the issues discussed above when exploring the results in Table 5. Our other findings based on inspection of Figure 6 remain largely the same when the number of factors is increased to 2 and then 6.

[Insert Figure 6 here]
[Insert Figure 7 here]
[Insert Figure 8 here]

Finally, Figures 9 - 11 plot MSFEs of selected MIDAS models, with $r = 1, 2$, and 6. In these figures, MIDAS results are presented with factors estimated using OPCA, RPCA, EM, and KF. Additionally, various benchmark models are included (i.e., AR(SIC), RW, CBADL, and BEX). Using these figures, we can compare the performance of factor estimation methods for a given MIDAS model and value of r. When $r = 1$, RPCA or OPCA are clearly preferred. However, when $r = 2$, Kalman filtering also works well at many forecast horizons. Finally, as previously observed, when the number of factors is increased, forecast performance worsens substantially for 'Basic MIDAS' and 'Unrestricted MIDAS', as seen in Figure 11. Interestingly, 'Smoothed MIDAS' continues to perform well, even when $r = 6$. This again points to the importance of smoothing when the number of factors is large.

[Insert Figure 9 here]
[Insert Figure 10 here]
[Insert Figure 11 here]

6 Concluding Remarks

We introduce a real-time dataset for Korean GDP, and analyze the usefulness of the dataset for forecasting, using a large variety of factor-MIDAS models, as well as linear benchmark models. In this context, various factor estimation schemes, data interpolation approaches, and data windowing methods are analyzed, and methodological recommendations made. For example, we find that only approximately 10% of the forecasting models examined are 'MSFE-best' when using VA interpolation instead of AR interpolation. Additionally, models estimated using rolling data windows are only 'MSFE-best' at 3 forecast horizons, when comparing real-time predictions to 'first available' data, and are never 'MSFE-best' when comparing predictions to 'most recent' data. Given the usual preference amongst empirical researchers to use 'most recent' data in predictive accuracy analyses, it is clear that, at least in the case of Korean GDP, recursive estimation is preferred. This is consistent with a conclusion that structural instability in our 'MSFE-best' models is mild, and also that factor-MIDAS models may serve to mitigate instability that arises in simpler linear specifications. With regard to the number of factors to specify in prediction models, either 1 or 2 factors, at most, are needed when the prediction horizon is more than 3 months ahead. On the other hand, for horizons -1 to 3 (i.e. all pastcasts and nowcasts), the evidence is more mixed, and 5 or 6 factors are also selected around 1/2 of the time. Interestingly, there is little evidence that using an intermediate number of factors is useful. One should either specify a very parsimonious 1 or 2 factor models, or one should go with our maximum of 5 or 6. In summary, forecast horizon matters, in the sense that when uncertainty is more prevalent (i.e., longer forecast horizons), then parsimony is the key ingredient to factor selection, and more than 1 or 2 factors leads to worsening predictive performance. This is consistent with the stylized notion that prediction multiple periods ahead becomes very uncertain when forecasting macroeconomic aggregates.

Finally, MIDAS models dominate at all forecast horizons, and autoregressive lags of the dependent variable (GDP in our case) are only useful at the our longest forecast horizons, when 'most recent' data are used in our predictive accuracy analyses. This suggests that the use of information rich real-time data negates the need for autoregressive lags when constructing short-term forecasts of Korean GDP. Namely, when properly revised and currently available GDP is modeled, and resulting real-time forecasts are compared with currently available (or 'most recent') data, the informational content of lags of GDP vanishes, when factor MIDAS is implemented.

References

Altissimo, F., Cristadoro, R., Forni, M., Lippi, M., and Veronese, G. (2010). New eurocoin: Tracking economic growth in real time. *The Review of Economic and Statistics*, 92(4):1024–1034.

Andreou, E., Ghysels, E., and Kourtellos, A. (2010). Should macroeconomic forecasters use daily financial data and how? Technical report, Manuscript, University of Cyprus.

Baffigi, A., Golinelli, R., and Parigi, G. (2004). Bridge models to forecast the euro area gdp. *International Journal of Forecasting*, 20(3):371–401.

Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.

Barhoumi, K., Benk, S., Cristadoro, R., Reijer, A. D., Jakaitiene, A., Jelonek, P., Rua, A., Rüstler, G., Ruth, K., and Nieuwenhuyze, C. V. (2008). Short-term forecasting of gdp using large monthly datasets: A pseudo real-time forecast evaluation exercise. ECB Occasional Paper 84, European Central Bank.

Boivin, J. and Ng, S. (2005). Understanding and comparing factor-based forecasts. *International Journal of Central Banking*, 1(3):117–152.

Clements, M. P. and Galvao, A. B. (2008). Macroeconomic forecasting with mixed frequency data. *Journal of Business & Economic Statistics*, 26:546–554.

Clements, M. P. and Galvao, A. B. (2009). Forecasting us output growth using leading indicators: An appraisal using midas mode. *Journal of Applied Econometrics Journal*, 24(7):1187–1206.

Diebold, F. X. and Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13(3):253–63.

Doz, C., Giannone, D., and Reichlin, L. (2012). A quasi-maximum likelihood approach for large, approximate dynamic factor models. *The Review of Economics and Statistics*, 94(4):1014–1024.

Ferrara, L. and Marsilli, C. (2013). Financial variables as leading indicators of GDP growth: Evidence from a MIDAS approach during the Great Recession. *Applied Economics Letters*, 20(3):233–237.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2005). The generalized dynamic factor model: One-sided estimation and forecasting. *Journal of the American Statistical Association*, 100:830–840.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2004). The midas touch: Mixed data sampling regression models. CIRANO Working Papers 2004s-20, CIRANO.

Giannone, D., Reichlin, L., and Small, D. (2008). Nowcasting gdp and inflation: The real time information content of macroeconomic data releases. *Journal of Monetary Economics*, 55:665–676.

Girardi, A., Golinelli, R., and Pappalardo, C. (2016). The role of iindicator selection in nowcasting euro area gdp in pseudo real time. *Empirical Economics*, forthcoming.

Golinelli, R. and Parigi, G. (2005). Short-run italian gdp forecasting and real-time data. CEPR Discussion Papers 5302, C.E.P.R. Discussion Papers.

Kim, H. H. (2013). Forecasting macroeconomic variables using data dimension reduction methods: The case of korea. BOK Working Paper 2013-26, Bank of Korea.

Kim, H. H. and Swanson, N. R. (2014). Forecasting financial and macroeconomic variables using data reduction methods: New empirical evidence. *Journal of Econometrics*, 178(2):352–367.

Kim, H. H. and Swanson, N. R. (2016). Mining big data using parsimonious factor machine learning, variable selection, and shrinkage methods. *International Journal of Forecasting*, forthcoming.

Koenig, E. F., Dolmas, S., and Piger, J. (2003). The use and abuse of real-time data in economic forecasting. *The Review of Economics and Statistics*, 85(3):618–628.

Kuzin, V., Marcellino, M., and Schumacher, C. (2011). MIDAS vs. mixed-frequency VAR: Nowcasting GDP in the euro area. *International Journal of Forecasting*, 27(2):529–542.

Marcellino, M. and Schumacher, C. (2010). Factor midas for nowcasting and forecasting with ragged-edge data: A model comparison for german gdp. *Oxford Bulletin of Economics and Statistics*, 72(4):518–550.

Peddaneni, H., Erdogmus, D., Rao, Y. N., Hegde, A., and Principe, J. (2004). Recursive principal components analysis using eigenvector matrix perturbation. In *Machine Learning for Signal Processing*, pages 83–92. IEEE.

Pettenuzzo, D., Timmermann, A. G., and Valkanov, R. (2014). A Bayesian MIDAS Approach to Modeling First and Second Moment Dynamics. CEPR Discussion Papers 10160, C.E.P.R. Discussion Papers.

Rünstler, G. and Sédillot, F. (2003). Short-term estimates of euro area real gdp by means of monthly data. Working Paper Series 0276, European Central Bank.

Schumacher, C. (2007). Forecasting german gdp using alternative factor models based on large datasets. *Journal of Forecasting*, 26(4):271–302.

Schumacher, C. and Breitung, J. (2008). Real-time forecasting of German GDP based on a large factor model with monthly and quarterly data. *International Journal of Forecasting*, 24(3):386–398.

Stock, J. H. and Watson, M. W. (2002). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, 97:1167–1179.

Stock, J. H. and Watson, M. W. (2012). Generalized shrinkage methods for forecasting using many predictors. *Journal of Business and Economic Statistics*, 30(4):481–493.

Swanson, N. and van Dijk, D. (2006). Are statistical reporting agencies getting it right? data rationality and business cycle asymmetry. *Journal of Business and Economic Statistics*, 24:24–42.

Wallis, K. (1986). Forecasting with an econometric model: the “ragged edge” problem. *Journal of Forecasting*, 5:1–13.

Zheng, I. Y. and Rossiter, J. (2006). Using monthly indicators to predict quarterly gdp. Working Papers 06-26, Bank of Canada.

Table 1: Summary of Models and Estimation Methods*

<table>
<tr><th>Estimation Scheme</th><th>MIDAS</th><th>Factor Estimation</th><th>Interpolation</th></tr>
<tr><td></td><td>Basic w/o AR term
Basic w/ AR term</td><td>OPCA

RPCA</td><td>AR

VA</td></tr>
<tr><td>Recursive

Rolling</td><td>Unrestricted w/o AR term
Unrestricted w/ AR term
Smoothed</td><td colspan="2">EM algorithm

Kalman Filtering</td></tr>
<tr><td></td><td colspan="3">AR
CBADL
BEX</td></tr>
</table>

Model	Description
AR(SIC)	Autoregressive model with length of lags determined by SIC
RW	Random Walk
CBADL	Combined Bivariate Autoregressive Distributed Lag model
BEX	Bridge Equation with Exogenous Variable
Basic w/o AR	Basic MIDAS model without AR terms
Basic w/ AR	Basic MIDAS model with AR terms
Unrestricted w/o AR	Unrestricted MIDAS model without AR terms
Unrestricted w/ AR	Unrestricted MIDAS model with AR terms
Smoothed	Smoothed MIDAS model

* Notes: Non-factor-MIDAS type models include AR(SIC), RW, CBADL and BEX. Three types of factor-MIDAS models are specified (‘Basic’, ‘Unrestricted’, and ’Smoothed’), and each of these are estimated using each factor estimation method (OPCA and RPCA), interpolation method (AR and VA), and factor-MIDAS estimation method (EM algorithm and Kalman filter). Finally, all of these permutations are implemented using each of recursive and rolling data windowing strategies. For complete details see Section 5.

Table 2: Comparison of Forecasting Performance with AR and VA Interpolation*

	Pastcast	Nowcast			Forecast					
	prev. qtr.	current quarter			1 quarter ahead			2 quarter ahead		
Horizon	-1	1	2	3	4	5	6	7	8	9
First available	0.285	0.337	0.110	0.145	0.151	0.134	0.105	0.134	0.093	0.186
Most Recent	0.058	0.093	0.110	0.058	0.180	0.157	0.209	0.337	0.262	0.326

* Notes: See notes to Table 1. Forecasting performance is evaluated by comparing MSFEs across all models which use interpolated missing values, including CBADL, BEX and factor-MIDAS models with OPCA and RPCA. Entries in the table report the proportion of times that the MSFE of models with AR interpolation is greater than 'like' models with VA interpolation. Thus, entries less than 0.5 indicate that AR interpolation performs better than VA, on average, across all model permutations. Prediction models are estimated in real-time using either 'first available' or 'most recent' historical data, and MSFEs are constructed by comparing these predictions with actual 'first available' or 'most recent' data, corresponding to the type of data used in estimation.

Table 3: Comparison of Forecasting Performance by Estimation Type*

	Pastcast	Nowcast			Forecast					
	prev. qtr.	current quarter			1 quarter ahead			2 quarter ahead		
Horizon	-1	1	2	3	4	5	6	7	8	9
First available	0.747	0.782	0.653	0.465	0.400	0.147	0.059	0.018	0.029	0.082
Most Recent	0.218	0.194	0.171	0.235	0.282	0.218	0.329	0.353	0.300	0.459

* Notes: See notes to Table 2. Forecasting performance is evaluated by comparing MSFEs across all models, with MSFEs calculated by estimating prediction models using either 'first available' or 'most recent' actual data, as discussed in the footnote to Table 2. In this table, entries report the proportion of times that the MSFEs of models estimated recursively are greater than when 'like' models are estimated using rolling data windows. Thus, entries less than 0.5 indicate that recursive estimation yields lower MSFEs, on average, across all model permutations.

Table 4: Comparison of Forecasting Performance Using Differing Numbers of Factors *

		Pastcast	Nowcast			Forecast					
		prev. qtr.	current quarter			1 quarter ahead			2 quarter ahead		
	Factor #	-1	1	2	3	4	5	6	7	8	9
First available	1	0.20	0.25	0.20	-	-	0.20	-	0.20	-	-
	2	0.20	-	0.20	-	0.60	0.60	1.00	0.60	1.00	1.00
	3	-	0.15	-	0.20	0.20	0.20	-	0.20	-	-
	4	-	-	0.20	0.40	-	-	-	-	-	-
	5	0.20	0.20	-	-	0.20	-	-	-	-	-
	6	0.40	0.40	0.40	0.40	-	-	-	-	-	-
Most Recent	1	0.40	0.20	0.40	0.60	0.60	0.80	1.00	1.00	1.00	1.00
	2	0.20	0.40	0.20	-	-	-	-	-	-	-
	3	-	-	-	-	0.20	-	-	-	-	-
	4	-	-	-	-	0.20	0.20	-	-	-	-
	5	0.20	0.40	0.20	0.20	-	-	-	-	-	-
	6	0.20	-	0.20	0.20	-	-	-	-	-	-

* Notes: See notes to Table 3. The proportion of factor-MIDAS 'MSFE-best' models, when comparing 'like' models with the number of factors varying from 1 to 6, is reported in this table. This using either 'first available' or 'most recent' data (as discussed in the footnote to Table 2), as well as for a number of pastcast, nowcast, and forecast horizons. See Section 6 for a detailed discussion of the different horizons reported on. All results are based on OPCA and RPCA using AR interpolation, under a recursive estimation scheme.

Table 5: Relative MSFEs When Pastcasting, Nowcasting, and Forecasting Korean GDP*

Panel (a): First Available

			Pastcast	Nowcast			Forecast					
			prev. qtr.	current quarter			1 quarter ahead			2 quarter ahead		
Factors	Recursive		-1	1	2	3	4	5	6	7	8	9
	RW		1.45	1.35	1.12	**0.94**	**0.94**	1.01	1.14	1.13	1.20	1.68
	CBADL		5.49^{*}	4.67^{*}	3.28^{*}	1.73	1.63	1.63	1.36	1.32	1.37	1.46
	BEX		3.48^{*}	3.25^{*}	2.16	**0.89**	**0.87**	**0.85**	$\mathbf{0.64}^{*}$	$\mathbf{0.62}^{**}$	$\mathbf{0.64}^{*}$	$\mathbf{0.71}^{**}$
1	Basic w/o AR	OPCA	3.01^{**}	2.46^{**}	1.70^{**}	**0.80**	**0.77**	**0.84**	**0.72**	$\mathbf{0.69}^{*}$	**0.73**	**0.87**
		RPCA	3.01^{**}	2.46^{**}	1.70^{**}	**0.80**	**0.77**	**0.84**	**0.72**	$\mathbf{0.69}^{*}$	**0.73**	**0.87**
		EM	3.25^{**}	2.96^{**}	2.00^{**}	1.07	1.08	1.09	**0.88**	**0.82**	**0.81**	**0.86**
		KF	2.72^{**}	2.32^{**}	1.73^{**}	**0.91**	**0.90**	**0.98**	**0.82**	**0.77**	**0.80**	**0.90**
	Basic w/ AR	OPCA	$\mathbf{0.70}_{\mathbf{GB}}$	$\mathbf{0.83}_{\mathbf{GB}}$	$\mathbf{0.70}^{**}$	$\mathbf{0.49}^{**}$	$\mathbf{0.57}^{**}$	**0.66**	**0.75**	**0.75**	**0.80**	**0.97**
		RPCA	**0.70**	**0.83**	$\mathbf{0.70}^{**}$	$\mathbf{0.49}^{**}$	$\mathbf{0.57}^{**}$	**0.66**	**0.75**	**0.75**	**0.80**	**0.97**
		EM	1.18	1.45	1.12	1.00	1.09	1.07	1.05	**0.95**	**0.97**	1.12
		KF	**0.90**	**0.99**	**0.93**	**0.71**	**0.78**	1.06	1.02	**0.98**	**0.98**	1.21
	Unrestricted w/o AR	OPCA	3.10^{**}	2.63^{**}	1.82^{**}	**0.83**	**0.75**	**0.75**	$\mathbf{0.61}^{*}$	$\mathbf{0.58}^{**}$	**0.62**	$\mathbf{0.74}^{**}$
		RPCA	3.10^{**}	2.63^{**}	1.82^{**}	**0.83**	**0.75**	**0.75**	$\mathbf{0.61}^{*}$	$\mathbf{0.58}^{**}$	**0.62**	$\mathbf{0.74}^{**}$
		EM	3.33^{**}	3.10^{**}	2.14^{**}	1.13	**0.98**	1.05	**0.78**	**0.69**	**0.78**	$\mathbf{0.80}^{*}$
		KF	2.81^{**}	2.45^{**}	1.80^{**}	**0.90**	**0.75**	**0.80**	**0.72**	$\mathbf{0.57}^{**}$	$\mathbf{0.62}^{*}$	$\mathbf{0.79}^{*}$
	Unrestricted w/ AR	OPCA	**0.76**	**0.88**	$\mathbf{0.68}^{*}_{\mathbf{FB}}$	$\mathbf{0.47}^{**}_{\mathbf{FB}}$	$\mathbf{0.49}^{**}$	$\mathbf{0.59}^{**}_{\mathbf{FB}}$	$\mathbf{0.49}^{**}_{\mathbf{FB}}$	$\mathbf{0.53}^{**}$	**0.70**	$\mathbf{0.73}^{**}_{\mathbf{FB}}$
		RPCA	**0.76**	**0.88**	$\mathbf{0.68}^{*}$	$\mathbf{0.47}^{**}$	$\mathbf{0.49}^{**}_{\mathbf{FB}}$	$\mathbf{0.59}^{**}$	$\mathbf{0.49}^{**}$	$\mathbf{0.53}^{**}$	**0.70**	$\mathbf{0.73}^{**}_{\mathbf{FB}}$
		EM	1.33	1.39	1.03	**0.80**	**0.75**	**0.85**	**0.75**	**0.82**	**0.73**	1.09
		KF	**0.91**	**0.99**	**0.76**	$\mathbf{0.57}^{*}$	$\mathbf{0.50}^{**}$	**0.66**	$\mathbf{0.58}^{**}$	$\mathbf{0.48}^{**}_{\mathbf{FB}}$	$\mathbf{0.55}^{**}_{\mathbf{FB}}$	1.07
	Smoothed	OPCA	2.47^{**}	2.16^{**}	1.51^{*}	**0.70**	**0.71**	**0.77**	$\mathbf{0.64}^{*}$	$\mathbf{0.65}^{*}$	**0.69**	$\mathbf{0.81}^{*}$
		RPCA	2.47^{**}	2.16^{**}	1.51^{*}	**0.70**	**0.71**	**0.77**	$\mathbf{0.64}^{*}$	$\mathbf{0.65}^{*}$	**0.69**	$\mathbf{0.81}^{*}$
		EM	2.44^{**}	2.43^{**}	1.75^{**}	**0.84**	**0.92**	**0.95**	**0.75**	**0.75**	**0.74**	$\mathbf{0.83}^{*}$
		KF	2.32^{**}	2.11^{**}	1.58^{**}	**0.78**	**0.81**	**0.89**	$\mathbf{0.72}^{*}$	$\mathbf{0.72}^{*}$	**0.74**	$\mathbf{0.84}^{*}$
2	Basic w/o AR	OPCA	3.10^{**}	2.28^{**}	1.53^{*}	**0.65**	$\mathbf{0.52}^{*}$	$\mathbf{0.51}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.40}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.57}^{**}_{\mathbf{GB}}$
		RPCA	3.10^{**}	2.28^{**}	1.53^{*}	**0.65**	$\mathbf{0.52}^{*}$	$\mathbf{0.51}^{**}$	$\mathbf{0.41}^{**}_{\mathbf{GB}}$	$\mathbf{0.40}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.57}^{**}$
		EM	3.34^{**}	3.11^{**}	1.74^{**}	**0.80**	**0.79**	**0.69**	$\mathbf{0.56}^{*}$	$\mathbf{0.56}^{*}$	$\mathbf{0.56}^{*}$	**0.71**
		KF	2.61^{**}	2.29^{**}	1.59^{**}	**0.61**	$\mathbf{0.54}^{**}$	$\mathbf{0.53}^{**}$	$\mathbf{0.43}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.47}^{*}$	$\mathbf{0.60}^{*}$
	Basic w/ AR	OPCA	$\mathbf{0.71}_{\mathbf{FB}}$	**0.85**	**0.69**	$\mathbf{0.41}^{**}_{\mathbf{GB}}$	$\mathbf{0.38}^{**}_{\mathbf{GB}}$	$\mathbf{0.42}^{**}$	$\mathbf{0.48}^{**}$	$\mathbf{0.39}^{**}_{\mathbf{GB}}$	$\mathbf{0.44}^{**}$	$\mathbf{0.57}^{**}$
		RPCA	**0.71**	$\mathbf{0.84}_{\mathbf{FB}}$	$\mathbf{0.67}^{*}$	$\mathbf{0.42}^{**}$	$\mathbf{0.38}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.48}^{**}$	$\mathbf{0.39}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.57}^{**}$
		EM	1.18	1.55	**0.88**	$\mathbf{0.56}^{*}$	$\mathbf{0.63}^{*}$	$\mathbf{0.58}^{*}$	**0.56**	$\mathbf{0.55}^{*}$	$\mathbf{0.54}^{*}$	**0.72**
		KF	**0.87**	**0.94**	$\mathbf{0.65}^{*}_{\mathbf{GB}}$	$\mathbf{0.41}^{**}$	$\mathbf{0.39}^{**}$	$\mathbf{0.40}^{**}_{\mathbf{GB}}$	$\mathbf{0.43}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.41}^{**}_{\mathbf{GB}}$	$\mathbf{0.66}^{*}$
	Unrestricted w/o AR	OPCA	3.08^{**}	2.78^{**}	1.72^{**}	**0.66**	$\mathbf{0.52}^{*}$	$\mathbf{0.52}^{**}$	$\mathbf{0.48}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.62}^{**}$
		RPCA	3.08^{**}	2.78^{**}	1.72^{**}	**0.66**	$\mathbf{0.52}^{*}$	$\mathbf{0.52}^{**}$	$\mathbf{0.48}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.62}^{**}$
		EM	3.72^{**}	3.20^{**}	1.88^{**}	**0.83**	**0.90**	1.02	**0.72**	**0.77**	**0.83**	**0.94**
		KF	2.72^{**}	2.43^{**}	1.56^{**}	**0.69**	**0.69**	**0.85**	$\mathbf{0.57}^{*}$	$\mathbf{0.46}^{**}$	$\mathbf{0.50}^{*}$	**0.65**
	Unrestricted w/ AR	OPCA	**0.73**	**0.89**	**0.76**	$\mathbf{0.49}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.59}^{**}$	$\mathbf{0.43}^{**}$	$\mathbf{0.56}^{*}$	**0.65**	$\mathbf{0.61}^{**}$
		RPCA	**0.73**	**0.89**	**0.76**	$\mathbf{0.49}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.59}^{**}$	$\mathbf{0.43}^{**}$	$\mathbf{0.56}^{*}$	**0.65**	$\mathbf{0.61}^{**}$
		EM	1.31	1.29	1.35	**0.78**	**0.82**	1.15	**0.76**	**0.97**	1.43	1.05
		KF	1.12	1.16	**0.96**	**0.57**	**0.68**	**0.83**	**0.65**	**0.60**	**0.58**	**0.85**
	Smoothed	OPCA	3.41^{**}	2.54^{**}	1.61^{**}	$\mathbf{0.60}^{*}$	$\mathbf{0.56}^{**}$	$\mathbf{0.53}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.45}^{**}$	$\mathbf{0.59}^{**}$
		RPCA	3.41^{**}	2.54^{**}	1.61^{**}	$\mathbf{0.60}^{*}$	$\mathbf{0.56}^{**}$	$\mathbf{0.53}^{**}$	$\mathbf{0.41}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.45}^{**}$	$\mathbf{0.59}^{**}$
		EM	3.18^{**}	2.76^{**}	1.62^{**}	**0.69**	**0.70**	$\mathbf{0.62}^{*}$	$\mathbf{0.50}^{**}$	$\mathbf{0.52}^{**}$	$\mathbf{0.52}^{**}$	$\mathbf{0.65}^{**}$
		KF	2.99^{**}	2.31^{**}	1.44^{*}	$\mathbf{0.58}^{*}$	$\mathbf{0.56}^{**}$	$\mathbf{0.54}^{**}$	$\mathbf{0.42}^{**}$	$\mathbf{0.44}^{**}$	$\mathbf{0.46}^{**}$	$\mathbf{0.60}^{**}$

	Basic w/o AR	OPCA	1.15	1.30	**0.87**	**0.57**	**0.75**	1.39	1.08	2.25	3.89	3.18
		RPCA	1.24	1.32	**0.94**	**0.58**	**0.78**	1.38	1.09	2.13	3.80	2.90
		EM	1.91	1.75	1.03	**0.89**	3.11	1.70	1.70	2.43	3.14	4.53
		KF	1.22	1.15	**0.79**	**0.51****	1.03	**0.99**	1.22	2.43	4.27	5.91
	Basic w/ AR	OPCA	**0.95**$_{\mathbf{FB}}$	1.16	**0.90**	**0.56***	**0.70**	1.38	**0.88**	1.61	5.41	4.72
		RPCA	**0.98**	1.16	**0.86**	**0.56***	**0.69**	1.37	**0.88**	1.56	4.95	4.76
		EM	1.49	1.65	**0.84**	**0.76**	1.66	1.85	1.53	4.09	4.43	2.46*
		KF	1.11	1.06	**0.72**$_{\mathbf{FB}}$	**0.64**	**0.86**	1.11	**0.91**	2.06	4.14	7.19
6	Unrestricted w/o AR	OPCA	3.17*	1.74	1.21	1.96	1.73	3.39	5.00	2.35	6.53**	18.97
		RPCA	3.17*	1.74	1.21	1.96	1.73	3.39	5.00	2.35	6.53**	18.97
		EM	3.13**	2.11**	2.33*	**0.94**	1.42	1.88*	2.36*	2.96**	3.33	8.79**
		KF	1.46	1.40	1.71	**0.65**	1.32	1.60	2.00	3.84	5.03**	3.84**
	Unrestricted w/ AR	OPCA	2.27	1.21	1.16	**0.67**	1.96	2.63	2.19	2.30	6.57*	9.20**
		RPCA	2.27	1.21	1.16	**0.67**	1.96	2.63	2.19	2.30	6.57*	9.20**
		EM	2.58**	1.90**	2.06	**0.92**	2.11	3.04**	2.57**	6.30*	3.79**	7.83**
		KF	1.36	1.66	1.90	**0.63**	2.28	5.44	2.11	6.27*	4.60*	5.70
	Smoothed	OPCA	1.47	1.43	**0.99**	**0.48****	**0.54**$^{**}_{\mathbf{FB}}$	**0.57**$^{**}_{\mathbf{FB}}$	**0.48**$^{**}_{\mathbf{FB}}$	**0.59**$^{*}_{\mathbf{FB}}$	**0.59**$_{\mathbf{FB}}$	**0.83**$_{\mathbf{FB}}$
		RPCA	1.47	1.43	**0.99**	**0.48**$^{**}_{\mathbf{FB}}$	**0.54**$^{**}_{\mathbf{FB}}$	**0.57**$^{**}_{\mathbf{FB}}$	**0.48**$^{**}_{\mathbf{FB}}$	**0.59**$^{*}_{\mathbf{FB}}$	**0.59**$_{\mathbf{FB}}$	**0.83**$_{\mathbf{FB}}$
		EM	1.60	1.87	1.04	**0.62***	**0.68**	**0.71**	**0.57***	**0.73**	**0.73**	**0.98**
		KF	1.45*	1.45	**0.95**	**0.52****	**0.57****	**0.60****	**0.51****	**0.60***	**0.62**	**0.85**

Panel (b): Most Recent

Factors	Recursive		Pastcast	Nowcast			Forecast					
			prev. qtr.	current quarter			1 quarter ahead			2 quarter ahead		
			-1	1	2	3	4	5	6	7	8	9
	RW		1.07	1.11	1.15	1.25**	1.29**	1.27**	1.41**	1.49**	1.49**	1.60**
	CBADL		**0.97**	**0.97**	1.10	1.16	1.12	1.21*	1.33**	1.29**	1.38**	1.51**
	BEX		**0.65****	**0.67****	**0.74****	**0.80**	**0.82**	**0.85**	**0.96**	**0.98**	1.02	1.11
1	Basic w/o AR	OPCA	**0.49****	**0.52****	**0.55****	**0.57****	**0.59****	**0.60****	**0.67****	**0.74****	**0.77****	**0.87**
		RPCA	**0.49****	**0.52****	**0.55****	**0.57****	**0.59****	**0.60**$^{**}_{GB}$	**0.67****	**0.74****	**0.77****	**0.87**
		EM	**0.67****	**0.69****	**0.69****	**0.74**	**0.73***	**0.68****	**0.77**	**0.82**	**0.82***	**0.94**
		KF	**0.57****	**0.60****	**0.64****	**0.66****	**0.67****	**0.66****	**0.74****	**0.80***	**0.81***	**0.92**
	Basic w/ AR	OPCA	1.02	1.05	**0.99**	1.10	1.18	**0.99**	**0.62****	**0.62****	**0.64****	**0.65****
		RPCA	1.02	1.06	**0.99**	1.10	1.18	**0.99**	**0.62****	**0.62****	**0.64****	**0.65****
		EM	1.02	**0.99**	**0.92**	1.03	1.21	**0.70***	**0.62****	**0.65***	**0.64****	**0.65****
		KF	**0.98**	**0.98**	**0.98**	1.05	1.08	**0.76**	**0.54**$^{**}_{GB}$	**0.57**$^{**}_{GB}$	**0.60**$^{**}_{GB}$	**0.59**$^{**}_{GB}$
	Unrestricted w/o AR	OPCA	**0.48**$^{**}_{FB}$	**0.49**$^{**}_{FB}$	**0.53**$^{**}_{GB}$	**0.55**$^{**}_{GB}$	**0.59**$^{**}_{GB}$	**0.61****	**0.67****	**0.79**	**0.86**	**0.94**
		RPCA	**0.48****	**0.49**$^{**}_{FB}$	**0.53**$^{**}_{GB}$	**0.55**$^{**}_{GB}$	**0.59**$^{**}_{GB}$	**0.61****	**0.67****	**0.79**	**0.86**	**0.94**
		EM	**0.65****	**0.67****	**0.66****	**0.72***	**0.70***	**0.64****	**0.76**	**0.83**	**0.81**	**0.94**
		KF	**0.57****	**0.59****	**0.61****	**0.63****	**0.66****	**0.65****	**0.71***	**0.84**	**0.84**	**0.91**
	Unrestricted w/ AR	OPCA	1.01	1.04	1.08	1.11	1.06	**0.92**	**0.92**	**0.93**	**0.85**	**0.94**
		RPCA	1.01	1.04	1.08	1.11	1.06	**0.92**	**0.92**	**0.93**	**0.85**	**0.94**
		EM	**0.98**	**0.98**	1.04	1.18	1.14	1.03	**0.98**	1.16	**0.98**	**0.72**
		KF	**0.99**	**0.99**	1.06	1.07	1.02	**0.93**	**0.82**	1.12	**0.94**	**0.65***
	Smoothed	OPCA	**0.53****	**0.55****	**0.58****	**0.60****	**0.61****	**0.62****	**0.71****	**0.76****	**0.80****	**0.92**
		RPCA	**0.53****	**0.55****	**0.58****	**0.60****	**0.61****	**0.62****	**0.71****	**0.76****	**0.80****	**0.92**
		EM	**0.62****	**0.65****	**0.67****	**0.71****	**0.68****	**0.68****	**0.77***	**0.80**	**0.84***	**0.95**
		KF	**0.56****	**0.59****	**0.63****	**0.65****	**0.66****	**0.66****	**0.75****	**0.80***	**0.83***	**0.94**
2	Basic w/o AR	OPCA	**0.51****	**0.56****	**0.58****	**0.64****	**0.72****	**0.76***	**0.91**	1.05	1.12	1.31**
		RPCA	**0.51****	**0.56****	**0.58****	**0.64****	**0.72****	**0.76***	**0.91**	1.05	1.12	1.31**
		EM	**0.66****	**0.71***	**0.71****	**0.81**	**0.84**	**0.84**	1.00	1.11	1.14	1.31*
		KF	**0.59****	**0.61****	**0.63****	**0.72****	**0.79**	**0.81**	**0.97**	1.07	1.11	1.29*
	Basic w/ AR	OPCA	1.03	1.05	1.08	1.02	1.07	1.04	1.20	1.16	1.11	1.17
		RPCA	1.03	1.05	1.09	1.03	1.07	1.04	1.20	1.16	1.11	1.17
		EM	1.00	**0.97**	**0.99**	1.10	1.15	1.12	1.26	1.26*	1.37*	1.25
		KF	**0.97**	**0.96**	**0.99**	1.07	1.09	1.09	1.25	1.26*	1.24	1.07
	Unrestricted w/o AR	OPCA	**0.48****	**0.48**$^{**}_{GB}$	**0.54**$^{**}_{FB}$	**0.60**$^{**}_{FB}$	**0.72***	**0.75***	**0.81**	1.07	1.11	1.24*
		RPCA	**0.48****	**0.48**$^{**}_{GB}$	**0.54**$^{**}_{FB}$	**0.60**$^{**}_{FB}$	**0.72***	**0.75***	**0.81**	1.07	1.11	1.24*
		EM	**0.60****	**0.65****	**0.67****	**0.78**	**0.80**	**0.74**	**0.93**	1.03	**0.99**	1.26
		KF	**0.56****	**0.58****	**0.63****	**0.68****	**0.70***	**0.70**$^{*}_{FB}$	**0.77**$_{FB}$	1.02	1.03	1.26
	Unrestricted w/ AR	OPCA	**0.96**	1.04	1.07	1.17	1.13	**0.97**	1.02	1.37**	1.17	1.15
		RPCA	**0.96**	1.04	1.07	1.17	1.13	**0.97**	1.02	1.37**	1.17	1.15
		EM	**1.00**	**0.95**	1.01	1.03	1.12	1.15	**0.90**	1.40	1.63	1.14
		KF	**0.92**	**0.96**	**0.94**	**0.97**	**0.89**	**0.79**	**0.86**	**0.83**$_{FB}$	**0.88**$_{FB}$	**0.91**$_{FB}$
	Smoothed	OPCA	**0.43**$^{**}_{GB}$	**0.50****	**0.56****	**0.65****	**0.69**$^{**}_{FB}$	**0.75***	**0.90**	**0.98**	1.04	1.17
		RPCA	**0.43****	**0.50****	**0.56****	**0.65****	**0.69**$^{**}_{FB}$	**0.75***	**0.90**	**0.98**	1.04	1.17
		EM	**0.56****	**0.61****	**0.68****	**0.73****	**0.76**	**0.81**	**0.95**	1.02	1.05	1.18
		KF	**0.52****	**0.57****	**0.64****	**0.70****	**0.74***	**0.79**	**0.93**	**1.00**	1.03	1.15

	Basic w/o AR	OPCA	**0.86**	**0.91**	**0.95**	1.03	1.16	1.40	1.57	2.36	2.97	3.27
		RPCA	**0.87**	**0.91**	**0.95**	1.03	1.18	1.41	1.57	2.28	2.94	3.10
		EM	1.11	1.12	1.04	1.05	1.97	1.41	1.34	2.22	2.61	3.31
		KF	**0.98**	1.05	1.05	1.19	1.20	1.16	1.62	2.33	3.12	4.44
	Basic w/ AR	OPCA	**0.90**	**0.92**	**0.96**	1.08	1.27	1.50	1.44	1.64	3.78	3.79
		RPCA	**0.90**	**0.92**	**0.95**	1.08	1.27	1.51	1.44	1.64	3.71	3.78
		EM	1.08	1.08	1.01	1.12	1.69	1.68	1.47	3.15	3.39	1.22
		KF	1.04	1.00	1.12	1.08	1.28	1.28	1.47	1.96	3.40	4.84
6	Unrestricted w/o AR	OPCA	1.03	**0.89**	**0.81**$_{\mathbf{FB}}$	1.52	1.62	1.65	3.43	2.15	4.49	10.69
		RPCA	1.03	**0.89**	**0.81**	1.52	1.62	1.65	3.43	2.15	4.49	10.69
		EM	1.17	1.08	1.19	1.28	1.76**	1.20	1.57	2.95*	2.06	4.83*
		KF	1.04	1.02	**0.92**	**0.98**	1.68*	1.46	1.37	2.82*	2.74	1.85
	Unrestricted w/ AR	OPCA	**0.89**	**0.94**	**0.83**	**0.88**	1.46	1.41	1.15	2.05	3.92	4.40**
		RPCA	**0.89**	**0.94**	**0.83**	**0.88**	1.46	1.41	1.15	2.05	3.92	4.40**
		EM	1.15	1.20	1.08	1.25	1.98*	**0.97**	1.69	4.95*	2.13	4.07*
		KF	**0.94**	1.07	**0.81**$_{\mathbf{FB}}$	1.17	1.83*	1.75	1.13	4.63*	1.94	1.77
	Smoothed	OPCA	**0.80***	**0.84**	**0.86**	**0.88**	**0.89**	**0.87**	**0.97**	1.04	**0.99**$_{\mathbf{FB}}$	1.20
		RPCA	**0.80**$^{*}_{\mathbf{FB}}$	**0.84**	**0.86**	**0.88**	**0.89**	**0.87**	**0.97**	1.04	**0.99**$_{\mathbf{FB}}$	1.20
		EM	**0.86**	**0.84**$_{\mathbf{FB}}$	**0.87**	**0.85**$_{\mathbf{FB}}$	**0.87**$_{\mathbf{FB}}$	**0.82**$_{\mathbf{FB}}$	**0.96**$_{\mathbf{FB}}$	1.01	1.01	1.17
		KF	**0.90**	**0.90**	**0.92**	**0.91**	**0.91**	**0.89**	1.00	1.04	1.03	1.17

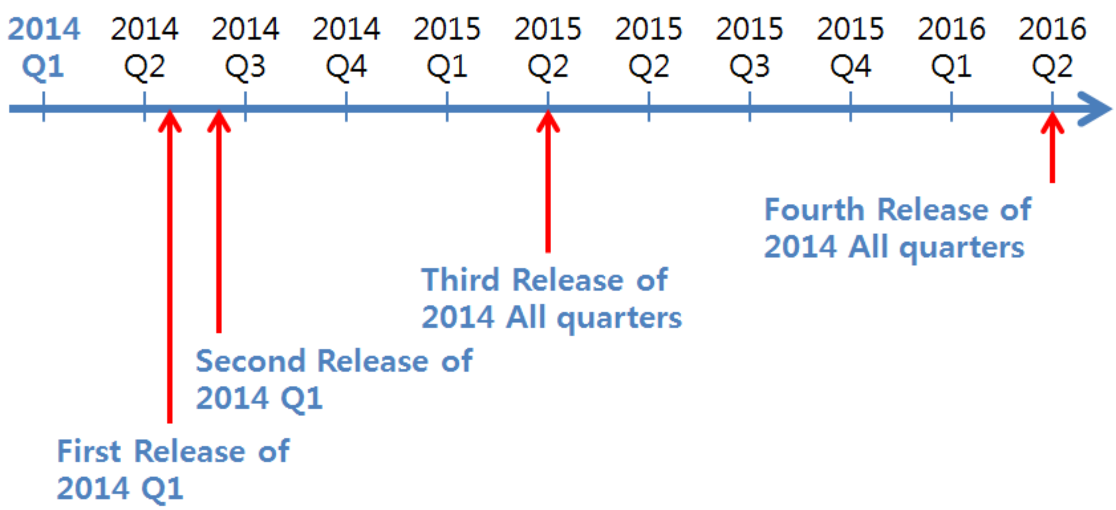
* Notes: See notes to Tables 1-4. Entries in this table are ratios of point MSFEs of our benchmark or 'strawman' AR(SIC) model to each other model, for various estimation methods and horizons. Panel (a) reports MSFEs based on experiments using 'first available' real-time quarterly historical data, and Panel (b) reports results based on the use of 'most recent' real-time quarterly historical data. All results are based on recursively estimated models. The column denoted by 'Pastcast' contains MSFEs for quarterly forecasts of GDP made 1-month prior to the calendar date of the quarterly GDP datum being predicted, and the columns denoted by 'Nowcast' contain MSFEs for forecasts of the first, second and third months of each quarterly calendar dated GDP observation. Finally, the columns denoted by 'Forecast' contain MSFEs based on 1-quarter ahead predictions made from 1 month after the end of the quarter (called month 4) to 3 month ahead (called month 6). Months 7-9 correspondingly refer to 2-quarter ahead predictions. Bold entires denote cases for which the point MSFE of a given model is lower than the point MSFE of the AR(SIC) model. Entries superscripted by a ** (5% level) and a * (10% level) are significantly better than the AR(SIC) model, based on application of the DM predictive accuracy test. Finally, entries subscripted with 'FB' denote the MSFE-best models for a given number of estimated factors and for each horizon, while entries subscripted with 'GB' denote MSFE-best models across all specification permutations, for a given horizon. See Section 5 for complete details.

Table 6: Summary of MSFE-Best Models Across All Modelling Permutations*

	Fac. No.	Pastcast prev. qtr. -1	1	Nowcast current quarter 2	3	4	Forecast 1 quarter ahead 5	6	7	2 quarter ahead 8	9
First Available	All	Basic w/ AR OPCA	Basic w/ AR OPCA	Basic w/ AR KF	Basic w/ AR OPCA	Basic w/ AR OPCA	Basic w/ AR KF	Basic w/o AR RPCA	Basic w/ AR OPCA	Basic w/ AR KF	Basic w/o AR PCA
	1	Basic w/ AR OPCA	Basic w/ AR OPCA	Unrestricted w/ AR OPCA	Unrestricted w/ AR OPCA	Unrestricted w/ AR RPCA	Unrestricted w/ AR OPCA	Unrestricted w/ AR OPCA	Unrestricted w/ AR KF	Unrestricted w/ AR KF	Unrestricted w/ AR Both PCAs
	2	Basic w/ AR OPCA	Basic w/ AR RPCA	Unrestricted w/ AR KF	Unrestricted w/ AR OPCA	Unrestricted w/ AR OPCA	Unrestricted w/o AR KF	Unrestricted w/ AR RPCA	Unrestricted w/ AR OPCA	Unrestricted w/o AR KF	OPCA
	6	Basic w/ AR OPCA	AR	Basic w/ AR KF	Smoothed RPCA	Smoothed Both PCAs	Smoothed Both PCAs	Smoothed Both PCAs	Smoothed Both PCAs	Smoothed Both PCAs	Smoothed Both PCAs
Most Recent	All	Smoothed PCA	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Basic w/o AR OPCA	Basic w/ AR KF	Basic w/ AR KF	Basic w/ AR KF	Basic w/ AR KF
	1	Unrestricted w/o AR OPCA	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Basic w/o AR OPCA	Basic w/ AR KF	Basic w/ AR KF	Basic w/ AR KF	Basic w/ AR KF
	2	Smoothed OPCA	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Unrestricted w/o AR Both PCAs	Smoothed Both PCAs	Unrestricted w/o AR KF	Unrestricted w/o AR KF	Unrestricted w/ AR KF	Unrestricted w/ AR KF	Unrestricted w/ AR KF
	6	Smoothed RPCA	Smoothed EM	Basic w/ AR KF	Smoothed EM	Smoothed EM	Smoothed EM	Smoothed EM	AR	Smoothed Both PCAs	AR

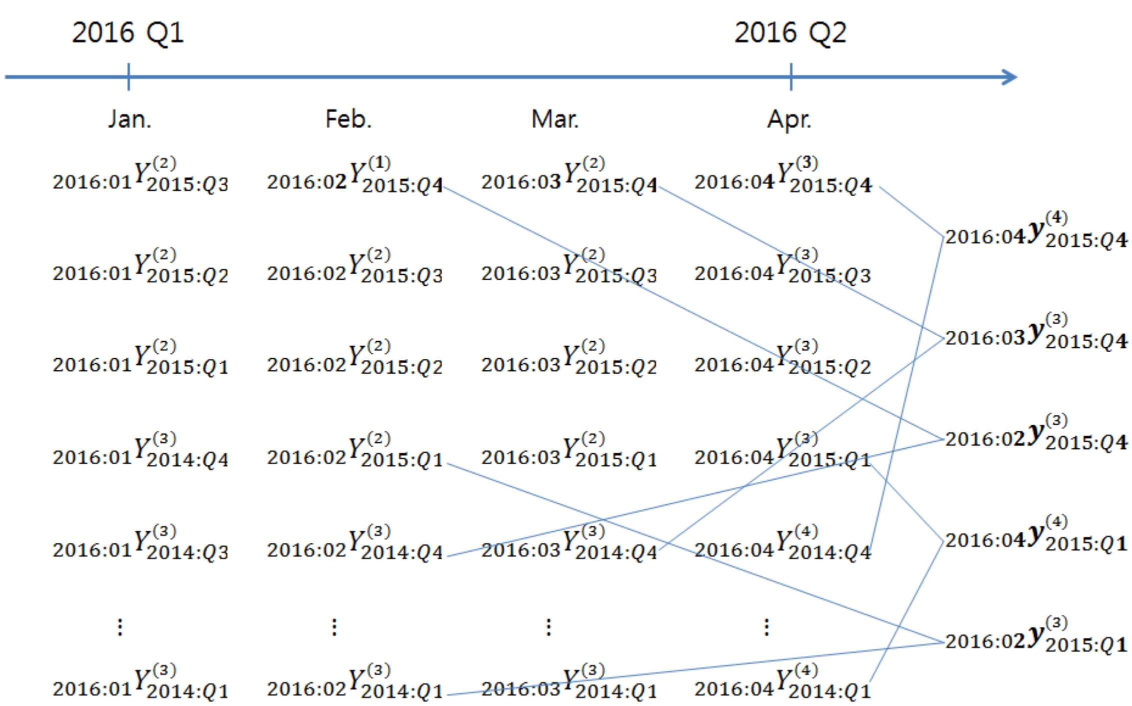
* Notes: See notes to Table 5. Entries indicate the model and estimation methods for all ‘MSFE-best’ specifications, by historical data type, number of factors used, and horizon. Entries in the row labeled ‘All’ are the ‘MSFE-best’ models across all factor specifications, for a given historical data type. All model estimation is done recursively and AR interpolation is used for missing value construction. For example, for the ‘Pastcast’ horizon, the ‘Basic factor-MIDAS’ model with AR terms and with factors estimated using OPCA is the ‘globally best’ performer when experiments are conducted using ‘first available’ real-time historical data. When MSFEs based on the use of OPCA and RPCA are the same up to three decimal places, the PCA method is denoted by ‘both PCA’.

Figure 1: Release Dates for Real-Time Korean GDP*



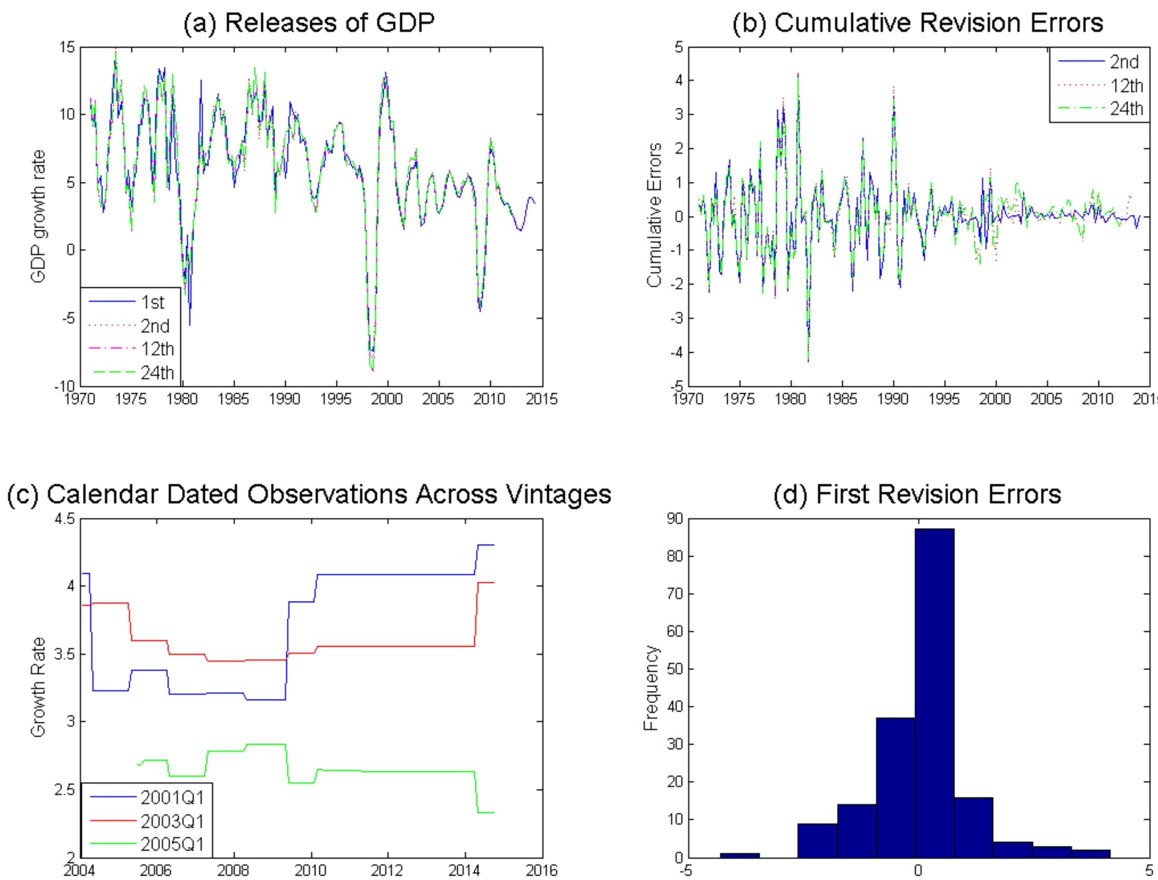
* Notes: See Sections 2 and 3 for complete details.

Figure 2: Depiction of Annualized GDP Growth Rates Based on Real-Time Data*



* Notes: See Section 2 for a detailed discussion of the dating conventions used in this diagram.

Figure 3: Historical Real-Time Data Releases for Korean GDP*



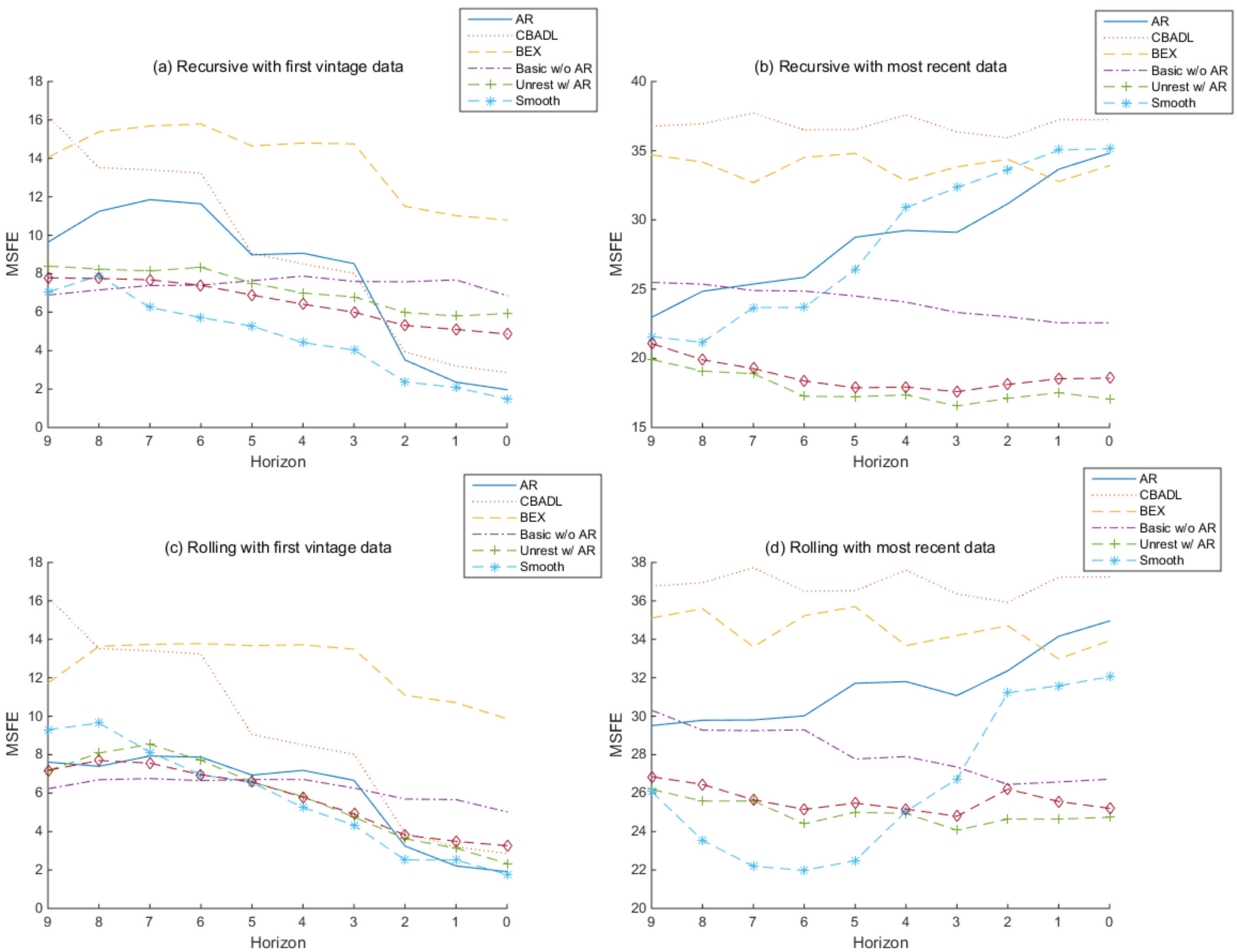
* Notes: In Figure (a), the solid line depicts 1st release GDP, the dotted line depicts 2nd release data, and the dot-dash line depicts final release data. Figure (b) depicts cumulative revision errors between the 1st and either the 2nd, 12th, or 24th releases. Figure (c) shows how the growth rate of 2001:1, 2003:1 and 2005:1 calendar dated GDP evolves as the series is revised. Figure (d) plots the distribution of first revision errors (i.e., the differences between 1st and 2nd data releases).

Figure 4: Structure of Monthly/Quarterly Prediction Experiments*

Month	Pastcast	Nowcast	Forecast		
2010:01	${}_{2010:01}\hat{Y}_{2009:Q4}$	${}_{2010:01}\hat{Y}_{2010:Q1}$	${}_{2010:01}\hat{Y}_{2010:Q2}$	${}_{2010:01}\hat{Y}_{2010:Q3}$	${}_{2010:01}\hat{Y}_{2010:Q4}$
2010:02	-	${}_{2010:02}\hat{Y}_{2010:Q1}$	${}_{2010:02}\hat{Y}_{2010:Q2}$	${}_{2010:02}\hat{Y}_{2010:Q3}$	${}_{2010:02}\hat{Y}_{2010:Q4}$
2010:03	-	${}_{2010:03}\hat{Y}_{2010:Q1}$	${}_{2010:03}\hat{Y}_{2010:Q2}$	${}_{2010:03}\hat{Y}_{2010:Q3}$	${}_{2010:03}\hat{Y}_{2010:Q4}$
2010:04	${}_{2010:04}\hat{Y}_{2010:Q1}$	${}_{2010:04}\hat{Y}_{2010:Q2}$	${}_{2010:04}\hat{Y}_{2010:Q3}$	${}_{2010:04}\hat{Y}_{2010:Q4}$	${}_{2010:04}\hat{Y}_{2011:Q1}$
2010:05	-	${}_{2010:05}\hat{Y}_{2010:Q1}$	${}_{2010:05}\hat{Y}_{2010:Q3}$	${}_{2010:05}\hat{Y}_{2010:Q4}$	${}_{2010:05}\hat{Y}_{2011:Q1}$
2010:06	-	${}_{2010:06}\hat{Y}_{2010:Q1}$	${}_{2010:06}\hat{Y}_{2010:Q3}$	${}_{2010:06}\hat{Y}_{2010:Q4}$	${}_{2010:06}\hat{Y}_{2011:Q1}$
⋮	⋮	⋮	⋮	⋮	⋮

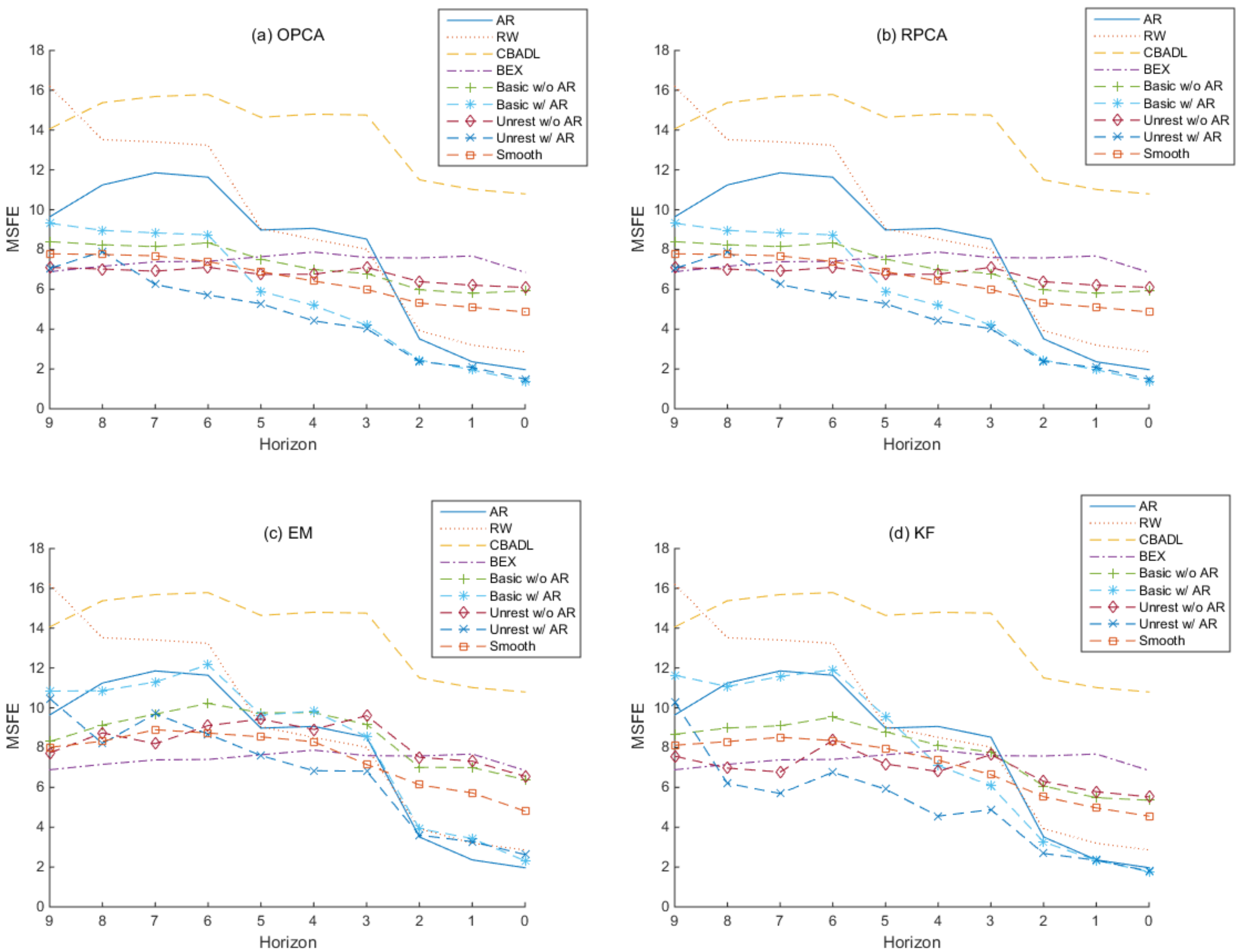
* Notes: This table describes the timing of monthly pastcasts, nowcasts, and forecasts of quarterly GDP. For example, in 2010:01, we pastcast the GDP growth of 2009:Q4, since its value is not available yet in 2009:Q4; and we nowcast all three months in 2010:Q1. Finally, at the same point in time, we also create monthly forecasts of GDP at 2010:Q2 and 2010:Q3. In the next month, 2010:02, we do not pastcast 2009:Q4, since its value is now available. For complete details, refer to Section 5.

Figure 5: Forecasting Using 'First Available vs. 'Most Recent' Real-Time Data*



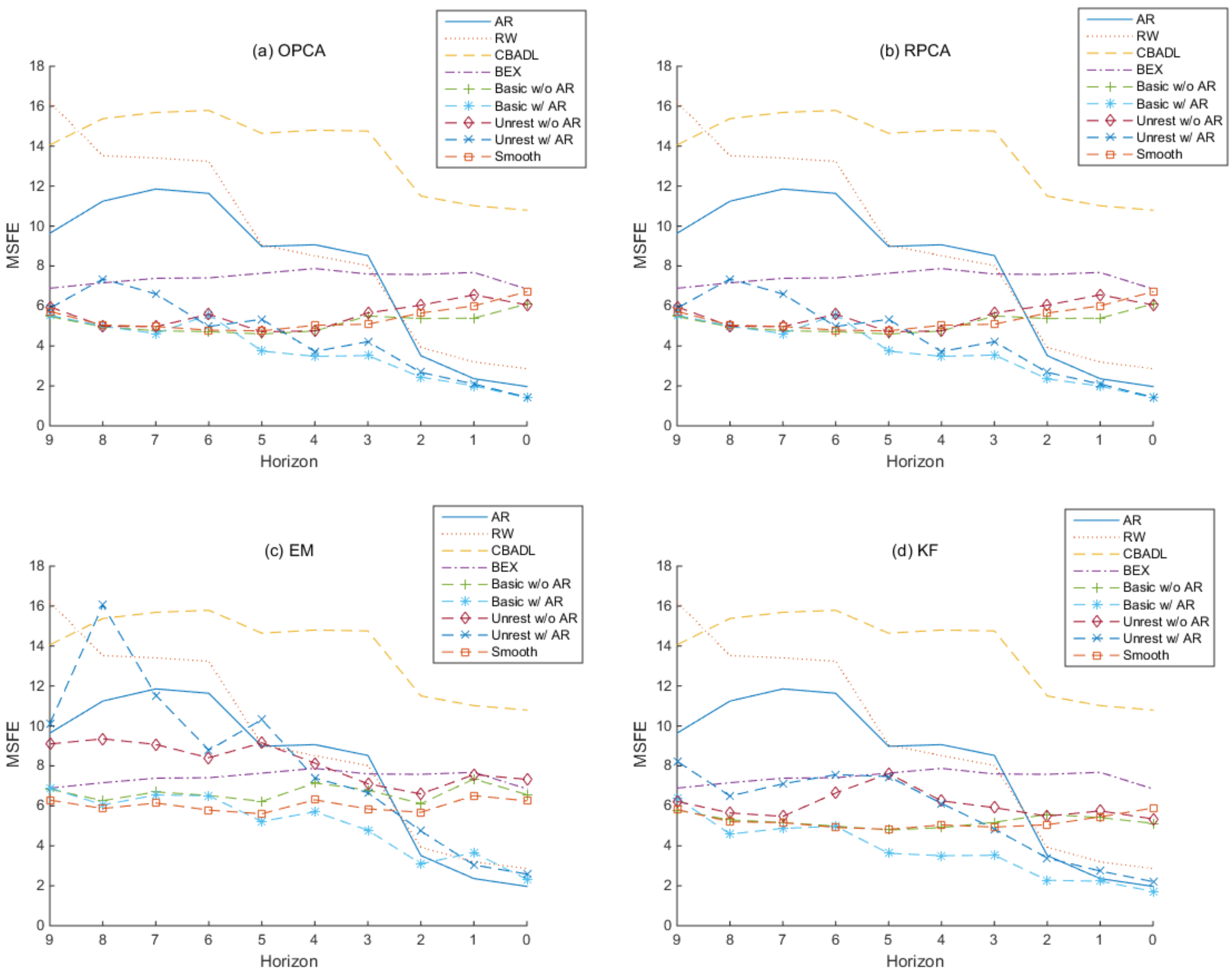
* Notes: This figure plots MSFEs for various models estimated using recursive and rolling data windows, based on either 'first available' or 'most recent' real-time historical data. Factor-MIDAS models are estimated using OPCA and AR interpolation, and horizons (depicted on the horizontal axes of the graphs) range from nine months ahead (forecasts) to zero months ahead (pastcasts). See Section 5 for complete details.

Figure 6: MSFEs of Forecasting Models Constructed Using One Factor ($r = 1$)*



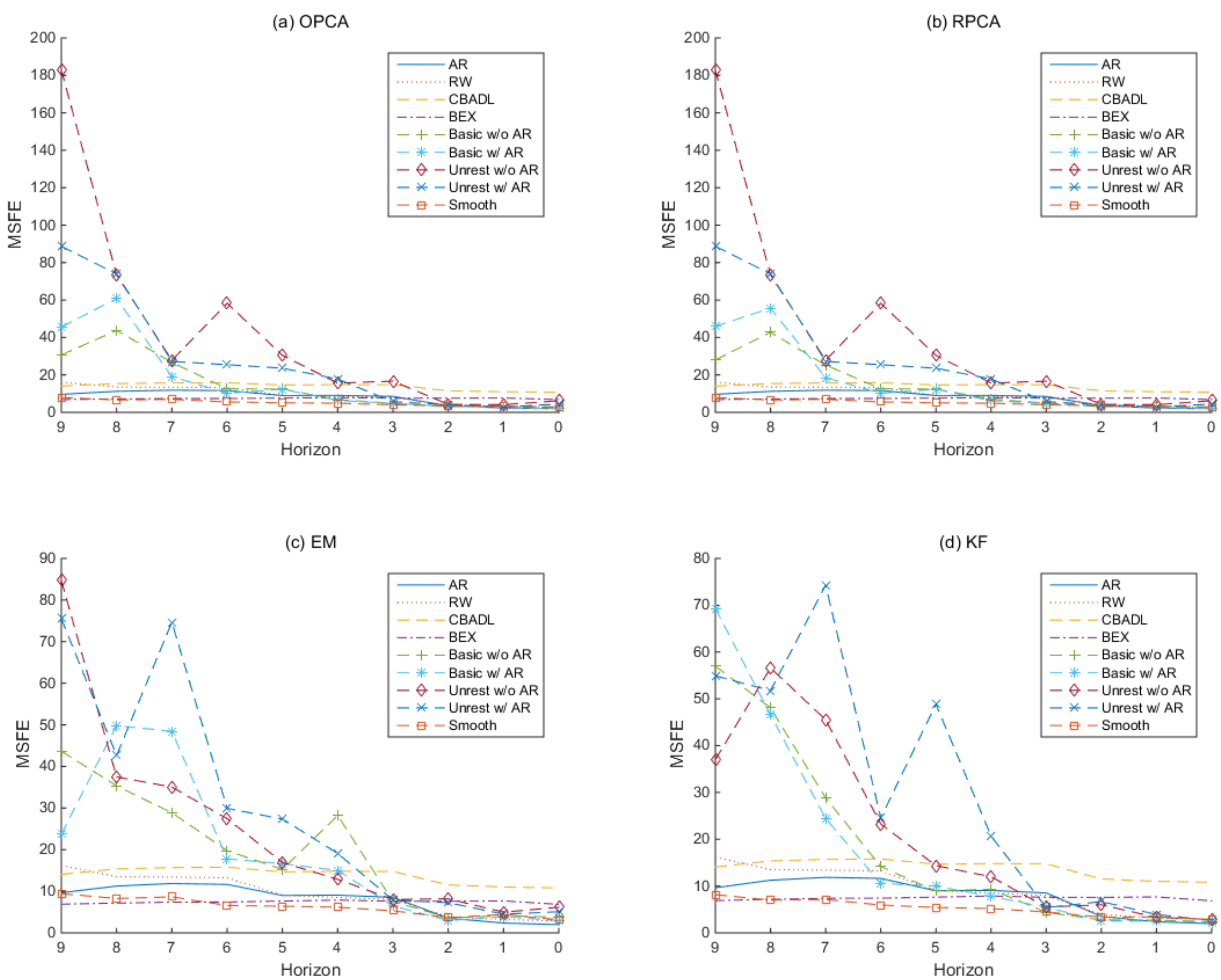
* Notes: See notes to Figure 5. Each panel plots the MSFEs of various models for a different estimation method. Benchmark models, including AR, CBADL and BEX, are redundantly included in all panels of this figure, for comparability. 'Basic w/o AR' and 'Basic w/ AR' are the basic factor-MIDAS models with and without AR terms. 'Unrest' and 'Smooth' denote alternative factor-MIDAS specifications (see Section 4). OPCA and RPCA are implemented with AR interpolation, and all forecasts are based on recursively estimated models. See Section 5 for complete details.

Figure 7: MSFEs of Forecasting Models Constructed Using Two Factors ($r = 2$)*



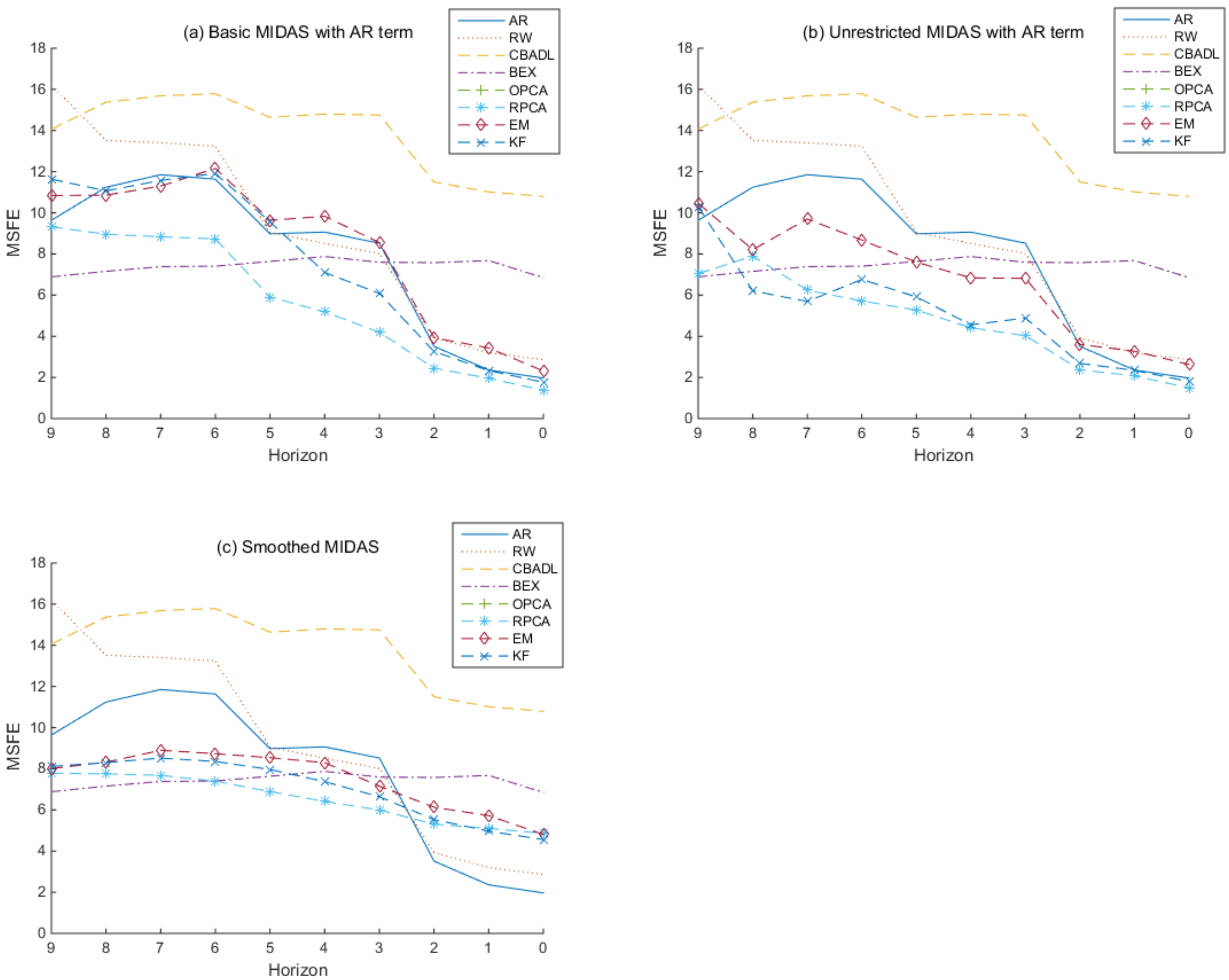
* Notes: See the Notes to Figure 6.

Figure 8: MSFEs of Forecasting Models Constructed Using Six Factors ($r = 6$)*



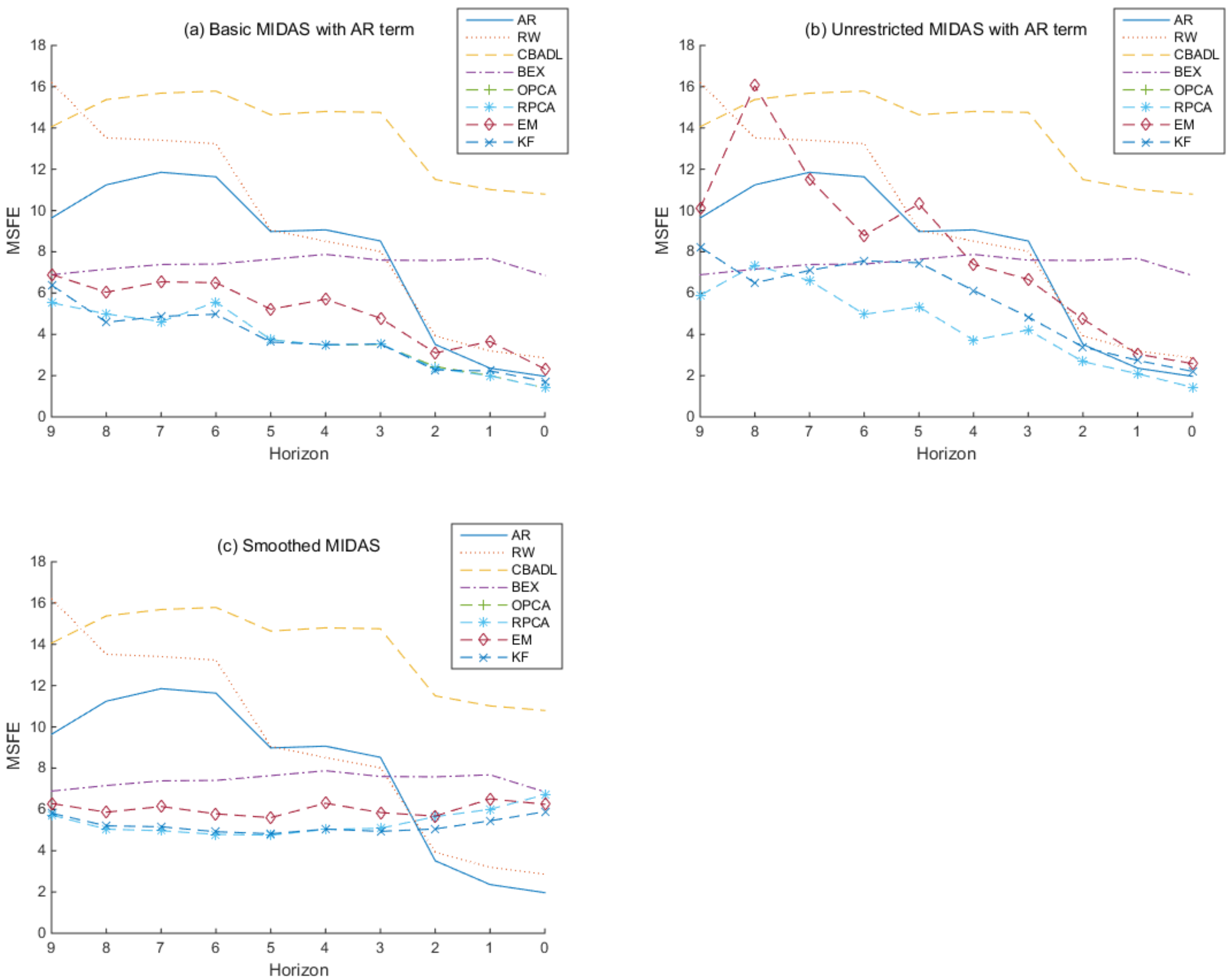
* Notes: See the Notes to Figure 6.

Figure 9: MSFEs of Factor-MIDAS Models with One Factor ($r = 1$)*



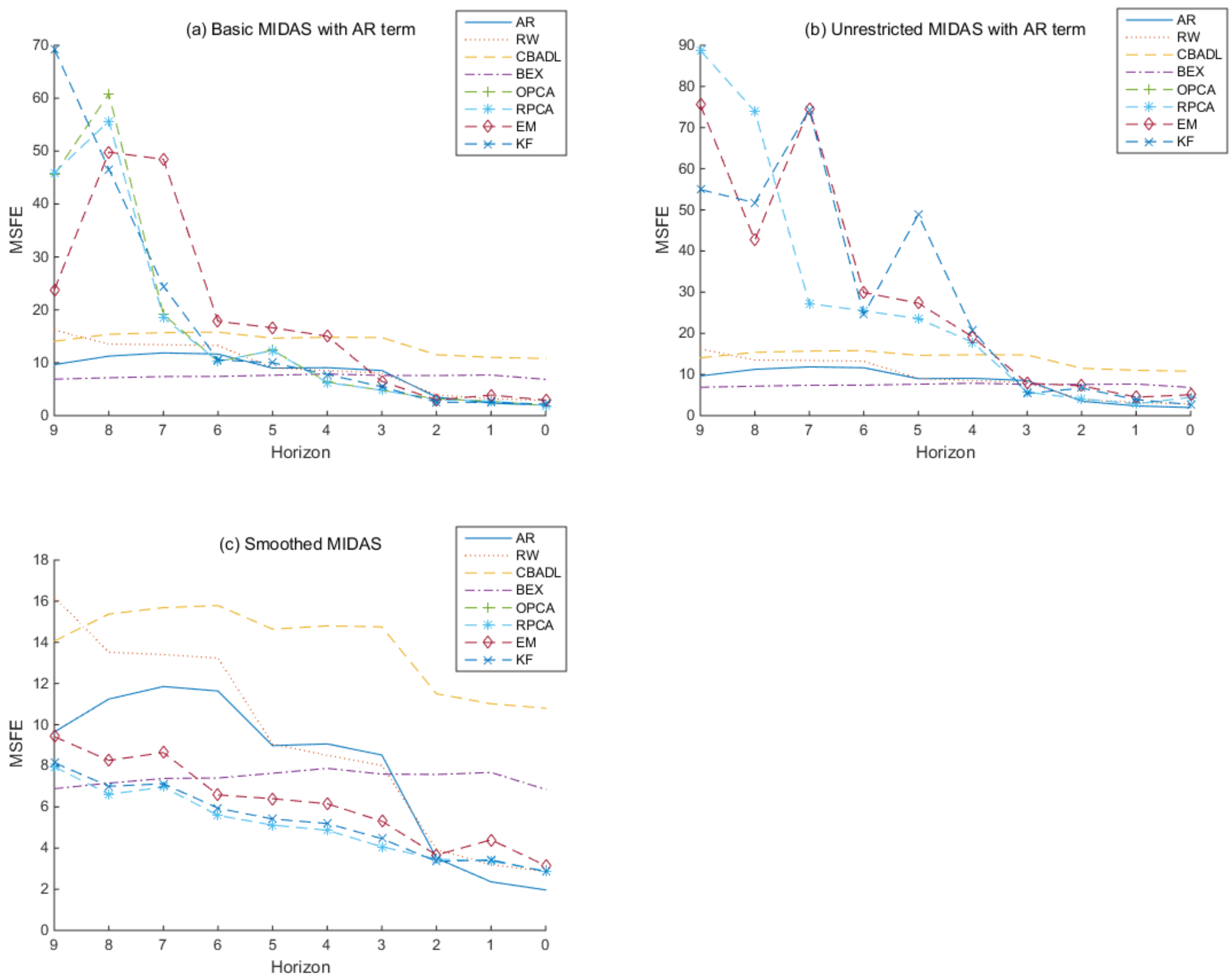
* Notes: See the Notes to 6.

Figure 10: MSFEs of Factor-MIDAS Models with with Two Factors ($r = 2$)*



* Notes: See the Notes to 6.

Figure 11: MSFEs of Factor-MIDAS Models with with Six Factors, $(r = 6)$*



*Notes: See the Notes to 6.