How Sticky Is Sticky Enough? A Distributional and Impulse Response Analysis of New Keynesian DSGE Models

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Abstract

In this paper, we add to the literature on the assessment of how well RBC simulated data reproduce the dynamic features of historical data. In particular, we evaluate a variety of new Keynesian DSGE models, including the standard sticky price model discussed in Calvo (1983), the sticky price with dynamic indexation model discussed in Christiano, Eichenbaum and Evans (2001), Smets and Wouters (2003), and Del Negro and Schorfheide (2005), and the sticky information model of Mankiw and Reis (2002). We carry out our evaluation by using standard impulse response and correlation measures and via use of a distribution based approach for comparing all of our (possibly) misspecified DSGE models via direct comparison of simulated inflation and output gap values with corresponding historical values. In this sense, our analysis can be thought of as an empirical model selection exercise. In addition, and given that one of our objectives is to choose the model which yields simulation distributions that are closest to the historical record, our analysis can be viewed as a type of predictive density model selection, where the "best" simulated distributions can be used as predictive densities whenever the starting values for the simulations correspond to those actual historical values which are most recently available. Some important precedents to our approach to accuracy assessment include DeJong, Ingram, and Whiteman (1996) and Geweke (1999a,b). One of our main findings is that for a standard level of stickiness (i.e. annual price or information adjustment), the sticky price model with indexation dominates other models. However, when models are calibrated using the lower level of information and price stickiness, there is much less to choose from between the models.

1 Introduction

Of critical importance in the analysis of stochastic dynamic general equilibrium models is the reconciliation of historical and simulation based empirical evidence. A partial list of recent advances in this area includes: (i) the examination of how RBC simulated data reproduce the covariance and autocorrelation functions of actual time series and second moments in general (see e.g. Watson (1993) and Cogley and Nason (1995a)); (ii) the comparison of RBC and historical spectral densities (see e.g. Diebold, Ohanian and Berkowitz (1998)); (iii) the evaluation of the difference between the second order time series properties of vector autoregression (VAR) predictions and out-of-sample predictions from RBC models (see e.g. Rotemberg and Woodford (1996) and Schmitt-Grohe (2000)); (iv) the construction of Bayesian odds ratios for comparing RBC models with unrestricted VAR models (see e.g. Schorfheide (2000), Chang, Gomes and Schorfheide (2002), and Fernandez-Villaverde and Rubio-Ramirez (2004)); (v) the comparison of historical and simulated data impulse response functions (e.g. Cogley and Nason (1993), (1995a)); and (vi) the formulation of "reality" bounds for measuring how close the density of an RBC model is to the density associated with an unrestricted VAR model (see e.g. Bierens and Swanson (2000) and Bierens (2003)). Of note is that the papers cited above are mainly concerned with the issue of model evaluation (i.e. with the problem of measuring how well a given model fits certain aspects of actual time series), and that the papers usually address the case in which the objective is to test for the "correct specification" of some aspects of a given candidate model. In the case of real business cycle models, however, we view it as crucial to account for the fact that all models may well be approximations, and so are misspecified (i.e. no models are "correctly specified"). Thus, we posit that the notion of "correct specification" may be inappropriate when comparing alternative DSGE models. For this reason, one of the tools that we use in our empirical analysis is the distributional accuracy assessment methodology of Corradi and Swanson (2005a,b). It should be noted, however, that the idea of allowing for misspecification is not new. For example, important papers by DeJong, Ingram and Whiteman (1996), and Geweke (1999a,b) allow for calibrator's uncertainty with respect to model specification, parameter values, and the actual data's support of the moments of interest. In this sense, their Bayesian calibration approach offers many of the same tools as those used in these

paper that are due to Corradi and Swanson.[1]

Our intent in this paper is to add to the first strand of the literature enumerated above. We do this in conjunction with the evaluation of a variety of currently available new Keynesian DSGE models. In particular, the models that we consider include the standard sticky price model discussed in Calvo (1983), the sticky price with dynamic indexation model discussed in Christiano, Eichenbaum and Evans (2001), Smets and Wouters (2003), and Del Negro and Schorfheide (2005), and the sticky information model of Mankiw and Reis (2002). We carry out our evaluation by: (i) using standard impulse response and correlation measures, and (ii) using a distribution based approach for comparing all of our (possibly) misspecified DSGE models via direct analysis of simulated and historical inflation and output gap distributions. In this sense, our analysis can be thought of as an empirical model selection exercise. In addition, and given that one of our objectives is to choose the model which yields simulation distributions that are closest to the historical record, our analysis can be viewed as a type of predictive density model selection, where the "best" simulated distributions can be used as predictive densities whenever the starting values for the simulations correspond to those actual historical values which are most recently available.

This paper also contributes to the discussion of shortcomings of new Keynesian Phillips curves. In particular, we use a series of theoretical experiments to show, among other things, that the Ball (1994) critique of the standard new Keynesian Phillips curve (derived under static sticky price assumptions) may be not robust to experiment design and parametrization. In this sense, our experiments add to earlier related evidence presented in Mankiw and Reis (2002) and Trabandt (2005).

The impetus for our study comes from the observation that new Keynesian Phillips curves derived under standard sticky price assumptions have several shortcomings. For example, Ball (1994) has found that such models yield the controversial result that an announced credible disinflation causes booms rather than recessions. Additionally, Fuhrer and Moore (1995) show that the New Keynesian Phillips curve falls short when used to explain inflation persistence, one of the stylized empirical facts describing US inflation. Furthermore, Mankiw and Reis (2002) note that such models have trouble explaining why shocks to monetary policy have delayed and gradual effects on

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[1]Furthermore, Smith (1993) and Gourieroux, Monfort and Renault (1993) fully account for the fact that DSGE models are misspecified. Additionally, the source of misspecification in these papers can be induced by both linear approximation solution methods and the fact that the DSGE model is an abstraction of the true model.

inflation.[2]

Some of the problems outlined in the previous paragraph are addressed in a series of important papers, including those of Christiano et al. (2001), Smets and Wouters (2003), and Del Negro and Schorfheide (2005) - sticky prices with dynamic indexation, and Mankiw and Reis (2002) - sticky information. For example, Mankiw and Reis posit that information about macroeconomic conditions spreads slowly because of information acquisition and/or re-optimization costs. Compared to the standard sticky price model, prices in this setup are always readjusted, but decisions about prices are not always based on the latest available information. The model is representative of the wider class of Rational Inattention (RI) models developed by Phelps (1970), Lucas (1973), and more recently by Mankiw and Reis (2002), Sims (2003), and Woodford (2003b). As might be expected, the three models that we consider have very different properties. For example, Ball, Mankiw and Reis (2003) show that implications with regard to optimal monetary policy are quite different for sticky price and sticky information models. In the sticky price model, inflation enters the loss function, which leads to inflation targeting. It is thus optimal to allow inflation drift in this model. On the other hand, in the sticky information model, inflation drift or inflation targeting is a suboptimal policy, as it is optimal to target the price level. These sorts of model implications suggest that the dynamic properties of the alternative models may be quite different, in turn implying that our distributional comparison of historical and simulated inflation and the output gap measures may uncover interesting new evidence concerning the relative merits of the models. Put another way, our approach allows us to shed light on the issue of whether theoretical advantages translate into a better empirical fit, and if not, then why not?

From a theoretical perspective, we follow the sticky information approach of Mankiw and Reis (2002), who suggest that a "more realistic" aggregated demand specification is desirable. In particular, we extend their sticky information model by specifying standard consumer preferences and money demand.[3] Thus, aggregate demand is derived from intertemporal household maximization, rather than from a static quantity-theory type of model. This is especially important since we are interested in assessing the empirical performance of alternative models. In addition, we assume

[2] Bernanke and Gertler (1995) and Christiano, Eichenbaum and Evans (2000) present empirical evidence supporting this problem noted by Mankiw and Reis (2002).

[3] Our approach to incorporating consumer preferences and money demand is similar to the approach used by Gali (2002) and Woodford (2003a).

dynamic inflation indexation, as is also commonly done in empirical applications.

In addition to shedding light on the relative merits of various new Keynesian DSGE models, one of the contributions of this paper is the implementation of a distribution-based approach for comparing DSGE models. In particular, it is our intent to add to the model evaluation literature by introducing a measure of "goodness of fit" of DSGE models that is based on applying standard notions of Kolmogorov distance and drawing on recent advances in the theory of the bootstrap. As opposed to the common practice of testing for the correct specification of some aspects of a given candidate model, we evaluate the overall distributional fit of our candidate models, assuming that all models are potentially misspecified. We thus follow the approach recently elucidated by Corradi and Swanson (2005a,b,c). To be more precise, the approach we take begins by fixing a given DSGE model as the "benchmark" model, against which all "alternative" models are compared. We then form statistics based on the comparison between the empirical distribution of the historical series and that of the simulated series. These statistics can be viewed as distributional analogs of the mean square error based statistical tests discussed in Diebold and Mariano (1995) and White (2000). The limiting distribution of the test statistics is a Gaussian process with a covariance kernel that reflects the effect of (possible) dynamic misspecification, simulation error, and measurement error in the actual data.[4] This limiting distribution is thus not nuisance parameter free; thus critical values cannot be tabulated. In order to obtain valid asymptotic critical values, we outline two block bootstrap procedures that depend on the relative rate of growth of the actual and simulated sample sizes.

Our findings can be summarized as follows. First, we question the extent to which the sticky information model has better theoretical properties than the sticky price model. In particular, we show in our theoretical experiments that the Ball (1994) critique is to some extent limited to the special case of anticipated disinflation with low semi-elasticity of money demand with respect to interest rate. Output booms that proceed disinflation do not occur in experiments with unantici- pated disinflation or in the case if the interest rate semi-elasticity is very high. These findings are in accord with results presented in Mankiw and Reis (2002) and Trabandt (2005).

Second, for a standard level of stickiness (i.e. annual price or information adjustment), we find

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[4]In general, the testing procedure used in this paper (see discussion in Section 5 and the Appendix) can also accommodate parameter estimation error, for cases where parameters are estimated rather than calibrated (see Corradi and Swanson (2005b)).

that the sticky price model with indexation dominates other alternatives. For example, the joint distribution of inflation and the output gap simulated from a sticky price model with indexation is "closest" to the historical distribution. Informally, we define the level of "closeness" to be the distance between distribution and density graphs and distributional quantile differences when comparing both historical and simulated data. More formally, we evaluate "closeness" using the distributional accuracy testing framework of Corradi and Swanson (2005b) that is discussed above. Of note is that data simulated using the sticky price model with indexation also yields the "closest" fit to historical data based on other "goodness of fit" measures, like auto- and cross-correlations of inflation and the output gap, contingency tables, and the relationship between level of economic activity and inflation growth (i.e. the so called acceleration phenomena). We conclude that these results arise because the model has the largest response of inflation and the smallest response of the output gap, after a shock occurs.

Third, in our empirical analysis we find evidence for a lower level of stickiness (i.e. twice annual price or information adjustment) than the commonly assumed annual adjustment. For example, simulated inflation and output gap data from all models are much "closer" to historical levels when an adjustment occurs twice annually.

Fourth, when the alternative models are calibrated using the lower level of information and price stickiness, there is much less to choose from between the sticky price, sticky price with indexation, and sticky information models. One reason why the sticky information model does not dominate both versions of the sticky price model (i.e. with higher and lower degrees of stickiness) is because we use what we view as realistic models of the persistence of exogenous shocks. In addition, the lower level of stickiness reduces the delays in the response of inflation to monetary policy shocks, so that the effect of sluggish inflation responses generated by the sticky price model with indexation and the sticky information model is no longer strong enough to result in overall dominance of the sticky price model.

The rest of the paper is organized as follows. Section 2 outlines our DSGE models, in which the New Keynesian Phillips curve is derived under sticky price, sticky price with indexation, and sticky information assumptions. In Section 3, we describe the data used to construct historical measures of inflation and the output gap. Baseline calibration of the DSGE model is discussed, and theoretical impulse response functions are evaluated in Section 4. Section 5 briefly outlines our distributional accuracy testing framework, with further details given in an appendix. The empirical results of

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our informal and formal comparisons of the sticky price, sticky price with indexation and sticky information models are gathered in Section 6. Finally, concluding remarks are given in Section 7.

2   New Keynesian DSGE Models for Inflation and the Output Gap

In this section we outline the sticky price, sticky information and sticky price with indexation models that will be compared and contrasted via impulse response, correlation, and, most importantly, distributional comparison. Our presentation of the models follows closely along the lines of Gali (2002) and Woodford (2003a).

2.1   Households

Assume that the representative consumer's preferences are represented by the following utility function:

$$U(C_t, N_t(i)) = \frac{C_t^{(1-\sigma)}}{1 - \sigma} - \int_0^1 \frac{N_t(i)^{(1+\varphi)}}{1 + \varphi} di, \tag{1}$$

where $N_t(i)$ denotes the quantity of labor supplied by a consumer of type "i", and C_t is an index of the different goods consumed. We assume a factor specific labor market, so that production of good i requires labor of type i to be used. The parameter σ is the inverse of the intertemporal elasticity of substitution, and the parameter φ is the inverse of the elasticity of labor supply. Assume further that C_t is a constant-elasticity-of-substitution index, namely:

$$C_t = \left(\int_0^1 C_t(i)^{(\frac{\varepsilon-1}{\varepsilon})} di \right)^{(\frac{\varepsilon}{\varepsilon-1})},$$

where $\varepsilon < 0$. The corresponding price index, P_t, is given by:

$$P_t = \left(\int_0^1 P_t(i)^{(1-\varepsilon)} di \right)^{(\frac{1}{1-\varepsilon})},$$

where $P_t(i)$ denotes the price of good $i \in [0, 1]$.

Subject to a standard sequence of budget constraints and a solvency condition, the solution to the consumer's optimization problem can be summarized in log-linear ($x_t = \ln X_t$) form by two static conditions:

$$c_t(i) = -\varepsilon \left(p_t(i) - p_t \right) + c_t \tag{2}$$

and

$$w_t(i) - p_t = \sigma\, c_t + \varphi\, n_t(i), \tag{3}$$

where $w_t(i)$ is the log nominal wage paid for labor type i; and by the intertemporal Euler equation:

$$c_t = -\frac{1}{\sigma}(r_t - E_t\, \pi_{t+1} - \rho) + E_t\, c_{t+1}, \tag{4}$$

where r_t is the yield on a nominal riskless one period bond (i.e. the nominal interest rate), π_{t+1} is the rate of inflation between t and $t+1$, $\rho = -\ln\beta$ represents the time discount rate (as well as the steady state real interest rate, given the absence of secular growth), and β is the subjective discount factor.

Following Gali (2002) we postulate (without derivation) a standard money demand equation:

$$m_t - p_t = y_t - \eta\, r_t, \tag{5}$$

which has unit income elasticity.

2.2   Firms

Assume that there exists a continuum of firms, each producing a differentiated good:

$$Y_t(i) = A_t\, N_t(i)^\alpha,$$

where log of productivity evolves according to the following process:

$$\triangle a_t = \rho_a\, \triangle a_{t-1} + \varepsilon_{a,t}, \tag{6}$$

which is an exogenous, difference-stationary stochastic process. Assume further that the producer is a wage taker, so that the real marginal cost of supplying good i is equal to:

$$MC_t(i) = \frac{1}{\alpha}\frac{W_t(i)}{P_t A_t}\left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{\alpha}-1}. \tag{7}$$

Total demand for good i is thus given by:

$$Y_t(i) = C_t(i).$$

Now, let $Y_t = \left(\int_0^1 Y_t(i)^{\left(\frac{\varepsilon-1}{\varepsilon}\right)} di\right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}$ denote aggregate output. Then equilibrium in the goods market implies that:

$$Y_t = C_t.$$

Combining the real marginal cost equation together with a market clearing condition and the static first order condition from the consumer optimization problem (i.e. see equation (3)) and taking a log transformation yields the equilibrium real marginal cost of the individual firm in terms of output produced by the individual firm, aggregate output and productivity. Namely:

$$mc_t(i) = \sigma\, y_t + \omega y_t(i) - (1 + \omega)\, a_t - \ln(\alpha), \tag{8}$$

where $\omega = \frac{\psi}{\alpha} + \frac{1}{\alpha} - 1$. We also can combine the Euler equation with the market clearing condition to get another equilibrium condition, as follows:

$$y_t = -\frac{1}{\sigma}(r_t - E_t\,\pi_{t+1} - \rho) + E_t\, y_{t+1}. \tag{9}$$

2.3 Optimal Pricing

In deriving equilibrium behavior it remains to discuss how firms set prices. In this section we describe four alternative models of price setting behavior, the final three of which will be examined in the sequel.

I. Flexible Prices: First, suppose that all firms choose the price of good i each period, independent of prices that were charged in the past, and with full information about current demand and cost. Due to the fact that real marginal costs are increasing in $y_t(i)$, the same quantity of each good is supplied, and it is equal to Y_t. This implies that all firms will choose a common constant markup given by $\mu = \frac{\epsilon}{(\epsilon-1)}$. The flexible price equilibrium process for output, consumption, and the expected real rate is given by:

$$y_t^n = \gamma + \psi_a\, a_t, \tag{10}$$

$$c_t^n = \gamma + \psi_a\, a_t, \tag{11}$$

$$r_t^n = \rho + \sigma\, \phi_a\, \rho_a\, \triangle a_{t-1}, \tag{12}$$

where $\psi_a = \frac{1+\omega}{\sigma+\omega}$ and $\gamma = \frac{\ln\alpha - \mu}{\sigma+\omega}$. We will refer to the above equilibrium conditions as a natural levels of the corresponding variables.

II. The Sticky Price Model: Following Calvo (1983), assume that in every period, a fraction, $(1 - \theta_1)$, of firms can set a new price, independent of the past history of price changes. This set-up implies that the expected time between price changes is $\frac{1}{1-\theta_1}$. Also assume that firms that cannot set their prices optimally have to keep last periods' price (i.e. $P_t(i) = P_{t-1}(i)$).

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III. The Sticky Price Model with Indexation: Modifications of the standard sticky price model have been shown by numerous authors to perform better in empirical applications. For example, we follow Christiano et al. (2001), Smets and Wouters (2003), and Del Negro and Schorfheide (2005), who use dynamic inflation indexation. In this model, as in Calvo (1983), only a proportion of firms, $(1 - \theta_2)$, can reset their prices during the current period; but other firms, unable to set prices optimally, set their price equal to: $P_t(i) = \Pi_t P_{t-1}(i)$.

IV. The Sticky Information Model: Following Mankiw and Reis (2002), assume that all firms reset prices each period. A fraction of firms, $(1 - \theta_3)$, use current information in pricing decisions, so that the probability that a firm acts upon the newest information available in a given quarter is $1 - \theta_3$, independent of the past history of price changes. The remaining fraction of firms use past or outdated information when they set prices. The sticky information model can be interpreted as a model where firms, which are unable to set prices optimally, use even more complex updating schemes than in the case of the sticky price model with indexation. Instead of using past inflation for indexation, when they have opportunity to use current information, firms in the sticky information model solve not only for the optimal current price, but also for the infinite path of future prices. Later, when firms do not have the opportunity to update information, they set price equal to the appropriate value in their solution set; a set which was calculated based on the old information set.

In these models, the fact that a fraction of firms is not able to adjust prices optimally implies a difference between the actual and the potential (natural) level of output. We denote this difference by $y_t^g = y_t - y_t^n$, and refer to it as the output gap. Now, solving the associated optimization problems and using a log-linear transformation, we can write expressions for the Phillips curve for each model.[5] In particular, the dynamics of inflation in the sticky price economy is characterized by New Keynesian Phillips Curve:

$$\pi_t = \beta \, E_t \, \pi_{t+1} + \lambda_1 \, y_t^g, \tag{13}$$

where $\lambda_1 = \frac{(1-\theta_1)\,(1-\beta\,\theta_1)\,\xi}{\theta_1}$ and $\xi = \frac{\omega+\sigma}{1+\varepsilon\,\omega}$. In the sticky price model with indexation the above equation has a hybrid New Keynesian Phillips Curve analog:

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta} \, E_t \, \pi_{t+1} + \frac{\lambda_2}{1+\beta} \, y_t^g, \tag{14}$$

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[5]For a detailed derivation for the sticky price and the sticky price with indexation models, see Woodford (2003a). For derivation using the sticky information model, see Khan and Zhu (2002).

where $\lambda_2 = \frac{(1-\theta_2)(1-\beta\theta_2)\xi}{\theta_2}$. Finally, in the sticky information model, dynamics of inflation are governed by a sticky information Phillips Curve:

$$\pi_t = \frac{(1-\theta_3)\xi}{\theta_3} y_t^g + (1-\theta_3) \sum_{k=0}^{\infty} E_{t-k-1}\theta_3^k(\pi_t + \xi \triangle y_t^g). \tag{15}$$

Finally, notice that the Euler equation above can be written in terms of the output gap. Namely:

$$y_t^g = -\frac{1}{\sigma}(r_t - E_t \pi_{t+1} - r_t^n) + E_t y_{t+1}^g. \tag{16}$$

2.4 Equilibrium Dynamics

To close our models, we specify a monetary policy rule by assuming that an exogenous path for the growth rate of the money supply is given by the following stationary process:

$$\triangle m_t = \rho_m \triangle m_{t-1} + \epsilon_{m,t}, \tag{17}$$

where $\rho_m \in [0,1]$.

This yields the desired outcome that: (i) the money demand equation (5), (ii) the equilibrium Euler equation (16), (iii) one of three of the Phillips curve equations: (13), (14) or (15), and (iv) the specification of an exogenous process for technology (6), and (v) an exogenous process for the money supply (17) fully describe the equilibrium dynamics of the economy, and in particular, the dynamics of the (endogenous) output gap and inflation variables in the models.

3 Data

Our empirical investigation is based upon the use of quarterly U.S. data for the period 1964:1 and 2003:4. For our measure of inflation, we use the consumer price index (CPI) (the GDP deflator is also used in order to check for the robustness of our results). We construct our measures of the output gap using real GDP.[6] Our approach to output decomposition is to apply the widely used Hodrick-Prescott (H-P) filter (for a detailed discussion, see Hodrick and Prescott (1997)).[7]

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[6]We also used the output gap measure constructed by the OECD and the one-sided optimal bandpass filter in order to check the robustness of our results. Empirical results were qualitatively the same as those reported here, and are available upon request.

[7]The H-P filter minimizes the sum of squared deviations of the actual output, y_t, from the estimated trend, τ_t, subject to a smoothness constraint. Formally, it minimizes:

$$\min_{\tau_t} \sum (y_t - \tau_t)^2 + \lambda \sum ((\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}))^2,$$

All data were obtained from the OECD Main Economic Indicators database (Database Edition (ISSN 1608-1234)), where both yearly and quarterly data are available. We report results based on quarterly data, although results based on yearly data were compiled, and yield qualitatively similar conclusions. Plots of the raw and adjusted data are given in Figure 1, where quarterly inflation and de-meaned inflation are depicted in the top graph. Of note is that we remove the mean of the historical inflation data in order to make the data directly comparable to analogous inflation data simulated using the DSGE models. The historical output gap data depicted in the bottom graph is directly comparable with data simulated from the DSGE models.

4 Calibration and Impulse Response Analysis

In this section we discuss calibration of the models and present the results of a preliminary impulse response analysis of the alternative DSGE models.

With regard to calibration, we follow Gali (2002). Namely, assume log utility of consumption, so that $\sigma = 1$. Also, set the labor wage elasticity as $\psi = 1$, and set the value of the elasticity of money demand with respect to the interest rate as $\eta = 1$, which is consistent with the interest rate elasticity found in empirical work and used in other calibration studies (see e.g. Mankiw and Summers (1986), Stock and Watson (1993)). The Dixit-Stiglitz elasticity of substitution is set to $\epsilon = 11$, which implies a 10% markup of price over marginal cost; and the consumer discount factor is set to $\beta = 0.99$, which implies an average annual interest rate 4%. We set the labor share parameter to $\alpha = 2/3$.

The degree of information and price stickiness, θ, was chosen to be common across all models and is initially set to $\theta = 0.75$.[8] This implies yearly price or information updating. This choice is common in many theoretical (see e.g. Gali (2002), Gali, Lopez-Salido and Valles (2003), and Woodford (2003a)) and empirical studies (see e.g. Blinder et. al. (1998), Gali and Gertler (1999), Khan and Zhu (2002), Korenok (2004), Sbordone (2002), and Smets and Wouters (2003)). In

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where λ is a parameter that is usually set to 1600 for quarterly data. Formally, the output gap, y_t^g, is defined as $y_t - \tau_t$. Of note is that the H-P filter was also used to compare historical and artificial (model simulated) data by Backus, Kehoe and Kydland (1992), Cooley and Hansen (1989), Hansen (1985), and Kydland and Prescott (1982), among others.

[8]Our motivation for common value for information and price stickiness comes from the fact that empirical estimates of information and price stickiness are quite close.

addition, we subsequently compare models with a lower degree of information and price stickiness, namely $\theta = 0.5$. The motivation for this lower level of stickiness comes from Bils and Klenow (2004), who study price stickiness by examining 350 categories of goods and services, constituting about 70% of consumer spending, and find evidence of more frequent price changes than hitherto suspected.

Finally, the exogenous processes are calibrated in the following way. For the technology growth rate, we set the value of the autoregression coefficient, ρ_a, equal to zero, and the standard deviation equal to $\sigma_a = 0.007$. The low value of ρ_a accounts for the low autocorrelation of output growth and common measures of the output gap. Of further note is that the usual standard deviation for the technology growth rate is at or below 1% (see e.g. Gali (2002) or Gali et al. (2003)). The autoregression coefficient of growth in the money supply is set equal to $\rho_m = 0.5$, and the standard deviation is set equal to $\sigma_m = 0.007$; a value which is close to the estimated parameters for autoregressive processes describing M0 or M1 growth rates in the United States.[9]

We now turn to a discussion of impulse response functions in the sticky price, sticky price with indexation and sticky information models. In the discussion, we use our "baseline" calibration, where all parameters are as given above, and where $\theta_i = 0.75$. Conclusions from the "baseline" calibration also apply to our alternative calibration where $\theta_i = 0.5$, which is motivated and discussed in Section 6.3.

4.1 Experiment: Response to An Anticipated Disinflation

We can replicate the theoretical experiment of Mankiw and Reis (2002): namely, an announced and credible shift in the money growth rate. The purpose of their experiment was to illustrate a problem in sticky price models pointed out by Ball (1994), in that an announced credible disinflation causes booms rather than recessions, and their purpose was to show that sticky information models address this problem. We replicate their experiment using the DSGE model specified and calibrated as discussed above.

Inflation responses to announced (8 quarters in advance) and credible disinflations in sticky price, sticky information, and sticky price with indexation models are presented in Figure 2. In the sticky price model, inflation moves in anticipation of demand. It falls in the announcement

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[9]See Mankiw and Reis (2002), Cooley and Hansen (1989), Walsh (1998), and Yun (1996) for further justification of this calibration.

period and then slowly decreases to 0 after 9 years. In the sticky price model with indexation there is no initial fall; inflation decreases smoothly, and reaches 0 after 3.5 years. Furthermore, after the 4th year, it oscillates around 0. In the sticky information model, inflation does not respond immediately, and the eventual response is very small, although it accelerates, reaching a peak during the implementation period. Such behavior does not necessarily mean that agents do not take into account the announcement. Indeed, we should expect such behavior if agents update their information sets on average every 4 quarters, and if the announcement is made 8 quarters in advance. The timing of updates means that at the date of actual policy implementation, half of the agents have already included the new policy in their information sets.

The response of the output gap to an announced disinflation is presented in the bottom of Figure 2. In the sticky price model, the anticipated disinflation results in an increase in the output gap. This increase can be explained by the money demand equation. The output gap increases because inflation falls between announcement and implementation of the disinflation policy, while money growth remains constant. This leads to an increase in real money balances and to higher output, while the natural output level remains constant.[10] The output gap also increases in the sticky price model with indexation. However, the increase is much lower than in the sticky price model, and the output gap returns to 0 after 2 years. Thereafter, it remains negative for 4 years. Indeed, the cumulative response in this case is negative. The slower increase, and then decrease, in the output gap is due to the fact that there is an inflation inertia built into the sticky price model with indexation model. The fall in inflation is lower than in the sticky price model, and with constant money growth leads to a lower increase in real money balances and a lower increase in the output gap. In contrast to the sticky price model, the output gap declines in the sticky information model from the beginning. Inflation responds very slowly because most of the agents set prices based on an old information set, in which they did not expect inflation to change. After the announcement takes place, inflation declines more slowly than money growth, because not all agents have had the opportunity to introduce the announcement into their information sets. Thus, real money balances and output fall, while the natural output level remains constant.

In summary, experiment supports Ball's (1994) argument. However, it should be noted that implementation of the experiment is somewhat unusual given that we assume a permanent shock

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[10]We do not discuss interest rate effects because the size of the interest rate change is small in our experiment relative to the change in output.

to inflation and a credible announced disinflation.

4.2 Robustness of Experiment Results

Ball (1994) and Mankiw and Reis (2002) conducted an equivalent of the above experiment for permanent shock anticipated 1 period ahead and 8 periods ahead respectively. Both experiments resulted in disinflationary boom, i.e. boom after announced, credible decline of money supply. On the other hand, Mankiw and Reis (2002) show that after transitory unanticipated shock there is no disinflationary boom. Trabandt (2005) in a different setup finds that both anticipated 8 periods ahead and unanticipated shocks result in recession. He concludes that previous outcomes were driven by the fact that both Ball (1994) and Mankiw and Reis (2002) assumed zero semi-elasticity of money demand with respect to interest rate, parameter η in equation 5. Interest rate semi-elasticity determines the relative size of interest rate decrease compared to increase in real money balances after disinflationary shock, thus it determines change in output in equation 5. Finally, in our experiment we find that permanent anticipated 8 periods ahead negative shock to money supply growth results in boom, even though η is equal to one.

Here we show why outcomes in all these experiments are different. In particular, we check robustness of the disinflationary boom to changes in: (i) persistence of the shock (ρ_m), (ii) anticipation period (k) and (iii) semi-elasticity of money demand with respect to interest rate (η). All other parameters are at their baseline calibrated values (since values of other parameters influence the value of the output we focus on ordinal relationships). For each experiment design and parameters choice we report maximum of the output gap response to negative money supply shock. We take positive maximum as indication of disinflationary boom.

Figure 3 reports robustness of disinflationary boom to changes in η, k and ρ_m. We report results only for sticky price model.[11] There is little dependence of disinflationary boom on persistence of money supply shock, Graph A. The maximum value of the output gap somewhat increases as persistence of the process increases from very persistent to permanent. For $\eta = 0$ and any anticipation period, k from 1 to 8 on Graph B, disinflationary boom exists which is in line with Ball (1994) and Mankiw and Reis (2002) results. While for $k = 0$ there is no disinflationary boom which

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[11]Results for sticky price model with indexation are very close to the sticky price model results, while results for sticky information model are trivial, i.e. there are no disinflationary booms for any experiment design or reparametrization.

is in line with Mankiw and Reis (2002) and Trabandt (2005) experiments with unanticipated shocks. For high values of η, Graphs A and B, disinflationary boom occurs independent of anticipation period or persistence of the shock which is in line with Trabandt (2005) findings. For his choice of utility function and baseline calibration $\eta = 50$ and interest rate effect dominates change in demand for real money balances. On the other hand, most of the empirical estimates of semi-elasticity are below or close to 1 as we pointed out in our calibration section.[12]

Our robustness exercise demonstrates that disinflationary boom is not robust to design of experiment or parameters choice, i.e. if shocks are unanticipated or if semi-elasticity of money demand with respect to interest rate is very high, negative shock to money demand growth is followed by recession. Since most of empirical research concentrated on unanticipated shocks and we are using empirical findings to motivate our calibrations, we consider only this kind of shocks in the paper (our calibration is in the low-right corner of Graph B). In this we follow Mankiw and Reis (2002), who comment that they "take a step toward greater realism" when they analyze transitory monetary shocks.

4.3   Response to a Contractionary Monetary Policy Shock

The left column of Figure 4 reports impulse response functions for inflation (top graph) and the output gap (bottom graph), given that a contractionary monetary policy shock is imposed in our baseline calibration of the three DSGE models (recall that the baseline model sets $\theta = 0.75$). Notice that inflation responds immediately in the sticky price model, with the highest response in the initial period. Furthermore, notice that the lack of lags in the response to the monetary policy shock for a model with sticky prices was also pointed out by Mankiw and Reis (2002). The dynamics of inflation after the initial shock is very persistent for all three models, in contradiction to the point made by Fuhrer and Moore (1995) that inflation is not persistent in sticky price models. Furthermore, the response of inflation in the sticky price model with indexation and the sticky information model displays inflation inertia. The maximum impact of the monetary policy shock on inflation occurs after 5 quarters for the sticky price model with indexation and after 7 quarters for the sticky information model. Finally, note that the inflation response is the highest for the sticky price model with indexation, while for the sticky price model and the sticky information

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[12]With exception of Ireland (2004) estimate of semi-elasticity in general equilibrium framework, which is 7.75 with standard error 1.75.

model, the size of the response is comparable.

In all three models the output gap response to a transitory, unanticipated shock is similar. Namely, the response is negative and hump-shaped, and there is a fast initial response. The response decreases over the first two quarters and then increases. On further note is that the size of the responses for the different models is very close, while the speed of recovery is much faster for the sticky price model with indexation than for the other models.

To summarize, in a more standard setups it appears that disinflation dynamics are much closer amongst the competing models. In all cases, both inflation and the output gap decrease. However, the sticky price model with indexation and the sticky information models have more inflation inertia than the sticky price model. It is worth noting, however, that the decline in inflation in our calibrated models may also be occurring in the current context because of contractionary technology shocks. Thus, we next consider an experiment where there is a contractionary technology shock.

4.4   Response to a Contractionary Technology Shock

The left column of Figure 5 reports the impulse response functions for inflation (top graph) and output (bottom graph), given that a contractionary technology shock is imposed in our baseline calibration of the three DSGE models.

As in the case of the monetary policy shock, a technology shock leads to an immediate inflation response, with the highest response in the initial period. The response of inflation in the sticky price model with indexation and the sticky information models, however, displays inflation inertia. Furthermore, the size of the response is somewhat lower than the size of response to monetary policy shock.

The output gap response to a technology shock is similar for all three models. Namely, the response is negative, with a fast initial response. Unlike the output gap response to a monetary policy shock, the responses to a technology shock are not hump shaped and the size of response is lower for all models. On further note is that the size of the responses for the models is very close, while the speed of recovery is much faster for the sticky price model with indexation than for the other models.

To summarize, the response to a technology shock is somewhat smaller in size and the output gap response is not hump shaped. Otherwise, the dynamics of the response of inflation and the output gap after a contractionary shock to technology is very close to the dynamics of the response

after contractionary monetary shock.

5 A Distribution Comparison Test for DSGE Models

In this section we briefly discuss the distributional accuracy test discussed in Corradi and Swanson (2005b) (CS), which shall be used in our subsequent empirical analysis. Of note is that the following approach to DSGE model evaluation is but one of many approaches currently in use by economists (see discussion in the introduction for further remarks in this regard).[13]

Assume that our objective is to compare the joint distribution of the historical data with the joint distribution of the simulated series. Following CS, and for the sake of simplicity (but without loss of generality), we limit our attention in the section to the evaluation of the joint empirical distribution of (actual and model-based) current and previous period output. In principle, if we have a model driven by k shocks, then we can consider the joint CDF of k variables plus an arbitrary (but finite) number of lags of each variable.

Consider m DSGE models, and set model 1 as the benchmark model. We require at least one of the competing models (e.g. model j for $j = 2, ..., m$) to be nonnested with respect to the benchmark, a requirement which is satisfied in the current context when model parameters such as θ_i, $i = 1, 2, 3$ are calibrated and are not estimated (when freely estimated all parameters could be identically zero, leading to nestedness). For the sake of notational ease of expression, let $\Delta \log X_t$, $t = 1, ..., T$ denote the actual historical (output) series, and let $\Delta \log X_{j,n}$, $j = 1, ..., m$ and $n = 1, ..., S$, denote the output series simulated under model j, where S denotes the length of the simulated sample. In general, some parameters in the DSGE models may be kept fixed (at their calibrated values), while others may be estimated.

Along these lines, denote $\Delta \log X_{j,n}(\widehat{\Theta}_{j,T})$, $n = 1, ..., S$, $j = 1, ..., m$ to be a sample of length S drawn (simulated) from model j and evaluated at the parameters estimated (or calibrated) under

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[13]The test discussed in this section is not meant to be informative about the source of the rejection. In this sense, our distributional comparisons are weak tests of DSGE theories. Alternative approaches to that taken here include the study of ARMA models, wherein restrictions implied by the different theories can be imposed, and the construction of tests of cross-equation restrictions. Much of this is done in the papers cited at the beginning of the introduction, and key papers along these lines include Cogley and Nason (1995a) and Geweke (1999b). Adaptation of the distributional approach taken here to the examination of the auto- and cross-correlation functions discussed earlier, for example, are left to future research.

model j, using the T available historical observations.[14] We assume stationarity in our subsequent analysis.[15] On the other hand, endogenous business cycles models that predict persistent, but stationary, fluctuations should not be treated in this manner in order to avoid potential problems associated with overdifferencing. In general, we require both the actual and the simulated series to be (strictly) stationary and mixing.

For ease of exposition, and in keeping with our focus on current and lagged values of the variable of interest, let $Y_t = (\log \Delta X_t, \log \Delta X_{t-1})$, $Y_{j,n}(\widehat{\Theta}_{j,T}) = (\Delta \log X_{j,n}(\widehat{\Theta}_{j,T}), \Delta \log X_{j,n-1}(\widehat{\Theta}_{j,T}))$. Also, let $F_0(u; \Theta_0)$ denote the distribution of Y_t evaluated at u and $F_j(u; \Theta_j^\dagger)$ denote the distribution of $Y_{j,n}(\Theta_j^\dagger)$, where $\Theta_j^\dagger$ is the probability limit of $\widehat{\Theta}_{j,T}$, taken as $T \to \infty$, when parameters are estimated, and where $u \in U \subset \Re^2$, possibly unbounded. Accuracy is measured in terms of square error. The squared (approximation) error associated with model i, $i = 1, ..., m$, is measured in terms of the (weighted) average over U of $E\left(\left(F_i(u; \Theta_i^\dagger) - F_0(u; \Theta_0)\right)^2\right)$, where $u \in U$, and U is a possibly unbounded set on $\Re^2$. The rule is to choose Model 1 over Model 2 if

$$\int_U E\left(\left(F_1(u; \Theta_1^\dagger) - F_0(u; \Theta_0)\right)^2\right) \phi(u) du < \int_U E\left(\left(F_2(u; \Theta_2^\dagger) - F_0(u; \Theta_0)\right)^2\right) \phi(u) du$$

where $\int_U \phi(u) du = 1$ and $\phi(u) \geq 0$ for all $u \in U \subset \Re^2$. For any evaluation point, this measure defines a norm and it implies a usual goodness of fit measure. The hypotheses of interest are:

$$H_0 : \max_{j=2,...,m} \int_U E\left(\left(F_0(u; \Theta_0) - F_1(u; \Theta_1^\dagger)\right)^2 - \left(F_0(u) - F_j(u; \Theta_j^\dagger)\right)^2\right) \phi(u) du \leq 0$$

versus

$$H_A : \max_{j=2,...,m} \int_U E\left(\left(F_0(u; \Theta_0) - F_1(u; \Theta_1^\dagger)\right)^2 - \left(F_0(u) - F_j(u; \Theta_j^\dagger)\right)^2\right) \phi(u) du > 0.$$

Thus, under H_0, no model can provide a better approximation (in square error sense) to the distribution of Y_t than the approximation provided by model 1. In order to test H_0 versus H_A, the

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[14]In practical applications, $\widehat{\Theta}_{j,T}$ is usually a combination of estimated and calibrated parameters. However, in our empirical analysis, all parameters are calibrated, and hence there is no parameter estimation error.

[15]In our analysis, we use the H-P filter to induce stationarity on the output gap measure, while inflation is assumed stationary. Thus, the output gap data that we examine is potentially problematic, as the filter ignores cointegration, for example. However, it should be noted that there is mixed empirical evidence of the usefulness of imposing cointegration restrictions when forecasting. Furthermore, we carried out our analysis replacing our output gap with alternative measures of the output gap, and results were qualitatively the same. Finally, it should be stressed that an excellent frequency domain based approach to the problem of constructing goodness of fit tests that are invariant to linear filtering is explored in Cogley and Nason (1995b).

relevant test statistic is $\sqrt{T}Z_{T,S}$, where:

$$Z_{T,S} = \max_{j=2,\ldots,m} \int_U Z_{j,T,S}(u)\phi(u)du, \tag{18}$$

and

$$
\begin{aligned}
Z_{j,T,S}(u) &= \frac{1}{T}\sum_{t=1}^{T}\left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{1,n}(\widehat{\Theta}_{1,T}) \le u\}\right)^2 \\
&\quad -\frac{1}{T}\sum_{t=1}^{T}\left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{j,n}(\widehat{\Theta}_{j,T}) \le u\}\right)^2.
\end{aligned}
$$

From equation (18), it is immediate to see that the computational burden increases with the dimensionality of U, that is with the number of variables and/or lagged values we are considering. In fact, we need to approximate the integral by taking an average over a fine grid of U.[16] The limiting distribution of $\sqrt{T}Z_{T,S}$ is a zero mean Gaussian process with a covariance kernel that reflects the contribution of parameter estimation error (see appendix for further details), the time series structure of the data and, for $\delta > 0$, the contribution of simulation error. Bootstrap critical values for the above test can be obtained in straightforward manner, as outlined in CS, and as discussed in the appendix.

6 Empirical Results

6.1 Basic Data Analysis ($\theta = 0.75$ - High Degree of Price Stickiness)

In order to form an initial impression of the performance of our alternative models, we first plot empirical distribution and density functions for simulated and historical inflation and output gap observations, where all simulation based calculations are based on samples of length 100T (T is the historical sample size). In a subsequent section (Section 6.3), we consider the case where $\theta = 0.5$, at which point we compare all models and calibrations using various other statistical measures, including autocorrelation and cross correlation functions, directional accuracy, and the well known acceleration phenomenon.

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[16]For example, if U is a two-dimensional subset of $\Re^2$, and ϕ is uniform on U, then

$$Z_{T,S} = \frac{1}{N_1 N_2} \max_{j=2,\ldots,m} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} Z_{j,T,S}(u_{i,j}).$$

Actual and simulated distributions of inflation and the output gap are plotted in Figure 6 (see plots for which $\theta = 0.75$). Of note is that for inflation, the sticky price model with indexation appears "closest" to the historical distribution. For short region, between 0.4 and 0.7 quantiles the sticky price model is the closest to historical distribution. Clearly, though, none of the models perform well at mimicking the left tail of the historical distribution, as is apparent upon inspection of the cumulative distributions beyond the 0.8 quantile given on the vertical axis of the plots. Interestingly, for the output gap, the sticky price model with indexation is still the "closest" to the historical distribution. However, in this case none of the simulated distributions are particularly accurate, as evidenced by the poor fit at both tails of the historical distribution.

As an alternative way to compare historical and simulated distributions, we also plot marginal densities in Figure 7. In this figure the relevant plots are again the two corresponding to the case where $\theta = 0.75$. These plots illustrate much more clearly that the sticky price with indexation model appears to outperform the other models, at least based on this initial measure of accuracy. Interestingly, inflation values simulated using all models, however, appear to have distributions with tails that are too thin, while the opposite can be said for the output gap.

6.2 Distributional Accuracy Tests ($\theta = 0.75$ - High Degree of Price Stickiness)

We now turn to a more formal discussion of the results discussed in the previous sub-section. In particular, we apply the distributional accuracy test discussed above. Results are gathered in Tables 1 - 3. The tables are organized as follows. The first column gives S, the length of the simulation sample used, and l, the block length used in the construction of test critical values. The second column of entries reports the numerical values of the test statistic ($Z = Z_{T,S}$) discussed above, while the next four columns report 5% and 10% bootstrap critical values based on a bootstrap statistics for the cases where $T/S \to \delta > 0$ (Z^{**}) and $T/S \to 0$ (Z^*), where T denotes the historical sample size. The last three columns report the Corradi and Swanson (2005a) distributional loss measure associated with model i, $i = 1, 2, 3$, (i.e. $\int_U \frac{1}{T} \sum_{t=1}^{T} \left(1\{Y_t \leq u\} - \frac{1}{S} \sum_{n=1}^{S} 1\{Y_{i,n}(\widehat{\theta}_{1,T}) \leq u\} \right)^2 \phi(u)du$) - see Section 5 for further details. As noted previously, T denotes the historical sample size. For the case where $S = T$, we set the block length used in the bootstrap as follows: $l1 = 5$, $l2 = 8$, $l3 = 10$, $l4 = 16$ and $l5 = 20$. For all other cases, where $S = aT$, say, we set l equal to 'a' times the corresponding value of l when $S = T$. All statistics are based on grids of 20x20 values for u, distributed uniformly across the historical data ranges of π_t and y_t^g. Bootstrap empirical distributions

are constructed using 500 bootstrap replications.

We begin by setting the sticky price model as the benchmark, against which we compare our two alternative models - the sticky price with indexation model and the sticky information model. Additionally, tests are constructed using three different joint distributions. Namely (i) inflation and the output gap, (ii) inflation and lagged inflation, and (iii) output gap and lagged output gap. In Table 1, results for case (i) are given. Of note is that the benchmark sticky price model is almost always rejected at conventional 5 and 10% levels, regardless of simulation sample size and bootstrap block length. This constitutes strong evidence in favor of the alternative models. Interestingly, upon inspection of Tables 2 and 3, the same result holds (for virtually all sample size/block length permutations). Thus, for cases (i), (ii), and (iii), the sticky price model appears to be dominated by the alternative models, in accord with our earlier interpretation of the distributional and density plots of simulated and historical data. Furthermore, inspection of the last three columns in the three tables indicates that the CS distributional loss measure is always lowest for the sticky price with indexation model, suggesting that the sticky price with indexation model is the "best" based on our point mean square error type distributional loss measure, and is thus likely to be the model driving the rejections of the test.

In order to shed further light on the issue of whether our test is rejecting because of the dominant performance of the sticky price with indexation model, we also performed several bi-model tests (results are available upon request from the authors). In particular, we tested: (i) the sticky price model against an alternative of the sticky price with indexation model, (ii) the sticky price model against an alternative of the sticky information model, and (iii) the sticky information model against an alternative of the sticky price with indexation model. Interestingly, in all cases the null model is rejected in favor of the alternative model, suggesting an ordering as follows: the sticky price with indexation model, followed by the sticky information model, followed by the sticky price model. As we shall shortly see, however, this result is highly dependent upon the level of price stickiness.

6.3 Basic Data Analysis ($\theta = 0.5$ - Low Degree of Price Stickiness)

From comparison of simulated and historical density functions (see above discussion) for the case where $\theta = 0.75$, it was found that the distributions of simulated inflation for all three models have thinner tails than in the historical record. Also, the distributions of the simulated output gap for all three models have thicker tails than in the historical record. It follows that in order to improve

the fit of the models, the response of inflation to shocks should be larger, and the response of the output gap should be smaller. Unfortunately, it is not possible to achieve a larger response for inflation and a smaller response for the output gap by choosing different values for the volatility of the shocks in the models. On the other hand, a larger response of inflation and a smaller response of the output gap is expected for a given shock volatility if the level of price stickiness is reduced. For example, a decrease in price stickiness, allows a larger fraction of agents in the economy to adjust prices, which in turn produces a larger response of inflation. At the same time, a larger inflation response coupled with a money growth shock that is the same size as prior to the decrease in price stickiness leads to a decrease in the response of real money balances and thus output. These facts, coupled with the results of the study by Bils and Klenow (2004) discussed above, provide the impetus for our second calibration, where we set $\theta = 0.5$. Of note is that the decrease from $\theta = 0.75$ (i.e. 12 months between price changes) to $\theta = 0.5$ (i.e. 6 months between price changes) still results in price stickiness slightly higher than the 4.3 months suggested in Bils and Klenow.

We begin our basic data analysis with a discussion of impulse response functions analogous to those discussed in Section 4. In particular, compare the right hand side panels in Figures 4 and 5 to the left side panels in the same figures. The following conclusions emerge. First, the size of the inflation response to both shocks increases when information and price stickiness is reduced. In fact, for the monetary shock the inflation response is twice as big. Additionally, the size of the output gap response to both shocks decreases with the largest decrease due to monetary policy shock. Finally, the decrease in stickiness also leads to a decrease in the length of response.

Next we discuss the simulated distributions and densities, as denoted in the right hand side panels of Figures 6 and 7. Interestingly, the simulated distributions of inflation for all three models are much closer to the historical record. However, all three models fail to reproduce the right tail asymmetry of the historical distribution of inflation. Furthermore, the simulated distributions of the output gap for all three models are much closer to the historical record. Nevertheless it is again worth noting that all three models continue to fail to capture the small left tail asymmetry of the historical distribution of the output gap. Overall, there appears to be little to choose between the models based on inspection of empirical densities and distributions.

To further elucidate our distributional comparison, note that Table 4 contains quantiles for both measures of information and price stickiness, for all three models, and for the historical data. In particular, quantiles for the historical and simulated distributions of inflation are given in Panel A

and those for the output gap are given in Panel B. Different columns represent different quantiles, while each row gives results for a different model (the exception is the first row, which contains results for the historical data). For our higher level of information and price stickiness (i.e. see Part I of the table), the sticky price model with indexation is the closest to historical inflation for 7 out of 9 quantiles, and to the output gap for 7 quantiles. On the other hand, for lower levels of stickiness (i.e. see Part II of the table), there is no clear winner among the three models. For example, note that although the simulated distribution for the sticky price model with indexation is closest to the historical distribution of inflation for 4 quantiles, the simulated distribution for the sticky price model is the closest to historical distribution of the output gap for 7 quantiles. Furthermore, and even more importantly, note that upon comparison of the distributions for high and low levels of stickiness, it is clear that the simulated distributions for all three models are closer to the historical record when $\theta = 0.5$.

Contingency tables are reported in Table 5. The table is organized in a manner similar to that used to present results in Table 4, with the exception that the last row of entries in each panel of the table reports p-values to the classical χ^2 test for independence based on 2x2 contingency tables (see Swanson and White (1995) for further discussion). Of note is that for our high level of stickiness, the results in Part I of the table indicate that none of the models are able to mimic historical behavior. For the low level of stickiness (see Part II of the table), performance does not improve suggesting that there is little to choose between either the models or the level of stickiness based on the evaluation of contingency tables.

Table 6 contains auto- and cross-correlations for inflation and the output gap. Entries in the table correspond to autocorrelations of inflation ($corr(\pi_t, \pi_{t-1})$) and the output gap ($corr(y_t, y_{t-1})$), as well as the cross correlation between inflation and the output gap. Corresponding historical values are: $corr(\pi_t, \pi_{t-1}) = 0.8363$, $corr(y_t, y_{t-1}) = 0.8690$, $corr(y_t, \pi_t) = 0.2582$. Values are computed for different simulation samples of length S = 5T, 10T, 20T, 30T, 50T, 100T, where T is the historical data sample size. For our high level of stickiness (i.e. $\theta = 0.75$), the sticky price model with indexation better reproduces historical cross-correlations between inflation and the output gap, while the sticky price model fares better at reproducing autocorrelation features for inflation and the output gap. For our lower level of stickiness, the sticky price model better reproduces autocorrelation features for the output gap and cross-correlation between inflation and the output gap, while the sticky price model with indexation better reproduces historical autocorrelation of

inflation.

Finally, we compare the ability of the models to reproduce the so-called acceleration phenomenon, which is commonly defined to be the positive relationship between level of economic activity and changes in the inflation rate.[17] Table 7 reports simulated cross correlations between the output gap and changes in inflation, i.e. $corr(y_t, \pi_{t+2} - \pi_{t-2})$. Corresponding historical values are: $corr(y_t, \pi_{t+2} - \pi_{t-2}) = 0.4491$ and $corr(y_t, \pi_{t+4} - \pi_{t-4}) = 0.5458$. As in the previous table, values are given for simulation samples of length S = 5T, 10T, 20T, 30T, 50T, 100T, where T is the historical data sample size. Of note is that results are independent of the level stickiness (compare Parts I and II of the table). Additionally, note that the sticky price model fails to reproduce the positive relation between inflation and the output gap. The sticky information model yields values that are closest to historical values for both measures. Thus, the sticky price model appears inadequate based upon these measures, sticky information model is preferred based on acceleration measure.

6.4 Distributional Accuracy Tests ($\theta = 0.5$ - Low Degree of Price Stickiness)

The distributional accuracy tests tell a very different story in this case, than when $\theta = 0.75$. In particular note in Tables 8-10 that the null hypothesis is never rejected. This suggests that when the degree of price stickiness is decreased, there is little to choose between the three models. Put differently, notice that the CS distributional accuracy measure for the sticky price with indexation model is still (often) lower than the comparable values for the other two models (see last three columns of the tables). However, the numerical values are so close together that, for a statistical perspective, there is nothing to choose between the models. This result is made more interesting by the fact that all models are clearly performing better in the case where $\theta = 0.5$, as discussed above, and as is apparent upon noting that the CS distributional loss measures reported in the distributional accuracy tables are always lower when θ is reduced from 0.75 to 0.5.

7 Concluding Remarks

In this paper, we compare the theoretical and empirical performance of the sticky price model with two important alternatives, namely the sticky information model and the sticky price model with indexation. Based upon empirical observation of distributional fits (relative to the historical record)

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[17]See Stock and Watson (1999), Fuhrer and Moore (1995), and Mankiw and Reis (2002) for further discussion.

and based upon examination of various correlation features of simulated and historical inflation and output gap data, we find evidence that lower levels of price stickiness than hitherto suggested may be appropriate. Additionally, we find that impulse response functions of the alternative models are very close, with the important exception that there is no initial delay for inflation in the sticky price model. This is opposed to the other models, which yield delayed and gradual response of inflation to monetary policy shock. However, these responses have maximums after only 3 periods, so that the models are not significantly different from the sticky price model, as noted by the inability of our distributional accuracy tests to distinguish between the models when the level of price stickiness is low. Furthermore, the sticky price model is good at reproducing auto- and cross-correlations for low levels of price stickiness. Overall, we thus conclude that there is mild evidence in favor of the sticky information and sticky price with indexation models, although further investigation is needed in order to ascertain whether the newer sticky information and sticky price with indexation models will continue to dominate sticky price model upon both theoretical and empirical grounds.

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Appendix: Corradi and Swanson Distributional Comparison Test

In this appendix, we discuss the CS distributional accuracy test in somewhat more detail. Recall that the hypotheses of interest are:

$$H_0 : \max_{j=2,...,m} \int_U E\left(\left(F_0(u;\Theta_0) - F_1(u;\Theta_1^\dagger)\right)^2 - \left(F_0(u) - F_j(u;\Theta_j^\dagger)\right)^2\right)\phi(u)du \leq 0$$

versus

$$H_A : \max_{j=2,...,m} \int_U E\left(\left(F_0(u;\Theta_0) - F_1(u;\Theta_1^\dagger)\right)^2 - \left(F_0(u) - F_j(u;\Theta_j^\dagger)\right)^2\right)\phi(u)du > 0.$$

If interest focuses on confidence intervals, so that the objective is to "approximate" $\Pr(\underline{u} \leq Y_t \leq \overline{u})$, then the null and alternative hypotheses can be stated as:

$$H_0' : \max_{j=2,...,m} E\left(\left(\left(F_1(\overline{u};\Theta_1^\dagger) - F_1(\underline{u};\Theta_1^\dagger)\right) - (F_0(\overline{u};\Theta_0) - F_0(\underline{u};\Theta_0))\right)^2\right.$$
$$\left. - \left(\left(F_j(\overline{u};\Theta_k^\dagger) - F_j(\underline{u};\Theta_k^\dagger)\right) - (F_0(\overline{u};\Theta_0) - F_0(\underline{u};\Theta_0))\right)^2\right) \leq 0.$$

versus

$$H_A' : \max_{j=2,...,m} E\left(\left(\left(F_1(\overline{u};\Theta_1^\dagger) - F_1(\underline{u};\Theta_1^\dagger)\right) - (F_0(\overline{u};\Theta_0) - F_0(\underline{u};\Theta_0))\right)^2\right.$$
$$\left. - \left(\left(F_j(\overline{u};\Theta_k^\dagger) - F_j(\underline{u};\Theta_k^\dagger)\right) - (F_0(\overline{u};\Theta_0) - F_0(\underline{u};\Theta_0))\right)^2\right) > 0.$$

The relevant statistic for testing H_0 is $\sqrt{T}Z_{T,S}$, as discussed above.[18] The following assumptions are used in our first proposition.

Assumption A1: Y_t is stationary-ergodic $\beta-$mixing processes with size -4, for $j = 1, ..., m$. [19]

Assumption A2: For $j = 1, ..., m : \sqrt{T}\left(\widehat{\Theta}_{j,T} - \Theta_j^\dagger\right) = A_j(\Theta_j^\dagger)\frac{1}{\sqrt{T}}\sum_{t=2}^T q_j(Y_t, \Theta_j^\dagger) + o_P(1)$, where $\frac{1}{\sqrt{T}}\sum_{t=2}^T q_j(Y_t, \Theta_j^\dagger)$ satisfies a central limit theorem and $A_j(\Theta_j^\dagger)$ is positive definite.[20]

Assumption A3: For $j = 1, ..., m :$ (i) $\forall \Theta_j \in \Xi_j$, with Ξ_j a compact set in $\Re^{p_j}$ and $Y_{j,n}(\Theta_j)$ is a strictly stationary ergodic $\beta-$mixing process with size -4, where p_j is the number of estimated parameters in model j; (ii) $Y_{j,n}(\Theta_j)$ is continuously differentiable in the interior of Ξ_j, for $n = 1, ..., S$; (iii) $\nabla_{\Theta_j} Y_{j,n}(\Theta_j)$ is $2r-$dominated in Ξ_j, uniformly in n for $r > 2$;[21] (iv) $F_j(u;\theta_j^\dagger)$ is twice continuously differentiable in u; and (v) for at least one j, $F_j(u;\Theta_j^\dagger) \neq F_1(u;\Theta_1^\dagger)$ for $u \in \widetilde{U}$, where $\widetilde{U}$ is a subset of U of non-zero Lebesgue measure.

A2 requires that $\sqrt{T}\left(\widehat{\theta}_{j,T} - \theta_j^\dagger\right)$ is asymptotically normal with a positive definite covariance matrix. Thus, given the size condition in **A1**, **A2** is satisfied by OLS, NLS, and QMLE, under mild conditions, such as finite $(4+\delta)$th moments and unique identifiability. It is satisfied for the GMM-type estimator of Christiano and Eichenbaum (1992) and the estimator of Bierens (2005). With regard to **A3**(i), whenever the production function is a Cobb-Douglas type, and the shock to technology follows a unit root process in logs, then output follows a unit root process in logs,

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[18] H_0' versus H_A' can be tested in a similar manner (see e.g. Corradi and Swanson (2005a)).

[19] $\beta-$mixing is a memory requirement stronger than $\alpha-$mixing, but weaker than (uniform) $\phi-$mixing.

[20] Given the size condition in A1, A2 is satisfied by the LS, NLS, QMLE estimator, under mild conditions, such as finite $(4+\delta)$th moments and unique identifiability.

[21] This means that $|\nabla_{\theta_j} Y_{j,n}(\theta_j)| \leq D_{j,n}$, with $\sup_n E(D_{j,n}^{2r}) < \infty$ (see e.g. Gallant and White (1988), p.33).

and the growth rate is stationary. This is not necessarily true in the case of more general CES production functions.[22] **A3**(ii) need only hold for estimated parameters. When solving RBC models, we often obtain a (linear) ARMA representations for the variables of interest, in terms of final (or reduced form) parameters. Therefore, because of linearity, **A3**(ii) holds straightforwardly for the final parameters. Hence, if the structural (deep) parameters are smooth functions of the final parameters, as is often the case **A3**(ii) is satisfied. **A3**(iii) is a standard assumption (see e.g. Duffie and Singleton (1993)), and **A3**(iv) is always satisfied for linearized solutions of RBC models. Finally, **A3**(v) ensures that at least one competing model is nonnested with the benchmark model. This in turn ensures that the covariance matrix of the statistic is positive semi-definite. Hereafter, for notational simplicity, let $F_j(u) = F_j(u; \Theta_j^\dagger)$.

Proposition 1 (CS (2005b)): Let Assumptions A1-A3 hold. (i) Assume that as $T, S \to \infty$: $T/S \to \delta$, $0 < \delta < \infty$, then:

$$\max_{j=2,...,m} \sqrt{T} \int_U \left(Z_{j,T,S}(u) - \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \right) \phi(u) du \qquad (19)$$

$$\xrightarrow{d} \max_{j=2,...,m} \int_U Z_j(u) \phi(u) du,$$

where $Z_j(u)$ is a zero mean Gaussian process with covariance kernel, $K_j(u, u')$, as given in CS (2005b).

(ii) Assume that as $T, S \to \infty$: $S/T^2 \to 0$ and $T/S \to 0$, then:

$$\max_{j=2,...,m} \sqrt{T} \int_U \left(Z_{j,T,S}(u) - \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \right) \phi(u) du$$

$$\xrightarrow{d} \max_{j=2,...,m} \int_U \widetilde{Z}_j(u) \phi(u) du,$$

where $\widetilde{Z}_j(u)$ is a zero mean Gaussian process with covariance kernel, $\widetilde{K}_j(u, u')$, as given in CS (2005b).

Notice that when $T/S \to 0$, then $\frac{1}{\sqrt{S}} \sum_{n=1}^S \left(1\{Y_{j,n}(\Theta_1^\dagger) \le u\} - F_j(u) \right) \xrightarrow{pr} 0$, uniformly in u, and so the covariance kernel of the limiting distribution does not reflect the contribution of the error term due to the fact we replace the "true" distribution of the simulated series with its empirical counterpart; in other words, in this case, the simulation error vanishes. Also, notice that we require S to grow at a rate slower than T^2; such a condition is used in order to show the stochastic equicontinuity of the statistic.

From Proposition 1, we see that when all competing models provide an approximation to the true joint distribution that is as accurate (in terms of square error) as that provided by the benchmark, then the limiting distribution is a zero mean Gaussian process with a covariance kernel that reflects the contribution of parameter estimation error, the time series structure of the data and, for $\delta > 0$, the contribution of simulation error. This is the case where

$$\int_U \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \phi(u) du = 0, \text{ for all } j.$$

It follows that in this case, the limiting distribution of

$$\max_{j=2,...,m} \sqrt{T} \int_U \left(Z_{j,T,S}(u) - \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \right) \phi(u) du$$

---

[22]It remains to establish whether or not **A3**(i) can be relaxed to weak stationarity. Given this fact, and given that strict stationarity is not generally ensured, results of the test should be viewed with caution.

is the same as that of $\sqrt{T}Z_{T,S}$, and so the critical values of the limiting distribution on the RHS of equation (19) provide valid asymptotic critical values for $\sqrt{T}Z_{T,S}$. On the other hand, when $\int_U \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \phi(u)du < 0$ for some j, so that at least one alternative model is less accurate than the benchmark, then these critical values provide upper bounds for critical values for $\sqrt{T}Z_{T,S}$. Also, when all competing models are less accurate than the benchmark model, then the statistic diverges to minus infinity.

Finally, under the alternative, $\int_U \left((F_0(u) - F_1(u))^2 - (F_0(u) - F_j(u))^2 \right) \phi(u)du > 0$ for some j, so that $\sqrt{T}Z_{T,S}$ diverges to infinity. Therefore, the test has correct asymptotic size if all models are equally good, is conservative when some model is strictly dominated by the benchmark, and has unit power under the alternative. Note that the above testing procedure can in principle be modified to allow for the evaluation of predictive densities under rolling and/or recursive estimation strategies (see e.g. Corradi and Swanson (2005d,e)).

Bootstrap critical values for the above test can be obtained in straightforward manner, as outlined in CS. Namely, one can rely on an empirical process version of the block bootstrap that properly captures the contribution of parameter estimation error, simulation error, when present, and the time series structure of the data to the covariance kernel given in Proposition 1.

Begin by resampling b blocks of length l, $bl = T - 1$, from the actual sample. Let $Y_t^* = (\log \Delta X_t^*, \log \Delta X_{t-1}^*)$ be the resampled series, such that $Y_2^*, ..., Y_{l+1}^*, Y_{l+2}^*, ..., Y_{T-l+2}^*, ..., Y_T^*$ is equal to $Y_{I_1+1}, ..., Y_{I_1+l}, Y_{I_2+1}, ..., Y_{I_b+1}, ..., Y_{I_b+l}$, where I_i, $i = 1, ..., b$ are independent, discrete uniform on $1, ..., T - l + 1$, that is $I_i = i$, $i = 1, ..., T - l$ with probability $1/(T - l)$. We use the resampled series Y_t^* to compute the bootstrap estimator $\widehat{\Theta}_{j,T}^*$ for $j = 1, ..., m$.

We now use $\widehat{\Theta}_{j,T}^*$ to simulate samples under model j, $j = 1, ..., m$; let $Y_{j,n}(\widehat{\Theta}_{j,T}^*)$, $n = 2, ..., S$ be the series simulated under model j. At this point, we need to distinguish between the case of $\delta = 0$, vanishing simulation error and $\delta > 0$, nonvanishing simulation error. In the former case, we do not need to resample the simulated series, as there is no need of mimicking the contribution of simulation error to the covariance kernel. On the other hand, in the latter case, we do need to resample the simulated series. More precisely, we draw $\widetilde{b}$ blocks of length $\widetilde{l}$, with $\widetilde{b}\widetilde{l} = S - 1$, let $Y_{j,n}^*(\widehat{\Theta}_{j,T}^*)$, $j = 1, ..., m$, $n = 2, ..., S$ denote the resample series under model j. Notice that $Y_{j,2}^*(\widehat{\Theta}_{j,T}^*), ..., Y_{j,l+1}^*(\widehat{\Theta}_{j,T}^*), ..., Y_{j,S}^*(\widehat{\Theta}_{j,T}^*)$ is equal to $Y_{j,\widetilde{I}_1}(\widehat{\Theta}_{j,T}^*), ..., Y_{j,\widetilde{I}_1+l}(\widehat{\Theta}_{j,T}^*), ..., Y_{j,\widetilde{I}_{\widetilde{b}}+l}(\widehat{\Theta}_{j,T}^*)$, where $\widetilde{I}_i$, $i = 1, ..., \widetilde{b}$ are independent discrete uniform on $1, ..., S - \widetilde{l}$. Notice that, for each of the m models, and for each bootstrap replication, we draw $\widetilde{b}$ discrete uniform $\widetilde{I}_i$ on $1, ..., S - \widetilde{l}$, draws are independent across models, we have just suppressed the dependence of $\widetilde{I}_i$ on j, for notational simplicity.

We consider two different bootstrap analogs of $Z_{T,S}$, the first of which is valid when $T/S \to \delta > 0$ and the second of which is valid when $T/S \to 0$. Notice that in the second version, simulation error vanishes so that $Y_{j,n}^*(\widehat{\Theta}_{j,T}^*)$ in the first statistic is replaced with $Y_{j,n}(\widehat{\theta}_{j,T}^*)$, $j = 1, ..., m$. Define:

$$Z_{T,S}^{**} = \max_{j=2,...,m} \int_U Z_{j,T,S}^{**}(u)\phi(u)du,$$

where

$$
\begin{aligned}
Z^{**}_{j,T,S}(u) \;=\;& \frac{1}{T}\sum_{t=1}^{T}\left(\left(1\{Y_t^* \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y^*_{1,n}(\widehat{\Theta}^*_{1,T}) \le u\}\right)^2\right. \\
& \left. - \left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{1,n}(\widehat{\Theta}_{1,T}) \le u\}\right)^2\right) \\
& - \frac{1}{T}\sum_{t=1}^{T}\left(\left(1\{Y_t^* \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y^*_{j,n}(\widehat{\Theta}^*_{j,T}) \le u\}\right)^2\right. \\
& \left. - \left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{j,n}(\widehat{\Theta}_{j,T}) \le u\}\right)^2\right)
\end{aligned}
$$

and

$$
Z^*_{T,S} = \max_{j=2,\dots,m}\int_U Z^*_{j,T,S}(u)\phi(u)du,
$$

where

$$
\begin{aligned}
Z^*_{j,T,S}(u) \;=\;& \frac{1}{T}\sum_{t=1}^{T}\left(\left(1\{Y_t^* \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{1,n}(\widehat{\Theta}^*_{1,T}) \le u\}\right)^2\right. \\
& \left. - \left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{1,n}(\widehat{\Theta}_{1,T}) \le u\}\right)^2\right) \\
& - \frac{1}{T}\sum_{t=1}^{T}\left(\left(1\{Y_t^* \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{j,n}(\widehat{\Theta}^*_{j,T}) \le u\}\right)^2\right. \\
& \left. - \left(1\{Y_t \le u\} - \frac{1}{S}\sum_{n=1}^{S}1\{Y_{j,n}(\widehat{\Theta}_{j,T}) \le u\}\right)^2\right).
\end{aligned}
$$

CS (2005b) prove the first order validity of critical values constructed using the above bootstrap statistics; and they suggest proceeding in the following manner. For any bootstrap replication, compute the bootstrap statistic, $\sqrt{T}Z^{**}_{T,S}\left(\sqrt{T}Z^*_{T,S}\right)$. Perform B bootstrap replications (B large) and compute the quantiles of the empirical distribution of the B bootstrap statistics. Reject H_0 if $\sqrt{T}Z_{T,S}$ is greater than the $(1-\alpha)th$-quantile. Otherwise, do not reject. Now, for all samples except a set with probability measure approaching zero, $\sqrt{T}Z_{T,S}$ has the same limiting distribution as the corresponding bootstrapped statistic, when $\int_U\left((F_0(u)-F_1(u))^2 - (F_0(u)-F_j(u))^2\right)\phi(u)du = 0$ for all $j = 2,\dots,m$. In this case, the above approach ensures that the test has asymptotic size equal to α. On the other hand, when one (or more) competing models is (are) strictly dominated by the benchmark, the approach ensures that the test has an asymptotic size between 0 and α. Finally, under the alternative, $Z_{T,S}$ diverges to (plus) infinity, while the corresponding bootstrap statistic has a well defined limiting distribution. This ensures unit asymptotic power.

Table 1: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated π_t and y_t^g: Price and Information Stickiness ($\theta = 0.75$)**

S,l	Z	Crit. val. (Z^{**})		Crit. val. (Z^*)		CS Distributional Loss		
		5%	10%	5%	10%	sp	spi	si
5T,l1	0.1525	0.1543	0.0707	0.0744	0.0359	1.3623	1.2097	1.2926
5T,l2	0.1525	0.1797	0.1209	0.1047	0.0462	1.3623	1.2097	1.2926
5T,l3	0.1525	0.1610	0.0950	0.0997	0.0393	1.3623	1.2097	1.2926
5T,l4	0.1525	0.1644	0.0913	0.0978	0.0469	1.3623	1.2097	1.2926
5T,l5	0.1525	0.1659	0.0946	0.1036	0.0479	1.3623	1.2097	1.2926
10T,l1	0.0995	0.1343	0.0941	0.0798	0.0462	1.3041	1.2046	1.2914
10T,l2	0.0995	0.1352	0.0929	0.0939	0.0530	1.3041	1.2046	1.2914
10T,l3	0.0995	0.1345	0.0820	0.0860	0.0550	1.3041	1.2046	1.2914
10T,l4	0.0995	0.1385	0.0986	0.0824	0.0558	1.3041	1.2046	1.2914
10T,l5	0.0995	0.1541	0.1082	0.0908	0.0533	1.3041	1.2046	1.2914
20T,l1	0.0855	0.0969	0.0722	0.0788	0.0548	1.2884	1.2030	1.3046
20T,l2	0.0855	0.1112	0.0776	0.0763	0.0555	1.2884	1.2030	1.3046
20T,l3	0.0855	0.1087	0.0807	0.0778	0.0573	1.2884	1.2030	1.3046
20T,l4	0.0855	0.1034	0.0775	0.0747	0.0550	1.2884	1.2030	1.3046
20T,l5	0.0855	0.0967	0.0722	0.0715	0.0563	1.2884	1.2030	1.3046
30T,l1	0.0753	0.0646	0.0478	0.0489	0.0300	1.2775	1.2022	1.2805
30T,l2	0.0753	0.0765	0.0481	0.0486	0.0343	1.2775	1.2022	1.2805
30T,l3	0.0753	0.0761	0.0479	0.0481	0.0353	1.2775	1.2022	1.2805
30T,l4	0.0753	0.0705	0.0476	0.0479	0.0341	1.2775	1.2022	1.2805
30T,l5	0.0753	0.0694	0.0472	0.0489	0.0356	1.2775	1.2022	1.2805
50T,l1	0.0578	0.0590	0.0434	0.0425	0.0295	1.2675	1.2097	1.2699
50T,l2	0.0578	0.0603	0.0481	0.0413	0.0283	1.2675	1.2097	1.2699
50T,l3	0.0578	0.0618	0.0458	0.0451	0.0295	1.2675	1.2097	1.2699
50T,l4	0.0578	0.0666	0.0456	0.0377	0.0294	1.2675	1.2097	1.2699
50T,l5	0.0578	0.0582	0.0423	0.0413	0.0296	1.2675	1.2097	1.2699
100T,l1	0.0346	0.0565	0.0437	0.0422	0.0343	1.2558	1.2212	1.2698
100T,l2	0.0346	0.0564	0.0466	0.0419	0.0366	1.2558	1.2212	1.2698
100T,l3	0.0346	0.0586	0.0446	0.0446	0.0360	1.2558	1.2212	1.2698
100T,l4	0.0346	0.0592	0.0454	0.0472	0.0358	1.2558	1.2212	1.2698
100T,l5	0.0346	0.0539	0.0455	0.0440	0.0367	1.2558	1.2212	1.2698

Notes: Joint distributions of inflation and the output gap are compared using the simulation based test statistic discussed above. The historical data period is 1964:1-2003:4. The second column of entries reports the numerical values of the test statistic, while the next four columns report 5% and 10% bootstrap critical values based on a bootstrap statistics for the cases where $T/S \to \delta > 0$ (Z^{**}) and $T/S \to 0$ (Z^*), where T denotes the historical sample size and S the length of the simulated sample. The last three columns of entries in the table report the Corradi and Swanson (2005b) distributional loss measure associated with model i, which is an estimate of $E\left(\left(F_i(u;\theta_i^\dagger) - F_0(u;\theta_0)\right)^2\right)$.

Namely, we report $CS = \int_U \frac{1}{T} \sum_{t=1}^{T} \left(1\{Y_t \leq u\} - \frac{1}{S}\sum_{n=1}^{S} 1\{Y_{i,n}(\widehat{\theta}_{1,T}) \leq u\}\right)^2 \phi(u)du$ (see above for complete details). The CS distributional loss measure is calculated for the sticky price (sp), sticky price with indexation (spi) and sticky information (si) models. For the case where $S = T$ we set the block length used in the bootstrap as follows: $l1 = 5$, $l2 = 8$, $l3 = 10$, $l4 = 16$ and $l5 = 20$. For all other cases, where $S = aT$, we set li equal to 'a' times the corresponding value of li when $S = T$. All statistics are based on use of grids of 20x20 values for u distributed uniformly across the historical data ranges of π_t and y_t^g. Bootstrap empirical distributions are constructed using 500 bootstrap replications. Further details are given above.

Table 2: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated π_t and π_{t-1}: Price and Information Stickiness ($\theta = 0.75$)**

S, l	Z	Crit. val. (Z^{**})		Crit. val. (Z^*)		CS Distributional Loss		
		5%	10%	5%	10%	sp	spi	si
5T,l1	-0.0017	0.3594	0.3039	0.2483	0.1981	1.2928	1.2944	1.3318
5T,l2	-0.0017	0.3686	0.2854	0.2377	0.2030	1.2928	1.2944	1.3318
5T,l3	-0.0017	0.4120	0.3212	0.2553	0.2093	1.2928	1.2944	1.3318
5T,l4	-0.0017	0.4723	0.3303	0.2936	0.2114	1.2928	1.2944	1.3318
5T,l5	-0.0017	0.4645	0.3136	0.3109	0.2276	1.2928	1.2944	1.3318
10T,l1	0.0209	0.2016	0.1606	0.1613	0.1324	1.3056	1.2847	1.3187
10T,l2	0.0209	0.2168	0.1704	0.1590	0.1341	1.3056	1.2847	1.3187
10T,l3	0.0209	0.2163	0.1669	0.1700	0.1334	1.3056	1.2847	1.3187
10T,l4	0.0209	0.2635	0.1830	0.1837	0.1366	1.3056	1.2847	1.3187
10T,l5	0.0209	0.2691	0.1968	0.1755	0.1357	1.3056	1.2847	1.3187
20T,l1	0.0370	0.1380	0.1124	0.0977	0.0792	1.3134	1.2764	1.3088
20T,l2	0.0370	0.1486	0.1087	0.1000	0.0830	1.3134	1.2764	1.3088
20T,l3	0.0370	0.1499	0.1186	0.0980	0.0830	1.3134	1.2764	1.3088
20T,l4	0.0370	0.1584	0.1197	0.1085	0.0847	1.3134	1.2764	1.3088
20T,l5	0.0370	0.1643	0.1203	0.1194	0.0855	1.3134	1.2764	1.3088
30T,l1	0.0400	0.1059	0.0821	0.0772	0.0601	1.3210	1.2810	1.3107
30T,l2	0.0400	0.1086	0.0838	0.0785	0.0656	1.3210	1.2810	1.3107
30T,l3	0.0400	0.1093	0.0874	0.0832	0.0607	1.3210	1.2810	1.3107
30T,l4	0.0400	0.1238	0.0961	0.0827	0.0671	1.3210	1.2810	1.3107
30T,l5	0.0400	0.1275	0.0974	0.0856	0.0717	1.3210	1.2810	1.3107
50T,l1	0.0460	0.0847	0.0618	0.0573	0.0466	1.3233	1.2773	1.3149
50T,l2	0.0460	0.0862	0.0665	0.0573	0.0485	1.3233	1.2773	1.3149
50T,l3	0.0460	0.0830	0.0663	0.0586	0.0476	1.3233	1.2773	1.3149
50T,l4	0.0460	0.0887	0.0689	0.0636	0.0485	1.3233	1.2773	1.3149
50T,l5	0.0460	0.1045	0.0772	0.0649	0.0495	1.3233	1.2773	1.3149
100T,l1	0.0581	0.0451	0.0344	0.0330	0.0228	1.3281	1.2700	1.3076
100T,l2	0.0581	0.0491	0.0373	0.0329	0.0231	1.3281	1.2700	1.3076
100T,l3	0.0581	0.0506	0.0360	0.0311	0.0247	1.3281	1.2700	1.3076
100T,l4	0.0581	0.0520	0.0405	0.0326	0.0236	1.3281	1.2700	1.3076
100T,l5	0.0581	0.0573	0.0393	0.0361	0.0222	1.3281	1.2700	1.3076

Notes: See notes to Table 1.

Table 3: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated y_t^g and y_{t-1}^g: Price and Information Stickiness ($\theta = 0.75$)**

S,l	Z	Crit. val. (Z^{**})		Crit. val. (Z^*)		CS Distributional Loss		
		5%	10%	5%	10%	sp	spi	si
5T,l1	0.2261	-0.0212	-0.0648	-0.0722	-0.1025	1.4334	1.2073	1.2577
5T,l2	0.2261	-0.0065	-0.0511	-0.0613	-0.0959	1.4334	1.2073	1.2577
5T,l3	0.2261	0.0177	-0.0469	-0.0685	-0.0939	1.4334	1.2073	1.2577
5T,l4	0.2261	-0.0029	-0.0436	-0.0787	-0.1069	1.4334	1.2073	1.2577
5T,l5	0.2261	-0.0165	-0.0593	-0.0857	-0.1101	1.4334	1.2073	1.2577
10T,l1	0.1551	0.0269	0.0043	0.0050	-0.0126	1.3509	1.1958	1.2940
10T,l2	0.1551	0.0431	0.0121	0.0022	-0.0093	1.3509	1.1958	1.2940
10T,l3	0.1551	0.0363	0.0050	0.0014	-0.0130	1.3509	1.1958	1.2940
10T,l4	0.1551	0.0375	0.0135	0.0017	-0.0134	1.3509	1.1958	1.2940
10T,l5	0.1551	0.0392	0.0057	-0.0026	-0.0196	1.3509	1.1958	1.2940
20T,l1	0.1259	0.0474	0.0296	0.0292	0.0179	1.3250	1.1991	1.3123
20T,l2	0.1259	0.0572	0.0331	0.0340	0.0190	1.3250	1.1991	1.3123
20T,l3	0.1259	0.0574	0.0405	0.0356	0.0195	1.3250	1.1991	1.3123
20T,l4	0.1259	0.0526	0.0285	0.0298	0.0178	1.3250	1.1991	1.3123
20T,l5	0.1259	0.0494	0.0277	0.0252	0.0117	1.3250	1.1991	1.3123
30T,l1	0.1044	0.0677	0.0520	0.0445	0.0369	1.2990	1.1946	1.3092
30T,l2	0.1044	0.0595	0.0491	0.0495	0.0356	1.2990	1.1946	1.3092
30T,l3	0.1044	0.0699	0.0497	0.0468	0.0370	1.2990	1.1946	1.3092
30T,l4	0.1044	0.0560	0.0450	0.0457	0.0356	1.2990	1.1946	1.3092
30T,l5	0.1044	0.0540	0.0403	0.0408	0.0346	1.2990	1.1946	1.3092
50T,l1	0.0708	0.0501	0.0412	0.0428	0.0326	1.2802	1.2094	1.2926
50T,l2	0.0708	0.0565	0.0450	0.0450	0.0327	1.2802	1.2094	1.2926
50T,l3	0.0708	0.0586	0.0423	0.0425	0.0339	1.2802	1.2094	1.2926
50T,l4	0.0708	0.0497	0.0387	0.0395	0.0301	1.2802	1.2094	1.2926
50T,l5	0.0708	0.0433	0.0383	0.0354	0.0292	1.2802	1.2094	1.2926
100T,l1	0.0340	0.0646	0.0555	0.0546	0.0489	1.2620	1.2279	1.2963
100T,l2	0.0340	0.0641	0.0542	0.0580	0.0494	1.2620	1.2279	1.2963
100T,l3	0.0340	0.0625	0.0544	0.0565	0.0502	1.2620	1.2279	1.2963
100T,l4	0.0340	0.0622	0.0559	0.0544	0.0491	1.2620	1.2279	1.2963
100T,l5	0.0340	0.0621	0.0531	0.0509	0.0474	1.2620	1.2279	1.2963

Notes: See notes to Table 1.

Table 4: **Historical and Simulated Distribution Quantiles for Inflation and the Output Gap**

I. Information and Price Stickiness $\theta = 0.75$

A. Inflation

	10%	20%	30%	40%	50%	60%	70%	80%	90%
data	-0.0064	-0.0047	-0.0031	-0.0022	-0.0014	-0.0004	0.0017	0.0055	0.0118
sp	-0.0043	-0.0028	-0.0017	-0.0007	0.0001	0.0010	0.0019	0.0031	0.0046
spi	-0.0066	-0.0044	-0.0028	-0.0014	0.0000	0.0013	0.0027	0.0042	0.0063
si	-0.0050	-0.0033	-0.0020	-0.0009	0.0000	0.0010	0.0021	0.0033	0.0050

B. Output Gap

	10%	20%	30%	40%	50%	60%	70%	80%	90%
data	-0.0167	-0.0104	-0.0072	-0.0034	0.0008	0.0047	0.0089	0.0125	0.0184
sp	-0.0381	-0.0244	-0.0149	-0.0065	0.0009	0.0085	0.0170	0.0275	0.0406
spi	-0.0349	-0.0232	-0.0147	-0.0071	-0.0003	0.0063	0.0137	0.0229	0.0343
si	-0.0407	-0.0271	-0.0167	-0.0076	0.0002	0.0085	0.0167	0.0270	0.0409

II. Information and Price Stickiness $\theta = 0.5$

A. Inflation

	10%	20%	30%	40%	50%	60%	70%	80%	90%
data	-0.0064	-0.0047	-0.0031	-0.0022	-0.0014	-0.0004	0.0017	0.0055	0.0118
sp	-0.0067	-0.0043	-0.0026	-0.0012	0.0001	0.0015	0.0029	0.0046	0.0070
spi	-0.0086	-0.0057	-0.0036	-0.0017	0.0000	0.0016	0.0033	0.0055	0.0083
si	-0.0076	-0.0050	-0.0031	-0.0015	0.0000	0.0015	0.0031	0.0051	0.0077

B. Output Gap

	10%	20%	30%	40%	50%	60%	70%	80%	90%
data	-0.0167	-0.0104	-0.0072	-0.0034	0.0008	0.0047	0.0089	0.0125	0.0184
sp	-0.0203	-0.0131	-0.0081	-0.0036	0.0003	0.0045	0.0089	0.0143	0.0214
spi	-0.0222	-0.0146	-0.0092	-0.0045	-0.0003	0.0041	0.0088	0.0142	0.0219
si	-0.0221	-0.0146	-0.0091	-0.0044	0.0000	0.0042	0.0090	0.0145	0.0221

Notes: Distribution quantiles are given for historical and simulated inflation and output gap. Simulated data are based on the sticky price (sp), sticky price with indexation (spi) and sticky information (si) models. Historical data are for the period 1964:1-2003:4. The simulation sample size is 100T, where T is the historical data sample size.

Table 5: **Directional Accuracy of Simulated Inflation and the Output Gap Paths**

I. Information and Price Stickiness $\theta = 0.75$

A. Inflation

(π_t, π_{t-1})	data	sp	spi	si
(down, down)	0.2025	0.2453	0.2518	0.2553
(up, down)	0.3038	0.2547	0.2512	0.2470
(down, up)	0.3038	0.2547	0.2482	0.2447
(up, up)	0.1899	0.2453	0.2488	0.2530
p-value	0.0068	0.0185	0.8868	0.0341

B. Output Gap

(y_t^g, y_{t-1}^g)	data	sp	spi	si
(down, down)	0.3228	0.2653	0.2483	0.2508
(up, down)	0.2215	0.2341	0.2537	0.2464
(down, up)	0.2152	0.2340	0.2510	0.2485
(up, up)	0.2405	0.2666	0.2470	0.2543
p-value	0.1293	0.0000	0.2297	0.2006

C. Inflation and Output Gap

(y_t^g, π_t)	data	sp	spi	si
(down, down)	0.2579	0.4589	0.3599	0.3848
(up, down)	0.2516	0.0411	0.1431	0.1174
(down, up)	0.2830	0.0404	0.1421	0.1123
(up, up)	0.2075	0.4596	0.3548	0.3854
p-value	0.3708	0.0000	0.0000	0.0000

II. Information and Price Stickiness $\theta = 0.5$

A. Inflation

(π_t, π_{t-1})	data	sp	spi	si
(down, down)	0.2025	0.2402	0.2477	0.2562
(up, down)	0.3038	0.2601	0.2513	0.2475
(down, up)	0.3038	0.2600	0.2525	0.2441
(up, up)	0.1899	0.2397	0.2484	0.2522
p-value	0.0068	0.0000	0.3270	0.0341

B. Output Gap

(y_t^g, y_{t-1}^g)	data	sp	spi	si
(down, down)	0.3228	0.2509	0.2486	0.2483
(up, down)	0.2215	0.2475	0.2543	0.2470
(down, up)	0.2152	0.2474	0.2497	0.2500
(up, up)	0.2405	0.2542	0.2473	0.2547
p-value	0.1293	0.1952	0.3052	0.4598

C. Inflation and Output Gap

(y_t^g, π_t)	data	sp	spi	si
(down, down)	0.2579	0.4642	0.3780	0.3523
(up, down)	0.2516	0.0361	0.1211	0.1514
(down, up)	0.2830	0.0341	0.1249	0.1430
(up, up)	0.2075	0.4656	0.3760	0.3533
p-value	0.3708	0.0000	0.0000	0.0000

Notes: Contingency tables are given for historical and simulated inflation (π_t) and the output gap (y_t^g) variables. Simulated data are based on the sticky price (sp), sticky price with indexation (spi) and sticky information (si) models. Historical data are for the period 1964:1-2003:4. The simulation sample size is 100T, where T is the historical data sample size. P-values reported in the last row of each panel in the table correspond to the χ^2 test of independence. The statistic has a χ^2 limiting distribution with 1 degree of freedom.

Table 6: **Autocorrelation and Cross Correlation: Inflation and the Output Gap**

I. Information and Price Stickiness $\theta = 0.75$

S	$corr(\pi_t, \pi_{t-1})$			$corr(y_t^g, y_{t-1}^g)$			$corr(y_t^g, \pi_t)$		
	sp	spi	si	sp	spi	si	sp	spi	si
5T	0.8470	0.9027	0.9543	0.8841	0.7294	0.8579	0.9151	0.8379	0.9424
10T	0.8473	0.9146	0.9518	0.8826	0.7517	0.8611	0.9142	0.8411	0.9447
20T	0.8348	0.9199	0.9502	0.8721	0.7524	0.8598	0.9073	0.8432	0.9435
30T	0.8286	0.9172	0.9452	0.8662	0.7474	0.8451	0.9040	0.8417	0.9379
50T	0.8316	0.9158	0.9423	0.8691	0.7514	0.8387	0.9055	0.8419	0.9355
100T	0.8322	0.9176	0.9412	0.8695	0.7571	0.8366	0.9063	0.8437	0.9346

II. Information and Price Stickiness $\theta = 0.5$

S	$corr(\pi_t, \pi_{t-1})$			$corr(y_t^g, y_{t-1}^g)$			$corr(y_t^g, \pi_t)$		
	sp	spi	si	sp	spi	si	sp	spi	si
5T	0.6584	0.7507	0.7475	0.7094	0.4958	0.5600	0.8075	0.8701	0.9442
10T	0.6468	0.7730	0.7602	0.6932	0.5065	0.5701	0.7972	0.8679	0.9437
20T	0.6467	0.7793	0.7585	0.6942	0.5095	0.5723	0.7974	0.8686	0.9459
30T	0.6403	0.7742	0.7435	0.6871	0.5075	0.5465	0.7945	0.8696	0.9425
50T	0.6415	0.7771	0.7397	0.6889	0.5088	0.5366	0.7945	0.8698	0.9405
100T	0.6457	0.7822	0.7368	0.6932	0.5141	0.5362	0.7977	0.8710	0.9412

Notes: Entries in the table correspond to autocorrelations of inflation ($corr(\pi_t, \pi_{t-1})$) and the output gap ($corr(y_t^g, y_{t-1}^g)$), as well as the cross correlation between inflation and the output gap. Simulated data are based on the sticky price (sp), sticky price with indexation (spi) and sticky information (si) models. Historical data are for the period 1964:1-2003:4. Values are given for simulation samples of S = 5T, 10T, 20T, 30T, 50T, 100T, where T is the historical data sample size. Corresponding historical values are: $corr(\pi_t, \pi_{t-1}) = 0.8363$, $corr(y_t^g, y_{t-1}^g) = 0.8690$, $corr(y_t^g, \pi_t) = 0.2582$.

Table 7: **Correlation Between Output Gap and Change in Inflation: Acceleration Phenomena**

I. Information and Price Stickiness $\theta = 0.75$

S	$corr(y_t^g, \pi_{t+2} - \pi_{t-2})$			$corr(y_t^g, \pi_{t+4} - \pi_{t-4})$		
	sp	spi	si	sp	spi	si
5T	0.1226	0.7854	0.4971	0.0765	0.7257	0.4466
10T	0.1213	0.7706	0.4984	0.0658	0.7132	0.4488
20T	0.1272	0.7600	0.5091	0.0743	0.7018	0.4599
30T	0.1323	0.7651	0.5292	0.0817	0.7068	0.4763
50T	0.1295	0.7693	0.5391	0.0776	0.7122	0.4851
100T	0.1302	0.7666	0.5450	0.0786	0.7103	0.4905

II. Information and Price Stickiness $\theta = 0.5$

S	$corr(y_t^g, \pi_{t+2} - \pi_{t-2})$			$corr(y_t^g, \pi_{t+4} - \pi_{t-4})$		
	sp	spi	si	sp	spi	si
5T	0.2044	0.7984	0.5966	0.1075	0.6552	0.4382
10T	0.2108	0.7844	0.5894	0.0975	0.6436	0.4317
20T	0.2085	0.7813	0.5964	0.1040	0.6431	0.4331
30T	0.2132	0.7852	0.6097	0.1138	0.6467	0.4368
50T	0.2118	0.7832	0.6110	0.1105	0.6417	0.4351
100T	0.2111	0.7803	0.6164	0.1107	0.6374	0.4419

Notes: See notes to Table 3. Corresponding historical values are: $corr(y_t^g, \pi_{t+2} - \pi_{t-2}) = 0.4491$, $corr(y_t^g, \pi_{t+4} - \pi_{t-4}) = 0.5458$.

Table 8: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated** π_t **and** y_t^g**: Price and Information Stickiness** ($\theta = 0.5$)

S, l	Z	Crit. val. (Z^{**})		Crit. val. (Z^*)		CS Distributional Loss		
		5%	10%	5%	10%	sp	spi	si
5T,l1	0.0309	0.0670	0.0428	0.0523	0.0336	1.1865	1.1556	1.1583
5T,l2	0.0309	0.0776	0.0482	0.0574	0.0309	1.1865	1.1556	1.1583
5T,l3	0.0309	0.0872	0.0589	0.0613	0.0323	1.1865	1.1556	1.1583
5T,l4	0.0309	0.0928	0.0575	0.0601	0.0372	1.1865	1.1556	1.1583
5T,l5	0.0309	0.1023	0.0654	0.0661	0.0408	1.1865	1.1556	1.1583
10T,l1	0.0124	0.0723	0.0451	0.0482	0.0289	1.1669	1.1545	1.1586
10T,l2	0.0124	0.0727	0.0512	0.0520	0.0349	1.1669	1.1545	1.1586
10T,l3	0.0124	0.0800	0.0537	0.0527	0.0346	1.1669	1.1545	1.1586
10T,l4	0.0124	0.0805	0.0585	0.0521	0.0383	1.1669	1.1545	1.1586
10T,l5	0.0124	0.0836	0.0610	0.0562	0.0386	1.1669	1.1545	1.1586
20T,l1	0.0127	0.0427	0.0297	0.0355	0.0239	1.1658	1.1531	1.1620
20T,l2	0.0127	0.0506	0.0404	0.0395	0.0269	1.1658	1.1531	1.1620
20T,l3	0.0127	0.0563	0.0374	0.0391	0.0272	1.1658	1.1531	1.1620
20T,l4	0.0127	0.0601	0.0429	0.0455	0.0338	1.1658	1.1531	1.1620
20T,l5	0.0127	0.0622	0.0436	0.0457	0.0343	1.1658	1.1531	1.1620
30T,l1	0.0119	0.0367	0.0267	0.0276	0.0201	1.1642	1.1523	1.1590
30T,l2	0.0119	0.0419	0.0293	0.0300	0.0212	1.1642	1.1523	1.1590
30T,l3	0.0119	0.0421	0.0321	0.0305	0.0217	1.1642	1.1523	1.1590
30T,l4	0.0119	0.0485	0.0346	0.0363	0.0269	1.1642	1.1523	1.1590
30T,l5	0.0119	0.0502	0.0329	0.0399	0.0262	1.1642	1.1523	1.1590
50T,l1	0.0083	0.0307	0.0213	0.0251	0.0182	1.1618	1.1535	1.1576
50T,l2	0.0083	0.0359	0.0229	0.0274	0.0203	1.1618	1.1535	1.1576
50T,l3	0.0083	0.0380	0.0252	0.0288	0.0209	1.1618	1.1535	1.1576
50T,l4	0.0083	0.0422	0.0301	0.0348	0.0233	1.1618	1.1535	1.1576
50T,l5	0.0083	0.0473	0.0302	0.0339	0.0240	1.1618	1.1535	1.1576
100T,l1	0.0000	0.0303	0.0235	0.0265	0.0220	1.1579	1.1585	1.1579
100T,l2	0.0000	0.0357	0.0278	0.0292	0.0247	1.1579	1.1585	1.1579
100T,l3	0.0000	0.0372	0.0288	0.0312	0.0262	1.1579	1.1585	1.1579
100T,l4	0.0000	0.0403	0.0294	0.0345	0.0303	1.1579	1.1585	1.1579
100T,l5	0.0000	0.0426	0.0327	0.0372	0.0292	1.1579	1.1585	1.1579

Notes: See notes to Table 1.

Table 9: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated π_t and π_{t-1}: Price and Information Stickiness ($\theta = 0.5$)**

| S, l	Z	Crit. val. (Z^{**})		Crit. val. (Z^*)		CS Distributional Loss		
		5%	10%	5%	10%	sp	spi	si
5T,l1	-0.0105	0.1598	0.1156	0.1005	0.0769	1.2548	1.2653	1.2676
5T,l2	-0.0105	0.1858	0.1303	0.1056	0.0782	1.2548	1.2653	1.2676
5T,l3	-0.0105	0.2096	0.1372	0.1164	0.0898	1.2548	1.2653	1.2676
5T,l4	-0.0105	0.2065	0.1477	0.1288	0.0921	1.2548	1.2653	1.2676
5T,l5	-0.0105	0.2209	0.1536	0.1464	0.1009	1.2548	1.2653	1.2676
10T,l1	-0.0008	0.1077	0.0750	0.0666	0.0460	1.2631	1.2639	1.2713
10T,l2	-0.0008	0.1037	0.0816	0.0763	0.0515	1.2631	1.2639	1.2713
10T,l3	-0.0008	0.1106	0.0885	0.0766	0.0559	1.2631	1.2639	1.2713
10T,l4	-0.0008	0.1380	0.0932	0.0910	0.0602	1.2631	1.2639	1.2713
10T,l5	-0.0008	0.1589	0.1080	0.1008	0.0745	1.2631	1.2639	1.2713
20T,l1	-0.0038	0.0581	0.0452	0.0425	0.0270	1.2608	1.2646	1.2660
20T,l2	-0.0038	0.0773	0.0502	0.0488	0.0332	1.2608	1.2646	1.2660
20T,l3	-0.0038	0.0888	0.0578	0.0518	0.0353	1.2608	1.2646	1.2660
20T,l4	-0.0038	0.0883	0.0593	0.0664	0.0402	1.2608	1.2646	1.2660
20T,l5	-0.0038	0.1036	0.0750	0.0730	0.0476	1.2608	1.2646	1.2660
30T,l1	-0.0016	0.0409	0.0275	0.0286	0.0156	1.2657	1.2673	1.2673
30T,l2	-0.0016	0.0634	0.0391	0.0382	0.0239	1.2657	1.2673	1.2673
30T,l3	-0.0016	0.0659	0.0369	0.0418	0.0300	1.2657	1.2673	1.2673
30T,l4	-0.0016	0.0767	0.0509	0.0483	0.0336	1.2657	1.2673	1.2673
30T,l5	-0.0016	0.0830	0.0587	0.0564	0.0418	1.2657	1.2673	1.2673
50T,l1	0.0006	0.0319	0.0212	0.0208	0.0102	1.2656	1.2650	1.2668
50T,l2	0.0006	0.0403	0.0272	0.0322	0.0172	1.2656	1.2650	1.2668
50T,l3	0.0006	0.0476	0.0268	0.0342	0.0194	1.2656	1.2650	1.2668
50T,l4	0.0006	0.0560	0.0352	0.0423	0.0253	1.2656	1.2650	1.2668
50T,l5	0.0006	0.0584	0.0435	0.0448	0.0284	1.2656	1.2650	1.2668
100T,l1	0.0067	0.0146	0.0061	0.0123	0.0035	1.2687	1.2653	1.2620
100T,l2	0.0067	0.0295	0.0119	0.0219	0.0064	1.2687	1.2653	1.2620
100T,l3	0.0067	0.0253	0.0130	0.0235	0.0110	1.2687	1.2653	1.2620
100T,l4	0.0067	0.0360	0.0197	0.0304	0.0163	1.2687	1.2653	1.2620
100T,l5	0.0067	0.0465	0.0261	0.0440	0.0226	1.2687	1.2653	1.2620

Notes: See notes to Table 1.

Table 10: **Distributional Accuracy Tests Based on the Comparison of Historical and Simulated y_t^g and y_{t-1}^g: Price and Information Stickiness ($\theta = 0.5$)**

| S,l	Z	*Crit. val. (Z^{**})*		*Crit. val. (Z^*)*		*CS Distributional Loss*		
		5%	10%	5%	10%	sp	spi	si
5T,l1	0.0200	0.0598	0.0462	0.0430	0.0351	1.1362	1.1224	1.1162
5T,l2	0.0200	0.0682	0.0511	0.0508	0.0375	1.1362	1.1224	1.1162
5T,l3	0.0200	0.0681	0.0498	0.0488	0.0395	1.1362	1.1224	1.1162
5T,l4	0.0200	0.0706	0.0535	0.0443	0.0350	1.1362	1.1224	1.1162
5T,l5	0.0200	0.0691	0.0507	0.0420	0.0339	1.1362	1.1224	1.1162
10T,l1	0.0026	0.0618	0.0492	0.0470	0.0371	1.1195	1.1169	1.1177
10T,l2	0.0026	0.0618	0.0509	0.0486	0.0404	1.1195	1.1169	1.1177
10T,l3	0.0026	0.0567	0.0445	0.0491	0.0375	1.1195	1.1169	1.1177
10T,l4	0.0026	0.0545	0.0462	0.0450	0.0367	1.1195	1.1169	1.1177
10T,l5	0.0026	0.0557	0.0435	0.0433	0.0373	1.1195	1.1169	1.1177
20T,l1	0.0027	0.0478	0.0395	0.0375	0.0320	1.1193	1.1165	1.1191
20T,l2	0.0027	0.0475	0.0394	0.0398	0.0336	1.1193	1.1165	1.1191
20T,l3	0.0027	0.0499	0.0405	0.0410	0.0333	1.1193	1.1165	1.1191
20T,l4	0.0027	0.0478	0.0392	0.0381	0.0330	1.1193	1.1165	1.1191
20T,l5	0.0027	0.0450	0.0380	0.0359	0.0318	1.1193	1.1165	1.1191
30T,l1	0.0005	0.0404	0.0351	0.0354	0.0306	1.1168	1.1162	1.1189
30T,l2	0.0005	0.0438	0.0362	0.0368	0.0317	1.1168	1.1162	1.1189
30T,l3	0.0005	0.0449	0.0362	0.0374	0.0327	1.1168	1.1162	1.1189
30T,l4	0.0005	0.0434	0.0366	0.0369	0.0314	1.1168	1.1162	1.1189
30T,l5	0.0005	0.0396	0.0356	0.0345	0.0300	1.1168	1.1162	1.1189
50T,l1	-0.0014	0.0393	0.0344	0.0339	0.0304	1.1154	1.1168	1.1180
50T,l2	-0.0014	0.0422	0.0352	0.0364	0.0317	1.1154	1.1168	1.1180
50T,l3	-0.0014	0.0417	0.0356	0.0367	0.0329	1.1154	1.1168	1.1180
50T,l4	-0.0014	0.0401	0.0339	0.0347	0.0310	1.1154	1.1168	1.1180
50T,l5	-0.0014	0.0375	0.0334	0.0333	0.0311	1.1154	1.1168	1.1180
100T,l1	-0.0040	0.0375	0.0344	0.0340	0.0318	1.1140	1.1185	1.1180
100T,l2	-0.0040	0.0375	0.0343	0.0356	0.0325	1.1140	1.1185	1.1180
100T,l3	-0.0040	0.0403	0.0353	0.0361	0.0325	1.1140	1.1185	1.1180
100T,l4	-0.0040	0.0390	0.0342	0.0348	0.0316	1.1140	1.1185	1.1180
100T,l5	-0.0040	0.0379	0.0331	0.0335	0.0315	1.1140	1.1185	1.1180

Notes: See notes to Table 1.

Figure 1: **Historical Inflation and the Output Gap**
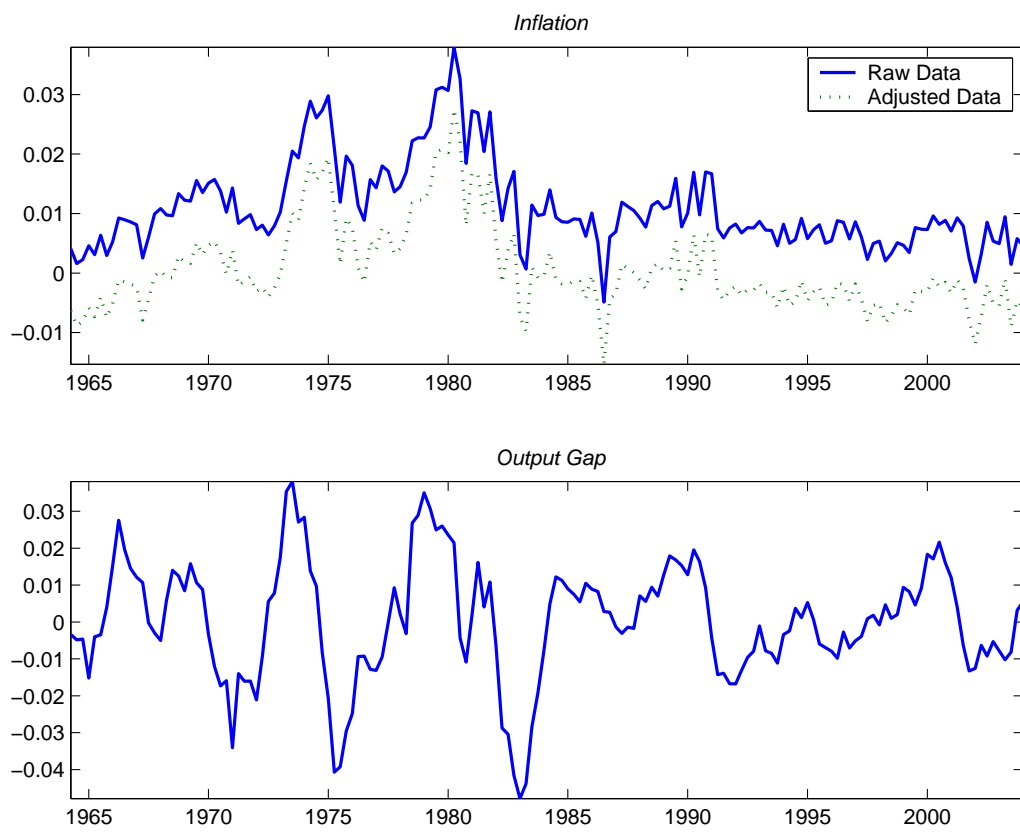
Notes: Historical data are for the period 1964:1 - 2003:4. Adjusted data correspond to the data simulated using the three alternative models, and are thus "de-meaned". The output gap is constructed using H-P filter.

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Figure 2: **Response to Anticipated Permanent Money Supply Shock**
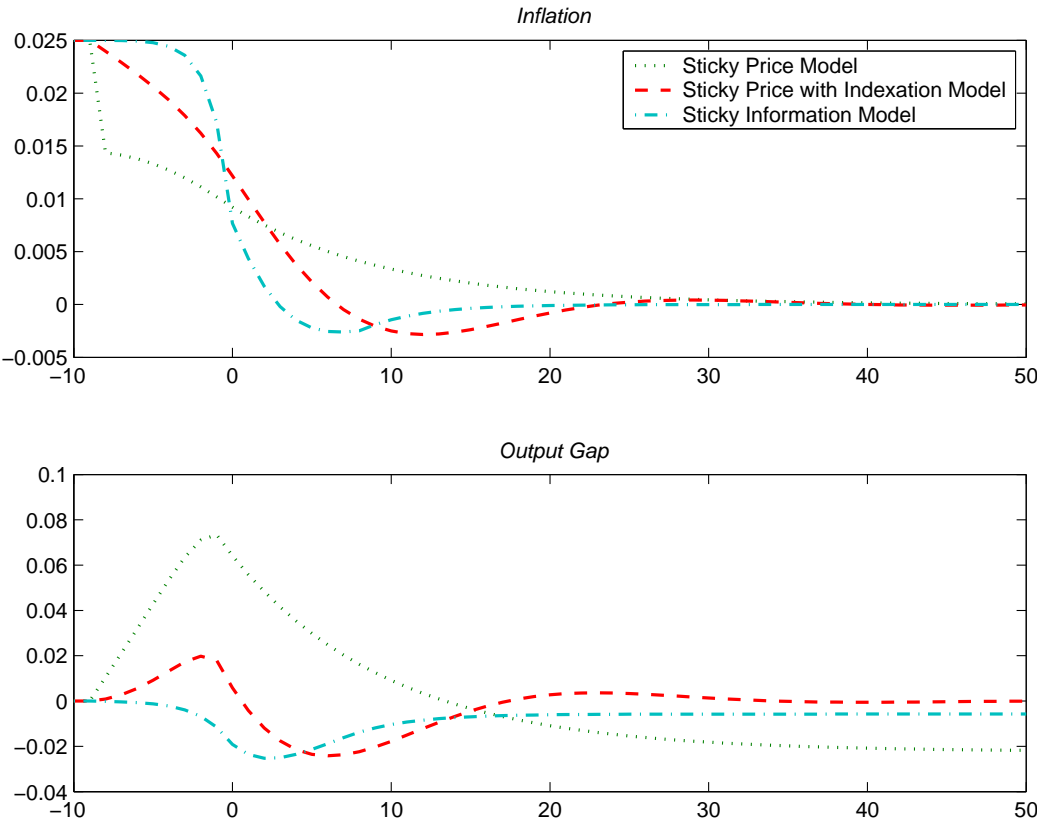
Notes: The size of the shock is equal to 0.025.

Figure 3: **Maximum Response of the Output Gap to Anticipated Money Supply Shock: Sticky Price Model**

A. Change in η and ρ_m


B. Change in η and Anticipation Period
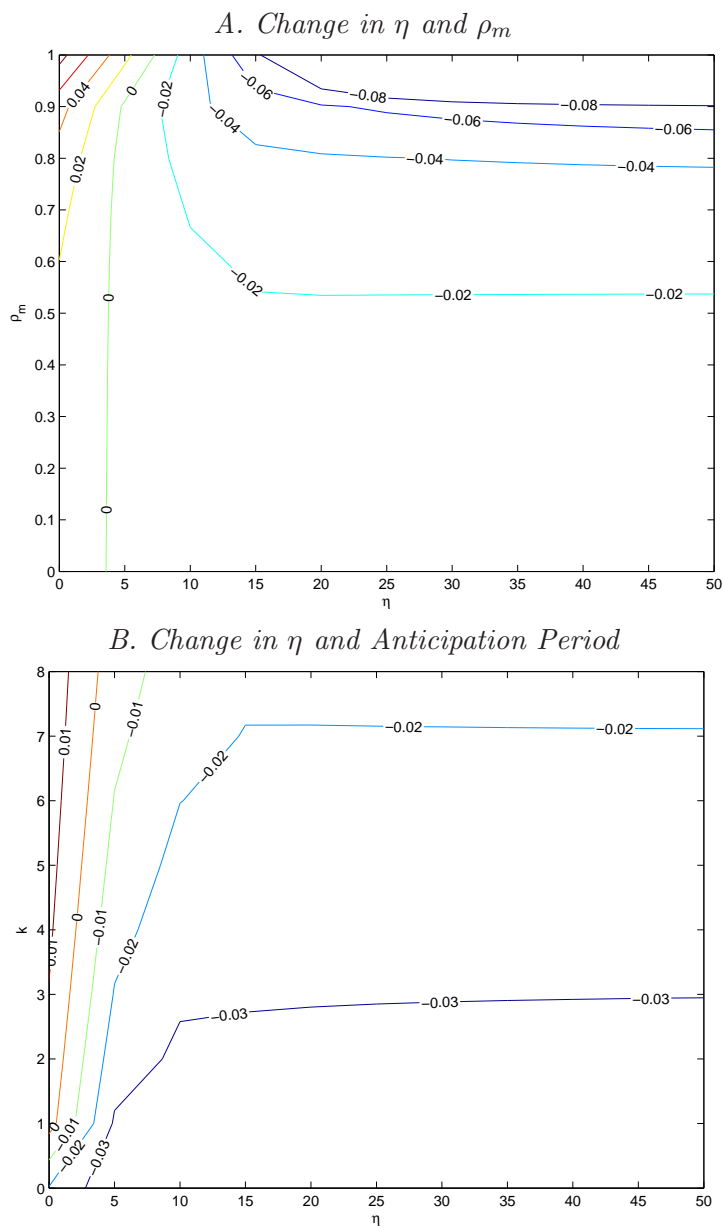
Notes: Graph A represent how maximum of the output gap response to anticipated money supply shock changes as interest rate semi-elasticity of money demand, η, and persistence of money shock, ρ_m, change. Graph B represents how maximum response of the output gap changes as η and anticipation period, k, change. All other parameter values at our baseline calibration. For example the maximum of the response to anticipated 8 quarters ahead ($k = 8$) permanent shock ($\rho_m = 1$) with other parameters at baseline values (e.g. $\eta = 1$), Figure 2, is in the upper left corner of the Graph A.

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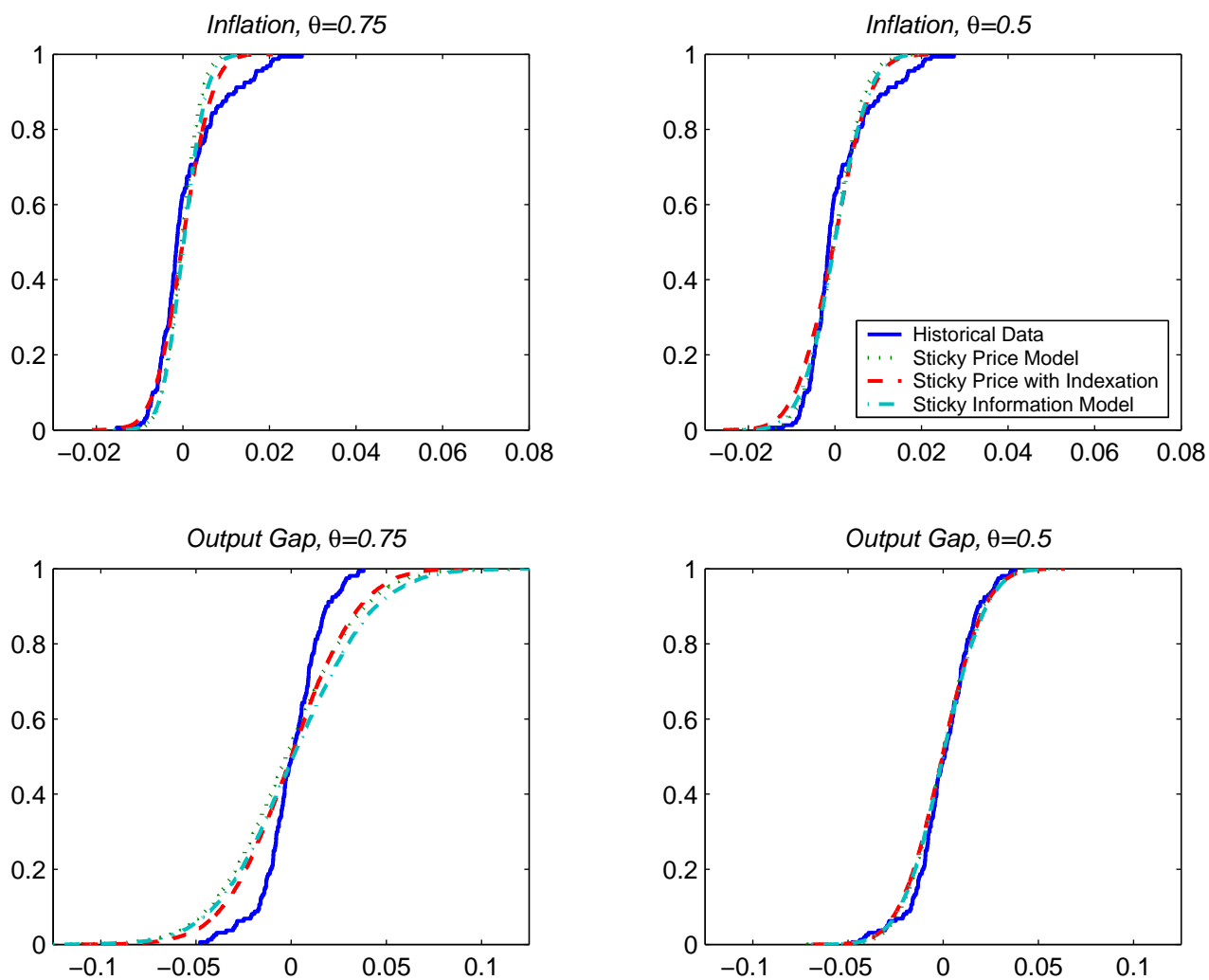Figure 4: **Impulse Response to Contractionary Money Supply Shocks**
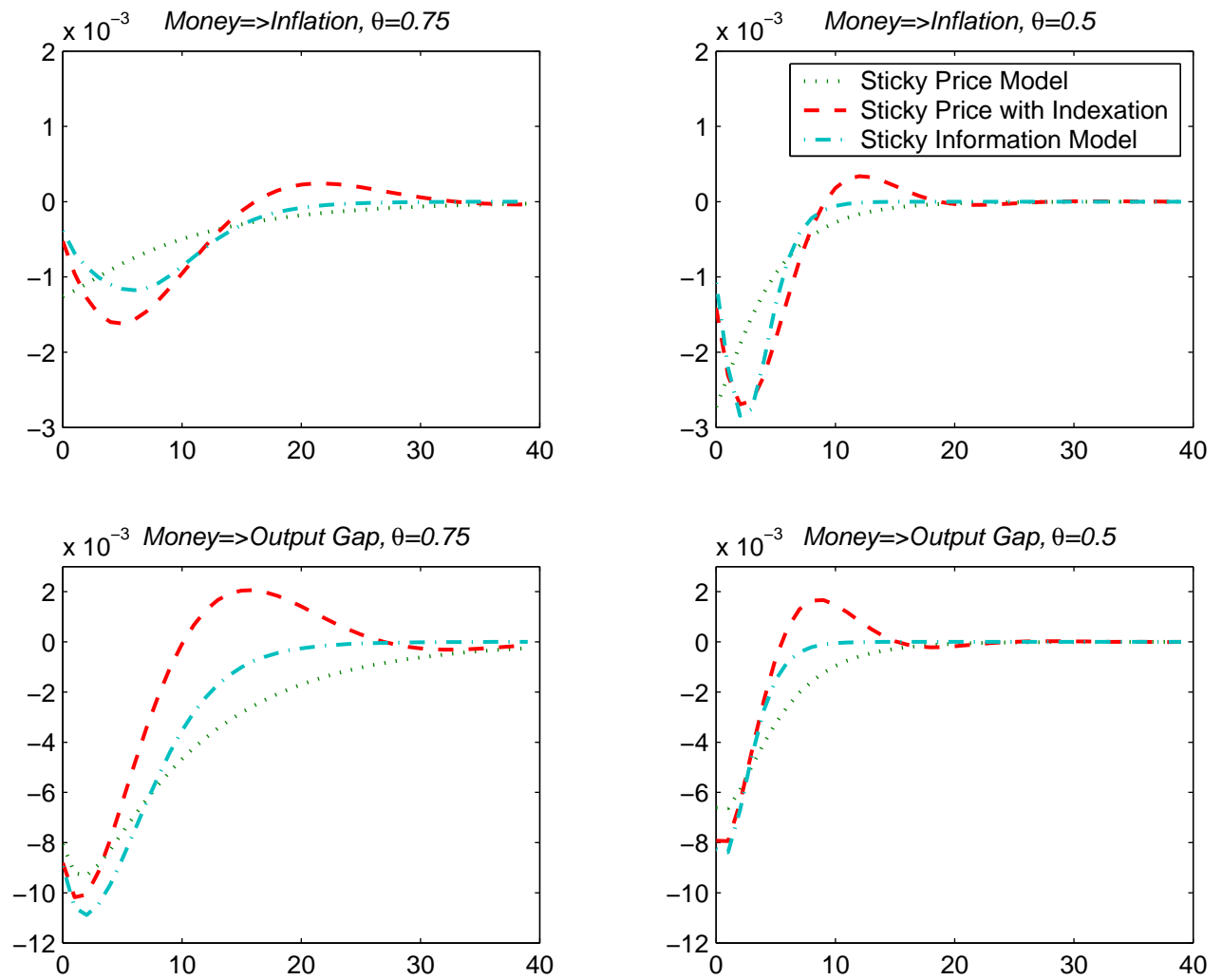
Notes: The size of the shock is equal to one standard deviation, $\sigma_m = 0.007$

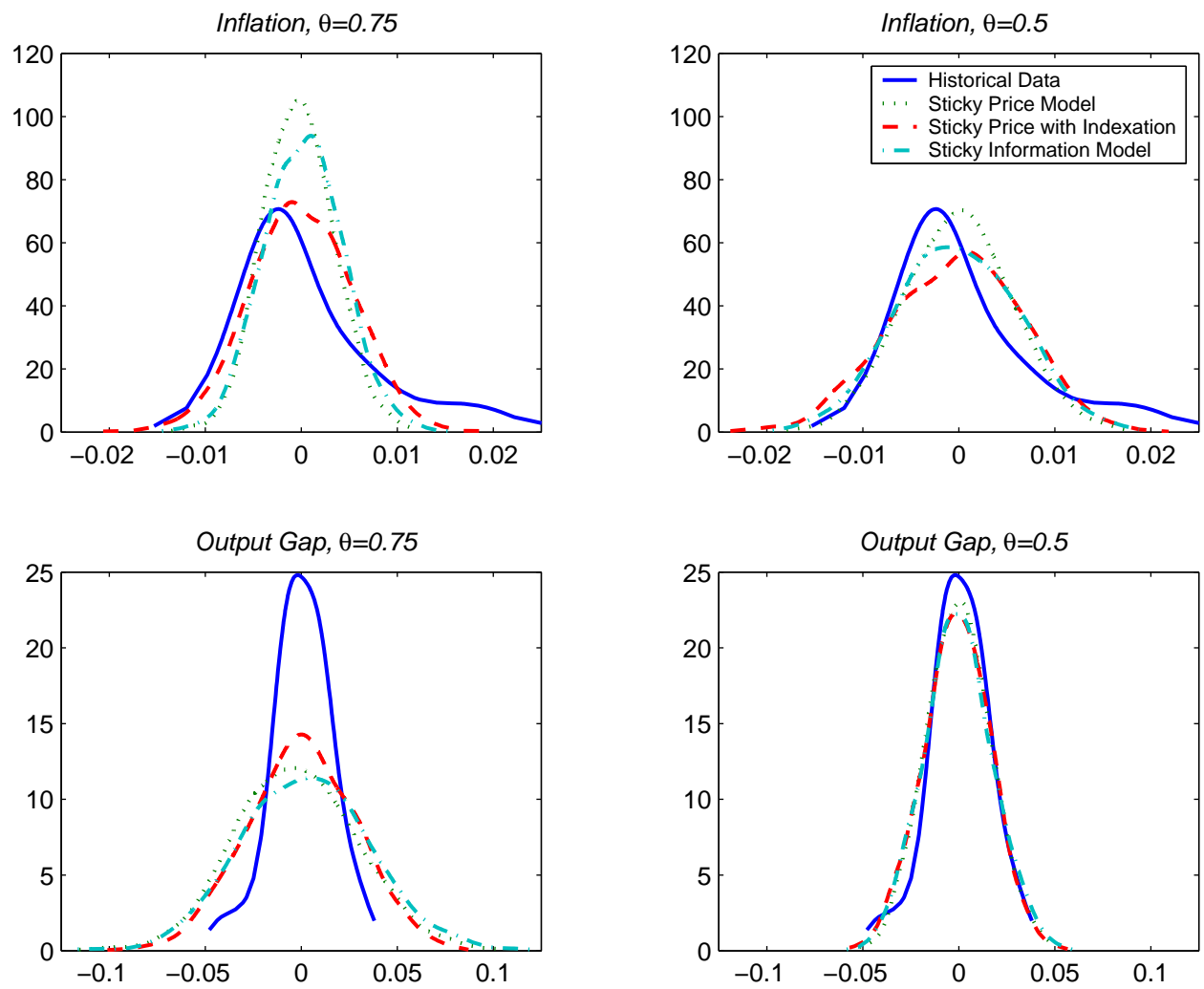Figure 5: **Impulse Response to Contractionary Technology Shocks**
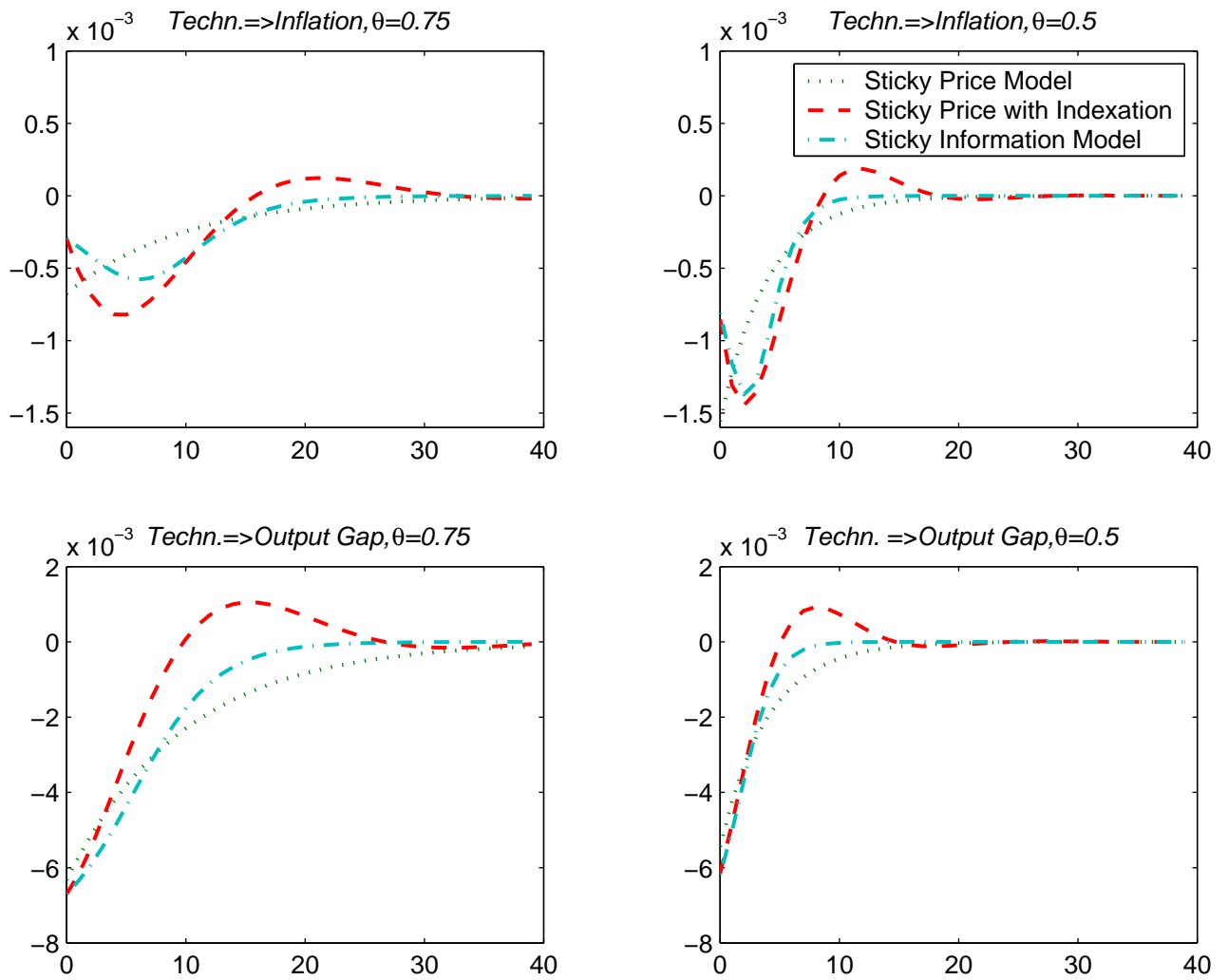
Notes: The size of the shock is equal to one standard deviation, $\sigma_a = 0.007$.

Figure 6: **Historical and Simulated Empirical Distributions of Inflation and the Output Gap**

Notes: See notes to Table 4. Plots of distributions corresponding to those used in the statistical comparison of the sticky price, sticky price with indexation, and sticky information models are given for π_t and y_t^g.

Figure 7: **Historical and Simulated Empirical Densities of Inflation and the Output Gap**

Notes: See notes to Figure 6.