→ Examen Parcial - Norman Daniel Vicente

Problema 1

a) $P(X < 0.5)$

$$\int_0^{1/2} \frac{3}{2}x^2 + x \, dx = 0.1875$$

b) $P(X > 0.5 \mid X \geq 0.25)$

$$\int_{1/2}^{1} \frac{3}{2}x^2 + x \, dx = 0.8125$$

$$\int_{1/4}^{1} \frac{3}{2}x^2 + x \, dx = 0.9609$$

$$\frac{P(X > 0.5)}{P(X \geq 0.25)} = \frac{0.8125}{0.9609} = 0.8455$$

Problema 11

a) $\mu = 100$

$\lambda = \frac{1}{100}$

$$P(X < 200) = \int_0^{200} \frac{1}{100} e^{-1/100\, x} dx = 0.864$$

b) $P(X > 150 \mid X > 100) =$

$$\int_{150}^{\infty} \frac{1}{100} e^{-1/100\, x} dx = 0.223$$

$$\int_{50}^{\infty} \frac{1}{100} e^{-1/100\, x} dx = 0.6065$$

$s + t = 150$

$s = 100$

50

$P(X > 150 \mid X > 100) = 0.6065$

Problema III

a) $\mu = 10$

$\sigma = 3.5$

$P(X \leq c) = 0.95$

$X = \mu + Z\sigma$

$X = (10) + (1.645)(3.5) = 15.75$

b) $\mu = 8$

$\sigma = 2$

$X = 15.75$

$X \sim N(8, 4)$

$P(X \leq 15.75) = 0.999$

Problema IV

$$f(x) = 2x \; ; \; 0 \leq x \leq 1$$

$$Y = g(x) = 3x - 1$$

$$g^{-1}(y) = \frac{y+1}{3} \qquad \left| \frac{dx}{dy} \right| = \frac{1}{3}$$

$$f_Y(y) = f_x\left(g^{-1}(y)\right) \left| \frac{d}{dy} g^{-1}(y) \right|$$

$$f_Y(y) = 2\left(\frac{y+1}{3}\right) * \frac{1}{3}$$

$$f_Y(y) = 2\left(\frac{y+1}{9}\right), \quad 0 \leq y \leq 1$$

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Problema V

a) falso

Porque son independientes si

$$F(x_1, x_2) = f(x_1) f(x_2)$$

y su valor es esperado es

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

b) falso

Esta propiedad se comple para la distribucion exponencial, se expresa como

$$P(X \geq s+t \,/\, X \geq s) = P(X \geq t)$$

c) Verdadero

Porque $E(x) = M_1' = \mu$

Si $t = 0$

$$m_x(0) = E(e^0) = 1$$

d) falso

Porque $Y = \mu + Z\sigma$

Problema VII

$$f(x_1, x_2) = 6(1 - x_2),\ 0 \le x_1 \le x_2 \le 1$$

a) $$\int_{1/2}^{1} \int_{1/2}^{x_2} 6(1 - x_2)\, dx_1\, dx_2 = \frac{1}{8}$$

b)

$$g(x_1) = \int_{x_1}^{1} 6(1 - x_2)\, dx_2 = -6x_1 + 3 + 3x_1^2$$

$$g(x_2) = \int_{0}^{x_2} 6(1 - x_2)\, dx_1 = 6x_2 - 6x_2^2$$

$$E(X_1) = \int_{0}^{1} x_1 (-6x_1 + 3 + 3x_1^2)\, dx_1 = 0.25$$

$$E(X_2) = \int_{0}^{1} x_2 (6x_2 - 6x_2^2) = 0.5$$