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CS 165A Homework #4

1a. $\neg$Write(Gershwin, Eleanor Rigby)

1b. $\exists$s Write(Joe, s)

1c. $\forall$s (Sings(McCartney, s, Revolver) $\Rightarrow$ Write(McCartney, s))

1d. $\forall$s (Write(Gershwin, s) $\Rightarrow$ $\exists$p $\exists$a Sings(p, s, a))

1e. $\exists$a ($\forall$s Write(Joe, s) $\Rightarrow$ $\exists$p Sings(p, s, a))

2a. (R $\wedge$ E) $\Leftrightarrow$ C : This is not the correct representation. This is saying a person is radical and electable if and only if they are conservative. This is wrong because this says that those who are radical and electable are equivalent to those who are conservative, which is not true because nothing is said about all conservatives being radical.

2b. R $\Rightarrow$ (E $\Leftrightarrow$ C) : This is correct. This is saying that if a person is radical, then they are electable if and only if they are conservative.

2c. R $\Rightarrow$ ((C $\Rightarrow$ E) $\vee$ $\neg$E) : This is incorrect. This representation is saying that if a person if radical, then they are conservative if they are electable, or they are not electable. However, this statement always returns true, as we end up using an or statement for E and $\neg$E.

3.

Smoke	Fire	($\neg$Smoke $\Rightarrow$ $\neg$F ire) = (Smoke V $\neg$Fire)	($\neg$F ire $\Rightarrow$ $\neg$Smoke) = (Fire V $\neg$Smoke)	($\neg$F ire $\Rightarrow$ Smoke) = Fire V Smoke
True	True	True	True	True
True	False	True	False	True
False	True	False	True	True
False	False	True	True	False

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4a. $[(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party]$
$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party]$
$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \Rightarrow [\neg (Food \land Drinks) \lor Party]$
$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \Rightarrow [(\neg Food \lor \neg Drinks) \lor Party]$
$[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \Rightarrow [(\neg Food \lor Party) \lor (\neg Drinks \lor Party)]$

4b. $[(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \Rightarrow [(\neg Food \lor Party) \lor (\neg Drinks \lor Party)]$
$\neg [(\neg Food \lor Party) \lor (\neg Drinks \lor Party)] \lor [(\neg Food \lor Party) \lor (\neg Drinks \lor Party)]$
$[(Food \land \neg Party) \land (Drinks \land \neg Party)] \lor [(\neg Food \lor Party) \lor (\neg Drinks \lor Party)]$
$Food \land Drinks \land \neg Party \lor \neg Food \lor \neg Drinks \lor Party$
This is a valid sentence, as by using tautology, the sentence is true under all possible interpretations.

6a. $7 \leq 3 + 9$.
For backwards-chaining, we start from the goal until we get to the axioms. First, we have the statement to prove, $7 \leq 3 + 9$. From axiom 7, we know that $\forall w, x, y, z\ w \leq y \land x \leq z \Rightarrow w + x \leq y + z$. We can also use axiom 5, in which $x + 0 \leq x$. Here, we can see that w could represent 0, x could represent 7, y could represent 3, and z could represent 9. Going further, we need to show that $w \leq y \land x \leq z$. Looking at axioms 1 and 2, we know that $0 \leq 3$ and $7 \leq 9$. This represents w, x, y, and z, respectively. Thus, this proves that $7 \leq 3 + 9$.

6b. $7 \leq 3 + 9$.
For a forward-chaining proof, we start from the axioms to reach the goal. We begin with axioms 1 and 2, $0 \leq 3$ and $7 \leq 9$. Now, using axiom 7, $\forall w, x, y, z\ w \leq y \land x \leq z \Rightarrow w + x \leq y + z$, we know that $0 \leq 3$ and $7 \leq 9$, so $0 + 7 \leq 3 + 9$. Using axiom 5, we can simply the left side of the equation, as $x + 0 \leq x$. Thus, the final solution is $7 \leq 3 + 9$.