

1a. i) The vacuum-cleaner agent function is rational because the definition of a rational agent is that the agent should do whatever action would help to maximize its performance. This being said, when the vacuum-cleaner agent is in execution, it is acting rationally. This is shown, as when the cleaner is on the left side, and it is clean, it will move to the right side, and vice versa. When the left side is dirty, it will clean the left side, and same for the right side. When the left side is clean, it moves to the right, as there is nothing left to do on the left, so the agent acts rationally. When a side is dirty, it cleans that stated side, so the agent behaves rationally.

ii) If each movement were to generate a unit cost, then rather than constantly switching between the left and right side, whether clean or dirty, the rational behavior might be to wait until something dirty comes into the sensor, then moving the agent to clean it up. For example, in the original behavior, if the left was dirty, it would be cleaned, then it would move to the right. If something dirty came up again on the left, it would move to the left, then clean, then move to the right again. This gives us a minimum unit cost of at least 5. In the new scenario stated above to minimize unit cost, the vacuum would clean the left, stay there, and when something dirty came on the left again, it would clean again, as it has not moved. Thus, this gives us a minimum unit cost of 2.

1b. i) Performance measures would be kicking the ball, running towards the ball, stealing the ball from opposing players, or blocking the ball from entering the goal. The environment would be the soccer field or grass, the weather, the borders of the soccer field, and the goal posts. The actuators are the legs to run and kick, the arms to block the ball as a goalie, and the cameras to detect where to go and where the ball is. The sensors would be the cameras, or even instruments that can detect the robot's surroundings.

ii) Performance measures would be going through different links, and buying the laptop. The environment would be the speed of the wifi, and the different webpages/shops visited. The actuators are the hands, to click through the different links, and the eyes to see the satisfaction rating or pleasing factor to the buyer. The sensors would be the eyes and the hands.

1c. i) No, a simple reflex agent cannot be perfectly rational for this environment, because it cannot memorize the obstacles that it encounters, which would lead to possibly getting stuck in a path.

ii) Yes, it could be outperformed with a randomized agent function, likely in the scenario where it gets lucky and doesn't manage to hit any obstacles. An example of such an agent would be as follows: If the current area is dirty, clean it. Else, randomly move to another area, up, down, left, or right.

iii) Yes, it is possible for the randomized agent function to perform poorly. If the environment was set up in which all directions except left were blocked with obstacles, so up down and right were blocked, then when the function is called, and the left direction isn't chosen, then the agent is stuck until the left direction is chosen.

iv) Yes, a reflex agent with state can outperform a simple reflex agent, as the reflex agent with state can act based off of its current location/state and which direction to go to avoid obstacles. An example of a reflex agent with state would be as follows:
if the current area is dirty, clean it. Else if, neighboring position to current position has an obstacle in its path, return a direction that is not towards the obstacle. Else, return the direction to the closest dirty area.

2a. i) $P(X,Y|Z) = P(X|Z)P(Y|Z)$ ii) $P(X|Y, Z) = P(X|Z)$

$P(X,Y|Z) \rightarrow P(X|Y,Z) * P(Y|Z)$ by chaining

Then, the $P(Y|Z)$ on both sides can cancel each other, which leaves us with $P(X|Y,Z)$, which is equal to $P(X|Z)$.

2b. We know that $P(A|V) = 0.95$, $P(A|\sim V) = 0.1$, $P(B|V) = 0.9$, $P(B|\sim V) = 0.05$, and $P(V) = 0.01$

We now must solve for $P(V|A)$ and $P(V|B)$.

For $P(V|A)$, we know it is equal to $P(A\cap V)/P(A)$. We also know, $P(A) = P(A|V)*P(V) + P(A|\sim V)*P(\sim V)$. Using the values given to us above, we can plug in and get $(0.95*0.01) + (0.1*0.99) = 0.1085$. Going back to the first equation, $P(A\cap V) = P(A|V)*P(V) = 0.0095$. Solving for $P(V|A)$, we get $0.0095/0.1085 = 0.0875$.

We now do a similar thing for $P(V|B)$, where it is equal to $P(B\cap V)/P(B)$. $P(B) = P(B|V)*P(V) + P(B|\sim V)*P(\sim V)$, which is equal to $(0.9*0.01) + (0.99*0.05) = 0.0585$. Solving for $P(B\cap V)$, we get $(0.9*0.01) = 0.009$. This, $P(V|B) = 0.1538$, so test B is more indicative of someone really carrying the virus.

2c. We are given that $P(D) = 0.0001$, and $P(T|D) = 0.99$.

First, it is good news because the disease is rare, which means that it is very unlikely that you have the disease, meaning that the test actually failed. The chances that you actually have the disease are as follows:

$$P(T) = P(T|D)*P(D) + P(T|\sim D)*P(\sim D) = 0.0101$$

$P(D|T) = P(T|D)*P(D)/P(T) = (0.99*0.0001)/0.0101 = 0.0098$. This means there is a 0.98% chance that you actually have the disease.

3a. No, it is not true. This is because Y has a causal effect on Z, and as a result they are not conditionally independent of each other. With the diagram, we know that X and Y are independent, and as a result we know that $P(X,Y) = 0$. Thus, the original statement cannot be true, as both X and Y affect Z, and $P(X|Y) = P(X,Y)*P(Y)/P(X) = 0$.

3b. Yes, it is true. This is because Y is independent of X, so Y has no effect on X. To calculate $P(X|Y)$, $P(X|Y) = P(X,Y,Z)/P(Y)$. Then, $P(X,Y,Z) = P(X|Z)*P(Y|Z)*P(Z)$. Thus, $P(X|Y) = P(X|Z)*P(Y|Z)*P(Z)/P(Y)$.

3c. i) $P(D) = P(D|A,B)*P(B|A)*P(A) + P(D|\sim A,B)*P(B|\sim A)*P(\sim A) + P(D|A,\sim B)*P(\sim B|A)*P(A) + P(D|\sim A,\sim B)*P(\sim B|\sim A)*P(\sim A) =$
 $(0.02*0.2*0.1) + (0.01*0.1*0.9) + (0.01*0.8*0.1) + (0.001*0.9*0.9) = 0.0029$

ii) $P(E|A) = P(E,A)/P(A) = (P(A,B,C,D,E) + P(A,\sim B,C,D,E) + P(A,B,\sim C,D,E) + P(A,B,C,\sim D,E) + P(A,\sim B,\sim C,D,E) + P(A,B,\sim C,\sim D,E) + P(A,\sim B,C,\sim D,E) + P(A,\sim B,\sim C,\sim D,E)) / P(A)$
 $P(A,B,C,D,E) = P(A)*P(B|A)*P(D|A,B)*P(C|B)*P(E|D) = 0.00018$
 $P(A,\sim B,C,D,E) = P(A)*P(\sim B|A)*P(D|A,\sim B)*P(C|\sim B)*P(E|D) = 0.0000072$
 $P(A,B,\sim C,D,E) = P(A)*P(B|A)*P(D|A,B)*P(\sim C|B)*P(E|D) = 0.00018$
 $P(A,B,C,\sim D,E) = P(A)*P(B|A)*P(\sim D|A,B)*P(C|B)*P(E|\sim D) = 0.00098$
 $P(A,\sim B,\sim C,D,E) = P(A)*P(\sim B|A)*P(D|A,\sim B)*P(\sim C|\sim B)*P(E|D) = 0.0007128$
 $P(A,B,\sim C,\sim D,E) = P(A)*P(B|A)*P(\sim D|A,B)*P(\sim C|B)*P(E|\sim D) = 0.00098$
 $P(A,\sim B,C,\sim D,E) = P(A)*P(\sim B|A)*P(\sim D|A,\sim B)*P(C|\sim B)*P(E|\sim D) = 0.0000792$
 $P(A,\sim B,\sim C,\sim D,E) = P(A)*P(\sim B|A)*P(\sim D|A,\sim B)*P(\sim C|\sim B)*P(E|\sim D) = 0.00784$
 Thus, $P(E|A) = 0.1096$

iii) $P(A|D) = P(A,D)/P(D)$
 $P(A,D) = P(A,D,B) + P(A,D,\sim B) = P(D|A,B)*P(B|A)*P(A) + P(D|A,\sim B)*P(\sim B|A)*P(A) = 0.0012$
 $P(A|D) = 0.4124$

$$\text{iv) } P(\sim D | \sim C, E) = P(\sim D, \sim C, E) / P(\sim C, E) = (P(A, B, \sim C, \sim D, E) + P(\sim A, B, \sim C, \sim D, E) + P(A, \sim B, \sim C, \sim D, E) + P(\sim A, \sim B, \sim C, \sim D, E)) / (P(\sim C, E))$$

$$P(A, B, \sim C, \sim D, E) = P(A) * P(B | A) * P(\sim D | A, B) * P(\sim C | B) * P(E | \sim D) = 0.00098$$

$$P(\sim A, B, \sim C, \sim D, E) = P(\sim A) * P(B | \sim A) * P(\sim D | \sim A, B) * P(\sim C | B) * P(E | \sim D) = 0.004455$$

$$P(A, \sim B, \sim C, \sim D, E) = P(A) * P(\sim B | A) * P(\sim D | A, \sim B) * P(\sim C | \sim B) * P(E | \sim D) = 0.00784$$

$$P(\sim A, \sim B, \sim C, \sim D, E) = P(\sim A) * P(\sim B | \sim A) * P(\sim D | \sim A, \sim B) * P(\sim C | \sim B) * P(E | \sim D) = 0.0089$$

$$P(\sim C, E) = P(A, B, \sim C, D, E) + P(\sim A, B, \sim C, D, E) + P(A, \sim B, \sim C, D, E) + P(A, B, \sim C, \sim D, E) + P(\sim A, \sim B, \sim C, D, E) + P(A, \sim B, \sim C, \sim D, E) + P(\sim A, B, \sim C, \sim D, E) + P(\sim A, \sim B, \sim C, \sim D, E)$$

$$P(A, B, \sim C, D, E) = P(A) * P(B | A) * P(D | A, B) * P(\sim C | B) * P(E | D) = 0.00018$$

$$P(\sim A, B, \sim C, D, E) = P(\sim A) * P(B | \sim A) * P(D | \sim A, B) * P(\sim C | B) * P(E | D) = 0.000405$$

$$P(A, \sim B, \sim C, D, E) = P(A) * P(\sim B | A) * P(D | A, \sim B) * P(\sim C | \sim B) * P(E | D) = 0.07057$$

$$P(A, B, \sim C, \sim D, E) = P(A) * P(B | A) * P(\sim D | A, B) * P(\sim C | B) * P(E | \sim D) = 0.00098$$

$$P(\sim A, \sim B, \sim C, D, E) = P(\sim A) * P(\sim B | \sim A) * P(D | \sim A, \sim B) * P(\sim C | \sim B) * P(E | D) = 0.000721$$

$$P(A, \sim B, \sim C, \sim D, E) = P(A) * P(\sim B | A) * P(\sim D | A, \sim B) * P(\sim C | \sim B) * P(E | \sim D) = 0.00784$$

$$P(\sim A, B, \sim C, \sim D, E) = P(\sim A) * P(B | \sim A) * P(\sim D | \sim A, B) * P(\sim C | B) * P(E | \sim D) = 0.004455$$

$$P(\sim A, \sim B, \sim C, \sim D, E) = P(\sim A) * P(\sim B | \sim A) * P(\sim D | \sim A, \sim B) * P(\sim C | \sim B) * P(E | \sim D) = 0.0089$$

$$P(\sim C, E) = .103$$

$$P(\sim D | \sim C, E) = 0.0222708 / (.103) = 0.216$$

$$\text{v) } P(A, B, \sim C | D) = P(A, B, \sim C, D) / P(D) = (P(A, B, \sim C, D, E) + P(A, B, \sim C, D, \sim E)) / P(D)$$

$$P(A, B, \sim C, D, E) = P(A) * P(B | A) * P(D | A, B) * P(\sim C | B) * P(E | D) = 0.00018$$

$$P(A, B, \sim C, D, \sim E) = P(A) * P(B | A) * P(D | A, B) * P(\sim C | B) * P(\sim E | D) = 0.00002$$

$$P(A, B, \sim C | D) = 0.0687$$

4a. $n-1$ free parameters

4b. i) $m*(n-1)$

ii) $n*(m-1)$

iii) $n*(m-1)$

iv) $(n*m) - 1$

4c. $P(Y) = P(Y|Z_k)*P(Z|X_n)*P(X)$, with the summation from (1 to k) and (1 to n)

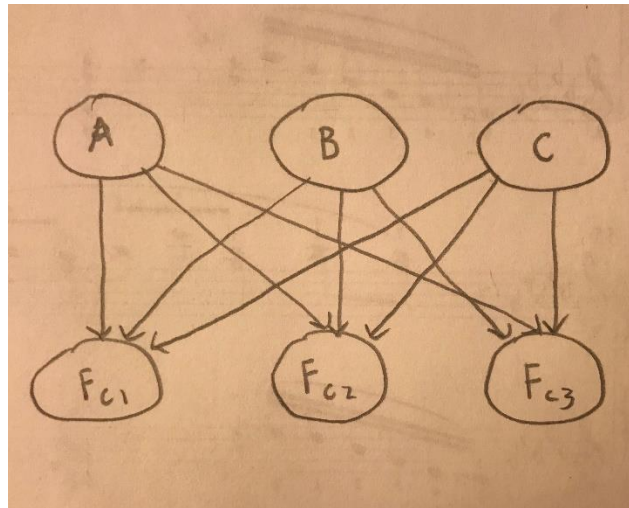
$P(Y|Z) = k*(m-1)$

$P(Z|X) = n*(k-1)$

$P(X) = n-1$

Thus, we would need $(k*(m-1) + n*(k-1) + n-1)$ free parameters for $P(Y)$

5a.



CPT for coin being chosen from bag:

COIN	P(COIN)
A	1/3
B	1/3
C	1/3

CPT for coin chosen being flipped and is heads:

COIN	F_c	$P(F_c)$
A	Heads	0.2
B	Heads	0.6
C	Heads	0.8

- 5b. $P(\text{COIN} | H, H, T) = P(H, H, T | \text{COIN}) * P(\text{COIN}) / P(H, H, T)$
 $P(H, H, T | \text{COIN}) = P(H | \text{COIN}) * P(H | \text{COIN}) * P(T | \text{COIN}) * P(\text{COIN})$
 For A: 0.01056
 For B: 0.04752
 For C: 0.04224

Thus, Coin B is the most likely to have been drawn.