ChatGPT

Research on Push-Your-Luck Dice Games (Risk vs. Reward)

Optimal Strategies in Jeopardy Dice Games (Pig and Variants)

The dice game *Pig* is a classic **push-your-luck** game where players roll dice to accumulate points but risk losing the turn's points if a "bust" outcome occurs. The optimal strategy for the basic two-player Pig (rolling a single die, where rolling a 1 ends your turn with 0 gain) was non-trivial and was solved using dynamic programming methods [1]. In particular, Neller and Presser (2004) derived the exact optimal policy for sequential Pig, overturning the old rule-of-thumb "hold at 20" with a more precise strategy [2]. Their analysis formalized Pig as a Markov decision process and computed the **optimal hold threshold** for each game state to maximize win probability [1].

Researchers have also extended this analysis to **Pig variants** involving two dice and special rules. Zhu and Ma (2022) presented an analytic solution for the standard two-dice Pig game *and* a variant called "Double Trouble," which forces additional risk when doubles are rolled [3] [4]. In Double Trouble Pig, rolling any double (except double-ones) compels the player to continue rolling (analogous to your rule where doubles **double the pot**, incentivizing continued play at higher risk). Using Markov decision processes and dynamic programming, Zhu and Ma derived optimal decisions that account for the doubled stakes on doubles and the increased bust probabilities [5] [6]. These analytic approaches confirm that **optimal push-or-hold strategies** can be computed even for complex dice rules, and they often differ from naive heuristics. For example, the optimal "hold" threshold in the Double Trouble variant is lower (more conservative) than in standard Pig, since the forced rolls on doubles increase risk [7] [8].

Multi-Player and Simultaneous Decision Dynamics

Your described game involves **N players deciding simultaneously** whether to bank the pot or continue, under a shared risk of rolling a 7 (which wipes out the pot). This is a multi-player push-your-luck scenario that introduces strategic interaction – each player's best choice may depend on the others' decisions. Recent work by Zhu *et al.* (2023) explicitly studied a *simultaneous-play* version of Pig with multiple players [9]. They formulated the game in game-theoretic terms and used a combination of dynamic programming and reinforcement learning to find **optimal strategies and Nash equilibria** in multi-player settings [9] [10]. Notably, they solved the 2-player simultaneous game analytically (deriving a mixed-strategy Nash equilibrium for the "press or bank" decision each turn) and extended this to N players by numerical methods [9]. One insight from their study is that when all players decide simultaneously, there can exist equilibrium strategies where each player randomizes their choice of banking or continuing, to avoid being too predictable [9]. They even characterized the equilibrium as the number of players grows large (approaching an infinite-player limit) [11]. This kind of analysis is highly relevant to your game's risk-vs-reward dilemma, since it treats the decision to continue or stop as a strategic move in a **non-cooperative game** among players, rather than an isolated choice.

In traditional sequential Pig with more than two players, it was observed that true optimal strategies can be very complex or may not even exist in simple "hold-at-n" form when the target score is high [12]. Bonnet, Neller, and Viennot (2019) explored the 3-player Pig game and found that the optimal policy might not be a fixed threshold strategy if the game goal is large, due to the changing tactical situation as scores evolve [13]. This further motivates studying simultaneous decision versions – as Zhu *et al.* did – to level the playing field and make analysis tractable by symmetry [10]. In a simultaneous push-your-luck game, every player faces the same risk at each turn, much like in your described rules where any player who remains risks the common pot being lost on a 7. The 2023 study provides a framework for such games by computing players' **best responses** and equilibrium probabilities of continuing vs. stopping, under various configurations of player count [9].

Game-Theoretic Analysis of Similar "Press Your Luck" Games

Outside of Pig, other press-your-luck games with simultaneous decisions have also been examined. *Incan Gold* (also known as **Diamant**) is a well-known multi-player push-your-luck game where each round players decide **simultaneously** to either "leave (bank their treasure)" or "continue (press onward)" as cards are revealed. Although its mechanics use cards (gems and hazards) instead of dice, the core decision is analogous to your description – everyone still in risks losing the accumulated pot if a hazard appears. Researchers have applied computational game theory techniques to Incan Gold to investigate optimal play. For example, De Jong (2021) used Monte Carlo Tree Search and **deep counterfactual regret minimization** to approximate optimal strategies in Incan Gold [14]. His analysis framed each decision as balancing *immediate reward* vs. *future risk*, noting that the **best choice depends strongly on the other players' tendencies** [14] [15]. If opponents are very risk-averse (leaving early), a bold strategy of pressing on can yield higher payoff (since you might claim the entire pot); conversely, if others are daredevils, a prudent exit can often be optimal [14] [15]. This kind of finding echoes the strategic considerations in your dice game: each player must weigh the chance of dramatically increasing the pot (especially with favorable rolls like doubles that double the stakes) against the probability that someone will eventually roll the fatal **7** and bust the round. Game-theoretic studies of these games often seek a **Nash equilibrium strategy**, where no player can unilaterally improve their long-term outcome by deviating [16] [15]. In practice, equilibrium might involve mixed (probabilistic) strategies – e.g. "continue with X% probability given the current pot size" – to keep opponents uncertain.

In summary, there is a growing body of analytical research on push-your-luck dice games and their multi-player extensions. Key references include the optimal stopping strategy for Pig [1], extensions to multi-dice and doubling rules [3] [4], and recent work combining **dynamic programming with game theory** to handle simultaneous decision-making among many players [9] [10]. These studies provide a mathematical foundation for understanding games like the one you described, where each player must decide to **"bank or press"** in the face of stochastic outcomes and the actions of others. The analytical approaches (Markov decision processes, Nash equilibrium analysis) and computational techniques (reinforcement learning self-play, MCTS, etc.) in these papers could be very useful if you are looking to derive or simulate optimal strategies for your risk-reward dice game.

Sources:

- Neller, T. & Presser, C. – *Optimal Play of the Dice Game Pig*, UMAP Journal (2004): First dynamic programming solution for classic Pig [1] [2].

- Zhu, T. & Ma, M. – *Deriving the Optimal Strategy for the Two-Dice Pig Game (Standard & "Double Trouble")*, **Stats**, 5(3):805–818 (2022): Solves two-dice Pig and a doubles-rolling variant via Markov decision processes [3] [4].
- Zhu, T. *et al.* – *Optimal Strategy of the Simultaneous Dice Game Pig for Multiplayers*, **Scientific Reports**, 13:8142 (2023): Game-theoretic and RL analysis of Pig with N players making simultaneous roll/hold decisions (finds Nash equilibria) [9] [10].
- De Jong, M. – *Incan Gold: Deep CFR with Monte Carlo Tree Search* (2021): Explores optimal play in the multi-player push-your-luck card game *Incan Gold*, using simulation-based game-solving methods [14] [15].

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[1] [2] [3] [9] [10] [11] [12] [13] Optimal strategy of the simultaneous dice game Pig for multiplayers: when reinforcement learning meets game theory | Scientific Reports
https://www.nature.com/articles/s41598-023-35237-x?error=cookies_not_supported&code=6e34dffc-6dd0-481e-aad0-5ead2e36eff9

[4] [5] [6] [7] [8] Deriving the Optimal Strategy for the Two Dice Pig Game via Reinforcement Learning
https://www.mdpi.com/2571-905X/5/3/47

[14] [15] [16] Incan Gold: Deep Counterfactual Regret with Monte Carlo Tree Search − Maxwell De Jong
https://maxwelldejong.com/machine_learning/incan_gold/