

<b>Student Number</b>	
<b>Signature</b>	

University of Cape Town ~ Department of Computer Science  
Computer Science 3003S Theory of Algorithms ~ 2017

## ToA Week Two Session One

### Procedure

Personal belongings are to be left at the front of the room. You may take a calculator, writing implements, and your student card to your assigned workstation.

This handout includes a sheet that you may use for rough work.

All materials must be returned before you leave.

You may consult the electronic Java and C/C++ API documentation ([docs.cs.uct.ac.za](http://docs.cs.uct.ac.za)), and submit to the automatic marker via Vula ([vula.uct.ac.za](http://vula.uct.ac.za)), but nothing else! You may NOT use your class notes, textbooks, internet or files on your flash disk, hard drive, etc.

### Submission

The automatic marker contains a submission entry bearing the name of this session.

Submit your source files within a single compressed, '.ZIP', archive.

Make sure you create a '.ZIP' archive, not a gzipped, '.gz', or tar-gzipped, '.tgz', or other kind of file.

Make sure your source file is the only item within the archive. Especially, avoid submitting an archive containing a folder containing the file.

When submitting a Java source file copied from an editor like Eclipse or Netbeans, please remove any package line that may appear at the beginning of the code.

## Question One

100 Marks

### File names

- Use `equalize.c` if you are writing your program in C.
- Use `equalize.cpp` if you are writing your program in C++.
- Use `Equalize.java` if you are writing your program in Java.
- Use `equalize.py` if you are writing your program in Python.

### Problem description

Given an array of  $n$  integers,  $[a_1, a_2, \dots, a_n]$ , you must find whether it is possible to make all elements of the array equal by multiplying each element by a power of 2. Each element can be multiplied by a different power of 2. All elements will contain large prime factors, making factorisation inefficient. Similarly, all elements will already contain a large power of 2 as a factor, making dividing each element out by 2 inefficient. If it is possible to make all elements equal, your program should output the smallest possible common value for all elements in the array. Otherwise, your program should output -1.

### Example

You are given the array [628, 1256, 628, 2512, 1256], and  $628 * 4 = 2512$  and  $1256 * 2 = 2512$ . Your program should, therefore, output 2512.

### Input and output

Program input and output will make use of stdio streams (System.in and System.out in Java) i.e. not file I/O.

### Input

The input will consist of a single line of  $n$  integers, separated by spaces.

Sample Input:

13 5 7 11 47 53 109

### Output

Your output should be a single integer, which is either the smallest possible common element, or -1.

Sample output:

-1

### Constraints

$1 \leq n \leq 100\,000$

$1 \leq a_i \leq 100\,000$

### Scoring

Each test case that is answered correctly will earn 5 points.

## Question Two

50 Marks

### File names

- Use `dominate.c` if you are writing your program in C.
- Use `dominate.cpp` if you are writing your program in C++.
- Use `Dominate.java` if you are writing your program in Java.
- Use `dominate.py` if you are writing your program in Python.

### Problem description

In 2 dimensional space, we say that point  $a$  dominates point  $b$  if each ordinate of  $a$  is strictly larger than the corresponding ordinate of  $b$ . That is, if  $a = (a_1, a_2)$  and  $b = (b_1, b_2)$ , then if  $a_1 \geq b_1$  and  $a_2 \geq b_2$  then  $a$  dominates  $b$ . Your job is to write a program which, when given a set of  $n$  points in 2 dimensional space, outputs the number of dominating pairs. That is, your program must output the number of pairs of points  $(a, b)$ , where  $a$  dominates  $b$ . You may wish to use the standard sorting functions in the language you are using and you are allowed to do so for this question.

### Example

You are given the set of points [1, 1], [-1, -4], [1, 5], [2, 3]. Here, the dominating pairs are:

[1, 1] dominates [-1, -4]

[1, 5] dominates [-1, -4]

[1, 5] dominates [1, 1]

[2, 3] dominates [-1, -4]

[2, 3] dominates [1, 1]

and so the output of the program will be 5.

#### Input and output

Program input and output will make use of stdio streams (System.in and System.out in Java) i.e. not file I/O.

#### Input

The input will consist of a single line of  $n$  integers, separated by spaces. Each consecutive pair of integers represents a single point.

Sample Input:

-5 -10 20 25 30 2 -1 -40

#### Output

Your output should be a single positive integer, which is the number of dominating pairs in the set of points provided.

Sample output:

4

#### Constraints

$1 \leq n \leq 100\,000$

$-10\,000 \leq a_i \leq 10\,000$

#### Scoring

Each test case that is answered correctly will earn 5 points.

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