Final Quiz

$$u2 = v2 - \frac{v2 \cdot u1}{u1 \cdot u1} u1$$

$$Var(X) = E[(X - \mu)^2] = 1/n \sum_{i=1}^{n} (x_i - \mu)^2$$

$$Cov(X, Y) = E[(XY)] - E[X]E[Y]$$
$$= 1/n \sum_{i=1}^{n} (x_i - E[X])(y_i - E[Y])$$

$$Q = 1/n \sum_{i=1}^{n} (u_i - Q)(u_i - Q)^T$$

Question

Work out the covariance matrix from this data:

Feature 1	Feature 2
1	3
4	6

$$\mu_1 = \frac{1 + 4}{2} = 2.5$$

$$\mu_2 = \frac{3 + 6}{2} = 4.5$$

$$\begin{matrix} \text{Feature 1} & \text{Feature 2} \\ 1 - 2.5 & 3 - 4.5 \\ 4 - 2.5 & 6 - 4.5 \end{matrix} = \begin{matrix} -1.5 & -1.5 \\ 1.5 & 1.5 \end{matrix}$$

$$Q = 1/n \sum_{i=1}^{n} (u_i - Q)(u_i - Q)^T$$

$$X = \begin{pmatrix} -1.5 & -1.5 \\ 1.5 & 1.5 \end{pmatrix}$$

$$X^T = \begin{pmatrix} -1.5 & 1.5 \\ -1.5 & 1.5 \end{pmatrix}$$

$$X^T X = \begin{pmatrix} -1.5 & 1.5 \\ -1.5 & 1.5 \end{pmatrix} \begin{pmatrix} -1.5 & -1.5 \\ 1.5 & 1.5 \end{pmatrix} = \begin{pmatrix} (-1.5 \cdot -1.5 + 1.5 \cdot 1.5) & (-1.5 \cdot -1.5 + 1.5 \cdot 1.5) \\ (-1.5 \cdot -1.5 + 1.5 \cdot 1.5) & (-1.5 \cdot -1.5 + 1.5 \cdot 1.5) \end{pmatrix} = \begin{pmatrix} 4.5 & 4.5 \\ 4.5 & 4.5 \end{pmatrix}$$

Finally, the covariance matrix is:

$$Q = \frac{1}{2} \begin{pmatrix} 4.5 & 4.5 \\ 4.5 & 4.5 \end{pmatrix}$$

$$Q = \begin{pmatrix} 2.25 & 2.25 \\ 2.25 & 2.25 \end{pmatrix}$$