

# Mathematical Logic Assignment 5

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- $\exists x \forall y (P(x) \wedge T(y) \wedge F(x, y))$
- $\exists y \forall x (P(x) \wedge T(y) \wedge F(x, y))$
- If the sentence means that fooling anyone at anytime is not feasible, the wff is  $\forall x \forall y (P(x) \wedge T(y) \wedge \neg F(x, y))$ . If the sentence means that fooling someone all of the time is feasible but not all of the people all of the time, the wff is  $\forall y \exists x (P(x) \wedge T(y) \wedge \neg F(x, y))$ .

2.

- In  $\forall y (P(x, y) \rightarrow \forall x P(x, y))$ , the  $x$  in the first  $P(x, y)$  is occurring free variable.
- In  $(\neg \exists y R(f(y, z)) \wedge (\forall x \forall y R(f(y, z)))$ , the  $z$  at the left side of  $\wedge$  is occurring free variable, and  $z$  at the right side of  $\wedge$  is occurring free variables.

3. The answer is at the left sub-graph of the Figure 1.

Left-3:

$$\begin{array}{c}
 3. \quad \frac{\frac{[\neg \exists x A(x)]'}{\neg \exists x A(x)} \quad \frac{[A(y)]^2}{\exists x A(x)} \exists\text{-I}}{\neg \exists x A(x) \wedge \exists x A(x)} \wedge\text{-I} \\
 \frac{\neg \exists x A(x) \wedge \exists x A(x)}{\neg \exists x A(x)} \neg\text{-I} \\
 2. \quad \frac{\{x=y\} \quad \neg A(y)}{\forall x \neg A(x)} \forall\text{I} \\
 1. \quad \frac{\neg \exists x A(x) \rightarrow \forall x \neg A(x)}{\neg \exists x A(x) \rightarrow \forall x \neg A(x)} \rightarrow\text{I} \\
 [\neg \exists x A(x)]' \quad [A(y)]^2
 \end{array}$$

Right-4:

$$\begin{array}{c}
 4. \quad \frac{\frac{[\forall x \neg A(x)]'}{\neg A(w)} \forall\text{E} \quad \frac{[A(w)]^2}{A(w)} \exists\text{E}}{\neg A(w) \wedge A(w)} \wedge\text{-E} \\
 3. \quad \frac{[\exists x A(x)]^2 \quad \neg A(w) \wedge A(w)}{\neg \exists x A(x)} \neg\text{-E} \\
 2. \quad \frac{\neg \exists x A(x)}{\neg \exists x A(x)} \neg\text{-I} \\
 1. \quad \frac{\forall x \neg A(x) \rightarrow \neg \exists x A(x)}{\forall x \neg A(x) \rightarrow \neg \exists x A(x)} \rightarrow\text{I} \\
 [\forall x \neg A(x)]' \quad [\exists A(x)]^2
 \end{array}$$

Figure 1: Left-3 Right-4

4. The wrong is the use of discharged assumption of  $\exists - E$   $[A(y)]$ . In the conclusion  $A(y) \wedge \neg A(y)$ , so  $y$  is not an occurring free variable, which can't be in discharged assumption. We should change it to  $[A(w)]$  as an example, and use  $\neg - E$  to prove  $A(y) \wedge \neg A(y)$ . The rational tree is at the right sub-graph of the Figure 1.

5. The answer is at the Figure 2.

$$\begin{array}{c}
 \frac{\frac{\frac{[\exists x \exists y (P(x) \wedge T(y, x))]^2}{T(v, w) \wedge T(v, w)} \exists - E}{\neg \exists x \exists y (P(x) \wedge T(y, x))} \neg - I}{\frac{1}{\forall x (P(x) \rightarrow \forall y (\neg T(y, x))) \rightarrow \neg \exists x \exists y (P(x) \wedge T(y, x))} \rightarrow I} \\
 \frac{3}{\frac{2}{\neg \exists x \exists y (P(x) \wedge T(y, x))} \neg - I} \\
 \frac{1}{\forall x (P(x) \rightarrow \forall y (\neg T(y, x))) \rightarrow \neg \exists x \exists y (P(x) \wedge T(y, x))} \rightarrow I
 \end{array}$$

Figure 2: Question 5