Mathematical Logic Assignment 3

522030910158 Song Yuanyi

November 2024

1. Let $\alpha = ((A \vee (B \wedge C)) \leftrightarrow ((A \vee B) \wedge (A \vee C)))$. Applying the parsing algorithm, check α from left to right.

1. The first symbol is (, and the second symbol is (, so scan for balanced expressions to find$(A \vee (B \wedge C)$ and $((A \vee B) \wedge (A \vee C))$ are connected by $\leftrightarrow$. Then we can create two child nodes for them.

2. All nodes are not sentence symbol. Check $(A \vee (B \wedge C)$ and the second symbol is not $\neg$, so scan for balanced expressions to find A and $(B \wedge C)$ are connected by $\vee$. Then we can create two child nodes for A and $(B \wedge C)$.

3. Check $((A \vee B) \wedge (A \vee C))$ and the second symbol is not $\neg$, so scan for balanced expressions to find $(A \vee B)$ and $(A \vee C)$ are connected by $\wedge$.Then we can create two child nodes for $(A \vee B)$ and $(A \vee C)$.

4. Now we only get one sentence symbol leaf A. By the similar method and process, in the following three times iterations, we can create leaf nodes B and C from $(B \wedge C)$, A and B from $(A \vee B)$, A and C from $(A \vee C)$.

5. Now all leaf nodes are sentence symbol. We get the parse tree of α.

2. Rewrite the set: $\Sigma = \{\neg(A_n \to A_{n-1} \to \cdots \to A_1) | n = 1, 2, \ldots\}$. The priority is group to the right, so $\neg(A_n \to A_{n-1} \to \cdots \to A_1) \equiv \neg(A_n \to (A_{n-1} \to \cdots \to A_1)) \equiv \neg(\neg A_n \vee (A_{n-1} \to \cdots \to A_1)) \equiv A_n \wedge \neg(A_{n-1} \to \cdots \to A_1)$Therefore, $\neg(A_n \to A_{n-1} \to \cdots \to A_1)$ is true if and only if A_n and $\neg(A_{n-1} \to \cdots \to A_1)$ are both true. If and only if A_1 is false and A_2 is true, $\neg A_1$ and $\neg(A_2 \to A_1)$ are both true. Suppose $\neg(A_i \to \cdots \to A_1)$ is true and A_{i+1} for all $i \geq 2$. By induction, we can get $\neg(A_{i+1} \to A_i \to \cdots \to A_1)$ is true.

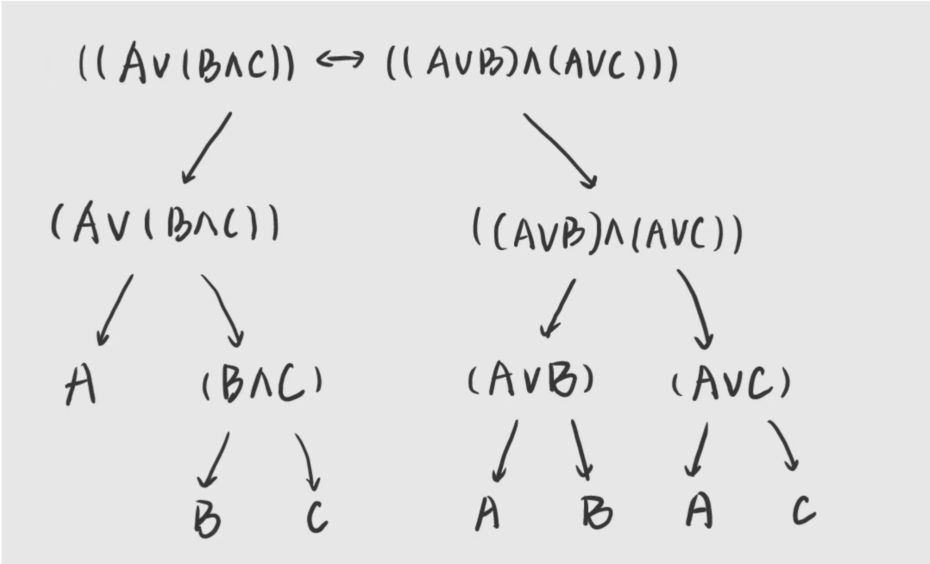

Figure 1: Parse Tree in Question 1

To sum up, the truth assignment that satisfy Σ is unique, which is $A_1 = F.A_i = T, i \geq 2$.

3.(a) First make sentence symbols: $A_1 =$ "Zhong Kui were able to catch ghosts.", $A_2 =$ "Zhong Kui were willing to catch ghosts.", $A_3 =$ "Zhong Kui does catch ghosts.", $A_4 =$ "Zhong Kui is powerless.", $A_5 =$ "Zhong Kui is malevolent.", $A_6 =$ "Zhong Kui does exist."

Therefore, KB= $\{(A_1 \wedge A_2) \to A_3, \neg A_1 \to A_4, \neg A_2 \to A_5, \neg A_3, A_6 \to (\neg A_4 \wedge \neg A_5)\}$.

3.(b) The question is "Does Zhong Kui exists?". $A_6 =$ "Zhong Kui does exist." We can analyse KB to find its truth assignments. $\neg A_3 = T$ and $((A_1 \wedge A_2) \to A_3) = T$ so $A_3 = F$ and $(A_1 \wedge A_2)$ is false. $A_6 \to (\neg A_4 \wedge \neg A_5)$ means as long as A_4 or A_5 is true,$(\neg A_4 \wedge \neg A_5)$ is false, so A_6 must be false. $\mathcal{M}(KB) = \{\{A_1 = T, A_2 = F, A_3 = F, A_4 = T/F, A_5 = T, A_6 = F\}, \{A_1 = F, A_2 = T, A_3 = F, A_4 = T, A_5 = T/F, A_6 = F\}, \{A_1 = F, A_2 = F, A_3 = F, A_4 = T, A_5 = T, A_6 = F\}\}$. Therefore, the answer is "Zhong Kui does not exist."

3.(c) The result of this action is rejection. It tells A_6 is true. From (b) we know $KB \cup \{A_6\}$ is not satisfiable, so it results in contradiction.

4.

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\begin{aligned}
&(\neg A \wedge (B \to C)) \to \neg(\neg B \vee C) \\
\equiv &\neg(\neg A \wedge (B \to C)) \vee \neg(\neg B \vee C) \\
\equiv &(A \vee \neg(B \to C)) \vee \neg(\neg B \vee C) \\
\equiv &(A \vee \neg(\neg B \vee C)) \vee \neg(\neg B \vee C)
\end{aligned}
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5. Let $\bar{v}$ be the set of truth assignments v that satisfies the set of well-formed formulas. Let $\Sigma : \bar{v}_1$, $\Delta : \bar{v}_2$, $\{\alpha\} : \bar{v}_3$, $\{\beta\} : \bar{v}_4$. Therefore, $\Sigma \models \alpha$ means $v_1 \subset v_3$, $\mathcal{M}(\Delta; \alpha) = v_2 \cap v_3$, $\Delta; \alpha \models \beta$ means $v_2 \cap v_3 \subset v_4$. $v_1 \subset v_3$ so $v_2 \cap v_1 \subset v_2 \cap v_3 \subset v_4$. The truth assignments of $\Sigma \cup \Delta$ need to satisfy both Σ and Δ, so $\mathcal{M}(\Sigma \cup \Delta) = v_2 \cap v_1$. $v_2 \cap v_1 \subset v_4$ means every truth assignments of $\Sigma \cup \Delta$ also satisfy β. Therefore, if $\Sigma \models \alpha$ and $\Delta; \alpha \models \beta$, then $\Sigma \cup \Delta \models \beta$.