University of Cape Town

EEE4118F

PROCESS CONTROL INSTRUMENTATION

GA Report

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1 Introduction

The aim of this project was to find and implement the controller for the servo motor system identified in early stages of the project for three cases:

1. When the motor is unconstrained (nominal case)
2. When the motor has some resistance applied in the form of eddy current (ie, a magnet is placed minimally over the disc which spins as the motor output)
3. When the motor has full (maximum available) resistance applied in the form of eddy current.

Because the plants are different for each case, Robust Tracking Design and using Quantitative Feedback Theory (QFT) Design become important for this application.

Feedback is important in control systems as it facilitates robustness in tracking, stability of the system, and disturbance rejection. This is important in systems needing to track a setpoint accurately. The setpoint that is being tracked is a voltage input to the motor which specifies its position. The designed controller needed to meet certain design specifications, and feedback was particularly useful in this design.

Additionally, the controlled system is of the form of a cascade controlled system, which incorporates an inner loop - representing motor velocity - and an outer loop which represents the motor position. The idea behind the project was to get the motor to track an input position whilst controlling both the velocity and subsequently the position.

This report follows a design procedure through a system identification, design of a controller for the identified system, and the implementation and success evaluation of the controller to meet the desired Graduate Attribute (2) of solving a complex problem using engineering skills and scientific knowledge.

2 Procedure

2.1 System Identification

The servo motor plant which needed identification was of the form of a first order system. The following equation is the transfer function's general form:

$$\frac{A}{1+\tau s} \tag{1}$$

The task of identifying the system is done by finding the amplitude (A) and time constant (τ) values of the first order system. This was done through finding the motor's dead-band where there is no linear response to any input, avoiding the dead-band and measuring and quantifying the response of the system to a step input. The following process was followed:

1. To find the region in which the motor has no speed output (deadband), the system input was a ramp function of $input = -2 + 0.1t$. The deadband is found where the motor velocity has no gradient for a range of input voltages.

2. After the deadband was identified, the process of collecting data for the system's response to a step input began.

   (a) The magnet was adjusted to the specified level of eddy current and recording of values was initiated.

   (b) A step of $0.5V$ was input to move the motor out of the identified deadband and then we waited for the speed to settle to a constant value.

   (c) Another step of $0.5V$ was input, and the response to this step is the important response for the system identification. The size of this step was chosen to keep the response in the linear region so that it does not saturate to $10V$.

   (d) the above steps were repeated for each different level of eddy current supplied by the magnet.

3. The data logged in the above steps was then used to identify the values of the system's transfer response for each level of eddy current.

   (a) The A value was calculated by taking the difference of the steady state values reached on the step response (the first steady state value being after the step out of the deadband, and the second after the step for which the response is recorded) and dividing it by the value of this step: $A = \frac{SS_{final} - SS_{initial}}{0.5}$.

   (b) The time constant was calculated with two methods, an interpolation method, and using a normalised average. For the purposes of the project, the interpolated value was used, therefore only that method is discussed in this report. Details of the normalised average method were in the previous system identification report. The interpolation method is as follows: the system's step response is of the form of a decaying exponential, and the following equation can be used to represent it: $y = AB(1 - e^{\frac{-t}{\tau}})$ where A is the steady state value, and B is the step size. The value of time which corresponds with this magnitude (X) is the τ value, and because the samples are 20ms apart, interpolation was used to find exact time values. The formula used with the step normalised for it's time to start at 0, was:

   $$\tau = \frac{AB_{after} - X}{AB_{after} - AB_{before}}(0.02) + time_{before} \quad (2)$$

   (c) The above 2 steps were repeated for the 3 eddy current cases.

   (d) The K value of the system represents the ratio of the motor speed output versus the overall system output, in volts per second. The value was found to be $K = 9.972368$ by finding the average of the gradient of the system output's linear region at steady state ($75.5978V$), and dividing the gradient by the steady state motor velocity ($7.58073V/s$).

4. The step responses measured were then compared to the step responses simulated using the identified system transfer functions for each resistance level to verify that the identified systems were correct.

2.1.1 MATLAB Simulink Model

This model was adjusted for each step input response test case, where the step size was $B = 0.5$, and A and τ values differed for each case.

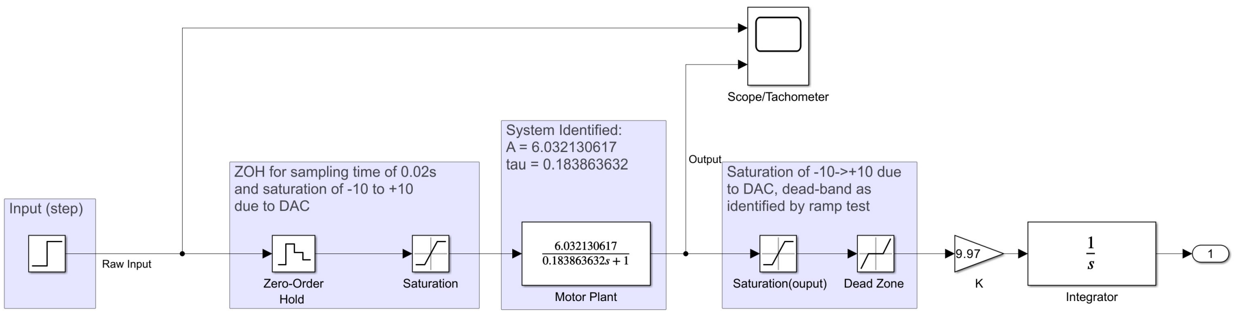
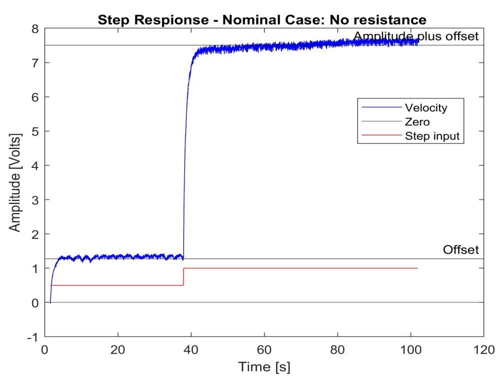


Figure 2.1.1: MATLAB Simulink Model for the Simulation of the Setup

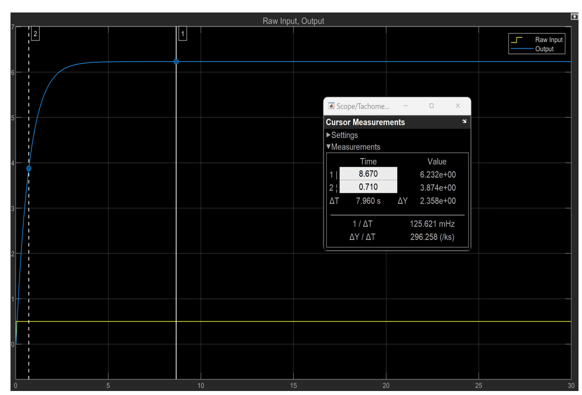
2.1.2 Identified Systems

The results of the step response tests were concluded with the final representation of the identified systems:

1. No eddy current resistance: $System = \frac{12.46406235}{1+0.704792628s}$, where $A = 12.46406235$ and $\tau = 0.704792628$



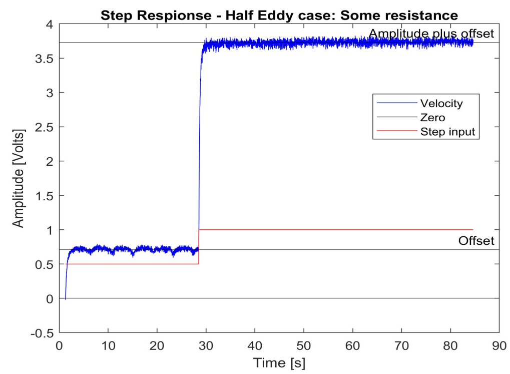
(a) Real Case Plot from which System Values were calculated



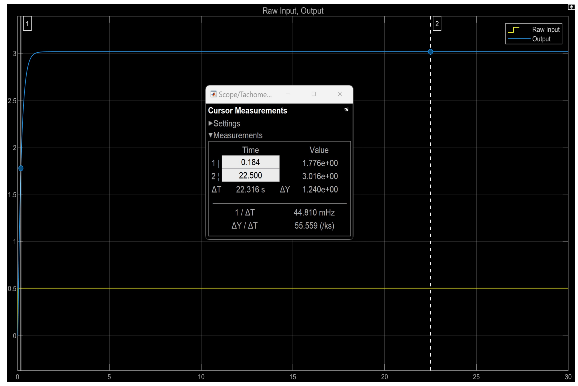
(b) Simulink Generated plot using identified values

Figure 2.1.2: The measured system response compared to the simulated system response when using no added resistance

2. Half eddy current resistance: $System = \frac{6.032130617}{1+0.183863632s}$, where $A = 6.032130617$ and $\tau = 0.183863632$



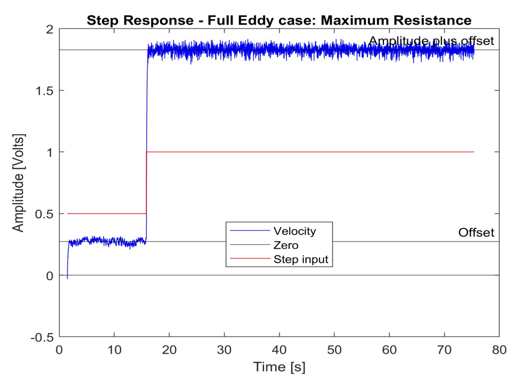
(a) Real Case Plot from which System Values were calculated



(b) Simulink Generated plot using identified values

Figure 2.1.3: The measured system response compared to the simulated system response when using half resistance level

3. Full eddy current resistance: $System = \frac{3.108844314}{1+0.15s}$, where $A = 3.108844314$ and $\tau = 0.15$



(a) Real Case Plot from which System Values were calculated

(b) Simulink Generated plot using identified values

Figure 2.1.4: The measured system response compared to the simulated system response when using maximum resistance

For all the systems, the simulated step response matched the actual step response withing a 10% error margin. This indicates that the systems identified are correct and accurate within 90% which is satisfactory for the purpose of identifying the systems, as the controller being designed to control the system will need to account for changing disturbances too.

2.2 Cascade Controller Design

2.2.1 Design Procedure

Plant templates

For the Quantitative Feedback Theory (QFT) Design, the first important step is to create the plant templates from the Plant specifications. To find these specifications in the frequency domain, the following steps were taken:

The plant specifications A and τ were converted from $P = \frac{A}{\tau s+1}$ to values for k and a to change the form of the plant function into $P = \frac{k}{s+a}$. Vectors were created using these values, which are evenly spaced for 10 items between the minimum and maximum plant values.

The plant template was created by looping through each value in the a vector, for a maximum value of k and similarly, the minimum value - and then iterating through each value in the k vector for the minimum and maximum a values. Hence, creating the outer bounds of the plant template for each frequency. These templates were applied to frequencies $\omega = \in [0.5, 3, 5, 10, 30, 100]$

The plant templates were plotted on an Inverse Nichols Chart

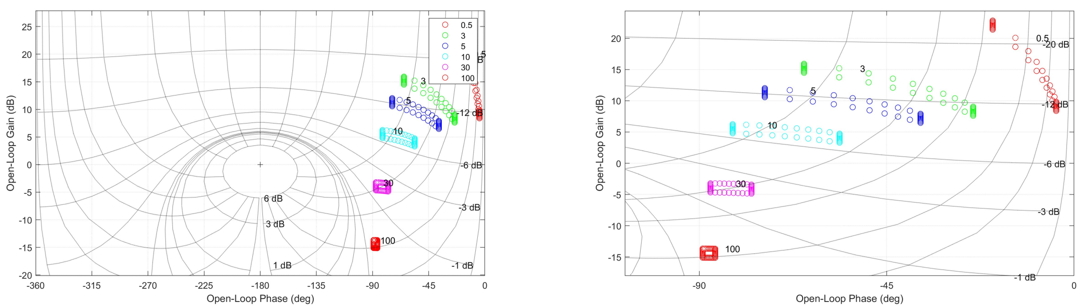


Figure 2.2.1: Plant Templates Shown for Multiple Frequencies on Inverse Nichols Chart, and a close-up version

Frequency Domain Specifications

To begin designing a controller for the plant, the bounds for regions in which the controlled system can and cannot be within needed establishment. To accomplish this, the time domain specifications (% overshoot and settling time) were converted into frequency domain specifications in the following way:

The inner loop should have a maximum % overshoot of 20%. From this, it could be calculated through the following formula that our desired system has a damping factor of $\zeta = 0.45595$

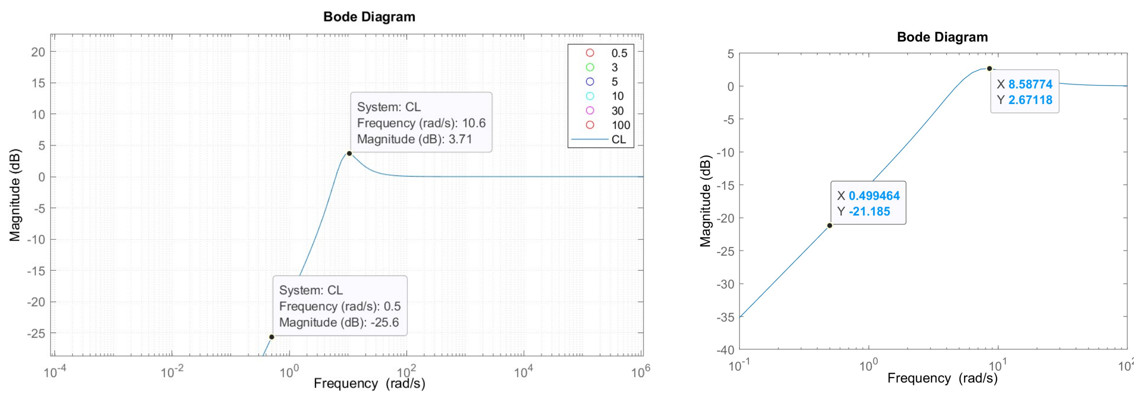
$$\zeta = \sqrt{\frac{ln(0.2)^2}{\pi^2 + ln(0.2)^2}} \tag{3}$$

The inner loop system was required to settle within 1 second. From the damping factor given above and this specification, the natural frequency of the inner loop system is calculated to be $\omega_n = 8.7729 rad/s$ through use of the formula:

$$\omega_n = \frac{4}{\zeta \times t2\%} = \frac{4}{0.45595 \times 1} \tag{4}$$

The ideal transfer function of the controlled loop is calculated as $L_1 = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ where sensitivity $= 1 - L_1$ The sensitivity function of L_1 was plotted on a Bode plot and the frequency domain specifications were taken from the plot to be

$$\left|\frac{1}{1+L}\right| \leq 3.71\text{dB}\ \forall\omega\ and\ \left|\frac{1}{1+L}\right| \leq -25.6\text{dB}, \omega = 0.5\frac{\text{rad}}{s}$$



(a) Bode Plot showing the Frequency Specifications for the Inner Loop

(b) Bode Plot showing the Frequency Specifications for the Outer Loop

The same procedure was completed for the outer loop, except it had a maximum overshoot of 10%, so the values of ζ and ω_n changed to $\zeta = 0.59$ and $\omega_n = 6.7797 rad/s$. The new frequency domain specifications for the outer loop design are:

$$\left|\frac{1}{1+L}\right| \leq 2.672\text{dB}\ \forall\omega\ and\ \left|\frac{1}{1+L}\right| \leq -21.2\text{dB}, \omega = 0.5\frac{\text{rad}}{s}$$

Loop Shaping and Controller Design

Once the frequency domain specifications are established from the time domain specifications, they can be converted into actual boundaries on an Inverse Nichols Chart (INC), using the plant templates to define the boundaries for each frequency. The bounds were then used as design boundaries around which the nominal plant was plotted and controlled.

Inner Loop

By plotting the nominal plant P_1 on the INC using the QFT toolbox, it was found that the only controller necessary for the inner loop was a gain controller. For this, the point representing $\omega = 0.5 rad/s$ needed to be moved up by approximately $7.1 dB$, which corresponds with a linear gain of approximately 2.3. The nominal plant before and after being controlled by gain to meet the specifications is shown below:

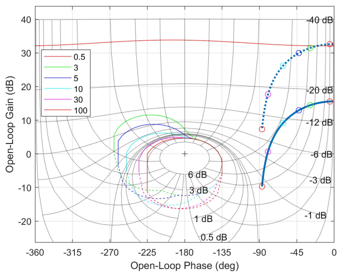


Figure 2.2.3: INC with Bounds Showing Nominal Plant before (solid) and after (dotted) $7.1dB$ Gain on QFT Design toolbox

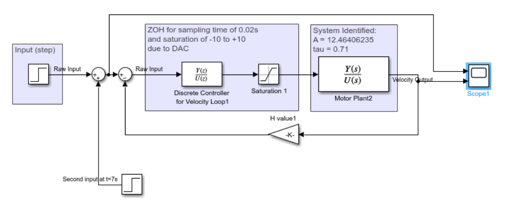
Once this plant is controlled by the gain: $G_1 = 2.3$, there was a difference in the magnitude of the setpoint and the response of a factor of 0.9663 that was accounted for by adding 0.9663 units of gain as the sensor value in the feedback loop: $H_1 = 0.9663$.

Further examining the sensor value for the feedback loop, the final value theorem of the system to a step input was used to obtain the value mathematically:

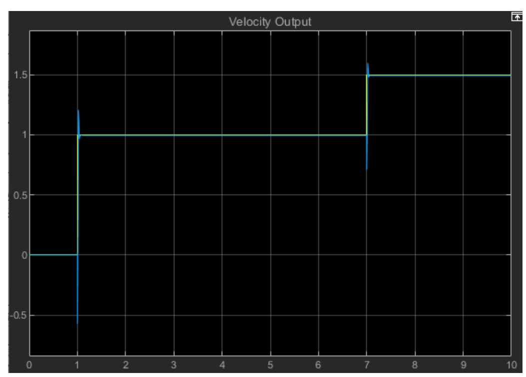
$$\lim_{s\to 0} \frac{s \cdot (2.3) \cdot \frac{12.46406235}{0.71s+1}}{s \cdot \left(1 + (2.3) \cdot \frac{12.46406235}{0.71s+1}\right)} = \frac{28.67}{29.67} = 0.9663 \tag{5}$$

Simulation of Inner Loop (Velocity) Gain Controller

The response of the velocity loop to test inputs such as a step input, input disturbance, output disturbance and a sine wave input worked for the gain controller implemented. It tracked the setpoint (input) within the specifications of 20% overshoot and settled within 1 second too.



(a) Simulink Model for Inner Loop Simulations



(b) Response of Inner Loop Showing Tracking within Specifications

The above Figure (a), shows the Simulink model used to simulate the velocity loop, where the discrete controller was a linear gain of 2.3 and the plant is the nominal case transfer function derived from system identification (no eddy current resistance). Figure (b) shows the output of the Simulink model response to a step input and a further step of 0.5V that occurs at 7 seconds. In the above plot, the output had the desired overshoot of just under 20% and the settling time was well within 1 second.

From the above calculations in equation 5, the value of 0.9963 was used as the sensor value.

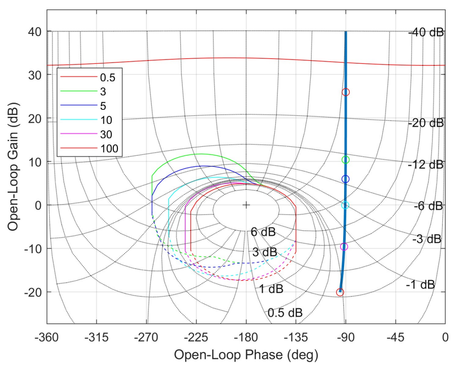
The inner closed loop transfer function now becomes

$$P_2 = \frac{P_1 G_1}{1 + P_1 G_1 H_1} \tag{6}$$

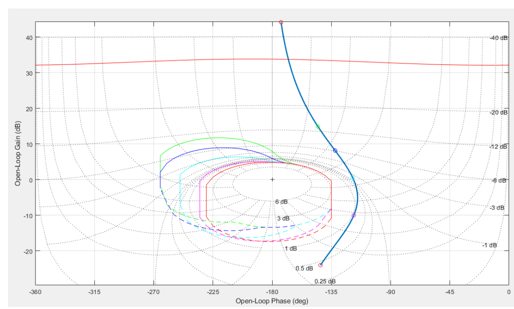
Which is treated as P_2 in the outer loop.

Outer Loop

The Plant P_2 as above in equation 6 alongside the gear ratio scale factor as calculated at the end of the system identification section (2.1), $K = 9.97$, and an integrator to take the velocity value into a position value - with a transfer function of $\frac{1}{s}$. The plant to be controlled is now $P_2 \times \frac{1}{s} \times 9.97$ which includes the controlled velocity loop. Once the frequency specifications for this outer loop were defined (**??**), the new bounds were shown and plotted on the INC alongside the nominal plant for the position.



(a) Bounds and Uncontrolled Nominal Plant for the Outer Loop



(b) Controlled Open Loop (Nominal Plant) Shown on the INC with specification bounds

After the nominal plant was plotted it was clear that the integrator would shift the plant up and left into the bounds, the gain would shift it up, and the lead term would be required to shift the plant to the right and move it out the bounds. 15° of additional phase lead is required to compensate for the effects of discretization. The controller found with the help of the QFT toolbox was:

$$G_2(s) = \frac{4.3 \cdot \left(\frac{s}{4} + 1\right)}{s} \tag{7}$$

this was further discretized to be of the form:

$$G_2(z) = \frac{1.118z - 1.032}{z - 1} \tag{8}$$

For future implementation this controller would take the form of the following difference equation:

$$u_{i+1} = u_i - 1.032e_i + 1.118e_{i+1} \tag{9}$$

2.3 Implementation of Controller

The controller described by the difference equation (equation 9) needed to be implemented on the software which controls the overall operation of the plant. It needed to be in the form of the difference equation to ensure ease of coding. The following image shows the implementation of the controller in code:

```
// Controller ////////////////////////////////////////////////

//CONSTRAINTS

double Error = rt_SetPoint - yt_PlantOutput ;
double U1 = Uprev - (1.032 * Eprev) + (1.118 * Error);
double ut_Output = U1 - (0.9663 * Velocity);
ut_Output = ut_Output * 2.1;

//CONSTRAINTS
if (ut_Output > 9.8) { ut_Output = 9.8; }
if (ut_Output < -9.8) { ut_Output = -9.8; }

//////////////////////////////////////////////////////////////
```

Figure 2.3.1: Code showing the overall controller implementation

The lines of code are explained in detail:

1. The error of the system was found.
2. Earlier in the code, the previous U and Error values were set to zero, and subsequently as the new U1 and Error values are found, the 'prev' values are set accordingly to keep memory in the system. This line of code is the line representing the controller's difference equation.
3. The sensor value was incorporated in the inner loop.
4. The velocity gain controller was implemented on the inner loop.
5. The output was set to saturate at an upper level of 9.8V.
6. The output was set to saturate at a lower limit of -9.8V.

The controller worked extremely well the first time it was implemented, except for the amount of overshoot. The overshoot did not sit within 20% as found in the simulation, it was slightly above, however the settling time was well within 1 second for a step input response.

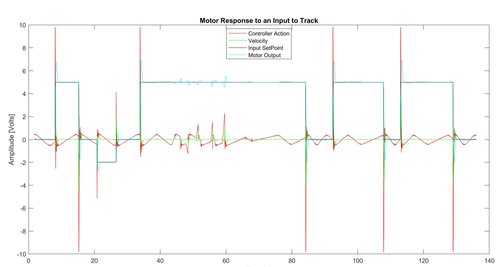


Figure 2.3.2: All outputs of the System, Including the controller action

The following graph is the response of the controlled plant to certain input tests:

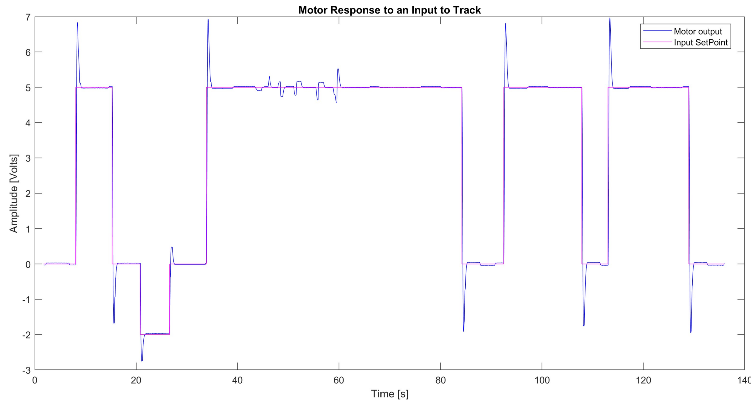


Figure 2.3.3: Motor Input and Setpoint Tracking for Different Test Cases

The test cases were as follows:

1. For 0-40s: A number of step inputs of different sizes were input to test the response to them, including a negative step of -2V.
2. Between 40s and 60s, we interfered with the equipment do emulate output disturbance, which was successfully rejected by the controller.
3. Between 60s and 80s, the system's attenuation was increased from 1 all the way to 6 and back down, with no change in the input tracking, demonstrating the robustness of the controller.
4. Between 80s and 100s, the eddy current level was changed to the middle level (half eddy current) and tested for a step input of 5V.
5. Between 100s and 130s, the eddy current level was changed to the maximum and tested for a step input of 5V. These responses for changing eddy current levels show the robustness again, as the controlled system still meets the specifications except for the overshoot.

3 Analysis and Discussion

In figure 2.3.2 it can be seen that the controller action (red) is higher for disturbances and input changes and is not overly compensating, and so it can be considered a successful design. Other reasons the controller was successful was through it's perfect set point tracking, output disturbance rejection, settling time of less than 1 second to the set point and minimal change for the input disturbance (rejects disturbance of eddy current). The only criteria the controller does not meet is to track the set point with less than 20% initial overshoot. The overshoot for the controller implemented was approximately 30% which was too high, and there is a significant trade-off between overshoot and response time. If further steps were to be taken to reduce this overshoot, they could include compensation with more derivative or integral components in the control loop. Overall, the design procedure followed was considered succesful and the overshoot can be remedied in the future.