EE141L: Lab 2 – Traffic Analysis

Sept. 7, 2018

1 Matlab exercises: traffic analysis

You are expected to submit both 1) your matlab code 2) and answers in a lab report. Both are to be uploaded on Blackboard every Friday before 11:59PM. Hard copies of your report and or scripts will not be accepted in class. Lab 2 will be due September 14th. Important things to keep in mind:

- MATLAB scripts should run without any errors.
- Figures and results from MATLAB calculations should exactly match those stated in your report (we will check)
- Provide documentation for any function you write (i.e. inputs, outputs, algorithm)
- Lab reports are to be written in Q&A Format
- For all calculations, you will be asked to provide observations as well as justifications for answers
- Figures should have proper labels and titles or captions

Failure to complete any of the above will negatively impact your grade for lab assignments.

2 Systems of Linear Equations

This lab is devoted to the study of systems of linear equations, which arise naturally in engineering and the physical sciences. The following system of equations where a_{ij} and b_i ($1 \leq i \leq m$ and $1 \leq j \leq n$) are scalars in $\mathbb{R}$ and $x_1, x_2, \ldots, x_n$ are n variables taking values in R, is called a **system of m linear equations in n unknowns over $\mathbb{R}$** .

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned} \tag{1}
$$

The $m \times n$ matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

is called the **coefficient matrix** of the system. Similarly, if we let

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

then the system of equations can be rewritten as a single matrix equation

$$\mathbf{Ax} = \mathbf{b} \tag{2}$$

where the operation of multiplying a matrix by a vector (to be discussed next week in class) is:

$$\mathbf{Ax} = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \ldots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

The **augmented matrix** of the system of equations is denoted as $[\mathbf{A}|\mathbf{b}]$ is:

$$[\mathbf{A}|\mathbf{b}] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array}\right]$$

In lecture, we covered **Gaussian Elimination**, an important technique for solving systems of linear equations (or $\mathbf{Ax} = \mathbf{b}$) which utilizes three elementary row operations on $[\mathbf{A}|\mathbf{b}]$:

- interchange any two rows in the system ($R_i \leftrightarrow R_j$)
- multiplying any rows by a nonzero constant ($R_i \leftarrow cR_i$)
- adding a multiple of one row to another ($R_j \leftarrow R_j + cR_i$)

By using elementary row operations (**EROS**), we transform the augmented matrix into an upper triangular matrix in which the first nonzero entry of each row is 1, and it occurs in a column to the right of the first nonzero entry of each preceding row.

Q1 Use **EROS** to solve the following system of equations. Label each step accordingly (i.e. add 4 times row 3 to row 1 is $R_1 \leftarrow R_1 + 4R_3$). Provide the general solution space of the system of equations. Provide the reduced row echelon form (RREF) of the augmented matrix. Which of the unknown variables (if any) correspond to pivots? Which variables (if any) correspond to free variables?

$$\begin{aligned} 3x_1 + 2x_2 + 3x_3 - 2x_4 &= 1 \\ x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 2x_2 + x_3 - x_4 &= 2 \end{aligned} \tag{3}$$

3 Traffic Flow

For this lab assignment, you will be asked to solve for unknown traffic flows along some of LA's main highways. Highway intersections are highlighted in red. You will use MATLAB for your calculations. Your script should be broken into different sections so that each section corresponds to different answers of your question. Answers and explanations should be included in your report.

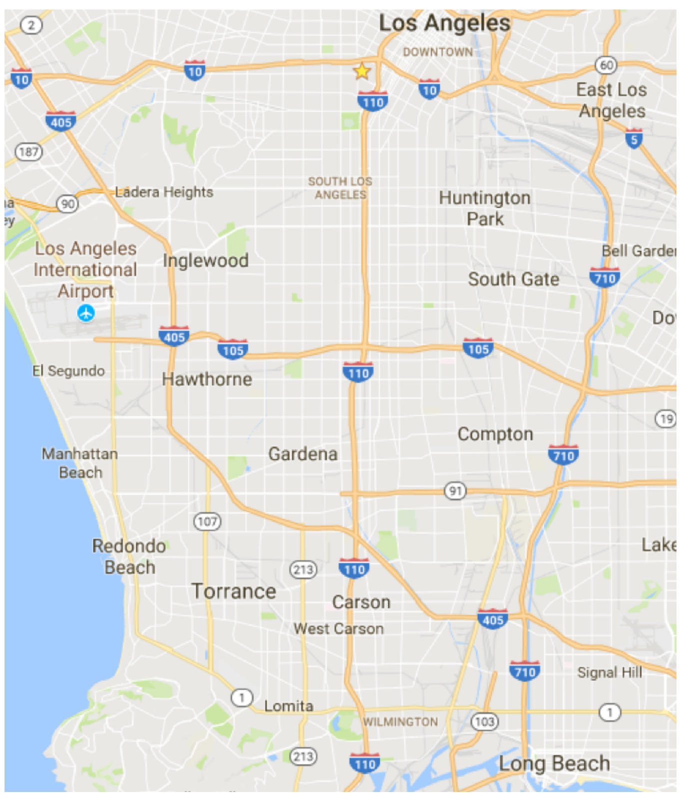

Figure 1: Highway Map of Los Angeles

Important assumptions for this lab assignment include:

- Roads have no leaks (no sinkhole in the middle of an intersection!)
- # of cars entering Intersection = # of cars leaving Intersection (Conservation of traffic flow)
- Arrows point in direction of traffic

The details of the problem statement are the following for each highway intersection :

- 405/105 **100** car/min enter from Santa Monica
- 110/105 **60** car/min enter from USC/DTLA
- 110/105 **40** car/min enter from Downey
- 405/107 ?
- 110/405/91 **25** car/min enter from Lakewood
- 110/405/91 **15** car/min enter from San Pedro

Your goal is to find unknown traffic flows t, u, v, w, x, y, z where t, u, v, w represent traffic between intersections. On the other hand, x, y, z represent traffic going to LAX, Manhattan beach, and San Pedro, respectively.

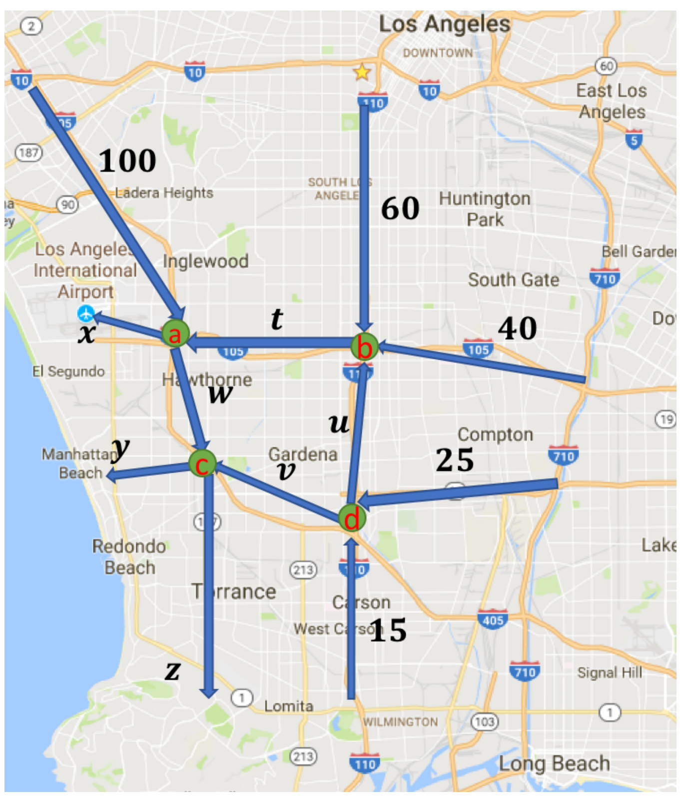

Figure 2: Flow of Traffic along major highways

Q2 (a) Derive flow conservation equations for all intersections

(b) Suppose $t = 100$ and $w + y + z = 100$, then write system of equations in the form $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{x}$ is the vector of flows

(c) Input the matrix $\mathbf{A}$, and vector $\mathbf{b}$ into MATLAB

(d) Construct the augmented matrix $[\mathbf{A}|\mathbf{b}]$

(e) Transform the augmented matrix into **reduced row echelon form** *(doc ref)*.

(f) Use the row echelon form to find the solution in parametric form

(g) What is the total traffic entering the network?

(h) Find $F = x + y + z$, the total flow exiting the network

Now we modify the previous scenario in order to allow traffic from intersections (d) to (a), and from (c) to (b) (green arrows).

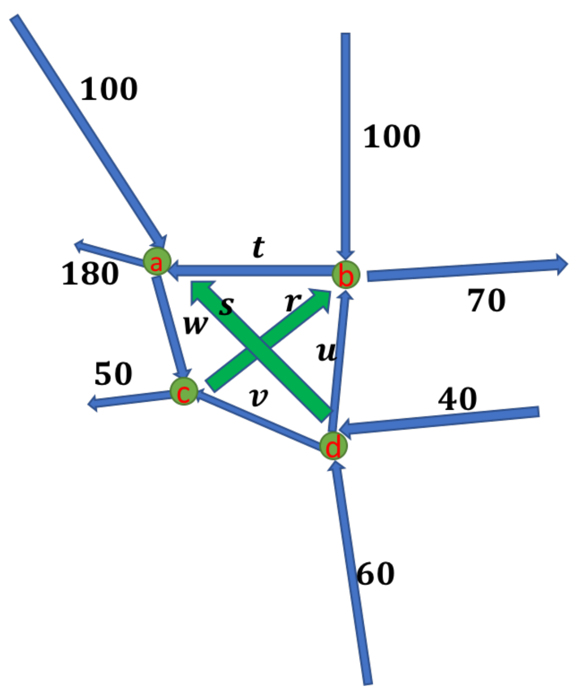

Figure 3: Second Scenario

Q3 (a) Derive the new flow conservation equations and formulate matrix equation

(b) Repeat steps in problem 1 parts (c)-(e)

(c) Report solution in parametric form