POINTS TO PONDER

1.1 A matrix with n rows and n columns has n^2 elements. So to initialize such a matrix requires n^2 assignments. To multiply two n by n matrices together, or to solve n linear equations in n unknowns, involves on the order of n^3 operations by classical methods. (These are the sorts of things that matrix manipulation programs do.) Give arguments to support the following conjectures:

If $n \geqslant 10$, the time required to initialize a matrix is not very important.

If $n < 10$, the time required to initialize a matrix is not very important. (Hint: input and output conversions are more time consuming than arithmetic.)

1.2 In the first edition of this book, we wrote the square root routine this way:

```
C            COMPUTE SQUARE ROOTS BY NEWTON'S METHOD
   10 READ(5,11) X
   11 FORMAT (F10.0)
      IF (X .GE. 0.0) GOTO 20
         WRITE(6,13) X
   13    FORMAT (' SQRT(', 1PE12.5, ') UNDEFINED')
         GOTO 10
   20 IF (X .GT. 0.0) GOTO 30
         B = 0.0
         GOTO 50
   30 B = 1.0
   40    A = B
         B = (X/A + A)/2.0
         IF (ABS((X/B)/B - 1.0) .GE. 1.0E-5) GOTO 40
   50 WRITE(6,51) X, B
   51 FORMAT(' SQRT(', 1PE12.5, ') = ', 1PE12.5)
      GOTO 10
      END
```

This is "more efficient" because there are no repeated tests. Which version do you prefer, and why? How much time and space difference does the change make? What deficiencies of the Fortran language are illustrated by both versions?

1.3 In the square root routine, we saw that testing for convergence against an absolute threshold like `1.0E-5` is perilous. We recommended testing instead against some sort of relative standard. How can the function

```
      REAL FUNCTION RELDIF(X, Y)
      RELDIF = ABS(X - Y) / AMAX1(ABS(X), ABS(Y))
      RETURN
      END
```

be used in the example? (`AMAX1` is the Fortran function that returns the maximum of two or more floating point numbers as floating point.) This function is relatively well-behaved for values that might be encountered in the square-root routine. In more general applications, are there any values of `X` and `Y` that might cause trouble?