Universal Sequential Search Problems

L. A. Levin

Abstract

The article examines several well-known problems of the "sequential search type" and proves that these problems can only be solved in the time it takes to solve any problem of the specified type in general.

1 Introduction

[Introduction remains the same as in the previous version]

2 Definitions and Problems

[Definitions and Problems section remains the same as in the previous version]

3 Main Results

Let $f(n)$ be a monotonic function.

Theorem 1. *If there exists any problem of sequential search (quasi-sequential search) type that cannot be solved in time less than $f(n)$ for argument length comparable to n, then problems 1-6 also have this property.*

Proof. Let P be a sequential search problem that cannot be solved in time less than $f(n)$ for argument length comparable to n. We will show that each of the problems 1-6 is at least as hard as P.

By Lemma 1 (proven below), problems 1-6 are universal sequential search problems. This means that P can be reduced to each of these problems. Let's consider the reduction of P to problem i (where i is any number from 1 to 6).

There exist three algorithms $r(x)$, $p(y)$, and $s(y)$, working in time comparable to the length of the argument, such that:

1) $P(x, p(y)) \Leftrightarrow \text{Problem}_i(r(x), y)$ 2) $P(x, y) \Leftrightarrow \text{Problem}_i(r(x), s(y))$

Suppose, for contradiction, that problem i can be solved in time less than $f(n)$. Then we could solve P as follows:

1) Given input x for P, compute $r(x)$. 2) Solve $\text{Problem}_i(r(x), y)$ in time less than $f(n)$. 3) If a solution y is found, compute $p(y)$ to get a solution for P.