The total time for this procedure would be less than $f(n)$ (plus some time comparable to the input length for computing $r(x)$ and $p(y)$). This contradicts our assumption that P cannot be solved in time less than $f(n)$.

Therefore, problem i cannot be solved in time less than $f(n)$. Since i was arbitrary, this applies to all problems 1-6. □

The idea of the proof is that problems 1-6 are "universal sequential search problems".

Definition 1. *Let $A(x, y)$ and $B(x, y)$ define sequential search problems A and B respectively. We say that problem A reduces to B if there are three algorithms $r(x)$, $p(y)$, and $s(y)$, working in time comparable to the length of the argument, such that $A(x, p(y)) \Leftrightarrow B(r(x), y)$ and $A(x, y) \Leftrightarrow B(r(x), s(y))$ (i.e., from an A-problem x, it's easy to construct an equivalent B-problem $r(x)$). A problem to which any sequential search problem reduces is called "universal".*

Thus, the essence of the proof of Theorem 1 consists in the following lemma.

Lemma 1. *Problems 1-6 are universal sequential search problems.*

Proof Sketch. We need to show that any sequential search problem can be reduced to each of the problems 1-6. We'll outline the reduction for Problem 2 (finding a DNF for a partial Boolean function).

Let $A(x, y)$ be any sequential search problem. We can encode the computation of $A(x, y)$ as a Boolean circuit. This circuit can be represented as a partial Boolean function f_x where:

- The input represents y - The output is 1 if and only if $A(x, y)$ is true - The function is undefined for inputs that don't correspond to valid encodings of y

Now, finding a y such that $A(x, y)$ is true is equivalent to finding a satisfying assignment for f_x, which in turn is equivalent to finding a DNF representation of f_x.

The reduction algorithms would work as follows: - $r(x)$ constructs the partial Boolean function f_x - $p(y)$ extracts y from the satisfying assignment - $s(y)$ encodes y as an input to the Boolean function

Similar reductions can be constructed for the other problems, showing that each of them is universal. □

The described method apparently allows easy obtaining of results like Theorem 1 and Lemma 1 for most interesting sequential search problems. However, the problem remains to prove the condition present in this theorem. Numerous attempts have long been made in this direction, and a number of interesting results have been obtained (see, for example, [3, 4]). However, the universality of various sequential search problems can be established without solving this problem. In the system of Kolmogorov-Uspensky algorithms, the following can also be proved:

Theorem 2. *For an arbitrary sequential search problem $A(x, y)$, there exists an algorithm that solves it in time optimal to within multiplication by a constant and addition of a value comparable to the length of x.*