A Systematic Evaluation of the Observed Degradation of Typesetting Technology in the 20th Century

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Abstract

We systematically evaluate typesetting technology over the course of the 20th century and discover an astonishing degradation. We hypothesize on the potential causes of this observed degradation and conclude that it is the work of malicious time-travelling monkeys.

Keywords: type, systems

1 Introduction

In this work, we advance the hypothesis that typesetting technology took a dangerously steep downward turn in the latter half of the 20th century. This is not a new hypothesis; the world-renowned computer science genius Knuth made similar observations in the thick of the matter 30 years ago in March 1977, saying of his gloriously comprehensive compendium *The Art of Computer Programming* [2], "I had spent 15 years writing those books, but if they were going to look awful, I didn't want to write any more" [3].

We take Knuth's hypothesis and validate it with a systematic and unbiased evaluation of over three academic papers selected at random from between the years 1900 and 1999. In Section 1, we introduce our hypothesis. Section 2 discusses our raw data in detail. Finally, in Section 3 we draw the startling conclusion that typesetting technology actually degraded over the course of the 20th century!

2 Experimental data

2.1 1936: the heady days of the decision problem

Hilbert's 23 problems began a century of glory in mathematics. In 1936, Alonzo Church published a note [1] on what is widely regarded as "by far the coolest of Hilbert's 23" [8], the Entscheidungsproblem. We excerpt this note in Figure 2.

(Church is also well-known and highly regarded for his work on the hat calculus; see [4] for a contemporary tutorial introduction.)

This publication represents the pinnacle of publishing quality in the 20th century, with its fully-typeset mathematics and proportionally spaced fonts. No characters are hand-drawn, and the kerning is superb. The footnotes aren't even fragile! An exemplary exemplar of style and quality—precisely what we've come to expect from an academician as talented as Church.

Figure 1: Alonzo Church, a very talented academician. (Cheerful, too!)

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A NOTE ON THE ENTSCHEIDUNGSPROBLEM

ALONZO CHURCH

In a recent paper[1] the author has proposed a definition of the commonly used term "effectively calculable" and has shown on the basis of this definition that the general case of the Entscheidungsproblem is unsolvable in any system of symbolic logic which is adequate to a certain portion of arithmetic and is ω-consistent. The purpose of the present note is to outline an extension of this result to the engere Funktionenkalkül of Hilbert and Ackermann.[2]

In the author's cited paper it is pointed out that there can be associated recursively with every well-formed formula[3] a recursive enumeration of the formulas into which it is convertible.[3] This means the existence of a recursively defined function a of two positive integers such that, if y is the Gödel representation of a well-formed formula $\boldsymbol{Y}$ then $a(x, y)$ is the Gödel representation of the xth formula in the enumeration of the formulas into which $\boldsymbol{Y}$ is convertible.

Consider the system L of symbolic logic which arises from the engere Funktionenkalkül by adding to it: as additional undefined symbols, a symbol 1 for the number 1 (regarded as an individual), a symbol = for the propositional function = (equality of individuals), a symbol s for the arithmetic function $x+1$, a symbol a for the arithmetic function a described in the preceding paragraph, and symbols $b_1, b_2, \cdots, b_k$ for the auxiliary arithmetic functions which are employed in the recursive definition of a; and as additional axioms, the recursion equations for the functions $a, b_1, b_2, \cdots, b_k$ (expressed with free individual variables, the class of individuals being taken as identical with the class of positive integers), and two axioms of equality, $x=x$, and $x=y\rightarrow[F(x)\rightarrow F(y)]$.

The consistency of the system L follows by the methods of existing proofs.[4] The ω-consistency of L is a matter of more difficulty, but for our present purpose the following weaker property of L is sufficient: *if* P *contains no quantifiers and* (Ex)P *is provable in* L *then not all of* $\overline{\mathrm{P}}_1, \overline{\mathrm{P}}_2, \overline{\mathrm{P}}_3, \cdots$ *are provable in* L (*where* $\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3, \cdots$ *are respectively the results of substituting* 1, 2, 3, $\cdots$ *for x throughout* P). This property has been proved by Paul Bernays[5] for any one of a class of systems of which L is one. Hence, by the argument of the author's cited paper, follows:

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[1] *An unsolvable problem of elementary number theory*, **American journal of mathematics**, vol. 58 (1936).

[2] ***Grundzüge der theoretischen Logik***, Berlin 1928.

[3] Definitions of the terms *well-formed formula* and *convertible* are given in the cited paper.

[4] Cf. Wilhelm Ackermann, *Begründung des "tertium non datur" mittels der Hilbertschen Theorie der Widerspruchsfreiheit*, **Mathematische Annalen**, vol. 93 (1924–5), pp. 1–136; J. v. Neumann, *Zur Hilbertschen Beweistheorie*, **Mathematische Zeitschrift**, vol. 26 (1927), pp. 1–46; Jacques Herbrand, *Sur la non-contradiction de l'arithmétique*, **Journal für die reine und angewandte Mathematik**, vol. 166 (1931–2), pp. 1–8.

[5] In lectures at Princeton, N. J., 1936. The methods employed are those of existing consistency proofs.

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Figure 2: 1936 publication.

2.2 1974: a less innocent age

Flash forward to the year 1974. Nixon faces impeachment for the Watergate scandal. India successfully detonates its first nuclear weapon. Polymorphism is in its fledgling stages.

Enter John Reynolds.

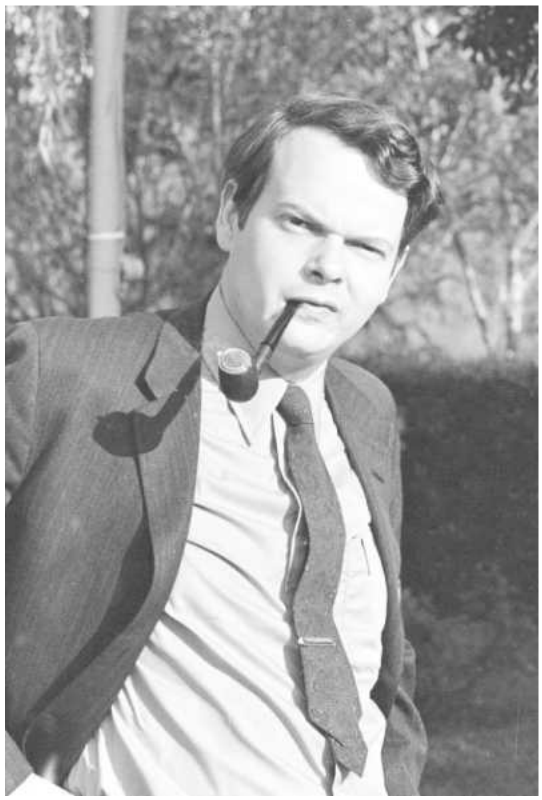

Figure 3: Bright-eyed and bushy-tailed John Reynolds, with a pipe.

It was in this year that Reynolds published his monumental manuscript on the polymorphic λ-calculus [5]. Despite being an academic work of the highest quality [XX cites???], its typesetting left much to be desired.

As one can see clearly from the scan in Figure 4, 1974 marked an age of "digital typography"—characters unavailable on the standard typewriter were drawn in by hand. Examples include the characters $\forall$, $\subseteq$, $\bigsqcup$, and (particularly damningly) $\mathcal{D}$. The careful reader will note the scribbly nature of the tail on the $\mapsto$ arrow. But at least we have a full complement of Greek letters, and everything is sufficiently well-spaced to be legible ...

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The definition of the functor delta is less obvious. For all functors θ from C to C, delta(θ) is the complete lattice with elements

$$\{ f \mid f \in \prod_{D \in \mathcal{D}} \theta(D) \text{ and } (\forall D, D' \in \mathcal{D})(\forall \rho \in \mathrm{rep}(D,D'))\, \theta(\rho)\colon f(D) \mapsto f(D') \}$$

with the partial ordering $f \sqsubseteq g$ iff $(\forall D \in \mathcal{D})\ f(D) \sqsubseteq_{\theta(D)} g(D)$. For all natural transformations η from θ to θ',

$$\begin{aligned} &\mathrm{delta}(\eta) = \\ &\quad < \lambda f \in \mathrm{delta}(\theta).\ \lambda D \in \mathcal{D}.\ [\eta(D)]_1(f(D)), \\ &\qquad\qquad \lambda f \in \mathrm{delta}(\theta').\ \lambda D \in \mathcal{D}.\ [\eta(D)]_2(f(D)) > \quad . \end{aligned}$$

At this point, we must admit a serious lacuna in our chain of argument. Although delta(θ) is a complete lattice (with $(\bigsqcup F)(D) = \bigsqcup_{\theta(D)} \{f(D) \mid f \in F\}$), it is not known to be a domain, i.e., the question of whether it is continuous and countably based has not been resolved. Nevertheless there is reasonable hope of evading the set-theoretic paradoxes. Even though $\prod_{D \in \mathcal{D}} \theta(D)$ is immense (since $\mathcal{D}$ is a class), the stringent restrictions on membership in delta(θ) seem to make its size tractable. For example, if $f \in$ delta(θ), then the value of $f(D)$ determines its value for any domain isomorphic to D.

The definition of delta and the properties of representations give the lemma:

Let η be a natural transformation from θ to θ', $f \in$ delta(θ) and $f' \in$ delta(θ'). Then

$$\mathrm{delta}(\eta)\colon f \mapsto f'$$

if and only if, for all $D, D' \in \mathcal{D}$, and $\rho \in \mathrm{rep}(D, D')$,

$$\eta(D') \cdot \theta(\rho)\colon f(D) \mapsto f'(D') \quad .$$

which, with the definition of B, gives:

Let $t \in T$, $w \in W$, $\overline{\rho} \in \mathrm{rep}(\overline{D}, \overline{D}')$, $f \in B[\Delta t.\ w](\overline{D})$, and $f' \in B[\Delta t.\ w](\overline{D}')$. Then

$$B[\Delta t.\ w](\overline{\rho})\colon f \mapsto f'$$

if and only if, for all $D, D' \in \mathcal{D}$, and $\rho \in \mathrm{rep}(D, D')$,

$$B[w][\overline{\rho}|t|\rho]\colon f(D) \mapsto f'(D') \quad .$$

From the final lemmas obtained about arrow and delta, the representation theorem can be proved by structural induction on r.

Figure 4: 1974 publication.

2.3 1983: oh the burning!

Advance the clock 9 years to 1983. Ronnie Reagan leads the United States in their epic battle with the Evil Empire; the Star Wars project[1] plays a critical role. Meanwhile Luke and Leia lead the rebellion in *Return of the Jedi.* Reynolds does his part by proving the glorious Abstraction Theorem [6]. Reynolds has

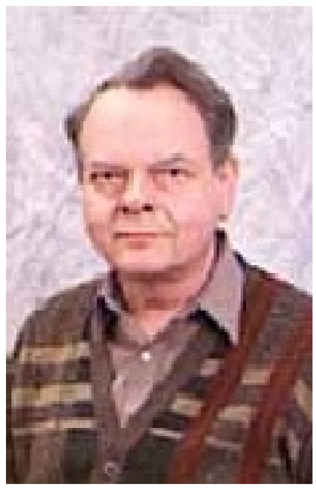

Figure 5: Older, wiser John Reynolds.

grown older, and perhaps wiser, but the quality of his typesetting has diminished dramatically.

Figure 6 demonstrates the terrible turn taken by typesetting technology in this war-torn era. Although typewriting technology has acquired certain key characters ($\forall$, for example), this is only at the expense of the all-important capital Π, which is more hand-drawn, larger, and uglier than it's ever been in the history of life on earth [citation needed]! Agghhh! The math, it burns my eyes!

[1] More properly referred to as the Strategic Defense Initiative

where $s \to s'$ denotes the set of all functions from s to s' and $s \times s'$ denotes the set of pairs $\langle x, x' \rangle$ such that $x \in s$ and $x' \in s'$. Then the set $S^{\#}\omega$ is the meaning of the type expression ω under the set assignment S. Note that, when $\omega \in \Omega_c$, $S^{\#}\omega$ is independent of the set assignment S.

We use $*$ to denote a further extension from type expressions to type assignments: If S is a set assignment then $S^{\#*}$ is the mapping from Ω^* to the class of sets such that

$$S^{\#*}\pi = \prod_{v \in \mathrm{dom}\,\pi} S^{\#}(\pi v) . \qquad (S*)$$

Then $S^{\#*}\pi$ is the set of environments appropriate to π and S.

A conventional semantics for ordinary expressions would be obtained by fixing some set assignment S and defining a family of semantic functions from $E_{\pi\omega}$ to $S^{\#*}\pi \to S^{\#}\omega$. However, to capture abstraction properties we will need to relate the meanings of an expression under different set assignments. For this reason, we will treat set assignments as explicit parameters of the semantic functions. Specifically, for each $\pi \in \Omega^*$ and $\omega \in \Omega$ we will define a semantic function

$$\mu_{\pi\omega} \in E_{\pi\omega} \to \prod_{S \in \mathcal{S}} (S^{\#*}\pi \to S^{\#}\omega) ,$$

where $\mathcal{S}$ denotes the class of all set assignments.

We assume we are given, for each $\omega \in \Omega_c$, a function

$$\alpha_\omega \in K_\omega \to S^{\#}\omega$$

providing meanings (independent of S) to the ordinary constants of type ω. Then the semantic functions are defined by

If $k \in K_\omega$ then $\mu_{\pi\omega}\llbracket k \rrbracket\, S\, \eta = \alpha_\omega\, k$, (Ma)

If $v \in \mathrm{dom}\,\pi$ then $\mu_{\pi,\pi v}\llbracket v \rrbracket\, S\, \eta = \eta\, v$, (Mb)

If $e_1 \in E_{\pi,\omega\to\omega'}$ and $e_2 \in E_{\pi\omega}$ then
$\mu_{\pi\omega'}\llbracket e_1(e_2) \rrbracket\, S\, \eta =$ (Mc)
$\mu_{\pi,\omega\to\omega'}\llbracket e_1 \rrbracket\, S\, \eta\, (\mu_{\pi\omega}\llbracket e_2 \rrbracket\, S\, \eta)$,

If $e \in E_{[\pi|v:\omega],\omega'}$ then
$\mu_{\pi,\omega\to\omega'}\llbracket \lambda v{:}\omega.\ e \rrbracket\, S\, \eta = f$ (Md)
where $f \in S^{\#}\omega \to S^{\#}\omega'$ is such that
$f\, x = \mu_{[\pi|v:\omega],\omega'}\llbracket e \rrbracket\, S\, [\eta|v{:}x]$,

If $e \in E_{\pi\omega}$ and $e' \in E_{\pi\omega'}$ then
$\mu_{\pi,\omega\times\omega'}\llbracket \langle e, e' \rangle \rrbracket\, S\, \eta =$ (Me)
$\langle \mu_{\pi\omega}\llbracket e \rrbracket\, S\, \eta,\ \mu_{\pi\omega'}\llbracket e' \rrbracket\, S\, \eta \rangle$,

If $e \in E_{\pi,\omega\times\omega'}$ then
$\mu_{\pi\omega}\llbracket e.1 \rrbracket\, S\, \eta = [\mu_{\pi,\omega\times\omega'}\llbracket e \rrbracket\, S\, \eta]_1$ (Mf)
$\mu_{\pi\omega'}\llbracket e.2 \rrbracket\, S\, \eta = [\mu_{\pi,\omega\times\omega'}\llbracket e \rrbracket\, S\, \eta]_2$,

If $b \in E_{\pi,\underline{\mathrm{Bool}}}$ and $e, e' \in E_{\pi\omega}$ then
$\mu_{\pi\omega}\llbracket \underline{\mathrm{if}}\ b\ \underline{\mathrm{then}}\ e\ \underline{\mathrm{else}}\ e' \rrbracket\, S\, \eta =$ (Mg)
$\underline{\mathrm{if}}\ \mu_{\pi,\underline{\mathrm{Bool}}}\llbracket b \rrbracket\, S\, \eta = \underline{\mathrm{true}}$
$\underline{\mathrm{then}}\ \mu_{\pi\omega}\llbracket e \rrbracket\, S\, \eta\ \underline{\mathrm{else}}\ \mu_{\pi\omega}\llbracket e' \rrbracket\, S\, \eta$.

3. THE ABSTRACTION THEOREM

We now want to formulate an abstraction theorem that connects the meanings of an ordinary expression under different set assignments. The underlying idea is that the meanings of an expression in "related" environments will be "related" values. But here "related" must denote a different relation for each type expression and type assignment. Moreover, while the relation for each type variable is arbitrary, the relations for compound type expressions and type assignments must be induced in a specified way. In other words, we must specify how an assignment of relations to type variables is extended to type expressions and type assignments.

This can be formalized by defining a "relation semantics" for type expressions that parallels their set-theoretic semantics. For sets s_1 and s_2, we introduce the set

$$\mathrm{Rel}(s_1, s_2) = \{r \mid r \subseteq s_1 \times s_2\}$$

of binary relations between s_1 and s_2, and we write

$$I(s) = \{\langle x, x \rangle \mid x \in s\} \in \mathrm{Rel}(s, s)$$

for the identity relation on a set s. For $r \in \mathrm{Rel}(s_1, s_2)$ and $r' \in \mathrm{Rel}(s_1', s_2')$, we write $r \to r'$ for the relation in $\mathrm{Rel}(s_1 \to s_1', s_2 \to s_2')$ such that

$$\langle f_1, f_2 \rangle \in r \to r' \text{ iff } (\forall \langle x_1, x_2 \rangle \in r)\ \langle f_1\, x_1, f_2\, x_2 \rangle \in r' ,$$

and $r \times r'$ for the relation in $\mathrm{Rel}(s_1 \times s_1', s_2 \times s_2')$ such that

$$\langle\langle x_1, x_1' \rangle, \langle x_2, x_2' \rangle\rangle \in r \times r' \text{ iff } \langle x_1, x_2 \rangle \in r \text{ and } \langle x_1', x_2' \rangle \in r' .$$

In other words, functions are related if they map related arguments into related results, and pairs are related if their corresponding components are related.

For set assignments S_1 and S_2, a member of

$$\prod_{\tau \in T} \mathrm{Rel}(S_1\tau, S_2\tau)$$

is called a (binary) <u>relation assignment</u> between S_1 and S_2. Having defined $\to$ and $\times$ for relations we can extend relation assignments from T to Ω and Ω^* in essentially the same way as we extended set assignments. If R is a relation assignment between S_1 and S_2 then

$$R^{\#} \in \prod_{\omega \in \Omega} \mathrm{Rel}(S_1^{\#}\omega, S_2^{\#}\omega)$$

is such that

If $\kappa \in C$ then $R^{\#}\kappa = I(CS\,\kappa)$, (R1)

If $\tau \in T$ then $R^{\#}\tau = R\,\tau$, (R2)

If $\omega, \omega' \in \Omega$ then
$R^{\#}(\omega \to \omega') = R^{\#}\omega \to R^{\#}\omega'$, (R3)

If $\omega, \omega' \in \Omega$ then
$R^{\#}(\omega \times \omega') = R^{\#}\omega \times R^{\#}\omega'$, (R4)

Figure 6: 1983 publication.

2.4 1990: hell on earth

Six years later, the Soviet Union collapses, and Ringard publishes his seminal preprint on mustard watches [7], shown in Figure 8.

Figure 7: Yann-Joachim Ringard—no relation to Jean-Yves “mad dog” Girard.

Although this superficially *seems* to represent an increase in publishing quality, our unbiased opinion is that this apparent increase is merely illusory. Figures are now entirely hand-drawn, and ”smart quotes” are conspicuously absent. “QED” replaces the traditional $\Box$. Mathematical quality has taken a similarly downward turn; Ringard’s so-called “proofs” can barely be called sketches.

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1. Mustard watches : definitions and main results

Unless otherwise stated we use the term "watch" as an abbreviation for "analog pocket watch".

DEFINITION 1

Let W be a classical watch ; a *mustard watch* derived from W is any W' obtained from W by adding a certain amount of mustard in the mechanism. (see fig. 1)

It is immediate from the definition that a given classical watch may have several extensions into a mustard watch, whereas given a mustard watch W' there is only one underlying classical watch W from which W' is derived.

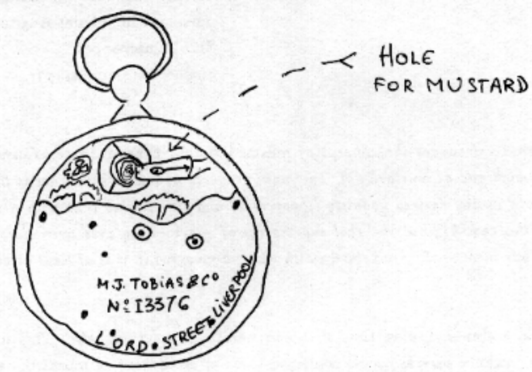

figure 1 : mustard watch just before refill

DEFINITION 2 :

A mustard watch is said to be *proper* when the amount of mustard in it is non zero ; it is said to be *degenerated* otherwise.

THEOREM 1 :

It is possible to get "as much mustard as wanted" from a mustard watch. More precisely, given any amount m of mustard, there is a mustard watch $W(m)$ containing mustard in quantity m.

PROOF : first observe that there are classical watches in any size ; in particular let W_m be one with enough room in the mechanism to harbour m grams of mustard. Consider the *completion* $W(m)$ of W achieved by adding m grams of mustard to W. QED

mustard watches

Figure 8: 1990 publication.

2.5 A graph

Any good experimental systems paper needs a graph. The graph in Figure 9 shows undeniably that typesetting quality has decreased between 1900 and 1999.

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   +------------------------------------->
   1900                               1999
                t i m e
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Figure 9: A graph

3 Conclusions

Undeniable! QED.

References

[1] Alonzo Church. A note on the Entscheidungsproblem. *Journal of Symbolic Logic*, 1:40–41, 1936.

[2] Donald E. Knuth. *The Art of Computer Programming.* Addison-Wesley, Reading, MA, 1968–Present.

[3] Donald E. Knuth. *Digital Typography.* Center for the Study of Languages and Information, Stanford, CA, 1999.

[4] Akiva Leffert. The letter before lambda is hat: A reconstruction of Church's hat calculus. In *Proceedings of the 6th Binarennial ACH Conference in Celebration of Harry Q. Bovik's 0x40th Birthday (SIGBOVIK'07)*, Pittsburgh, PA, March 2007. ACH Press.

[5] John C. Reynolds. Towards a theory of type structure. In B. Robinet, editor, *Programming Symposium*, volume 19 of *Lecture Notes in Computer Science*, pages 408–425, Berlin, 1974. Springer-Verlag.

[6] John C. Reynolds. Types, abstraction and parametric polymorphism. In R. E. A. Mason, editor, *Information Processing 83*, pages 513–523, Amsterdam, 1983. Elsevier Science Publishers B. V. (North-Holland).

[7] Y. J. Ringard. Mustard watches: an integrated approach to time and food. Preprint, Université Paris VII, Paris, France, Octobre 1990.

[8] SIGBOVIK Program Committee. Discussion of how the Entscheidungsproblem is by far the coolest of Hilbert's 23 problems. Committee meeting, March 2007.

A Appendix: an incomplete waste of paper

Figure 10: “Mad dog” hitting the San Pellegrino.