Dynamic Time-Frequency Decompositions as Unique Fingerprints for Time Series Feature Extraction

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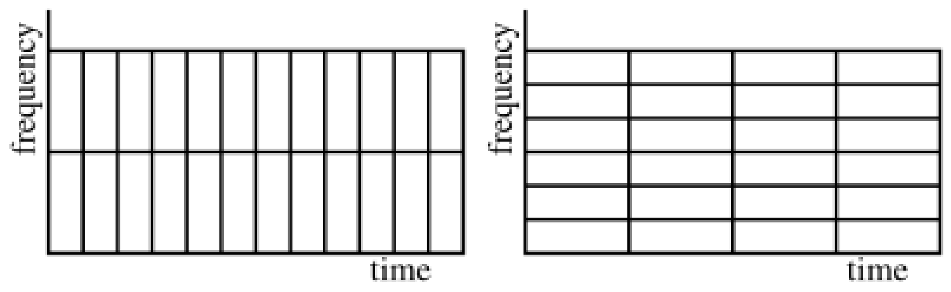
1 INTRODUCTION

Time-frequency decompositions for time-series analysis are lucrative but sensitive implementations, that come with significant compromises between time and frequency resolutions.

$$\sigma_x \cdot \sigma_p \geq \frac{h}{4\pi} \quad \Rightarrow \quad \Delta t \cdot \Delta f \geq \frac{1}{4\pi}$$

Mirroring the uncertainty principle from quantum physics, we are always given a trade-off between Fourier duals[1].

- Localization in one domain results in high spread in the other.
- Any *Short-Time Fourier Transform* is ineffective at tradeoff.
- Windowing function is static
- Fails to capture frequencies across large range of scales as they change rapidly.



AIM:

By efficiently employing the dynamic decomposition wavelet transform, we retain the vital time and frequency information in our data to practically and sufficiently identify key features of the signal. In this this example, the aim is to prove the method's efficacy by using the wavelet transform to categorize and name traditional Irish tunes.

OBJECTIVES:

1) Investigate techniques for deploying wavelet transforms.
2) Use wavelet coherence to pattern match recorded music against score data.

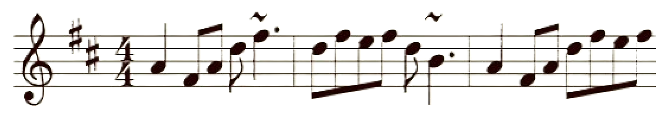
2 METHODS

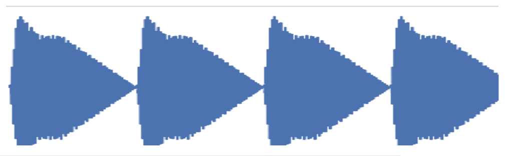
2.1. Data Description

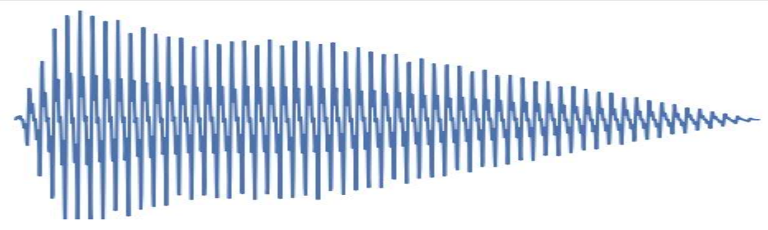
- Live recorded audio
- 8000 Hz sampling rate
- Instrumentation from fiddle, flute, concertina, lilting, and others

Score Sheets from TheSession.org

- ABC notation is saved in file
- Converted to frequency series, then constructed into waveform with harmonic profile of a piano



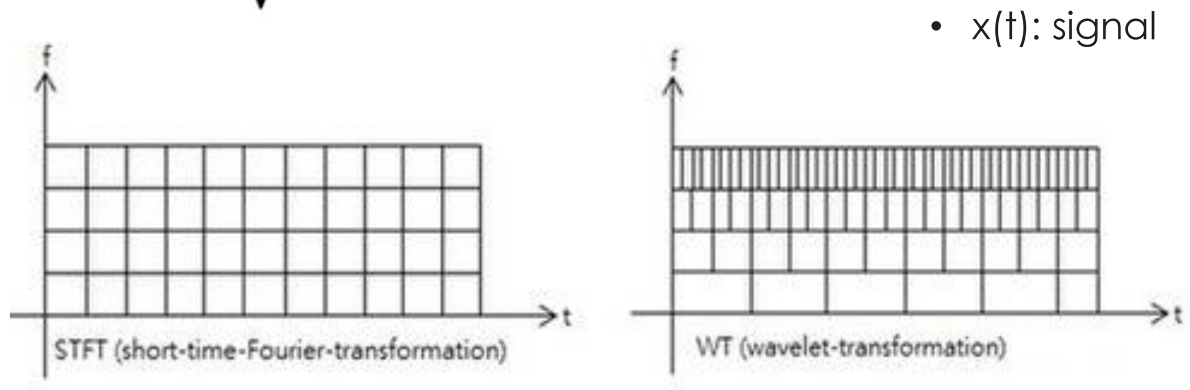




2.2. Continuous Wavelet Transform[3]

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\, \psi^* \frac{(t-b)}{a} dt$$

- a: location in frequency domain (scale)
- b: location in time domain
- Ψ*: scaled wavelet function
- x(t): signal



Model computes the integral using the *Fast Continuous Wavelet Transform* algorithm[2]

2.3. Wavelet Coherence[4]

$$Coh(x,y) = \frac{|S\left(T_{xy}(a,b)\right)|^2}{S(|T_x(a,b)|^2)S\left(|T_y(a,b)|^2\right)}$$

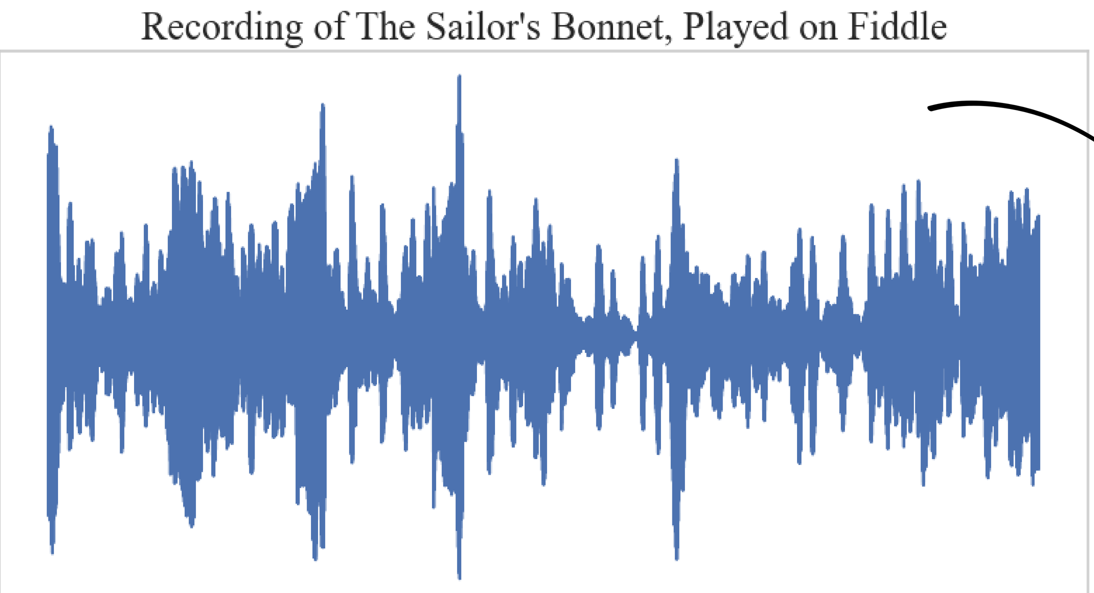
$T_{xy}(a,b) = T_x(a,b) * \bar{T}_y(a,b)$ is the cross wavelet transform

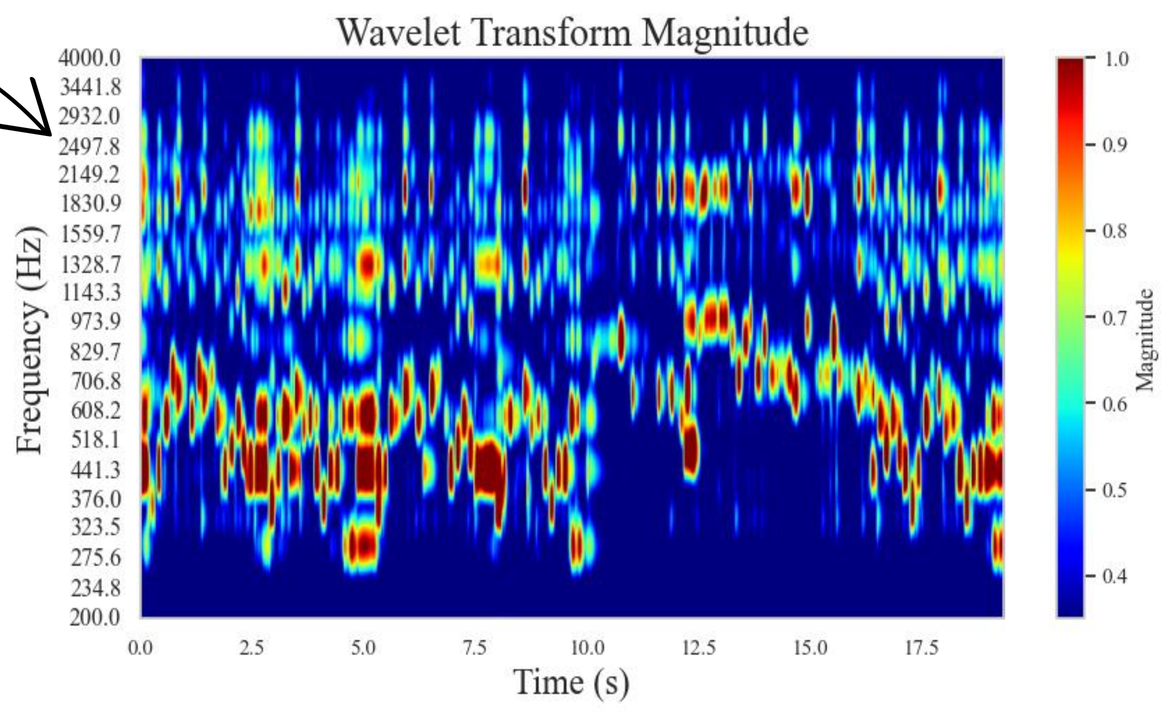
$S(\cdot)$ represents a personalized smoothing operator

3 RESULTS

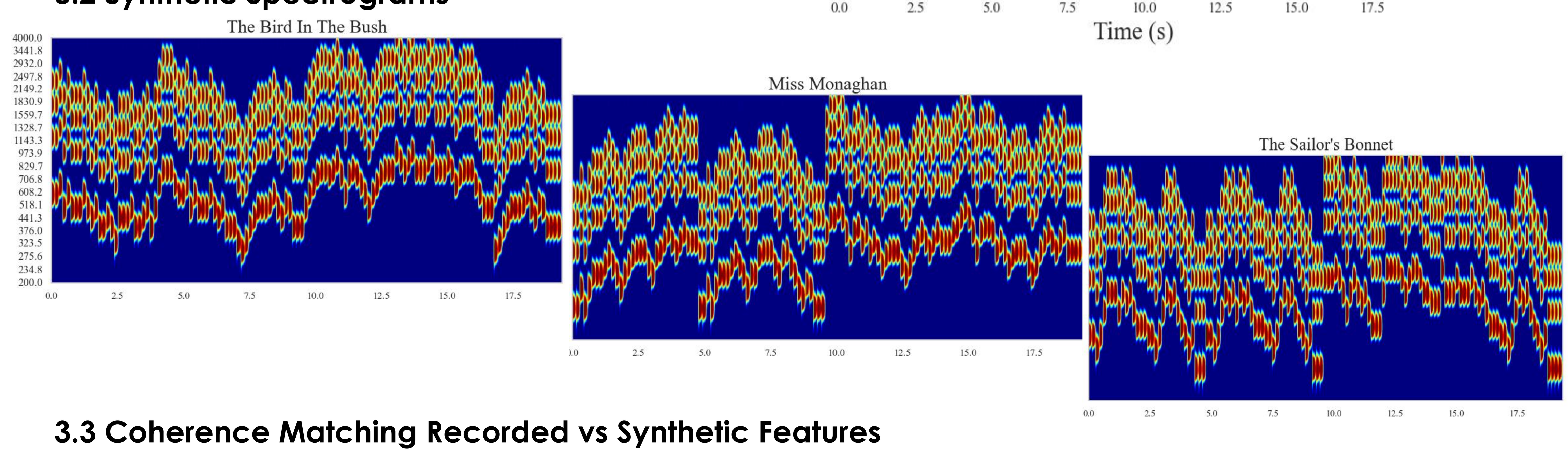
3.1. Continuous Wavelet Transform of Recording

Recording of The Sailor's Bonnet, Played on Fiddle

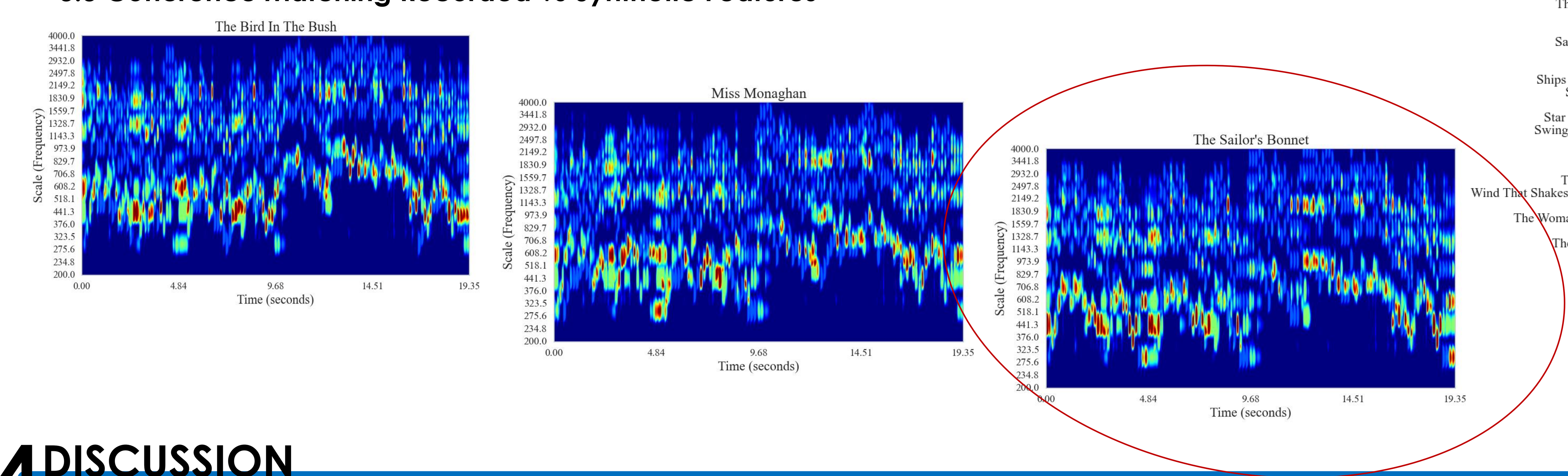




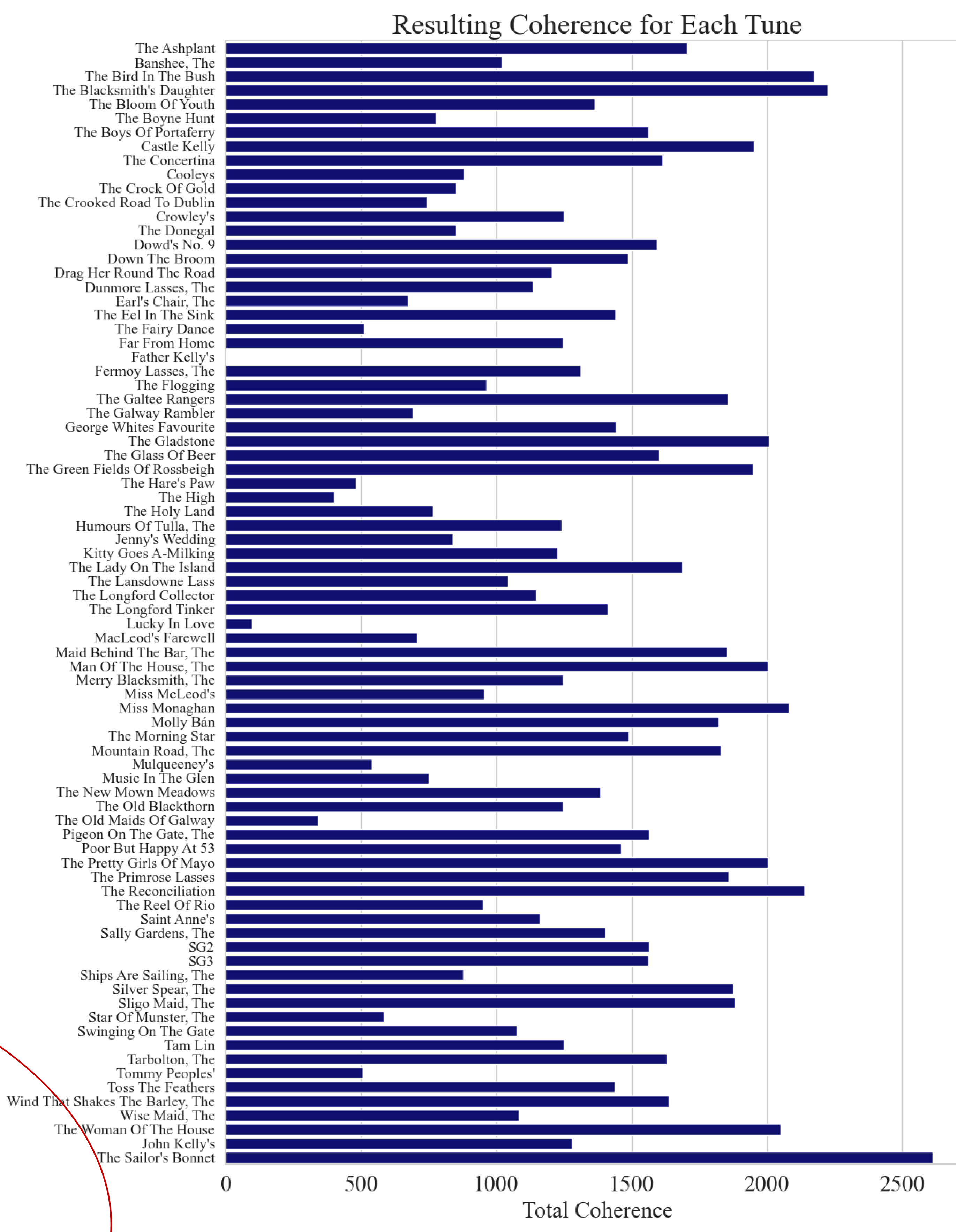
3.2 Synthetic Spectrograms



3.3 Coherence Matching Recorded vs Synthetic Features



3.4 Identification



4 DISCUSSION

Without the deployment of deep learning or using the internet to speed up compute time, we can reliably use the model to decide what tune is being played.

- Irish reels are used a vessel to showcase because they are all equal in length and can be easily represented symbolically.
- Coherence models like this one can also be used to fetch the transform for EEG or stock market data.
- The database of reels were selected from Mc Gettrick's *Estimations of Kolmogorov Complexity in Irish Music*[5] paper.
  - A slight inverse correlation was observed between the likelihood of prediction and the tune's *Kolmogorov Complexity* estimate.
- The harmonic profile of the instrument influences results as well:
  - Flute and Low whistle appear to generate less overtones, making the spectrogram cleaner for these instruments than the corresponding fiddle spectrogram.
  - This in turn provides more decisive results for the wind instruments.

- This model, as do most CWT models, uses the *Morlet Wavelet* to compute the wavelet transform.

$$\Psi(t) = \frac{1}{\sqrt{\pi f_0}} e^{2\pi i f_0 t} e^{-t^2}$$

f_0 is the wavelet's central frequency

Limitations:

1. Unequal frequency resolution: as the scale increases, the discernability becomes worse. The coherence model uses log scaling on the y axis, smushing all the higher frequencies together.
2. Data must be preprocessed so that the synthetic and recorded transforms contain information about corresponding time-series.

CONCLUSION:

This model showcases the wavelet transform's ability to serve as a unique fingerprint for time-series data while remaining computationally efficient. Its robustness in capturing both time-localized and frequency-dependent features makes it a powerful tool for pattern recognition, classification, and anomaly detection. Future work could explore optimizing the transform and extending its application to broader domains.

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