Assignment 01 Answer

January 1, 2024

1 TTK4225 Assignment 1

Instructions:

- submit not the file as a .ipynb, but rather as a .pdf (thus export the notebook accordingly before submitting it).
- if you have been doing this assignment together with somebody, then add a markdown cell after this one listing with whom you did this assignment.

1.0.1 1-1 Compute the equilibria of the mass-spring system, Lotka Volterra system, and the Van der Pol oscillator as functions of the parameters of the models;

Van der Pol oscillator
Mass-spring-damper
Lotka Volterra

Answer below :)

Van der Pol oscillator:

$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$
$\ddot{x} = \mu(1 - x^2)\dot{x} - x$

The equilibria is:

$\ddot{x} = 0$
$\dot{x} = 0$

$0 = 0 - x$
$x = 0$

Spring mass system:

$m\ddot{x} = -kx - c\dot{x} + F_{external}$

$\ddot{x} = -\frac{k}{m}x - \frac{c}{m}\dot{x} + \frac{F_{external}}{m}$

Equilibria:

$\ddot{x} = 0$
$\dot{x} = 0$

$0 = -\frac{k}{m}x + \frac{F_{external}}{m}$
$x = \frac{F_{external}}{k}$

Lotka-Volterra system:

$\dot{x} = \alpha x - \beta xy$
$\dot{y} = \delta xy - \gamma y$

Equilibrium:

$\dot{x} = 0$
$\dot{y} = 0$

$0 = \alpha x - \beta xy$
$\alpha x = \beta xy$
$y = \frac{\alpha}{\beta}$

$0 = \delta xy - \gamma y$
$\delta x = \gamma$
$x = \frac{\gamma}{\delta}$

Equilibrium is at: $(\frac{\gamma}{\delta}, \frac{\alpha}{\beta})$

[ ]:
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def van_der_pol_oscilator(x, dx, ddx, mu):
    return 0

def mass_spring_damper(x, dx, ddx, epsilon, k, c):
    # Assuming an external force of zero
    return 0/k

def lotka_volterra(x, dx, ddx, alpha, beta, gamma, delta):
    return [
        gamma/delta,
        alpha/beta,
    ]
```

1.0.2 1-2 Say which of these equilibria are simply stable, asymptotically stable, or unstable (note: simply stable is a synonym for marginally stable)

Answer below :)

- Lotka volterra is unstable because with no predators the population of prey will never stop growing
- Van der Pol is unstable because the system converges on the same trajectory regardless of inputs, meaning it is impossible to constrain the output to specific operating conditions by constraining the input.
- Spring mass system can be asymptotically stable if damped.

1.0.3 1-3 as for the asymptotically stable ones, describe in an intuitive way their basin of attraction.

Answer below :)

The basin of attraction is a region where the states tent to converge on the equilibrium point. For a damped spring mass system, the system constantly looses energy and will eventually stop moving.

1.0.4 2-1 Describe a physical system that has both an asymptotically stable equilibrium and an unstable one. Say why the equilibria are such that (hint: try to identify some sort of "energy" concept that can be used to characterize the system, e.g., kinetic or potential);

Answer below :)

A pendulum at rest has a maximum energy state when it points up and a minimum energy state when pointing down. A pendulum with friction, not starting in its unstable equilibrium (straight up), will always come to rest at its stable (convergent) equilibrium (straight down).

1.0.5 2-2 Say how one should modify the system so to make the asymptotically stable equilibrium only simply stable (note: simply stable is a synonym for marginally stable).

Answer here :)

By removing any friction from the pendulum it will, when not starting at an equilibrium, never converge on any point but rather oscillate forever. This lack of convergence makes the system marginally stable.

1.0.6 3-1 Describe how model predictive control works from intuitive perspectives;

Answer below :)

Lecture 028 Introduction to MPC

The concept of MPC is to use a mathematical model of a system to predict the future states of the system, and then control the inputs based on this information.

1.0.7 3-2 Describe the goal of this course.

Answer below :)

Identify the stability properties of LTI systems