Assignment 07 Answer

søndag 31. desember 2023 20:27

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Eigenvalues:

$$0 = \det(sI - A) = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \right| = \begin{vmatrix} s-2 & -5 \\ -1 & s+2 \end{vmatrix} = (s-2)(s+2) - 5 = (s^2 + 2s - 2s - 4) - 5$$

$$0 = s^2 - 4 - 5$$
$$s^2 = 9$$
$$s = \pm\sqrt{9}$$
$$s = \pm 3$$

$$\lambda_1 = 3 \quad \lambda_2 = -3$$

Eigenspace $\lambda_1 = 3$:

$$Av = \lambda_1 v$$
$$(A - \lambda_1 I)v = 0$$

For non-zero v:

$$A - \lambda_1 I = 0$$

$$\begin{bmatrix} 2-3 & 5 \\ 1 & -2-3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 1 & -5 \end{bmatrix} = 0$$

$$-V_1 + 5V_2 = 0$$
$$V_1 = 5V_2$$

Picking $V_2 = 1$

$$V_1 = 5(1) = 5$$

Verify: $(-1)5 + 5 \cdot 1 = 0$ OK!
$1 \cdot 5 - 5 \cdot 1 = 0$

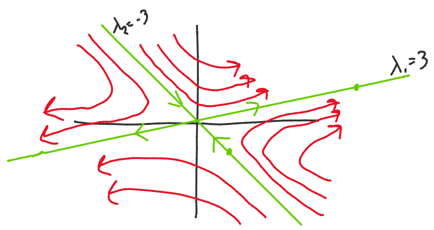
$$V_{\lambda_1 = 3} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Eigenspace $\lambda_2 = -3$:

$$\begin{bmatrix} 2-(-3) & 5 \\ 1 & -2-(-3) \end{bmatrix} = \begin{matrix} 5 & 5 \\ 1 & 1 \end{matrix}$$

$$5V_1 + 5V_2 = 0$$
$$V_1 = -V_2$$

$$V_{\lambda_2 = -3} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



The free evolution of the system converges on the origin along
The second eigenvector and diverges from the origin

Along the second eigenvector.
The system is asymptotically stable for all points that fall within
The span of the eigenspace of the first eigenvalue, and unstable
For all points that do not.

From a control perspective this is useful because it conveis
Information about what states are safe and what states are unsafe.

$$0 = \det(sI - A) = \underbrace{\begin{vmatrix} s+2 & -1 \\ 0 & s+2 \end{vmatrix}}_{\text{Linearly dependint?}} = (s+2)(s+2) = (s+2)^2$$

$\lambda = -2$

$(A - \lambda I)v = 0$

$$\begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -2+2 & 1 \\ 0 & -2+2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$

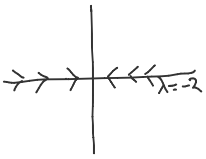
Only one non-zero row exists so only one eigenvector can be found.

$0v_1 + v_2 = 0$

$v_2 = 0$

$v_1 = R$ picking 1

Eigenvector: $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



Due to the columns being linearly dependent, we only have one
Eigenvector, meaning the span we can characterize using the previous
Method is one dimensional.

Using this method we can only say that the free evolution of the
system is convergent to the origin when y=0.

The stability properties of the origin depend on the kernel(A):

$Ax = 0$

$$\begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 = 0$

$-2x_2 = 0$

$\boxed{x_2 = 0}$

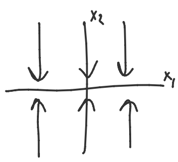
$-2x_1 + 0 = 0$

$\boxed{x_1 = 0}$

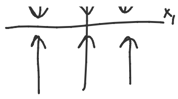
The kernel of A is zero dimensional, meaning the origin is an isolated
Equilibrium.

$\dot{x}_2 = -2x_2 \rightarrow$ convergent

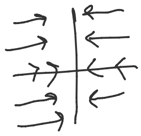
We know from the eiginspace



x_2 [illegible] convergent



We know from the eiginspace that x_1 is also convergent:



Based on this information, the system is asymptotically stable in the origin.