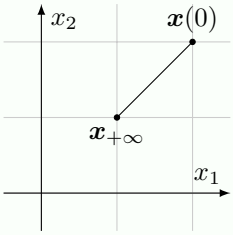
TTK4225 System theory, Autumn 2023

Assignment 6

The expected output is a .pdf written in LaTeX or a Python notebook exported to .pdf, even if photos of your handwritten notes or drawings will work. Every person shall hand in her/his assignment, independently of whether it has been done together with others. When dealing with mathematical derivations, unless otherwise stated, explain how you got your answer (tips: use programming aids like Python, Matlab, Maple, or compendia like Rottmann's to check if you have obtained the right answer).

Question 1

Consider the following trajectory, which starts from $\boldsymbol{x}(0)$ and asymptotically reaches $\boldsymbol{x}_{+\infty}$ (i.e., the system converges for $t \to +\infty$ to that point). Could this trajectory correspond to a free evolution of a linear time invariant state space system of dimension 2? Motivate the answer.



Question 2

The rank-nullity theorem is a central theorem in linear algebra, and states the following:

$$\text{rank}(A) + \dim(\ker(\text{A})) = y$$

where y is the number of columns in A.

Given a square matrix $A \in \text{R}^{2\times 2}$ and its eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 2$. What is its rank?

Question 3

Consider a generic $\mathbb{R}^{3\times 2}$ matrix.

1. How may one interpret it?
2. How may one interpret its range?
3. What is the usefulness of the range from control perspectives when analysing LTI systems?

Aid all your explanations through opportune drawings.

Question 4

Find, in the simplest way possible, the inverse of

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 3 & 2 & 1 \\ 0 & 5 & 4 \end{bmatrix} .$$

Be creative!

Question 5

Consider a generic $\mathbb{R}^{3\times 3}$ matrix.

1. How may one interpret its kernel?
2. What is the usefulness of the kernel from control perspectives when analysing LTI systems?
3. How may one interpret its determinant?
4. What is the usefulness of the determinant from control perspectives when analysing LTI systems?

Aid all your explanations through opportune drawings.