TTK4225 System theory, Autumn 2023

Assignments

The expected output is a .pdf written in LaTeX or a Python notebook exported to .pdf, even if photos of your handwritten notes or drawings will work. Every person shall hand in her/his assignment, independently of whether it has been done together with others. When dealing with mathematical derivations, unless otherwise stated, explain how you got your answer (tips: use programming aids like Python, Matlab, Maple, or compendia like Rottmann's to check if you have obtained the right answer).

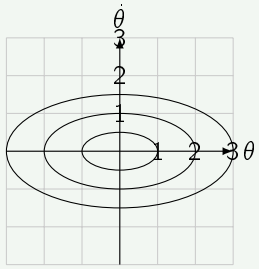
0.1 Assignment 4

Q1

Design a system, and write it as an ODE (even in discrete time, if you prefer), for which the origin is a convergent equilibrium but **not** a marginally stable equilibrium.

Q2

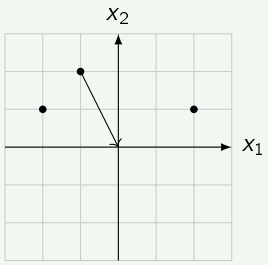
Consider the following phase portrait, corresponding to the trajectories of a pendulum without friction,



and the fact that the origin is a marginally stable equilibrium of the system. Considering the definition of marginal stability based on "ε, δ", what is the largest δ that can be considered by taking $\varepsilon = 2$?

Q3

Consider an autonomous system of the second order for which, through three distinct experiments, it was discovered that: *a*) the point $(-2, 1)$ is an equilibrium; *b*) also the point $(2, 1)$ is an equilibrium; *c*) the point $(-1, 2)$ is not an equilibrium. Indeed starting from that initial condition the evolved following a straight path that converges to the origin. Can the system be LTI? Why?



Q4

Consider a state space LTI system $\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$. Under what condition on A does such a system admit isolated equilibria? And how many isolated equilibria are admissible?

Q5

Consider the continuous-time autonomous system described as in the figure alongside, and consider the initial condition $y_0 = 1.9$. Where will the trajectory converge? And how long will it take to converge to a 0.1-neighborhood of such convergence point?