Assignment 03 Answer

January 1, 2024

1 TTK4225 Assignment 3

1.0.1 Question 1

After a nice evening in samfundet, you and your friend Tontolonius got a brilliant idea about how to build a new type of discombobulator, a gadget for the next iPhone 54, that will make you both ultra rich.

The first step is though to model its input output behavior, and while walking to Omega verksted to buy the stuff for building the discombobulator your friend says:

I think I understood how it will behave - it has to be a linear time-invariant homeogeneous first order partial differential equation. The process shall moreover have a constant forcing term of value 2 from time $t = 0$ on. If we say that $\mu(\cdot)$ is the step function, i.e., $\mu(t-\tau) = 0$ for $t < \tau$ and 1 otherwise, then the model we are looking for has to be

$$\ddot{y} = -\dot{y}t^2 + 2\sqrt{y} + 2\mu(t-1).$$

You are though a bit suspicious that Tontolonius drank a couple of beers more than enough, and fear the sentence above comes with some mistakes.

Write in the cell below, using it as a `markdown` cell, all the things are wrong in the sentence in italics assuming the ODE above is actually the correct one.

- It is not linear
- It is not time invarient
- Because of the step function, it is not homogenous
- It is not first order
- It contains no partial derivatives
- The step function activates at t=1, not t=0

1.0.2 Question 2

Now you just finished building the discombombulator, and want to characterize it. You start running an experiment, give it a Dirac delta (i.e., an impulse) as an imput, and get the following response:

Looking at the response, Tontolonius says "We can obviously model this as a first order scalar system". Is this the case? Why / Why not?

Write your thoughts in the cell below, using it as a `markdown` cell.

A first order system does not oscillate.

1.0.3 Question 3

You both want to understand if the discombombulator is fast enough in responding to iPhone 54 users' inputs.

How would you determine the time constant of the discombombulator starting from the impulse response above? And would you say it is 'X' seconds, if you were given only the graph above and no other information?

Write your thoughts in the cell below, using it as a `markdown` cell.

The exponential decay of the process is given by $e^{-\tau t}$. The maximum amplitude of the sine wave is determined at any given point by the state of the exponential decay. The time constant refers to the time required for the exponential decay to reach 38.8% of its initial value.

Looking at the graph above, the system appears to reach 0.388 at around $t = 1s$.

1.0.4 Question 4

You also fear the discombombulator may not be so innovative after all, after you notice that the impulse response seems oddly familiar.

Which examples of physical systems may produce such an impulse response? And how would changing their parameters change this response?

Write your thoughts in the cell below, using it as a `markdown` cell.

This system matches the impulse response of a harmonic oscillator. The parameters of the system would change the amplitude, frequency and decay of the oscillation.

An underdamped spring-mass system would have a similar response.

1.0.5 Question 5

Arrgh! Tontolonius forgot to plug all the cables in… You need to perform the test again, you after doing it you get a slightly more complicated impulse response.

What is the lowest possible order of a system that has an impulse response as this one? Can you tell which components that make up the impulse response $h(t)$? (hint: try to decompose it as a $h(t) = h_1(t) + h_2(t)$)

Write your thoughts in the cell below, using it as a `markdown` cell.

This is likely a system that matches the following pattern in the time domain:

$e^{-\tau t} + e^{-\tau t} \sin(\omega t + \phi)$

This would make it at least a third order system.

1.0.6 Question 6

Eventually you think that the discombombulator is too complex as it is thought right now, and decide to make a new version of it.

You also understand that this simplified version follows the dynamics of a cart with mass $m = 1$ kg, speed $x(t)$, a generic initial condition $x_0 = x(0)$, is affected by a friction $F_f(t) = -fx(t)$, and subject to an external commandable force $u(t) = F(t)$.

Derive a model that describes the dynamics of such a system. If you are solving this point via pen and paper, then write the procedure you followed to compute the solution in the cell below, using it as a `markdown` cell, and the solution itself. Otherwise write down the derivations directly below.

$\sum F = ma$
$F_f = -fx$
$a = \dot{x}$
$F = u$

$F + F_f = ma$
$F - fx = m\dot{x}$

$\dot{x} = -\frac{f}{m}x + \frac{1}{m}F$

$\dot{x} = -\frac{f}{m}x + \frac{1}{m}u$

$\dot{x}(t) = -\frac{f}{m}x(t) + \frac{1}{m}u(t)$

1.0.7 Question 7

Assuming F(t) = 0, how may one compute the trajectory of such a system? What is the name of the corresponding evolution? And what is the corresponding solution in this case?

Write your thoughts in the cell below, using it as a `markdown` cell.

For $u(t) = F(t) = 0$:

$\dot{x}(t) = -\frac{f}{m}x(t)$

When the input $u(t)$ is zero, the system is autonomous. This is called the free evolution. The solution is:

$x(t) = x_0 e^{-\frac{f}{m}t}$

1.0.8 Question 8

Write in the cell below a script that

- solves the dynamics of the system above for generic inputs $u(t)$, and
- plots the solution $x(t)$ for the specific case $f = 2$, $x_0 = 10$, and $u(t) = 0$ for $t < 3$, $u(t) = 1$ for $t \geq 3$.

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

m = 1
f = 2
x0 = 10
u = lambda t: 0 if t < 0 else 1
```

```python
def model(x, t):
    return -(f/m) * x + (1/m)*u(t)

t = np.linspace(0, 10, 1000)

x = odeint(model, x0, t)

plt.plot(t, x)
plt.show()
```