TTK4225 System theory, Autumn 2023

Assignment 5

The expected output is a .pdf written in LaTeX or a Python notebook exported to .pdf, even if photos of your handwritten notes or drawings will work. Every person shall hand in her/his assignment, independently of whether it has been done together with others. When dealing with mathematical derivations, unless otherwise stated, explain how you got your answer (tips: use programming aids like Python, Matlab, Maple, or compendia like Rottmann's to check if you have obtained the right answer).

Question 1

Explain from **both** graphical and mathematical perspectives why an harmonic oscillator (`https://en.wikipedia.org/wiki/Harmonic_oscillator`) is not a BIBO stable system.

Question 2

Find two first-order autonomous nonlinear dynamical systems (say, system S_1 and system S_2) that have both 0 as an equilibrium, and that for S_1 the origin is unstable, while for S_2 it is asymptotically stable, and for which when linearizing that two systems around 0 the origin becomes marginally stable for both the linearized versions of these systems.

Question 3

Assume that certain linear system has actually been obtained by linearizing an unknown nonlinear system around a given equilibrium. Assume the equilibrium for the linearized system to be marginally stable. Explain from **both** graphical and mathematical perspectives why given this information it is not possible to know which type of stability properties that equilibrium point has for the original nonlinear system.

Question 4

1. Consider the system
$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} .$$
Has the system a solution? If so, which one? If not, why?

2. Consider the system

$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Has the system a solution? If so, which one? If not, why?

Question 5

Compute the equilibria of the system $\dot{\boldsymbol{x}} = A\boldsymbol{x}$ with

$$A = \begin{bmatrix} J_{\lambda_1}^{(n_{1,1})} & & & & \\ & J_{\lambda_1}^{(n_{1,2})} & & & \\ & & \ddots & & \\ & & & J_{\lambda_2}^{(n_{2,1})} & \\ & & & & \ddots \end{bmatrix}$$

i.e., A block-diagonal with the various blocks being of the type

$$J_{\lambda_\star}^{(n_\star)} = \begin{bmatrix} \lambda_\star & 1 & & & \\ & \lambda_\star & 1 & & \\ & & \ddots & \ddots & \\ & & & \lambda_\star & 1 \\ & & & & \lambda_\star \end{bmatrix} \in C^{n_\star \times n_\star}$$