# TTK4225 System theory, Autumn 2023

# Assignment 9

The expected output is a .pdf written in LaTeX or a Python notebook exported to .pdf, even if photos of your handwritten notes or drawings will work. Every person shall hand in her/his assignment, independently of whether it has been done together with others. When dealing with mathematical derivations, unless otherwise stated, explain how you got your answer (tips: use programming aids like Python, Matlab, Maple, or compendia like Rottmann's to check if you have obtained the right answer).

**Question 1**

Consider the autonomous system

$$\begin{cases} \dot{\boldsymbol{x}} &= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x} \\ y &= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \boldsymbol{x} \end{cases}$$

Compute $y(T)$ for $T = 2$ assuming the initial condition for the system to be

$$\boldsymbol{x}_0 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

**Question 2**

Consider the system

$$\begin{cases} \dot{\boldsymbol{x}} &= \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \boldsymbol{x} \end{cases}$$

How may one do to compute its free evolution?

**Question 3**

Assume $A \in \mathbb{R}^{n \times n}$ to be so that its characteristic polynomial is

$$(s-5)^3 (s-4)^2$$

and its minimal polynomial, instead,

$$(s-5)^2(s-4).$$

Each element in the transition matrix $e^{At}$ will be then a combination of exponentials and exponentials multiplied by $t$ to some power, i.e.,

$$\left[e^{At}\right]_{ij} = \sum_k \alpha_k t^{(\beta_k)} e^{\lambda_k t}$$

where $i, j$ indicate the row and column of the element of the transition matrix. Which types of $t^(\beta_k) e^{\lambda_k t}$ do we expect to see in $e^{At}$? And which Jordan structure does $A$ have? And why?