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Hexapod Robot with Articulated Body

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Abstract
The paper describes kinematic control for hexapod robot with three segment articulated body. Forward and inverse kinematics for articulated body studied. Static stability studied in case of climbing so called cliff obstacle. Necessary and sufficient conditions for stability during climbing sequence provided.

Keywords: Multi legged robot, obstacle climbing, articulated body.

INTRODUCTION

Study of walking machines is rather old question. Starting in Ancient China [1] primitive mechanical designs and ending with state of the art reinforcement learning [2] and model predictive control [3] approaches. Multilleged walking robots are very complicated systems in terms of control and planning due to significant number of degrees of freedom (d.o.f.) and actuators, complexity of the environment and etc. Nowadays the most complex and robust walking machines are made by Boston Dynamics company[4]. Its robots are capable of working in human environment and outdoors. The basis of their solution is model predictive control and non-linear optimization. Their robots are based on the kinematics of mammals - dogs or horses. The kinematics of hexapod robots in turn is based on insects and spiders.

The main idea of the article – conventional mainstream hexapods have rigid body with legs attached symmetrically. This arrangement has its limitations but still can be surprisingly agile and overcomes complex obstacles as it was demonstrated in [5, 6]. Introducing additional degrees of freedom – making multiple rigid segments to be a body instead of one rigid body segment increases robot's geometric patency. In other words there is going to be trade off – the complexity of control will increase along with number of d.o.f. and actuators but the robot will climb up higher obstacles or do maneuvers that robots with rigid body are not capable of. For example, go through narrow passage, sharp ditch and windrows.

ROBOT KINEMATICS

Let us consider robot depicted on figure figure 1. It has six so-called insectomorphic legs, i.e. insect-like leg kinematics. Each leg has three degrees of freedom. Body consists of three rigid segments connected with hinges to each other. Body that consists of several segments connected to each other with controllable joints and can change its geometry is called articulated.

The total number of degrees of freedom (d.o.f.) for specified robot is 26:

- 3 d.o.f. for each leg, i.e. 18 d.o.f. for all legs
- 2 d.o.f. for body segments
- 6 d.o.f. for the whole system as one single body

Overall it is 26 d.o.f. and 20 of them can be controlled with an actuators installed in corresponding rotational joints. Leg kinematics is well known and was already studied in all details.

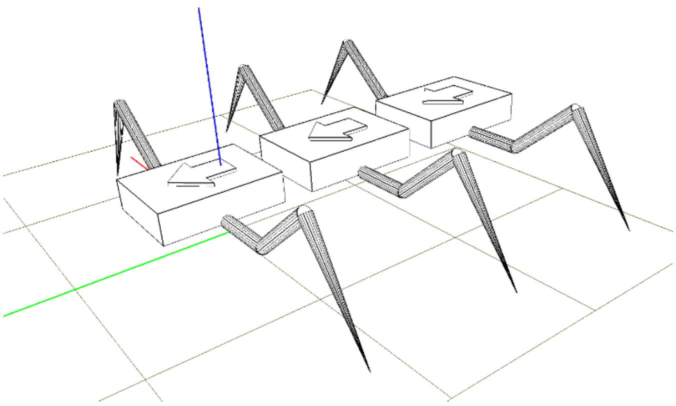

Figure 1. Hexapod robot with articulated body

CLIFF OBSTACLE

Cliff obstacle consists of three planes two of which are horizontal and one is vertical as depicted on figure 2.

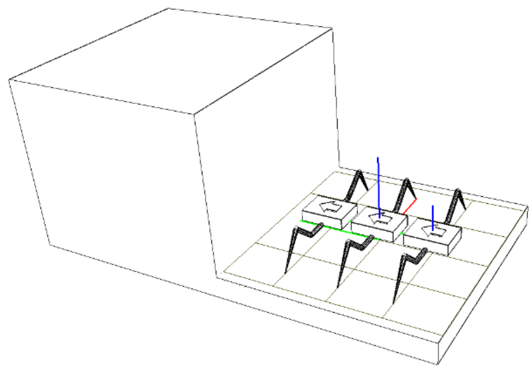

Figure 2. Cliff obstacle

The distance between two horizontal planes is equal to *H*. Robot starts from the lower horizontal plane and his goal is to climb up the higher horizontal plane using only the Coulomb friction.

To overcome cliff obstacle robot moves using so-called gallop gait when a pair of symmetrical legs from left and right sides of the robot are in transition state and the others are in support state, i.e. in every moment of time there are four legs in contact with obstacle. Body kinematics will be considered in next section.

BODY KINEMATICS

Robot's body consists of three equal rigid segments connected to each other with rotational joints with axes aligned in lateral direction of the body. Each segment has a pair of legs connected. Additional joints in the robots body allow to bend body and follow the surface shape and shift legs mounting points towards to supporting surface.

The following procedure is defined to calculated body joint angles. Initial and goal poses for middle segment are connected with a cubic spline curve, which represents target trajectory for middle segment. If segments and their trajectory are known then the taks is solved through simple linear approximation.

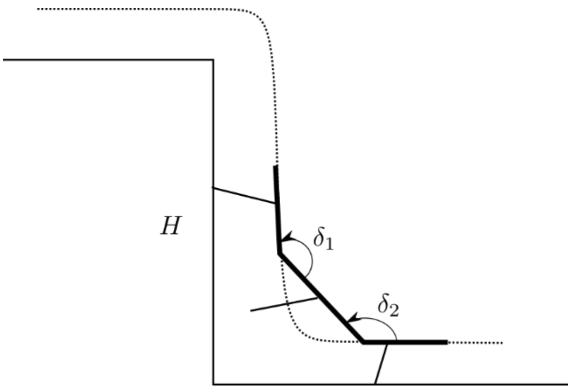

Figure 3. Articulated body kinematics

To keep the contact points on the goal trajectory all joints should act in a coordinated way. At every moment of time all joint coordinates must be updated to keep the end effector at the goal position. Additional mobility inside the robots body should be taken into account because all legs are connected to the different segments. Target point $\bar{R}_i$ for *i*-th leg is given in global reference frame. To obtain leg joint angles the inverse kinematic equations are used, point $\bar{R}_i$ must be translated into leg's reference frame. To manage all relative coordinate transformations of shifts and rotations between body segments, legs and joints, homogenous coordinates are used. Calculation of all coordinate transformation for each leg at every moment of time can be easily done automatically through well-known kinematics of a robot.

The main differences of articulated body from single segment body are:

- Higher ability to overcome obstacles – segments follow the surface;
- Articulated body is able to shift mounting points of its legs – service region is not constant, i.e. in some conditions robot can reach contact surface and put legs on it;
- Center of gravity is shifted in a wider range with all else parameters being equal – critical parameter in static stability preservation in extreme conditions.

STATIC STABILITY

The system is stable when sums of all external forces and all momentums are equal to zero.

$$\begin{cases} \sum_{i=1}^{N} \bar{R}_i + \bar{P} = 0 \\ \sum_{i=1}^{N} [\bar{r}_i \times \bar{R}_i] + [\bar{r}_c \times \bar{P}] = 0 \end{cases} \qquad (1)$$

The following configurations of supporting legs displacement should be studied for static stability:

- All legs on some horizontal plane. This case is already well studied in ;
- Front legs lean against the vertical plane and rear legs stand on the lower horizontal plane. Let us numerate this configuration as Number One.
- Front legs are placed at the upper horizontal plane, while the rear legs stand on the vertical plane. Let us numerate this configuration as Number Two.
- All legs stand on the upper horizontal plane – this case is similar to the initial one.

Considering the robot as a slow moving system at every moment of time let us find necessary conditions for static stability.

Reference frame *Oxyz* is defined as depicted on the figure 4.

Equations for first configuration

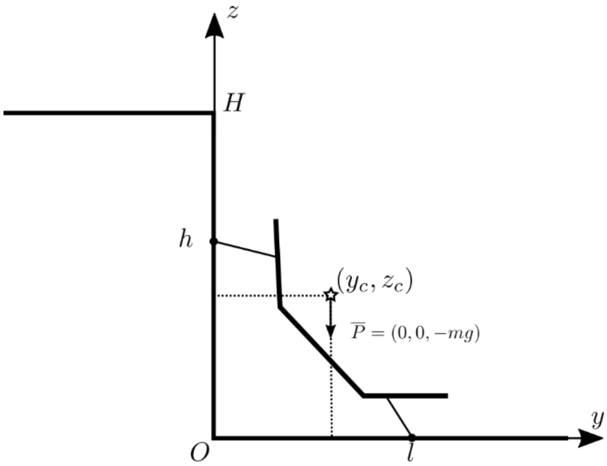

Figure 4. First static configuration

Contact points of the legs for first configuration are as follows:

$$\begin{aligned} \bar{r}_1 &= (d, 0, h), \\ \bar{r}_2 &= (-d, 0, h), \\ \bar{r}_3 &= (d, l, 0), \\ \bar{r}_4 &= (-d, l, 0). \end{aligned} \qquad (2)$$

There is a reaction $\bar{R}_i$ acting on the robot leg at each contact point:

$$\bar{R}_i = N_i \cdot \bar{n}_i + F_\tau^i \cdot \bar{\tau}_i + F_\nu^i \cdot \bar{\nu}_i \qquad (3)$$

The $\bar{\tau}_i$ and $\bar{\nu}_i$ vectors have the following coordinates:

$$\begin{aligned} &\bar{v}_1 = (1,0,0),\ \bar{\tau}_1 = (0,0,1),\\ &\bar{v}_2 = (1,0,0),\ \bar{\tau}_2 = (0,0,1),\\ &\bar{v}_3 = (1,0,0),\ \bar{\tau}_3 = (0,1,0),\\ &\bar{v}_4 = (1,0,0),\ \bar{\tau}_4 = (0,1,0). \end{aligned} \tag{4}$$

The center of gravity has coordinates:

$$\bar{r}_c = (0, y_c, z_c) \tag{5}$$

The gravity force $\bar{P}$ acts on the center of gravity of the robot:

$$\bar{P} = (0,0,-P) \tag{6}$$

The equations of static stability for first configuration are as follows:

$$\begin{cases} F_v^1 + F_v^2 + F_v^3 + F_v^4 = 0 \\ N_1 + N_2 + F_\tau^3 + F_\tau^4 = 0 \\ F_\tau^1 + F_\tau^2 + N_3 + N_4 = P \\ (N_1 + N_2)h + Py_c = l(N_3 + N_4) \\ d(F_\tau^1 - F_\tau^2 + N_3 - N_4) = h(F_v^1 + F_v^2) \\ d(N_1 - N_2 + F_\tau^3 - F_\tau^4) = l(F_v^3 + F_v^4) \end{cases} \tag{7}$$

The total number of equation is six. The number of unknown variables is twelve. Let us assume that the friction forces are modelled with Coulomb mathematical model:

$F_i = kN_i$, where k is coefficient of friction and $0 < k < 1$.

After substitution of the Coulomb friction model, the equations will transform into the following system:

$$\begin{cases} N_1 k_v^1 + N_2 k_v^2 + N_3 k_v^3 + N_4 k_v^4 = 0 \\ N_1 + N_2 + N_3 k_\tau^3 + N_4 k_\tau^4 = 0 \\ N_1 k_\tau^1 + N_2 k_\tau^2 + N_3 + N_4 = P \\ (N_1 + N_2)h + Py_c = l(N_3 + N_4) \\ d(N_1 k_\tau^1 - N_2 k_\tau^2 + N_3 - N_4) = h(N_1 k_v^1 + N_2 k_v^2) \\ d(N_1 - N_2 + N_3 k_\tau^3 - N_4 k_\tau^4) = l(N_3 k_v^3 + N_4 k_v^4) \end{cases} \tag{8}$$

The number of unknowns variables remains the same, and besides $N_i > 0$. Let us introduce additional assumptions that the left and the right side of the robot are loaded equally and coefficients of friction are the same between left and right legs:

$$\begin{cases} k_v^1 = -k_v^2 = k_v \\ k_v^3 = -k_v^4 = k_v \\ k_\tau^1 = k_\tau^2 = k_\tau^u \\ k_\tau^3 = k_\tau^4 = k_\tau^d \\ N_1 = N_2 = N_u \\ N_3 = N_4 = N_d \end{cases} \tag{9}$$

Finally, the system of three equations and four variables obtained:

$$\begin{cases} N_u + N_d k_\tau^d = 0 \\ 2N_d + 2N_u k_\tau^u = P \\ 2hN_u + Py_c = 2lN_d \end{cases} \tag{10}$$

Number of unknowns is still greater than number of equations. One more assumption must be introduced:

$$k_\tau^u = -k_\tau^d = k > 0 \tag{11}$$

Finally, the system of three equations and three unknowns obtained:

$$\begin{cases} N_u - N_d k = 0 \\ 2N_d + 2N_u k = P \\ 2hN_u + Py_c = 2lN_d \end{cases} \tag{12}$$

Let us find unknown reactions N_u, N_d and k. From first and second equations of (12) follows:

$$\begin{aligned} N_d &= \frac{P}{2(1+k^2)} \\ N_u &= \frac{kP}{2(1+k^2)} \end{aligned} \tag{13}$$

After substituting (13) to the third equation of (12) we have quadratic equation relative to k:

$$y_c k^2 + hk + (y_c - l) = 0 \tag{14}$$

There are two solutions for quadratic equation (14):

$$k_{1,2} = -\frac{h \pm \sqrt{h^2 - 4y_c^2 + 4ly_c}}{2y_c} \tag{15}$$

To make robot able to climb up the cliff without any hooks and adhesive forces the friction coefficient k must satisfy the condition $0 < k < 1$. It is easy to show that only solution k_1 fulfill the requirements of the accounted configuration:

$$k_1 = -\frac{h - \sqrt{h^2 - 4y_c^2 + 4ly_c}}{2y_c} > 0 \tag{16}$$

meets the requirement $k > 0$. The k_2 solution is always less than zero. Let us study when the $k < 1$ condition is fulfilled for k_1. To study this three dimensional parametric space the following dimensionless parameters will be introduced.

Dimensionless parameters

It is easy to see that the expression (16) for k depends on l, h and y_c parameters that are measured in meters – they all have the same physical dimension. Let us use this circumstance and define the following dimensionless parameters:

$$\left. \begin{aligned} p_1 &:= \frac{h}{y_c}, \\ p_2 &:= \frac{l}{y_c}. \end{aligned} \right., where\ y_c \neq 0 \tag{17}$$

After substitution of (17) into the expression (16) for k_1 we get the following inequality:

$$0 < -\frac{1}{2}p_1 + \sqrt{\frac{1}{4}p_1^2 + p_2 - 1} < 1 \qquad (18)$$

Inequality is equivalent to the following system:

$$\begin{cases} p_1^2 + 4p_2 - 4 \geq 0 \\ p_2 - 2 - p_1 < 0 \end{cases}, where\ p_1 > 0\ and\ p_2 > 0 \qquad (19)$$

The solution of system (19) is depicted on figure 5.

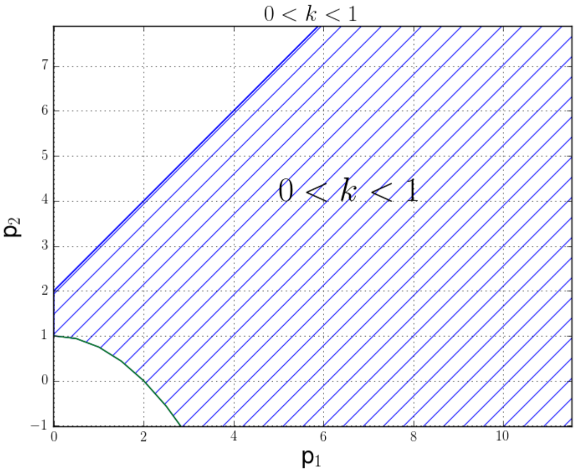

Figure 5. Solution for first configuration

From figure 5 it becomes clear that if there is lack of friction in contact points the robot should:

- move its center of gravity closer to the rear legs;
- choose contact points higher on the vertical plane for front legs;
- choose contact points closer to vertical plane for rear legs.

Equations for second configuration

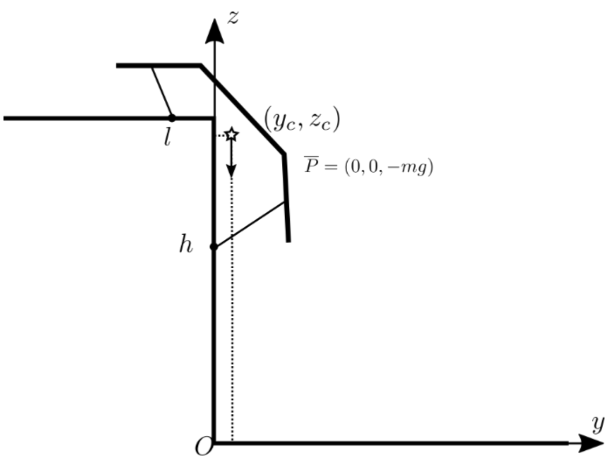

Figure 6. Second static configuration

Leg contact points for the second configuration are:

$$\begin{aligned} \bar{r}_1 &= (d, l, H), \\ \bar{r}_2 &= (-d, l, H), \\ \bar{r}_3 &= (d, 0, h), \\ \bar{r}_4 &= (-d, 0, h). \end{aligned}, where\ l < 0, H < h, y_c > l \qquad (20)$$

Similarly, for the second configuration we get the following system of three equations:

$$\begin{cases} N_u - N_d k = 0 \\ 2N_u + 2N_d k = P \\ 2N_d h + P y_c = 2N_u(l + Hk) \end{cases} \qquad (21)$$

There are two possible solutions for k:

$$\begin{aligned} k_1 &= \frac{(H-h) + \sqrt{(H-h)^2 - 4y_c^2 + 4ly_c}}{2y_c} \\ k_2 &= \frac{(H-h) - \sqrt{(H-h)^2 - 4y_c^2 + 4ly_c}}{2y_c} \end{aligned} \qquad (22)$$

It is easy to see that this time y_c can change its sign because of center of gravity transition above the upper edge of the cliff and because of that, the solution k_1 does not fulfill the $k_1 > 0$ requirement. On the other hand, solution k_2 meets the requirement $k_2 > 0$ for every $y_c > l$.
Let us find solution of inequality $k_2 < 1$ using the following dimensionless parameters:

$$\begin{aligned} p_1 &:= \frac{(H-h)}{y_c} \\ p_2 &:= \frac{l}{y_c} \end{aligned}, where\ y_c \neq 0 \qquad (23)$$

The solution for k_2 exists:

$$p_2 < 2 - p_1 \qquad (24)$$

Taking into account the domain of definition for square root from (22), the solution for inequality (24) in dimensionless space is depicted on the following figure:

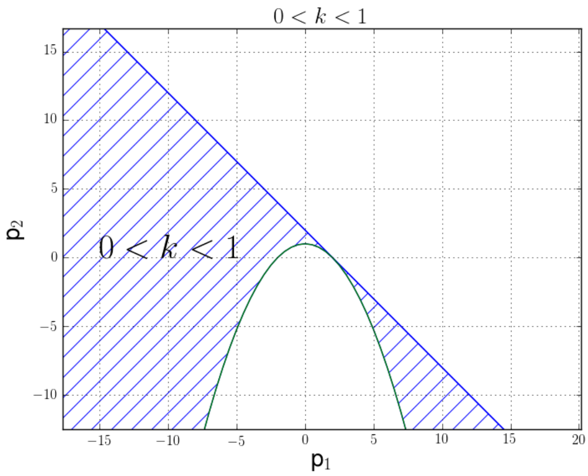

Figure 7. Solution for second configuration

From figure (7) it can be shown that to reduce the value of k the robot should:

- keep its center of gravity closer to front legs;
- keep rear legs as high~~er~~ as possible;
- keep front legs farther from edge of cliff.

We have considered the second configuration in assumption that $y_c \neq 0$. Let us see what happens when center of gravity is right above the cliffs edge in second configuration.

Second configuration. y_c equals to zero

If the $y_c = 0$ the equations of static stability equations (21) will transform into the following system:

$$\begin{cases} N_d - N_u k = 0 \\ 2N_u + 2N_d k = P \\ N_d h = N_u (l + Hk) \end{cases} \qquad (25)$$

There is only one solution for k for system (25):

$$k = -\frac{l}{(H-h)} \qquad (26)$$

Due to conditions (20) the expression for k is always greater than zero. From the other side, the requirement $k < 1$ is equivalent to the following inequality:

$$0 < -l < (H-h), where\ l < 0 \qquad (27)$$

Inequality (27) means that to provide stable configuration in case when $y_c = 0$ the contact points should be chosen in a way, that the front legs should be closer to cliff edge than the rear legs.

Reactions N_u and N_d will have the following expressions:

$$\begin{aligned} N_u &= \frac{P(H-h)^2}{2((H-h)^2 + l^2)} \\ N_d &= -\frac{Pl(H-h)}{2((H-h)^2 + l^2)} \end{aligned} \qquad (28)$$

CONCLUSION

The analysis of the robot configurations in different poses on the cliff proved that stable quasi static motion is possible for all steps, i.e. the robot is capable of climbing the cliff with static stability preservation using only Coulomb friction forces.

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