Computers & Industrial Engineering 145 (2020) 106458

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A two-stage stochastic programming framework for evacuation planning in disaster responses

Li Wang

School of Modern Post, Beijing University of Posts and Telecommunications, China

ARTICLE INFO

Keywords:
Evacuation
Two-stage stochastic programming
Recourse
Min-cost flow
Lagrangian relaxation

ABSTRACT

According to the information asymmetry, this paper focuses on a two-stage stochastic programming model to evacuate the affected people to safe areas during disaster response. A min-cost flow formulation is presented to portray the evacuation process of affected people in an urban transportation network. Nevertheless, it is difficult to predict the disaster magnitude and its impact on the urban road system, and thus travel time and capacity on the link are assumed to be scenario-based random variables. Then, this paper proposes a two-stage stochastic programming model to achieve the robust *a priori* evacuation plan. For solution convenience, the two-stage stochastic programming model is further deduced to a single stage formulation. Finally, a Lagrangian relaxation-based heuristic approach is designed to solve the proposed model. The experimental results indicate that the algorithm is able to solve the problem of interest efficiently and effectively.

1. Introduction

Extreme natural disasters (earthquakes, storms, fire, hurricanes, etc.) and unnatural ones (terrorist attacks, political issues, war, etc.) may strike a community with little warning and leave much damage and many casualties. The main goal of emergency response is to provide shelter and assistance to affected people as soon as possible. To achieve this goal, some emergency response researches focus on how can people at safe areas to be supported with necessary relief items and/or how to determine the locations of resource centers (e.g., Davoodi & Goli, 2019; Liu, Cui, & Zhang, 2019; Rawls & Turnquist, 2010; Rennemo, Rø, Hvattum, & Tirado, 2014; Ni, Shu, & Song, 2018; Zhang, Liu, Yu, Ruan, & Chan, 2019). Nevertheless, some scholars are interested in investigating the explicit evacuation plan for affected people from dangerous areas to safe areas (Bretschneider & Kimms, 2011; Li, Li, & Claramunt, 2018; Wang, Yang, Gao, Li, & Zhou, 2016). This research belongs to the latter and aims to address the issue of planning the evacuation path for the effected people by formulating a general modeling framework in occurrence of the disaster. As is known to all, it is much difficult to estimate the impact and damage on the urban roads from the disaster (such as earthquake, hurricane,etc.) since it is almost impossible to know the intensity in advance. Therefore, this evacuation problem should be generally regarded as a stochastic programming problem where randomness not only arises from travel time but also capacity. In other words, the possible damage on certain road that may prevent the people escaping from disaster areas results in stochastic link travel times and capacities.

With the development of internet and modern information technology, people can usually obtain real-time road network information after unexpected events through various channels. Therefore, how to find the optimal evacuation path for affected people under the condition of real-time information availability is the problem of interest in this paper. To this end, this paper will further consider the availability of real-time information, and thus the evacuation process can be divided into two stages. In the first time stage, it is assumed that the affected people cannot obtain the information of disaster level and road damage degree; and after a certain time period, they can obtain the accurate road network information through some real-time monitoring equipment. To the best of our knowledge, stochastic programming with recourse are used to find non-anticipative decisions that must be taken priori to know the realizations of some random variables such that the total expected costs of possible recourse actions are minimized. On the basis of above analysis, the evacuation problem in this paper is formulated as a scenario-based two-stage stochastic programming model to represent the randomness arising from earthquake magnitude and impact.

In literature, many methodologies have been provided for dealing with uncertainty of evacuees, road capacity, link travel times and so on in formulating the evacuation mathematical programming model (Miller-Hooks & Sorrel, 2008; Ng & Waller, 2010; Ng & Lin, 2015; Wang, Yang, Gao, Li, et al., 2016; Yazici & Ozbay, 2010). For instance, Miller-Hooks and Sorrel (2008) used the stochastic probability

E-mail address: wang_li@bupt.edu.cn.

https://doi.org/10.1016/j.cie.2020.106458
Received 18 May 2019; Received in revised form 11 February 2020; Accepted 3 April 2020
Available online 01 May 2020

distribution functions to represent the randomness of link travel time and capacity, finding the evacuation paths and associated maximum number of evacuees flows. Yao, Mandala, and Chung (2009) proposed a robust linear programming framework based on cell transmission model with considering the demand uncertainty. Kulshrestha, Wu, Lou, and Yin (2011) proposed a mathematical programming model with complementarity constraints under demand uncertainty. Ng and Lin (2015) only adopt the first and second moments to represent the random evacuation demand and road capacities, providing new techniques to determine optimal evacuate routes. Moreover, to determine the optimal evacuation path for affected people, Wang, Yang, Gao, Li, et al. (2016) used the stochastic scenario-based variables to represent the randomness of link travel times and capacities.

Besides, the stochastic programming with recourse (Dantzig, 1955) is another method for dealing with randomness of factors, and this method is to find non-anticipative decisions that have to be made before knowing the realizations of random variables. According to the number of stages, the stochastic programming with recourse problem is generally referred to as two-stage/multi-stage stochastic programming. For example, Powell and Cheung (1994) studied a class of two-stage dynamic networks with random arc capacities, and the Lagrangian relaxation method is used to decompose the problem into tractable tree subproblems. Cheung and Powell (1996) used a multi-stage dynamic network with random arc capacities to solve the dynamic fleet management problem, and a successive convex approximation approach was proposed to capture the future effects of current decisions under uncertainty. Barbarosoğlu and Arda (2004) proposed a two-stage stochastic programming model to deliver the first-aid commodities to disaster areas in emergency response. Rawls and Turnquist (2010) developed a two-stage stochastic mixed integer program (SMIP) that provided an emergency response pre-positioning strategy for hurricanes or other disaster threats. Grass and Fischer (2016) gave a detailed review on the two-stage stochastic programming in disaster management, and most literature focused on the pre-positioning of relief items. Rennemo et al. (2014) presented a three-stage mixed-integer stochastic programming model for disaster response planning, considering the facility location and last mile distribution decisions. Faturechi and Miller-Hooks (2014) proposed a bi-level, three-stage stochastic programming model to maximize roadway travel time resilience under non-recurring natural or human-induced disaster events. Zhang et al. (2019) addressed the emergency resource allocation problem by simultaneously considering primary and secondary disasters, formulating a multi-objective three-stage stochastic programming model to minimize transportation time, transportation cost and unsatisfied demand.

The optimal evacuation plan for affected people is one of the dominant components in emergency response after a disaster, and lots of scholars have denote their efforts into this interesting problem. Furthermore, the related research approaches can be roughly divided into simulation and optimization. The simulation approach addresses on investigating performance of an evacuation plan in multiple scenarios or strategies or finding out the main factors that influence the evacuation process (Chiu & Mirchandani, 2008; Helbing, Farkas, Molnár, & Vicsek, 2002; Li, Ozbay, & Bartin, 2015; Lu, Yang, Cimellaro, & Xu, 2019; Murray-Tuite & Mahmassani, 2003; Qu, Gao, Xiao, & Li, 2014). Nevertheless, the optimization technique is often adopted to acquire the concrete evacuation plan by formulating optimization models (Cova & Johnson, 2003; Hamacher & Tjandra, 2001; Li et al., 2018; Sbayti & Mahmassani, 2006; Xie & Turnquist, 2011). Scholars often regard the planning of evacuation paths as network flow problem (e.g., Bretschneider & Kimms, 2011; Cova & Johnson, 2003; Hamacher & Tjandra, 2001; Yamada, 1996; Zhang, Liu, Zhao, & Deng, 2018).

However, few papers have focused on the strategy of combining *a priori* (pre-disaster) and adaptive (post-disaster) path selection, which can be achieved by the two-stage stochastic programming, to determine the evacuation plan for affected people (see Fig. 1). The randomness can be reflected by a finite number of scenarios, each of which is associated with a known probability. This method of deal with randomness is often applied in various disaster relief management problems (e.g., Barbarosoğlu & Arda, 2004; Rawls & Turnquist, 2010; Rennemo et al., 2014). Additionally, robust optimization, e.g., min-max criterion, is another effective method to deal with the uncertain problem (e.g., Ben-Tal, Chung, Mandala, & Yao, 2011; Najafi, Eshghi, & Dullaert, 2013; Ni et al., 2018), which requires the most likely values together with upper and lower bounds of the random parameters. For simplicity, this paper uses a set of discrete scenarios to represents potential magnitudes of the disaster, which tries to formulate a model that combines pre-event emergency evacuation plan with scenario-based evacuation plan for affected people after an event. Specifically, a part of transportation roads may be destroyed during the event, causing stochastic travel times and capacities when traveling on the road. In other words, non-anticipative first-stage decisions are made in the advance of realization of uncertainty. The second-stage decisions (recourse), which are conditional on the first-stage decisions, are made after the realization of stochastic travel times and capacities. Therefore, the objective is to make the optimal pre-event evacuation plan in the first stage, which is under uncertainty conditions to be faced in the second stage (Birge & Louveaux, 1997). Moreover, instead of deciding the facility location and emergency commodity pre-positioning, this paper presents the specific evacuation path for the affected people.

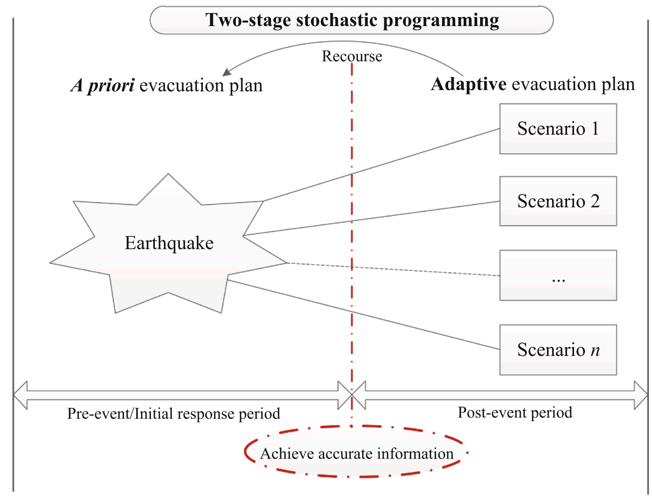

Fig. 1. An Illustrative Example for Occurrence of Earthquake.

A two-stage stochastic scenario-based programming model is proposed to evacuate affected people in disaster areas. The first-stage decisions are the robust and reliable evacuation plan for all levels of disaster. The second-stage decisions involve the evacuation plan for affected people in response to specific scenario-based road conditions. The main contributions of this paper can be summarized as follows:

(1) To the best of our knowledge, this paper firstly proposes the two-stage stochastic programming model that considers both *a priori* (pre-disaster) and adaptive (post-disaster) path selection to provide *a priori* evacuation plan for the affected people from dangerous areas to safe areas. Unlike the deterministic models, it involves the availability of scenario-based random link travel times and capacities. Therefore, this paper addresses on obtaining the *a priori* evacuation planning for affected people with the consideration of uncertain link travel times and capacities in the second stage.
(2) To formulate the explicit movement process of affected people when a disaster occurs, this paper proposed a min-cost flow model based on two-stage stochastic programming. Furthermore, this model is decomposed into two subproblems by means of Lagrangian relaxation approach. The two subproblems are respectively min-cost flow model and its time-dependent case. Hence, the proposed two-

stage stochastic programming model is relaxed as two tractable subproblems.

(3) To obtain the approximately optimal solution, the subgradient optimization algorithm framework is adopted to gradually reduce the relative gap of upper and lower bounds, i.e., the objective values of original model and relaxed model; and the two subproblems is solved by the successive shortest path algorithm. Numerical results reveal that the proposed model in this paper outperforms both the deterministic model and the wait and see model, demonstrating that the two-stage stochastic evacuation model provides a practical decision-making tool for evacuation of affected people in disaster areas.

The rest of this paper is organized as follows. In Section 2, a detailed description is presented to describe the evacuation process as a two-stage stochastic program. The min-cost flow model in the basis of two-stage stochastic programming model is proposed in Section 3. Based on the Lagrangian relaxation-based solution approach, Section 4 adopts the subgradient optimization algorithm framework embedded into successive shortest path algorithm to solve the relaxed model and reduce the relative gap between the upper and lower bounds. Section 5 demonstrates the performance analysis and applicability of the proposed model and algorithm by numerical examples. A conclusion and future research are given in the final section.

2. Problem statements

2.1. Representation of the evacuation problem as a two-stage stochastic programming problem

This section takes the occurrence of earthquake as an example to depict the evacuation process of people. In the evacuation process, it is assumed that the affected people can receive the early warning information and they escape from the dangerous area by their own cars. In the post-event period after receiving a dangerous signal, accurate information about the earthquake becomes available by advanced communication tools. However, emergency response and evacuation is initially implemented without knowing the affected scope and degree exactly. In this case, initial response will be dependent on a number of damage scenarios. Consequently, the evacuation phase should be divided into two stages according to the acquisition time of accurate information. In the non-anticipative first-stage, the evacuation decision is must be made before the uncertain future realizations, and the second-stage evacuation plan are determined after the realization of the uncertain information is known. Based on above analysis, the objective is to make the optimal evacuation planning in the first stage under uncertainty to be faced in the second stage.

In the following, the concept of two-stage stochastic evacuation planning problem is specifically illustrated by a simple network with 8 nodes and 15 links (see Fig. 2). Here, nodes 1 and 8 are assumed to be disaster area and safe area respectively, and four cars, namely **a**, **b**, **c** and **d**, are needed to be evacuated to safe area. In the pre-event and early period, these four cars are assumed to be evacuated by the *a priori* planning, i.e., cars **a** and **b** travel along the paths $1 \to 2 \to 3 \to 4 \to 6 \to 8$, **c** and **d** travel along the paths $1 \to 3 \to 5 \to 6 \to 7 \to 8$. In the post-event period that the accurate road information is received, i.e., after the time threshold $\widetilde{T}$, the latter

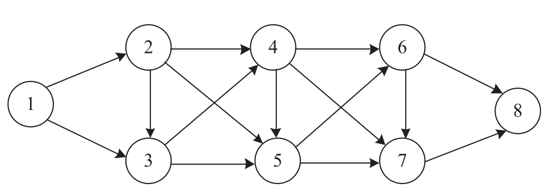

Fig. 2. An Illustrative Evacuation Road Network.

part of the plan may be changed adaptively. For simplicity, two random scenarios are taken into account to illustrate the planning generation. Meanwhile, Fig. 3 gives the time-space network of Fig. 2 by discretizing the time period into multiple unit time intervals. In this network, the horizontal axis represents the interested time period, and the vertical axis represents the nodes in the physical network. Obviously, the horizontal axis is divided into two stages by the time threshold $\widetilde{T}$. The *a priori* evacuation plan is given on the left of Fig. 3, and the adaptive plan in two scenarios are shown on the right of Fig. 3. The evacuation plan for these four cars are marked by color lines in different scenarios. In detail, the evacuation plan in scenario 1 is that cars **a** and **b** travel along the path $1 \to 2 \to 3 \to 4 \to 7 \to 8$, cars **c** and **d** travel along the path $1 \to 3 \to 5 \to 6 \to 8$; and in scenario 2, cars **a** and **b** travel along path $1 \to 2 \to 3 \to 4 \to 6 \to 8$, **c** travels along path $1 \to 3 \to 5 \to 6 \to 8$ and **d** travels along path $1 \to 3 \to 5 \to 6 \to 7 \to 8$.

From Fig. 3 it can be observed that, before the time threshold $\widetilde{T}$, the sub-trip in the adaptive evacuation plan under scenarios 1 and 2 are the same as that in the *a priori* plan, namely, cars **a** and **b** evacuate along the sub-path $1 \to 2 \to 3 \to 4$, **c** and **d** evacuate along the sub-path $1 \to 3 \to 5$. When the accurate information is available, i.e., after time threshold $\widetilde{T}$, the adaptive evacuation plans in two scenarios are different from the *a priori* evacuation plan. For example, the *a priori* evacuation plan for a and b is sub-path $6 \to 8$, while the adaptive plans for a and b are respectively sub-paths $7 \to 8$ and $6 \to 8$ in two scenarios.

A simple numerical example is given to analyze the scenario-based two-stage stochastic evacuation problem. The *a priori* evacuation plan in the first stage must be made before the random parameters are observed. Subsequently, the second stage decisions involve the evacuation plans with respect to specific scenario. This paper focuses on generating the robust *a priori* evacuation planning by considering the recourse in the second stage.

2.2. Multiple sources and sinks conversion

In the two-stage stochastic evacuation problem, there are more than one sources and sinks in the actual evacuation network. Therefore, the physical network with multiple sources should be converted to an equivalent network with a single supersource, even to multiple sinks. Practically speaking, a supersource k is added to the network, and meanwhile the dummy arcs (k, i) should be added with the travel time with value 0, i.e., $c_{ki} = 0$, $i \in K$, where K is the set of source nodes, and let the capacity on the dummy arc be the value of the supply at node i, i.e., $u_{ki} = d_i$, $i \in K$. Hence the supply value at the supersource can be set as $d_k = \sum_{i \in K} d_i$. Similarly, the supersink can be converted by the same way. Following is an example to illustrate how to convert multiple sources and sinks to the supersource and supersink (see Fig. 4).

For a time-dependent network, the arc travel time and capacity will vary with the departure time. When adding a supersource, the travel times on the dummy arcs are $c_{ki}(t) = 0$, $t \in \{0, 1, \cdots, T\}$, $i \in K$, and the capacity is equal to supply of node i at time t, i.e., $u_{ki}(t) = d_i(t)$, $t \in \{0, 1, \cdots, T\}$. Furthermore, the multiple sinks is converted to a supersink. Since the time of arriving at the sink for each flow may be different from each other, we first add a copy j' for each original sink j, $j \in D$, and then a supersink l is added to the network. For each sink, we add the arc (j, j') to the network with infinite capacity, i.e., $u_{jj'}(t) = \infty$, and the travel time for each dummy arc is assumed to be zero, $c^s_{jj'}(t) = 0$, $\forall\, t \in \{0, 1, \cdots, T\}$. In addition, a self-loop is added for each node j', in which $u_{j'j'}(t) = \infty$ and $c_{j'j'(t)} = 1$. Moreover, the capacity of arc (j', l) is equal to the demand of node j at time T, i.e., $u_{j'\nu}(T) = b_j(T)$, and the capacity at other time is set as 0, i.e., $u_{j'l}(t) = 0$, $t \in \{1, 2, \cdots, T\}/T$. Also, the travel time is assumed to be zero in the considered time horizon, $c_{j'l}(t) = 0$, $t \in \{1, 2, \cdots, T\}$.

Next an example is given to illustrate how to convert multiple sources and sinks as supersource and supersink (see Fig. 5).

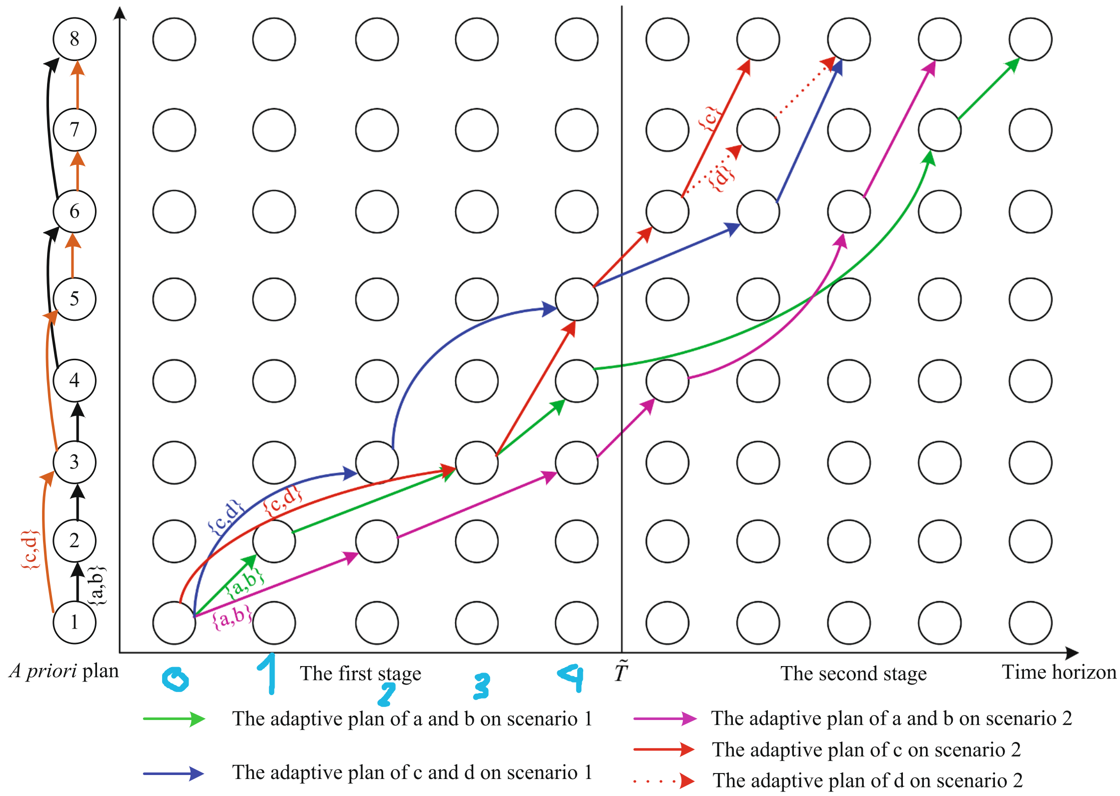

Fig. 3. Two-stage stochastic evacuation plan in time-dependent network.

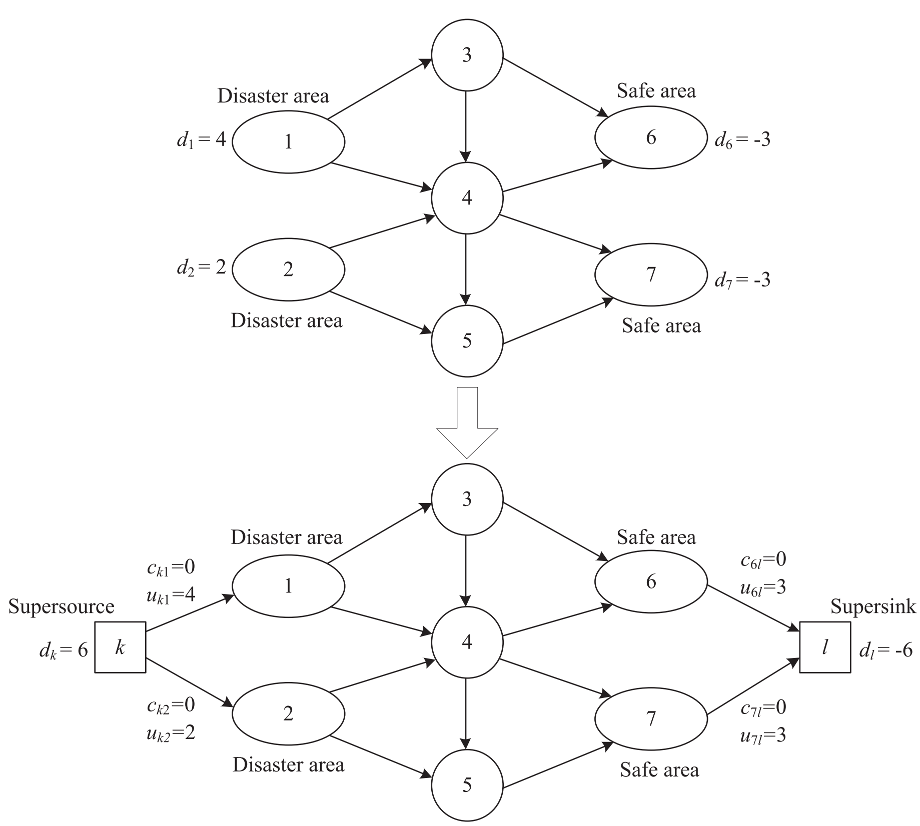

Fig. 4. Supersource and Supersink conversion example in physical network.

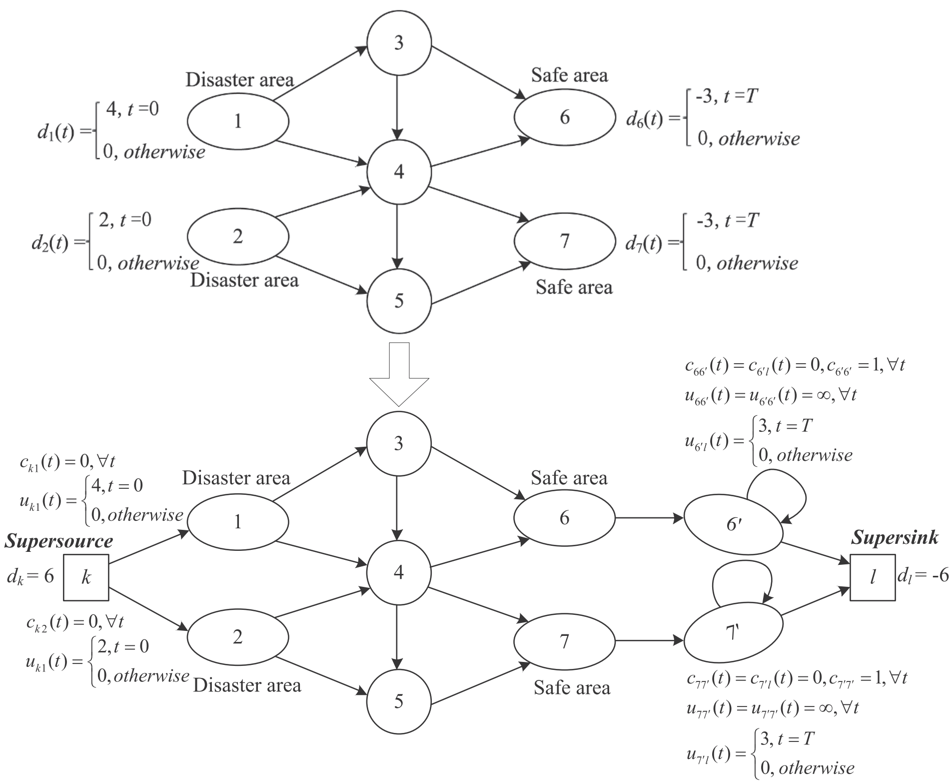

Fig. 5. Supersource and supersink conversion example in time-dependent network.

2.3. *Description of the two-stage stochastic evacuation problem by the min-cost flow problem*

The primary issue when a disaster occurs is to minimize the loss of life. In face of some disasters such as earthquake, the leak of dangerous chemical and hurricane, the people in the affected areas must be evacuated to the safe areas. Wang, Yang, Gao, Li, et al. (2016) have addressed the evacuation problems in disaster-response management by formulating three stochastic mathematical programming models according to different preferences of decision-makers. The decisions to make in this paper are involving not only the *a priori* optimization decision but also the adaptive selection strategy depending on a set of discrete scenarios. Hence, this paper formulates a two-stage stochastic mixed integer programming that provides an emergency response pre-evacuation strategy for the earthquake or other disasters.

This paper formulates the detailed movement planning of people from dangerous areas to safe areas as a min-cost flow problem with random link travel times and capacities. The objective is to move the people from dangerous areas to safe areas with the minimum evacuation time in a capacity-cost network $G(V, A, C, U, D)$, where V is the set of nodes, A is the set of links with random travel time and capacity, $C(i, j)$ represents the travel time on link $(i, j) \in A$, denoted by c_{ij}, $U(i, j)$ represents the capacity of link (i, j), denoted by u_{ij} and $D(i)$ represents the flow at node $i \in V$, denoted by d_i. It is assumed that the travel time on link $(i, j) \in A$ are the same if the affected people on a road is not beyond the capacity. That is, the link travel time does not change with the amount of affected people. In the context of stated problem, sources in the network represent the dangerous areas, sinks represent the safe areas, and other nodes are the intersections of the network. The links in the network represent the roads for moving affected people.

The presented model is different from the general min-cost flow problem. Specifically, the two-stage stochastic programming model with recourse represents a situation where both the first stage and second stage arise in different time phases in the same evacuation network. In the first stage, link travel times and capacities are only probabilistically known, but the affected people must be evacuated from source nodes to other nodes priori to realizing link travel times and capacities in the second stage. In the second stage, the evacuation plan will be determined with the realization of the link travel times and capacities. Note that the first-stage evacuation plan may not be feasible for the given realization, which is solved by allowing shorter evacuation times in the second stage. In this case, the objective function will consist of the first-stage punish costs and the expected value of the second-stage recourse costs.

3. Model formulation

3.1. *Notations*

The following provides a rigorous formulation for the problem of interest through discussing decision variables, system constraints and the objective function. For modeling convenience, Table 1 summarizes the relevant notations used in the mathematical formulation.

Table 1
Subscripts and Parameters Used in Mathematical Formulation.

Symbol	Definition
V	the set of nodes
A	the set of links
i, j	the index of nodes, $i, j \in V$
(i, j)	the index of directed links, $(i, j) \in A$
s	the index of scenario
S	the total number of scenarios
v	the supply value of source node
$\tilde{T}$	the time threshold
T	the total number of time intervals
u_{ij}	the capacity on physical link (i, j)
$u_{ij}^s(t)$	the capacity of link (i, j) in scenario s at time t
$c_{ij}^s(t)$	the travel time of link (i, j) in scenario s at time t
μ_s	the probability in scenario s

Table 2
Decision Variables Used in Mathematical Formulation.

Decision Variable	Definition
x_{ij}	the flow on link (i, j)
$y_{ij}^s(t)$	the flow on link (i, j) in scenario s at time t

3.2. Decision variables

Two types of decision variables are introduced to construct the two-stage stochastic evacuation model. In the first stage, a binary decision variable x_{ij} is used to represent the flow on link (i, j). The second decision variable $y_{ij}^s(t)$ is defined to represent the flow on link (i, j) in scenario s at time t. Meanwhile, Table 2 gives a straightforward overview for these two decision variables.

3.2.1. System constraints

The first stage

In the first stage, a feasible evacuation should be determined from super-source to super-sink. The flow on each link should satisfy the flow balance constraint below:

$$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = d_i \tag{1}$$

where d_i is a parameter with the following definition:

$$d_i = \begin{cases} v, & i = s \\ -v, & i = t \\ 0, & \text{otherwise} \end{cases}$$

Meanwhile, the flow on each link must also satisfy the capacity constraint:

$$0 \leqslant x_{ij} \leqslant u_{ij}, \forall\ (i, j) \in A \tag{2}$$

On a separate note, the flow balance constraint may generate a path with loops and sub-tours if there are potential loops in the evacuation network. To eliminate loops on the generated physical evacuation path, the link penalty $p_{ij}, (i, j) \in A$ is particularly introduced. With this consideration, the penalty function can be defined as

$$f(\mathbf{X}) = \sum_{(i,j)\in A} p_{ij} x_{ij} \tag{3}$$

where $\mathbf{X} := \{x_{ij}\}_{(i,j)\in A}$.

The second stage

The second stage is the evaluation of the first stage to obtain the robust optimal evacuation plan. In this stage, the affected people will receive the adaptive paths with the minimum evacuation time according to real-time disaster information. In advance of the time threshold, the affected people will be evacuated along the *a priori* evacuation plan in the first stage; in other words, the evacuation plan in different scenarios before the time threshold is the same as the *a priori* path. According to the above discussion, the coupling constraints for the evacuation plan of each scenario before the time threshold $\widetilde{T}$ can be formulated as follows:

$$\sum_{t\leqslant\widetilde{T}} y_{ij}^s(t) = x_{ij}, (i, j) \in A, s = 1, 2, \cdots, S. \tag{4}$$

The coupling constraint embodies the relationship between the physical link and the space-time arc in the evacuation plan, that is, if there are traffic flows on the link (i, j), such as $x_{ij} = 2$, then the traffic flow on each link (i, j) before the time threshold $\widetilde{T}$ is 2, i.e., $y_{ij}^s(t) = 2$. In other words, before the time threshold $\widetilde{T}$, the evacuation plan in each scenario of the second stage is the same as the priori evacuation plan in the first stage.

In the following, a second-stage model is established with the goal of minimizing the overall time of the affected people evacuated from the dangerous area to the safe area in each scenario:

$$Q(Y, s) = \min \sum\nolimits_{(i,j)\in A_s} c_{ij}^s(t)\cdot y_{ij}^s(t) \tag{5}$$

$$\begin{aligned} &s.\ t.\\ &\sum\nolimits_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum\nolimits_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t), \forall\ i \in V, t \in \{0, 1, \cdots, T\},\\ &s = 1, 2, \cdots, S \end{aligned} \tag{6}$$

$$0 \leqslant y_{ij}^s(t) \leqslant u_{ij}^s(t), \forall\ (i, j) \in A, t \in \{0, 1, \cdots, T\}, s = 1, 2, \cdots, S \tag{7}$$

$$\sum\nolimits_{t\leqslant\widetilde{T}} y_{ij}^s(t) = x_{ij}, (i, j) \in A, s = 1, 2, \cdots, S \tag{8}$$

The objective function (5) is the overall evacuation time of all traffic volumes in the minimization scenario s. Constraints (6) and (7) are flow balance constraint and traffic capacity constraint respectively. The constraint (8) is a coupling constraint to ensure that the evacuation scheme in each scenario in the second stage before the time threshold $\widetilde{T}$ is the same as the *a priori* plan in the first stage.

3.3. Two-stage stochastic evacuation planning model

The evacuation problem of interest is to obtain a robust plan in the first stage by the evaluation of adaptive plans in the second stage. To this end, this paper evaluates the evacuation plan of the first stage with the expected overall evacuation time of each scenario's adaptive evacuation path, and the probability of occurrence of each scenario s is assumed as $\mu_s, s = 1, 2, \cdots, S$. In order to minimize the penalty for the prior evacuation plan and the expected overall evacuation time of each scenario's adaptive evacuation plan, a two-stage evacuation planning model in time-dependent and random environment is formulated as:

$$\begin{cases} \min \sum\nolimits_{(i,j)\in A} p_{ij}x_{ij} + \sum\nolimits_{s=1}^{S} \mu_s\cdot Q(Y, s)\\ s.\ t.\\ \sum\nolimits_{(i,j)\in A} x_{ij} - \sum\nolimits_{(j,i)\in A} x_{ji} = d_i, \forall\ i \in V\\ 0 \leqslant x_{ij} \leqslant u_{ij}, \forall\ (i, j) \in A\\ \text{in which,}\\ Q(Y, s) = \min \sum\nolimits_{(i,j)\in A_s} c_{ij}^s(t)\cdot y_{ij}^s(t)\\ s.\ t.\\ \sum\nolimits_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum\nolimits_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t), \forall\ i \in V,\\ t \in \{0, 1, \cdots, T\}, s = 1, 2, \cdots, S\\ 0 \leqslant y_{ij}^s(t) \leqslant u_{ij}^s(t), \forall\ (i, j) \in A, t \in \{0, 1, \cdots, T\}, s = 1, 2, \cdots, S\\ \sum\nolimits_{t\leqslant\widetilde{T}} y_{ij}^s(t) = x_{ij}, (i, j) \in A, s = 1, 2, \cdots, S \end{cases} \tag{9}$$

This model is a time-dependent and stochastic two-stage evacuation planning model. The goal of the model is to develop a robust optimal evacuation plan to provide guidance to the affected people in the emergency event. Since the second stage of the model has a limited number of scenarios, the above time-dependent and stochastic two-stage evacuation planning model is equivalent to the following single stage model:

$$\begin{cases} \min \sum\nolimits_{(i,j)\in A} p_{ij}x_{ij} + \sum\nolimits_{s=1}^{S} \left(\mu_s\cdot \sum\nolimits_{(i,j)\in A_s} c_{ij}^s(t)\cdot y_{ij}^s(t)\right)\\ s.\ t.\\ \sum\nolimits_{(i,j)\in A} x_{ij} - \sum\nolimits_{(j,i)\in A} x_{ij} = d_i, \forall\ i \in V\\ 0 \leqslant x_{ij} \leqslant u_{ij}, \forall\ (i, j) \in A\\ \sum\nolimits_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum\nolimits_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t), \forall\ i \in V,\\ t \in \{0, 1, \cdots, T\}, s = 1, 2, \cdots, S\\ 0 \leqslant y_{ij}^s(t) \leqslant u_{ij}^s(t), \forall\ (i, j) \in A, t \in \{0, 1, \cdots, T\}, s = 1, 2, \cdots, S\\ \sum\nolimits_{t\leqslant\widetilde{T}} y_{ij}^s(t) = x_{ij}, (i, j) \in A, s = 1, 2, \cdots, S \end{cases} \tag{10}$$

Obviously, in comparison to the deterministic model, this model is particularly provided under the stochastic environment. That is, both travel time and capacity on each link are scenario-based random variables. If the link travel times and capacities are degenerated to constants, i.e., $S = 1$, this stochastic two-stage model is equivalent to a deterministic model.

In addition, if the real-time information has been updated from the original node, i.e., the *a priori* evacuation plan is not necessary to consider, then the evacuation model based on wait-and-see (WAS) can be formulated for minimizing the total expected travel times in multiple scenarios, shown as follows:

$$\begin{cases} \min \sum_{s=1}^{S}\left(\mu_s\cdot\sum_{(i,j)\in A_s} c_{ij}^s(t)\cdot y_{ij}^s(t)\right)\\ s.\ t.\\ \sum_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t),\ \forall\ i\in V,\\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\\ 0\leqslant y_{ij}^s(t)\leqslant u_{ij}^s(t),\ \forall\ (i,j)\in A,\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\end{cases} \tag{11}$$

Hereinafter, model (11) is denoted as WAS. In essence, the feasible domain of the WAS model obviously contains that of the original model, and thus this WAS model is the lower bound of original model. Since the constraints among different scenarios have no relations, the model can also be further decomposed into S sub-problems based on scenarios. For each scenario of this model, it is basically a time-dependent min-cost flow problem.

4. Solution algorithm

Model (10) is essentially an integer programming model which contains two types of decision variables, i.e., $\mathbf{X}: =\{x_{ij}\}_{(i,j)\in A}$ and $\mathbf{Y}: =\{y_{ij}^s(t)\}_{(i,j)\in A, t\in\{0,1,\cdots,T\}, s=1,2,\cdots,S}$. In this model, the coupling constraint is a complex constraint, which leads to the model cannot be solved in polynomial time. Therefore, this section intends to relax the coupling constraint into the objective function by Lagrangian relaxation approach. In the following, Section 4.1 provides a Lagrangian relaxation-based dual approach to decompose the original problem into two easy-solved subproblems, whose objective value is the lower bound of the optimal value of model (10). Concretely, the original model is decomposed into a standard min-cost flow problem and a time-dependent one, which can be solved efficiently by exact algorithms. The key issue of the Lagrangian relaxation approach is to found the upper bound of the problem of interest. The relaxation solutions, coincidentally, are feasible to the original problem. Therefore, Section 4.2 will give a detailed updated strategy of upper bound. Moreover, the solution procedure of the designed algorithm is presented in Section 4.3.

4.1. Model decomposition

From the original model it can be observed that the coupling constraint (4) is a hard constraint. This constraint characterizes the relationship between selection of a physical link and corresponding scenario-based time-dependent arcs. Hence, we introduce the Lagrangian multiplier $\alpha_{ij}^s(t), (i,j)\in A, s = 1, 2, \cdots, S, t\leqslant \widetilde{T}$ for the coupling constraint, and then this constraint can be relaxed into the objective function in the following form:

$$\sum_{s=1}^{S}\sum_{t\leqslant\widetilde{T}}\sum_{(i,j)\in A}\alpha_{ij}^s(t)(y_{ij}^s(t)-x_{ij}) \tag{12}$$

Abs( ) ?

Therefore, after the relaxation of the formula (12) to the objective function, the relaxed one of the model (10) can be formulated as follows:

$$\begin{cases} \min \sum_{(i,j)\in A} p_{ij}x_{ij} + \sum_{s=1}^{S}\left(\mu_s\cdot\sum_{(i,j)\in A_s} c_{ij}^s(t)\cdot y_{ij}^s(t)\right)+\\ \sum_{s=1}^{S}\sum_{t\leqslant\widetilde{T}}\sum_{(i,j)\in A}\alpha_{ij}^s(t)(y_{ij}^s(t)-x_{ij})\\ s.\ t.\\ \sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ij} = d_i,\ \forall\ i\in V\\ 0\leqslant x_{ij}\leqslant u_{ij},\ \forall\ (i,j)\in A\\ \sum_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t),\ \forall\ i\in V,\\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\\ 0\leqslant y_{ij}^s(t)\leqslant u_{ij}^s(t),\ \forall\ (i,j)\in A,\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\end{cases} \tag{13}$$

C1

It is worthwhile to note that the variables $\mathbf{X}$ and $\mathbf{Y}$ can be separated from each other in above relaxed model (13). That is, by combining similar terms, the relaxed model is decomposed into two subproblems as follows:

SubProblem 1: Min-cost Flow Problem

Obviously, the first sub-problem can be regarded as a min-cost flow problem, and its form is given as follows:

C2

$$\begin{cases} \min SP1(\alpha) = \sum_{(i,j)\in A}\left(p_{ij} - \sum_{s=1}^{S}\sum_{t\leqslant\widetilde{T}}\alpha_{ij}^s(t)\right)x_{ij}\\ s.\ t.\\ \sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ij} = d_i,\ \forall\ i\in V\\ 0\leqslant x_{ij}\leqslant u_{ij},\ \forall\ (i,j)\in A\end{cases} \tag{14}$$

The objective function of subProblem 1 can be defined as $g_{ij} := p_{ij} - \sum_{s=1}^{S}\sum_{t\leqslant\widetilde{T}}\alpha_{ij}^s(t)$ to represent the generalized cost of each link. Therefore, the sub-Problem 1 can be solved by the successive shortest path algorithm. For convenience, the optimal objective value of sub-Problem 1 is abbreviated as $Z_{SP1}^*(\alpha)$. The successive shortest path algorithm was proposed by Jewell (1962), Iri (1960), Busacker and Gowen (1961), et al. The goal of this algorithm is to calculate the minimum cost flow in the network $G = (V, A, C, U, D)$ (Xie, Xing, & Wang, 2009). Eventually, the successive shortest path algorithm for subProblem 1 is developed in Algorithm 1.

SubProblem 2: Time-Dependent Min-cost Flow Problem

The second subproblem of the relaxed model (13) is associated with the decision variables $\mathbf{Y}$, and its optimal objective value of the problem is abbreviated as $Z_{SP2}^*(\alpha)$, shown as follows:

$$\begin{cases} \min SP2(\alpha) = \sum_{s=1}^{S}\sum_{(i,j)\in A}\left(\sum_{t\in\{0,1,\cdots,T\}}\mu_s\cdot c_{ij}^s(t) + \sum_{t\leqslant\widetilde{T}}\alpha_{ij}^s(t)\right)y_{ij}^s(t)\\ s.\ t.\\ \sum_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_i^s(t),\ \forall\ i\in V,\\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\\ 0\leqslant y_{ij}^s(t)\leqslant u_{ij}^s(t),\ \forall\ (i,j)\in A,\ t\in\{0,1,\cdots,T\},\ s=1,2,\cdots,S\end{cases} \tag{15}$$

Algorithm 1: Successive shortest path algorithm for min-cost flow problem.

Step 1: Take variable x as a feasible flow between any OD and it has the minimum delivery cost in the feasible flows with the same flow value.

Step 2: The algorithm will terminate if the flow value of x reaches v or there is no minimum cost path in the residual network $(V, A(x), C(x), U(x), D)$; otherwise, the shortest path with the maximum flow is calculated by label-correcting algorithm, and then go to Step 3. The functions $A(x)$, $C(x)$, $U(x)$ in the residual network can be defined as:

$$A(x) = \{(i,j)|(i,j)\in A, x_{ij} < u_{ij}\}\cup\{(j,i)|(j,i)\in A, x_{ij} > 0\}$$

$$C(x) = \begin{cases} c_{ij}, (i,j)\in A, x_{ij} < u_{ij}\\ -c_{ji}, (j,i)\in A, x_{ji} > 0\end{cases}$$

$$U_{ij}(x) = \begin{cases} u_{ij}, (i,j)\in A, x_{ij} < u_{ij}\\ x_{ji}, (j,i)\in A, x_{ji} > 0\end{cases}$$

?

Step 3: Increase the flow along the minimum cost path. If the increased flow value does not exceed v, go to Step 2.

SubProblem 2 can be further decomposed into a total of S sub-problems, each of which can be referred to as the min-cost flow problem with time-dependent link travel times and capacities, namely:

$$\begin{cases} \min SP2(\alpha, s) = \sum_{(i,j)\in A}\left(\sum_{t\in\{0,1,\cdots,T\}}\mu_s\cdot c_{ij}^s(t) + \sum_{t\leqslant\widetilde{T}}\alpha_{ij}^s(t)\right)y_{ij}^s(t) \\ s.\ t. \\ \sum_{(i_t,j_{t'})\in A_s} y_{ij}^s(t) - \sum_{(j_{t'},i_t)\in A_s} y_{ij}^s(t') = d_t^s(t), \forall\ i\in V,\ t\in\{0, 1, \cdots, T\} \\ 0\leqslant y_{ij}^s(t)\leqslant u_{ij}^s(t), \forall\ (i, j)\in A,\ t\in\{0, 1, \cdots, T\} \end{cases} \tag{16}$$

For each scenario $s\in\{1, 2, \cdots, S\}$, subproblem (16) has a similar structure as subproblem (14) with time-dependent link cost $c_{ij}^s(t)$ and link capacity $u_{ij}^s(t)$ and the generalized cost $g_{ij}^s(t)$. Since the considered time period T is divided into two time stages, the generalized cost is defined as a piecewise function:

$$g_{ij}^s(t) = \begin{cases} \mu_s\cdot c_{ij}^s(t) + \alpha_{ij}^s(t),\ t\leqslant\widetilde{T} \\ \mu_s\cdot c_{ij}^s(t), \qquad \widetilde{T} < t\leqslant T \end{cases}$$

Since subproblem (16) is a time-dependent min-cost flow problem, and thus the algorithm 1 should be modified in Step 2. Firstly, parameters $A(y(t))$, $C(y(t))$, and $U(y(t))$ in the residual network $N(y(t))$ are defined as follows:

$$A_s(y(t)) = \{(i_t, j_{t'})|(i_t, j_{t'})\in A_s, y_{ij}^s < u_{ij}^s\}\cup\{(j_{t'}, i_t)|(j_{t'}, i_t)\in A_s, y_{ij}^s > 0\},$$
$$s = 1, 2, \cdots, S$$

$$c_{ij}^s(y(t)) = \begin{cases} c_{ij}^s(t),\ (i_t, j_{t'})\in A_s,\ y_{ij}^s(t) < u_{ij}^s(t),\ t\in\{0, 1, \cdots, T\} \\ -c_{ji}^s(t'),\ (j_{t'}, i_t)\in A_s,\ \forall\ \{t'\in\{0, 1, \cdots, T\}|y_{ji}^s(t') > 0\}, \\ s = 1, 2, \cdots, S \\ T,\ (j_{t'}, i_t)\in A_s,\ \forall\ \{t'\in\{0, 1, \cdots, T\}|y_{ji}^s(t') = 0\} \end{cases}$$

$$u_{ij}^s(y(t)) = \begin{cases} u_{ij}^s(t) - y_{ij}^s(t),\ (i_t, j_{t'})\in A_s,\ y_{ij}^s(t) < u_{ij}^s(t),\ t\in\{1, 2, \cdots, T\} \\ y_{ji}^s(t),\ (j_{t'}, i_t)\in A_s,\ \forall\ \{t'\in\{0, 1, \cdots, T\}|y_{ji}^s(t') > 0\}, \\ s = 1, 2, \cdots, S \\ 0,\ (j_{t'}, i_t)\in A_s,\ \forall\ \{t'\in\{0, 1, \cdots, T\}|y_{ji}^s(t') = 0\} \end{cases}$$

Secondly, the modified label-correcting algorithm (Ziliaskopoulos & Mahmassani, 1992) will be adopted to find the time-dependent min-cost path in the residual network.

By solving the subproblem (14) and (15) with the relaxation solution $\mathbf{X}$ and $\mathbf{Y}$, the optimal objective value Z_{LR}^* for the relaxed model (13) with a set of given Lagrangian multiplier vector α can be expressed as follows:

$$Z_{LR}^*(\alpha) = Z_{SP1}^*(\alpha) + Z_{SP2}^*(\alpha) \tag{17}$$

Obviously, the optimal objective value of the relaxed model (13) is the lower bound of the optimal objective value of the original model (10). In order to obtain a high-quality solution, it is needed to obtain a lower bound which is close to the optimal objective value of the original model. That is, the greatest possible lower bound should be obtained, and the expression is given as follows:

$$Z_{LD}(\alpha^*) = max_{\alpha\geqslant 0}Z_{LR}(\alpha) \tag{18}$$

4.2. Determination of potential optimal value

If the relaxed solutions obtained by solving the subproblems (14) and (15) are feasible to the original model (13), then they can be regarded as potential optimal values of the original model, each of which is also an upper bound to the optimal value of the original model. In the process of solving subproblem (14), $SP1(\alpha)$, an evacuate plan consisted of physical paths for each car is generated, denoted by $\check{UB}$. For solving the subproblem (16), $SP2(\alpha, s)$, $s = 1, 2, \cdots, S$, a total of S evacuate plans consisted of time-dependent paths for each car are generated, denoted by $\check{UB}^s$, $s = 1, 2, \cdots, S$. According to the coupling constraint (4), the generated evacuated plans by the subproblems (14) and (16) are all feasible solutions to the original model (13). Therefore, the objective value for each feasible solution obtained by subproblems (14) and (16) is an upper bound to the optimal value of the original model.

4.3. Solution procedure

If the lower bound (17) is just happened to be equal to the minimum upper bound obtained by subproblems (14) and (16), the returned feasible solution will be the exact optimal solution on the basis of the duality (Fisher, 1981). If not, we shall try to update the Lagrangian multipliers α to reduce the gap between the lower and upper bounds of the optimal value to improve the quality of current feasible solution. This research applies the subgradient optimization algorithm to iteration process. The subgradient optimization algorithm is a general approach to reduce the relative gap between the upper and lower bounds by iteration process, and the basic principle and detailed course of treatment of this algorithm can be referred to papers such as Yang and Zhou (2014, 2017) and Wang, Yang, and Gao (2016); Wang, Yang, Gao, Li, et al. (2016). Therefore, the main procedure for solving the proposed model of this paper is briefly given in the following.

Firstly, the Lagrangian relaxation solution approach is used to solve the classic minimum cost flow problem and time-dependent minimum cost flow problem presented in above Section 4.1. Secondly, the optimal value of the Lagrangian dual problem provides a lower bound for the original problem. In the process of finding the optimal solution for the Lagrangian dual problem, it is necessary to calculate the minimum cost flow plan and time-dependent minimum cost plan in each scenario s by the successive shortest path algorithm. Thirdly, each calculated minimum cost evacuation plans of the Lagrangian dual problem is a feasible solution to the original problem. Hence, the objective value of each minimum cost evacuation plan for the original problem can be regarded as an upper bound for this original problem. Following is the detailed procedure of the Lagrangian relaxation approach based on subgradient optimization algorithm.

Algorithm 2: Lagrangian relaxation-based approach combining with subgradient algorithm.

Step 1:
(a) Set iteration number $k = 1$;
(b) Select positive values to initialize the Lagrangian multipliers $\alpha_{ij}^s(t)$;
(c) Set the initial upper bound, i.e., $UB^0 = +\infty$;
Step 2:
(a) Solve subproblem (14) using the Algorithm 1, i.e., successive shortest path algorithm, and find a evacuation plan $\mathbf{X}$;
(b) Solve subproblem (16) using the modified Algorithm 1 to search for the time-dependent evacuation plans for each scenario s, denoted by $\mathbf{Y}$;
(c) Calculate the lower and upper bounds of problem (10) and their relative gap.
Step 3:
Update Lagrangian multipliers $\alpha_{ij}^s(t)$ by using subgradient $\alpha_{ij}^s(t) + \theta^k\times(y_{ij}^s(t) - x_{ij})$, where θ^k is the step length at iteration k.
Step 4:
Terminate this algorithm and return the current best feasible solution.

5. Numerical example

This section will verify the effectiveness of the Lagrangian relaxation-based heuristic algorithm for solving the two-stage stochastic evacuation planning model on two networks, i.e., a grid network and the Chicago sketch network.

5.1. Experiments on a grid network

5.1.1. Experiment design

This set of experiments is conducted on a grid network (shown in Fig. 6), which consisting of 100 nodes and 180 links. It is assumed that there are 500 cars need to be evacuated to a safe area, and the duration of the disaster is 120 min. The unit growth interval is one minute, and thus the disaster period is divided into 120 time intervals. In the data initialization phase, according to the link length, the corresponding

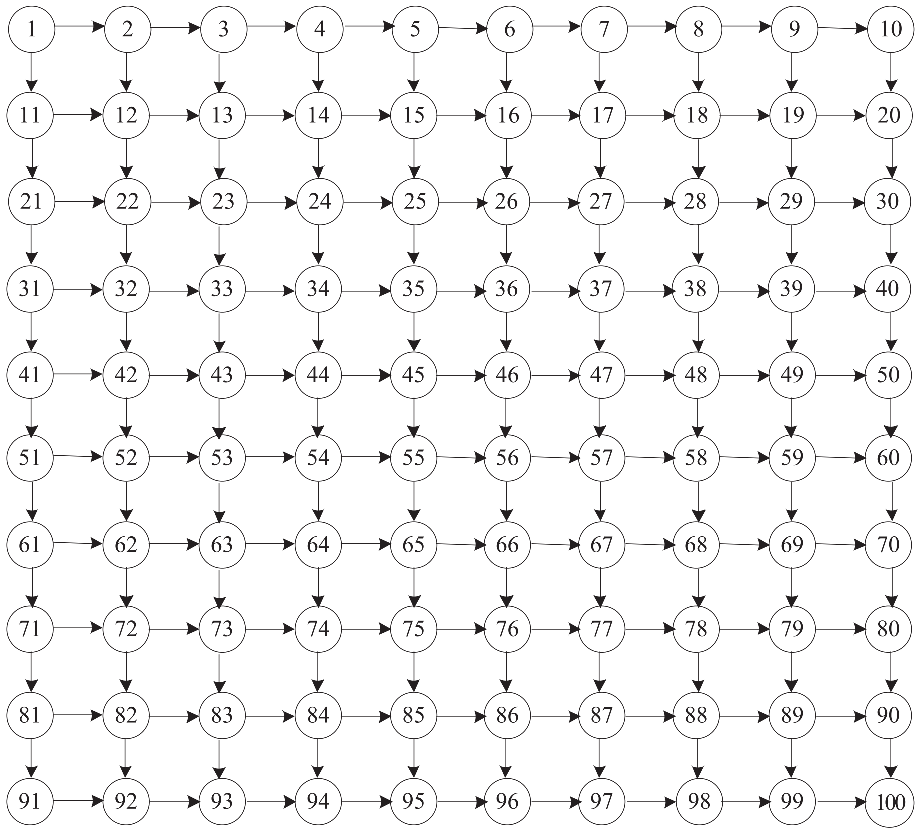

Fig. 6. A Grid Network.

scenario-based time-dependent link travel times and traffic capacities are randomly generated. This Lagrangian relaxation-based algorithm begins with a set of randomly generated multipliers $\alpha_{ij}^{s}(t)$, $(i, j) \in A$, $s = 1, 2, \cdots, S$, $t \leqslant \widetilde{T}$, and stops when the relative gap between upper and lower bounds is smaller than 0.01% or the number of iteration is up to 10.

5.1.2. Experiments of different time threshold examples

This experiment assumes that the number of scenarios is 10, and we set the link penalty value in first stage as $p_{ij} = 2$ min, $\forall\ (i, j) \in A$. This set of experiments with different time thresholds in three groups of OD pairs are analyzed below, assuming the time threshold $\widetilde{T} = 2, 3, 4, 5, 6, 7, 8$ (unit: min).

It can be seen from Table 3 that the relative differences between the upper and lower bounds of the three sets of OD pairs at different time thresholds are within 7%. For OD pairs $1 \rightarrow 25$ and $31 \rightarrow 66$, when the time threshold is 2, the relative differences are only 0.61% and 0.85%; and the maximum relative difference 6.58% occurs when OD pair $11 \rightarrow 44$ takes the time threshold 8. In practical applications, the relative difference is within an acceptable range. In order to analyze the relative differences of the three sets of OD pairs at different time thresholds more clearly, Fig. 7 depicts the curve of relative difference. As shown in Fig. 7, the relative difference of the third set of experiments increases with the increase of the time threshold, and the relative difference between the upper and lower bounds increases gradually, which indicates that the earlier the real-time information is obtained, the solution is closer to the optimal solution.

5.1.3. Experiments of different number of scenarios

The following is an analysis of the test results of the three sets of OD pairs in different scenarios, wherein the number of scenarios is 5, 10, 15, 20, and 25, respectively. Table 4 gives three groups of OD pair (i.e., $1 \rightarrow 25$, $11 \rightarrow 44$ and $31 \rightarrow 66$) about relative gaps and computational

Table 3
Experiment results in different scenarios.

OD pair	Time Threshold	Lower Bound	Upper Bound	Relative Gaps
	2	13815.1	13900	0.61%
	3	13807.2	13954.4	1.05%
	4	13814.4	13997	1.30%
$1 \rightarrow 25$	5	13799.4	13997	1.41%
	6	13795.8	14269.4	3.32%
	7	13785.7	14075.2	2.06%
	8	13792.4	14265.8	3.32%
	2	13611.6	13801.8	1.37%
	3	13585.6	13838.4	1.83%
	4	13548.9	14060	3.63%
$11 \rightarrow 44$	5	13496.5	14110.4	4.35%
	6	13496.3	14124.4	4.45%
	7	13473.5	14314	5.87%
	8	13376	14318	6.58%
	2	18154.9	18310.5	0.85%
	3	18092.9	18329.4	1.29%
	4	18042.4	18520	2.58%
$31 \rightarrow 66$	5	18007.8	18650	3.44%
	6	17944.7	18690	3.98%
	7	17911.5	18649	3.95%
	8	17896.2	18673.1	4.16%

times with different numbers of scenarios. As the model size increases with the number of scenarios, it can be seen from Table 4 that the solution time of the sub-gradient optimization algorithm increases gradually as the number of scenario increases. At the same time, Fig. 8 visually depicts the change of the relative gaps between the upper and lower bounds of the model under different numbers of scenarios. It can be seen from this figure that, the relative difference between the upper and lower bounds of the model increases as the number of scenes increases, which indicates that the increase in the number of scenarios

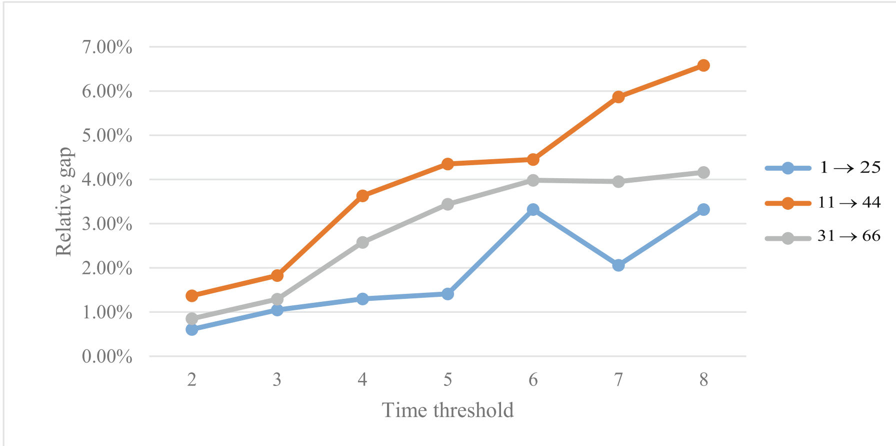

Fig. 7. Relative gaps in different time thresholds.

Table 4
Experiment results in different scenarios.

OD pair	Number of Scenarios	Relative Gaps	Run Time (hh:mm:ss)
	5	0.26%	00:00:23
	10	0.59%	00:00:55
1 → 25	15	0.37%	00:01:44
	20	2.18%	00:02:42
	25	3.35%	00:03:32
	5	0.10%	00:00:24
	10	1.40%	00:00:59
11 → 44	15	2.11%	00:01:41
	20	2.58%	00:02:36
	25	3.12%	00:03:44
	5	1.70%	00:00:24
	10	0.89%	00:01:00
31 → 66	15	1.91%	00:01:40
	20	1.95%	00:02:42
	25	3.42%	00:03:58

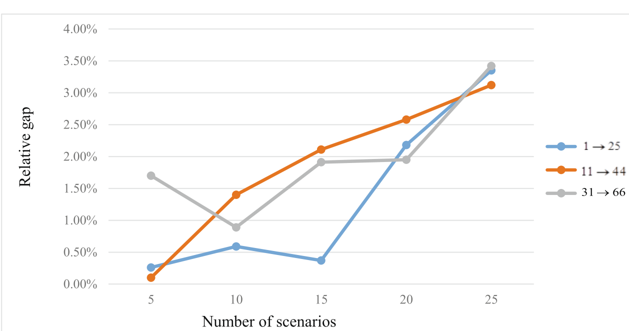

Fig. 8. Relative gaps in different number of scenarios.

leads to the increases in the randomness of the travel time and capacity on each link, thereby deteriorating the quality of the solution.

5.1.4. Comparisons on proposed model and its variants

In order to test the effectiveness of proposed model, this paper compares the deterministic model, WAS model and the two-stage stochastic evacuation model. For the deterministic model, the link travel times and capacities are assumed to be the average value of all scenarios. For instance, it is assumed that the number of scenarios is 10, time threshold $\widetilde{T} = 4$ on OD pair 1 → 25. The objective value of deterministic is then can be achieved, i.e., 14786.5, which is greater than the objective value (13997 in Table 3) of two-stage stochastic evacuation model. This observation implies that the two-stage stochastic evacuation model performs better than the deterministic model.

Furthermore, to testify the robust of optimal evacuation plan obtained by the proposed approach in this paper, a comparison is given to compare the original model and WAS model on three OD pairs, i.e., 1 → 15, 11 → 44 and 31 → 66, with 10 scenarios.

Firstly, the value of the second stage of original model can be achieved based on the optimal solution (abbreviated by VSOM). The second stage means that, the affected people evacuation according to the *a priori* plan before the time threshold $\widetilde{T}$, and the affected people will select the evacuation path according the real-time information after the threshold $\widetilde{T}$. The optimal values of WAS model on each scenario are also calculated (abbreviated by VWAS). Secondly, the standard deviation between the VSOM and VWAS is calculated, which represents the dispersion degree between the evacuation plan obtained by the proposed model and those achieved by the WAS model on each scenario, shown in Case 1 of Table 5. Thirdly, instead of two-stage stochastic programming model, if the decision-maker determines the evacuation plan by the WAS model, the standard deviations between VWASs (the VWAS on scenario s, $s = 1,2,..., S$) and VWAS are also given in Cases 2–11 of Table 5.

It can be observed from Table 5 that the standard deviation between VSOM and VWAS is smaller than that in Cases 2–11 expect for Case 6 on OD pair 11→44, which indicates that the *a priori* evacuation plans solved by the proposed approach in this paper is robust than those derived by WAS model.

5.2. Large-scale experiments

The proposed Lagrangian relaxation-based approach in this paper can further solve the large-scale problem. The experiments on large number of scenarios and large-scale network are implemented in the following.

Table 5
A comparison between the VSOM and VWAS.

Case	1 → 25	11 → 44	31 → 66
Case 1: VSOM &VWAS	411.51	661.24	703.68
Case 2: VWAS1&VWAS	490.87	733.06	839.12
Case 3: VWAS2&VWAS	434.55	1017.49	1281.76
Case 4: VWAS3&VWAS	471.72	1157.49	807.51
Case 5; VWAS4&VWAS	471.23	1121.29	766.68
Case 6: VWAS5&VWAS	854.95	653.75	1157.04
Case 7: VWAS6&VWAS	470.74	785.11	787.81
Case 8; VWAS7&VWAS	970.02	669.31	833.85
Case 9: VWAS8&VWAS	465.5	669.31	1294.36
Case 10: VWAS9&VWAS	417.87	1261.65	750.62
Case 11: VWAS10&VWAS	477.19	830.18	1035.57

Table 6
Experiment results with large number of scenarios.

OD pair	Number of Scenarios	Relative Difference	Run Time(hh:mm:ss)
	50	3.00%	00:11:45
	75	2.23%	00:24:40
1 → 25	100	2.34%	00:35:58
	150	2.66%	01:22:12
	200	2.50%	02:07:03

5.2.1. Large number of scenarios on grid network

Firstly, the experiments on large number of scenarios are given in Table 6. From the computational results we can observe that the proposed approach can still solve the problem with large number of scenarios effectively. For example, the run time with 100 and scenarios is 35’58” and 2:07’03” respectively, and the run time increases more significantly with the increasing number of scenarios. It is worthy to note that the algorithm is conducted on a PC platform with an i5-8250U 1.6 GHz CPU and 8 GB memory, nevertheless the computational efficiency will increase greatly if the algorithm is conducted on the enterprise server.

5.2.2. Experiments on large-scale network

The performance of proposed approach is conducted on a real network, Chicago Sketch network, consisting 933 nodes and 2950 links (see Fig. 9). In this experiment, the OD pairs are randomly selected, the set of random link travel times on 10 scenarios are generated according to link travel times derived by the actual data of link lengths and speeds, and the time threshold is assumed as 4. This algorithm begins with a set of randomly generated multipliers, and terminates with a total of 10 iterations or the relative gap that is less than 0.001 percent.

In Table 7 total of 10 cases are implemented and their relative gaps and run times are listed. Each of relative gap in these cases is able to converge to a small value (less than 8%), in which 8 cases are within 5%. The experiment results imply that the proposed approach can achieve the approximately optimal solutions with higher quality. Moreover, the average run time of these cases is 12’36” and it is an acceptable time. In comparison to the run time, 55 s, on above grid network with 10 scenarios, the run time in this network increases significantly since the number of nodes and links of this Chicago sketch network is much larger than that in the grid network.

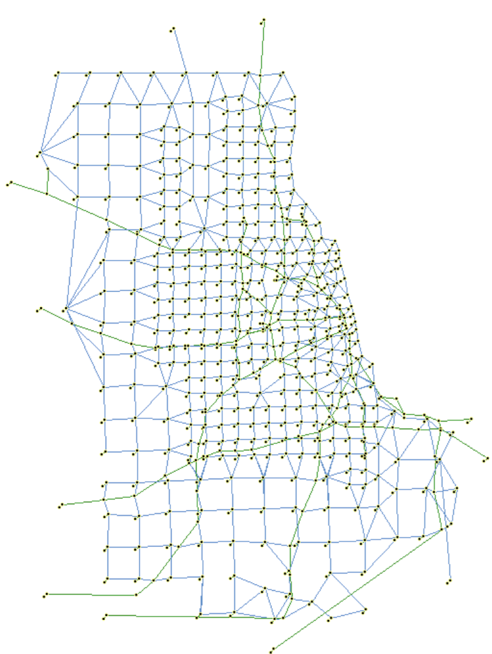

Fig. 9. Chicago sketch network.

Table 7
Experiment results on large-scale network.

OD pair	Lower Bound	Upper Bound	Relative Difference	Run Time
442 → 849	19338.2	20627	6.24%	00:11:52
639 → 570	12420.1	12664.5	1.93%	00:14:19
638 → 578	16146	16345.3	1.22%	00:12:41
681 → 592	14193.7	14803.6	4.12%	00:13:24
918 → 741	19196.6	19393.4	1.01%	00:10:11
551 → 615	15903.3	16623.3	4.33%	00:17:02
918 → 805	20473.3	21253	3.67%	00:10:26
921 → 752	9849.13	10038.2	1.88%	00:10:54
801 → 397	26975.9	27670.5	2.51%	00:13:47
860 → 669	26975.9	10763.2	1.21%	00:11:52
549 → 691	21316.9	23100	7.72%	00:13:22

6. Conclusions and future research

This paper applied the two-stage stochastic program to obtain a robust predetermined evacuation plan in the first stage by consideration of the uncertain scenarios in the second stage. To obtain the robust *a priori* evacuation plan, this paper firstly transformed the evacuation network with multiple sources and sinks to a network with single supersource and supersink, and then the two-stage stochastic network flow model were proposed. Since the proposed model was an NP-hard problem, and thus this paper adopted the Lagrangian relaxation approach to decompose the original model into two trackable subproblems. Furthermore, we designed the subgradient optimization algorithm embedded successive shortest path algorithm to obtain the approximate optimal solution.

This paper mainly discusses the theoretical model and algorithm, and the actual road network environment is more complicated than that considered in this paper. For example, the two-stage stochastic programming model proposed in this paper only from the point of view of decision-makers, but without considering the choice behavior of affected people in the evacuation process. Therefore, in the future, we can further consider the choice behavior of affected people in the actual evacuation process, and formulate a bi-level programming model by comprehensively considering the common interests of the decision-makers and affected persons. In addition, robust optimization is an effective method to deal with the problem interest of this paper. Therefore, robust optimization method based on two-stage stochastic programming framework for evacuating the affected people from dangerous areas to safe areas can be investigated in the future work, especially in the condition of the scenarios and the their probabilities are difficult to determine (Ni et al., 2018).

CRediT authorship contribution statement

Li Wang: Conceptualization, Data curation, Formal analysis, Funding acquisition, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing.

Acknowledgements

This research was supported by the National Natural Science Foundation of China (No. 71801018).

References

Barbarosogˇlu, G., & Arda, Y. (2004). A two-stage stochastic programming framework for transportation planning in disaster response. *Journal of the Operational Research Society, 55*(1), 43–53.
Ben-Tal, A., Chung, B. D., Mandala, S. R., & Yao, T. (2011). Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains.

Transportation Research Part B, 45(8), 1177–1189.
Birge, J. R., & Louveaux, F. (1997). *Introduction to Stochastic Programming.* New York: Springer.
Bretschneider, S., & Kimms, A. (2011). A basic mathematical model for evacuation problems in urban areas. *Transportation Research Part A: Policy and Practice, 45*(6), 523–539.
Busacker, R. G., & Gowen, P. J. (1961). A procedure for determining a family of minimum-cost flow patterns. *Operations Research Office Technical Report, 15*.
Cheung, R. K., & Powell, W. B. (1996). An algorithm for multistage dynamic networks with random arc capacities, with an application to dynamic fleet management. *Operations Research, 44*(6), 951–963.
Chiu, Y. C., & Mirchandani, P. B. (2008). Online behavior-robust feedback information routing strategy for mass evacuation. *IEEE Transactions on Intelligent Transportation Systems, 9*(2), 264–274.
Cova, T. J., & Johnson, J. P. (2003). A network flow model for lane-based evacuation routing. *Transportation Research Part A: Policy and Practice, 37*(7), 579–604.
Dantzig, G. (1955). Linear Programming under Uncertainty. *Management Science, 1*, 197–206.
Davoodi, S. M. R., & Goli, A. (2019). An integrated disaster relief model based on covering tour using hybrid Benders decomposition and variable neighborhood search: Application in the Iranian context. *Computers & Industrial Engineering, 130*, 370–380.
Faturechi, R., & Miller-Hooks, E. (2014). Travel time resilience of roadway networks under disaster. *Transportation Research Part B: Methodological, 70*, 47–64.
Fisher, M. L. (1981). The Lagrangian relaxation method for solving integer programming problems. *Management Science, 27*(1), 1–18.
Grass, E., & Fischer, K. (2016). Two-stage stochastic programming in disaster management: a literature survey. *Surveys in Operations Research and Management Science* S1876735416300162.
Hamacher, H., & Tjandra, S. (2001). *Mathematical modeling of evacuation problems: A state of art.* Berichte des Frauenhofer ITWM No. 24.
Helbing, D., Farkas, I. J., Molnár, P., & Vicsek, T. (2002). *Simulation of pedestrian crowds in normal and evacuation situations* (pp. 21–58). Springer.
Iri, M. (1960). A new method of solving transportation-network problems. *Journal of the Operations Research Society of Japan, 3*(1), 2.
Jewell, W. S. (1962). New methods in mathematical programming optimal flow through networks with gains. *Operations Research, 10*(4), 476–499.
Kulshrestha, A., Wu, D., Lou, Y., & Yin, Y. (2011). Robust shelter locations for evacuation planning with demand uncertainty. *Journal of Transportation Safety & Security, 3*(4), 272–288.
Li, J., Ozbay, K., & Bartin, B. (2015). Effects of Hurricanes Irene and Sandy in New Jersey: Traffic patterns and highway disruptions during evacuations. *Natural Hazards, 78*(3), 2081–2107.
Li, X., Li, Q., & Claramunt, C. (2018). A time-extended network model for staged evacuation planning. *Safety Science, 108*, 225–236.
Liu, Y., Cui, N., & Zhang, J. (2019). Integrated temporary facility location and casualty allocation planning for post-disaster humanitarian medical service. *Transportation Research Part E: Logistics and Transportation Review, 128*, 1–16.
Lu, X., Yang, Z., Cimellaro, G. P., & Xu, Z. (2019). Pedestrian evacuation simulation under the scenario with earthquake-induced falling debris. *Safety Science, 114*, 61–71.
Miller-Hooks, E., & Sorrel, G. (2008). Maximal dynamic expected flows problem for emergency evacuation planning. *Transportation Research Record: Journal of the Transportation Research Board, 44*(2089), 26–34.
Murray-Tuite, P., & Mahmassani, H. S. (2003). Model of household trip-chain sequencing in an emergency evacuation. *Transportation Research Record: Journal of the Transportation Research Board, 1831*, 21–29.
Ng, M. W., & Waller, S. T. (2010). Reliable evacuation planning via demand inflation and supply deflation. *Transportation Research Part E: Logistics and Transportation Review, 46*(6), 1086–1094.
Ng, M. W., & Lin, D. Y. (2015). Sharp probability inequalities for reliable evacuation planning. *Transportation Research Part C: Emerging Technologies, 60*, 161–168.
Ni, W., Shu, J., & Song, M. (2018). Location and emergency inventory pre-positioning for disaster response operations: Min-max robust model and a case study of Yushu earthquake. *Production and Operations Management, 27*(1), 160–183.
Najafi, M., Eshghi, K., & Dullaert, W. (2013). A multi-objective robust optimization model for logistics planning in the earthquake response phase. *Transportation Research Part E, 49*(1), 217–249.
Powell, W. B., & Cheung, K. M. (1994). A network recourse decomposition method for dynamic networks with random arc capacities. *Networks, 24*(7), 369–384.
Qu, Y., Gao, Z., Xiao, Y., & Li, X. (2014). Modeling the pedestrian's movement and simulating evacuation dynamics on stairs. *Safety Science, 70*(70), 189–201.
Rawls, C. G., & Turnquist, M. A. (2010). Pre-positioning of emergency supplies for disaster response. *Transportation research part B: Methodological, 44*(4), 521–534.
Rennemo, S. J., Rø, K. F., Hvattum, L. M., & Tirado, G. (2014). A three-stage stochastic facility routing model for disaster response planning. *Transportation Research Part E: Logistics and Transportation Review, 62*, 116–135.
Sbayti, H., & Mahmassani, H. S. (2006). Optimal scheduling of evacuation operations. *Transportation Research Record: Journal of the Transportation Research Board, 1964*(1), 238–246.
Wang, L., Yang, L., & Gao, Z. (2016). The constrained shortest path problem with stochastic correlated link travel times. *European Journal of Operational Research, 255*(1), 43–57.
Wang, L., Yang, L., Gao, Z., Li, S., & Zhou, X. (2016). Evacuation planning for disaster responses: A stochastic programming framework. *Transportation Research Part C: Emerging Technologies, 69*, 150–172.
Xie, C., & Turnquist, M. A. (2011). Lane-based evacuation network optimization: an integrated lagrangian relaxation and tabu search approach. *Transportation Research Part C: Emerging Technologies, 19*(1), 40–63.
Xie, J., Xing, W., & Wang, Z. (2009). *Network Optimization*. Tsinghua University Press.
Yamada, T. (1996). A network approach to a city emergency evacuation planning. *International Journal of Systems Science, 27*(10), 931–936.
Yang, L., & Zhou, X. (2014). Constraint reformulation and Lagrangian relaxation-based solution algorithm for a least expected time path problem. *Transportation Research Part B: Methodological, 59*, 22–44.
Yang, L., & Zhou, X. (2017). Optimizing on-time arrival probability and percentile travel time for elementary path finding in time-dependent transportation networks: Linear mixed integer programming reformulations. *Transportation Research Part B: Methodological, 97*, 182–213.
Yao, T., Mandala, S. R., & Chung, B. D. (2009). Evacuation transportation planning under uncertainty: a robust optimization approach. *Networks and Spatial Economics, 9*(2), 171–189.
Yazici, M. A., & Ozbay, K. (2010). Evacuation network modeling via dynamic traffic assignment with probabilistic demand and capacity constraints. *Transportation Research Record: Journal of the Transportation Research Board, 46*(2196), 11–20.
Zhang, J., Liu, Y., Zhao, Y., & Deng, T. (2018). Emergency evacuation problem for a multi-source and multi-destination transportation network: Mathematical model and case study. *Annals of Operations Research,* 1–29.
Zhang, J., Liu, H., Yu, G., Ruan, J., & Chan, F. T. (2019). A three-stage and multi-objective stochastic programming model to improve the sustainable rescue ability by considering secondary disasters in emergency logistics. *Computers & Industrial Engineering, 135*, 1145–1154.
Ziliaskopoulos, A., & Mahmassani, H. (1992). Design and implementation of a shortest path algorithm with time dependent arc costs. *In proceedings of the 5th advanced technology conference, DC. USA*.