1. Fill in the following table of numbers in decimal, binary, octal, and hexadecimal.

base	example				
decimal	10	256	512	32	512
binary	b1010	b10000000	b1000000000	b100000	b1000000000
octal	o12	o400	o1000	o40	o1000
hexadecimal	0xA	0x100	0x200	0x20	0x200

note: why are there duplicate columns?

base	example				
decimal	10	31582	153	196	65535
binary	b1010	b111101101011110	b10011001	b11000100	b1...1 (2^{16})
octal	o12	o75536	o231	o304	o177777
hexadecimal	0xA	0x7B5E	0x99	0xC4	0xFFFF

note: I used expansion steps in a calculator, e.g., o75536:

$$7 \cdot 8^4 + 5 \cdot 8^3 + 5 \cdot 8^2 + 3 \cdot 8^1 + 6 = 31582$$

but then it got tedious rather than insightful, so then I used python.

2. Complete the following: (A:10,B:11,C:12,D:13,E:14,F:15)

$$\begin{array}{r} \texttt{0x189} \\ + \ \texttt{0x345} \\ \hline \texttt{0x4CE} \end{array} \qquad \begin{array}{r} \texttt{b1010010100} \\ + \ \texttt{b0101101011} \\ \hline \texttt{b1111111111} \end{array} \qquad \begin{array}{r} \texttt{o743} \\ + \ \texttt{o265} \\ \hline \texttt{o1230} \end{array}$$

$$\begin{array}{r} \texttt{0xDEAD} \\ + \ \texttt{0xBEEF} \\ \hline \texttt{0x19D9C} \end{array} \qquad \begin{array}{r} \texttt{b1111111111} \\ + \qquad \texttt{b1} \\ \hline \texttt{b10000000000} \end{array} \qquad \begin{array}{r} \texttt{o100} \\ + \ \texttt{o777} \\ \hline \texttt{o1077} \end{array}$$

$$\begin{array}{r} \texttt{0x89} \\ \times \ \texttt{0xAB} \\ \hline \texttt{0x5B83} \end{array} \qquad \begin{array}{r} \texttt{b0111111111} \\ \times \ \texttt{b1000000001} \\ \hline \texttt{b1000000000} \\ + \ \texttt{b111111110000000000} \\ \hline \texttt{b11111111111111111} \end{array} \qquad \begin{array}{r} \texttt{o74} \\ \times \ \texttt{o26} \\ \hline \texttt{o2450} \end{array}$$

3. Compute the following sets. The universe for all of these sets is $\boldsymbol{Z}$, $\boldsymbol{P}$ is the set of prime numbers, $\boldsymbol{E}$ is the set of even numbers, and $\boldsymbol{O}$ is the set of odd numbers.

   (a) $\{1, 2, 3, 5, 8, 13, 21, 35\} \cup \{2, 3, 5, 7, 11, 13, 17, 19\} = \{1, 2, 3, 5, 7, 8, 11, 13, 17, 19, 21, 35\}$

   (b) $\{1, 2, 3, 5, 8, 13, 21, 35\} \cap \{2, 3, 5, 7, 11, 13, 17, 19\} = \{2, 3, 5, 13\}$

   (c) $\boldsymbol{E} \cap \boldsymbol{O} = \emptyset$

   (d) $\boldsymbol{P} \cap \boldsymbol{E} = \{x \in \boldsymbol{P} : x \in \boldsymbol{E}\}$

   (e) $\boldsymbol{N}' = \boldsymbol{N}$ ($\boldsymbol{N}$ has not been defined?, assuming $\mathbb{N}$)

   (f) $\boldsymbol{E}' \cap \boldsymbol{O}' = \emptyset$

   (g) $\{x : y \in \boldsymbol{N}, x = y^2\} \cap \{x : y \in \boldsymbol{N}, x = y^3\} = \{0, 1\}$

   (h) $P(\{1, 2, 3\}) \cap P(\{2, 3, 4\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\}\}$

4. Give the truth tables for the following. Which are tautologies, and which are satisfiable?

   (a) $a \wedge (b \vee a)$; **satisfiable**

a	b	$(b \vee a)$	(a)
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

   (b) $(a \wedge b) \vee (a \wedge a)$; **satisfiable**

a	b	$(a \wedge b)$	(b)
1	1	1	1
1	0	0	1
0	1	0	0
0	0	0	0

   (c) $a \vee \neg a$; **tautology**

a	$\neg a$	(c)
1	0	1
1	0	1
0	1	1
0	1	1

   (d) $\neg(a \vee b)$; **satisfiable**

a	b	(d)
1	1	0
1	0	0
0	1	0
0	0	1

   (e) $(\neg a) \wedge (\neg b)$; **satisfiable**

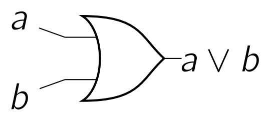
$\neg a$	$\neg b$	(e)
0	0	0
0	1	0
1	0	0
1	1	1

   (f) $\neg a \wedge (\neg b \vee c)$; **satisfiable**

a	b	c	$\neg a$	$\neg b \vee c$	(f)
1	1	1	0	1	0
1	1	0	0	0	0
1	0	1	0	1	0
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	1	1	1
0	0	0	1	1	1

5. Now we're going to learn how to add with logic. Instead of thinking of using logic with true and false ($\top$ or $\bot$), let's use 1 and 0. This yields the following truth table and circuit:

a	b	$(a \vee b)$
1	1	1
1	0	1
0	1	1
0	0	0



Now we're going to add 2 1-bit numbers, but there's a problem: adding 2 1-bit numbers might give us a two bit answer. Thus, we'll break the into two problems the bit on the right will be the sum bit s, and the bit on the left will be the carry bit c, .e.g,

$$b_1 + b_2 = cs$$

(a) give the truth table for the *sum* bit

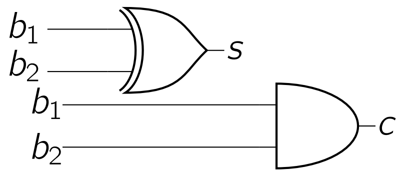
(b) give the truth table for the *carry* bit

b_1	b_2	c	s
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

(c) give the logical formulas for the sum and carry bits.

$$b_1 \wedge b_2 \implies c \qquad (b_1 \vee b_2) \wedge \neg(b_1 \wedge b_2) \text{ i.e., } b_1 \oplus b_2 \implies s$$

(d) give a circuit representing a *sum* gate. This gate should have 2 b_1, b_2 inputs and 2 outputs s, c



note: I'm not familiar with circuitikz, I'm not sure how to connect inputs.