

1. **4.35** The random variable  $X$ , representing the number of errors per 100 lines of software code, has the following probability distribution:

$x$	2	3	4	5	6
$f(x)$	0.01	0.25	0.4	0.3	0.04

Using Theorem 4.2 ( $\sigma^2 = E(X^2) - \mu^2$ ), find the variance of  $X$ .

$$\mu = \sum_{x=2}^6 xf(x) = 4.11, \quad E(X^2) = \sum_{x=2}^6 x^2 f(x) = 17.63$$

$$\Rightarrow \sigma^2 = 17.63 - 4.11^2 = \boxed{0.738}$$

2. **4.36** Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1, 2, or 3 power failures will strike a certain subdivision in any given year.

Find the mean and variance of the random variable  $X$  representing the number of power failures striking this subdivision.

$$x = [0, 1, 2, 3], \quad f(x) = [0.4, 0.3, 0.2, 0.1]$$

$$\mu = \sum_{x=0}^3 xf(x) = \boxed{1}, \quad E(X^2) = \sum_{x=0}^3 x^2 f(x) = 2$$

$$\Rightarrow \sigma^2 = 2 - 1 = \boxed{1}$$

3. **4.37** A dealer's profit, in units of \$5000, on a new automobile is a random variable  $X$  having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{else} \end{cases}$$

Find the variance of  $X$ .

$$\mu = \int_0^1 2x(1-x) dx = \frac{1}{3}$$

$$E(X^2) = \int_0^1 2x^2(1-x) dx = \frac{1}{6}$$

$$\sigma^2 = \frac{1}{6} - \frac{1}{3^2} = \frac{1}{18} \Rightarrow \text{var}(X) = \boxed{\frac{5000^2}{18}}$$

4. **4.38** The proportion of people who respond to a certain mail-order solicitation is a random variable  $X$  having the density function

$$f(x) = \begin{cases} \frac{2}{5}(x+2), & 0 < x < 1, \\ 0, & \text{else} \end{cases}$$

Find the variance of  $X$ .

$$\begin{aligned} \mu &= \int_0^1 \frac{2x(x+2)}{5} dx = \frac{8}{15} \\ E(X^2) &= \int_0^1 \frac{2x^2(x+2)}{5} dx = \frac{11}{30} \\ \sigma^2 &= \frac{11}{30} - \left(\frac{8}{15}\right)^2 = \boxed{0.082} \end{aligned}$$

5. **4.43 (Bonus)** The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable  $Y = 3X - 2$ , where  $X$  has the density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-\frac{x}{4}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean and variance of the random variable  $Y$ .

$$\begin{aligned} \mu_Y &= E(3X - 2) = \int_0^\infty \frac{1}{4}(3x - 2)e^{-\frac{x}{4}} dx = 10 \\ E(Y^2) &= \int_0^\infty \frac{1}{4}(3x - 2)^2 e^{-\frac{x}{4}} dx = 244 \\ \sigma_Y^2 &= 244 - 10^2 = \boxed{144} \end{aligned}$$

6. **4.50** For a laboratory assignment, if the equipment is working, the density function of the observed outcome  $X$  is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{elsewhere} \end{cases}$$

Find the variance and standard deviation of  $X$ .

$$\begin{aligned} \sigma^2 &= \boxed{\frac{1}{18}} && \text{by question 3 (4.37)} \\ \sigma &= \boxed{\sqrt{\frac{1}{18}}} \end{aligned}$$

7. **4.54** Using Theorem 4.5 and Corollary 4.6, i.e.,

$$E(aX + b) = aE(X) + b, \quad b = 0 \implies \sigma_{aX+c}^2 = a^2\sigma_X^2 = a^2\sigma^2,$$

find the mean and variance of the random variable  $Z = 5X + 3$ , where  $X$  has the probability distribution of Exercise 4.36 (Problem 2,  $\mu = 1$ ,  $\sigma^2 = 1$ ).

$$\sigma_{5X+3}^2 = 5^2(1) = \boxed{25}$$

8. **4.71 (Bonus)** The length of time  $Y$ , in minutes, required to generate a human reflex to tear gas has the density function

$$f(y) = \begin{cases} \frac{1}{4}e^{-\frac{y}{4}}, & 0 \leq y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- (a) What is the mean time to reflex?

$$\mu = \int_0^{\infty} \frac{1}{4}ye^{-\frac{y}{4}} dy = \boxed{4}$$

- (b) Find  $E(Y^2)$  and  $\text{var}(Y)$ .

$$E(Y^2) = \int_0^{\infty} \frac{1}{4}y^2e^{-\frac{y}{4}} dy = \boxed{32}$$

$$\sigma^2 = 32 - 4^2 = \boxed{16}$$

9. **4.101** Consider Review Exercise 3.73 on page 108. It involved  $Y$ , the proportion of impurities in a batch, and the density function is given by

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \leq y < 1, \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Find the expected percentage of impurities.

$$\mu = \int_0^1 10y(1-y)^9 dy = \frac{1}{11} = \boxed{0.09}$$

- (b) Find expected value of proportion of quality material, i.e.,  $E(1 - Y)$ .

$$E(1 - Y) = 1 - 0.09 = \boxed{0.91}$$

- (c) Find the variance of the random variable  $Z = 1 - Y$ .

$$\sigma_Z^2 = \sigma_{1-Y}^2 = \sigma_Y^2 = \int_0^1 10y^2(1-y)^9 dy - \frac{1}{11^2} = \frac{1}{66} - \frac{1}{11^2} = \frac{5}{726} = \boxed{0.0068}$$

10. **4.62** If  $X$  and  $Y$  are independent random variables with variances  $\sigma_X^2 = 5$  and  $\sigma_Y^2 = 3$ , find the variance of the random variable  $Z = -2X + 4Y - 3$ .
11. **4.63** Repeat Exercise 4.62 if  $X$  and  $Y$  are not independent and  $\sigma_{XY} = 1$ .
12. Let  $X$  and  $Y$  be random variables with the following information:
- $$E(X) = 6, \quad E(Y) = -\frac{1}{2}, \quad \text{var}(X) = 4, \quad \text{var}(Y) = 6, \quad \text{cov}(X, Y) = 2$$
- (a) Compute  $E(3X - 4Y)$
  - (b) Compute  $\text{var}(3X - 4Y)$
  - (c) Compute  $E(2X - Y^2)$

13. Let  $X$  and  $Y$  be independent random variables with the following information:

$$E(X) = -1, \quad E(Y) = 4, \quad \text{var}(X) = 6, \quad \text{var}(Y) = 8$$

- (a) Compute  $E(9X + 2Y)$
- (b) Compute  $\text{var}(9X + 2Y)$

14. **6.3** The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable  $X$  having a continuous uniform distribution with  $A = 7$  and  $B = 10$ .

Find the probability that on a given day the amount of coffee dispensed by this machine will be

- (a) at most 8.8 liters;
- (b) more than 7.4 liters but less than 9.5 liters;
- (c) at least 8.5 liters.

15. **6.4** A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform

- (a) What is the probability that the individual waits more than 7 minutes?
- (b) What is the probability that the individual waits between 2 and 7 minutes?