1. For parts (a) and (b) below, let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(a) Common notation for the vector with a 1 in the third entry and zeros elsewhere (with the total number of entries given via context) is $\boldsymbol{e}_3$. That is, in this case

$$\boldsymbol{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Calculate $\boldsymbol{Ae}_3$

$$\boldsymbol{Ae}_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

(b) Calculate $\boldsymbol{Ae}_4$

$$\boldsymbol{Ae}_4 = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

(c) Write a sentence that explains what happens when you multiply an arbitrary matrix $\boldsymbol{B}$ by $\boldsymbol{e}_j$ where $\boldsymbol{B}$ and $\boldsymbol{e}_j$ have compatible sizes.

- Multiplication of a matrix $\boldsymbol{B}$ with a vector $\boldsymbol{e}$ where the number of columns in $\boldsymbol{B}$ is equal to the number of rows in $\boldsymbol{e}$ results in the weighted combination of columns in $\boldsymbol{B}$.
- In the case of standard basis vectors, where $\boldsymbol{e}_j$ denotes the vector with a 1 in the j-th row and 0s elsewhere, then the result is simply the j-th column of $\boldsymbol{B}$.

2. In the parts below, let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Calculate $\boldsymbol{AB}$. Write what you observed in a sentence.

$$\boldsymbol{AB} = \begin{bmatrix} 1 & 2 & -9 & 4 \\ 1 & 2 & -9 & 4 \\ 1 & 2 & -9 & 4 \\ 1 & 2 & -9 & 4 \end{bmatrix}$$

- The row of the matrix $\boldsymbol{B}$ with the scaled basis vector $(1 \to -3)$ results in the scaling of the column of the original matrix $\boldsymbol{A}$ by that same amount $(3 \to -9)$.
- In general, post-multiplication of a diagonal matrix (all elements outside the main diagonal are zero) results in the **scaling of the columns** of the original matrix.

(b) Calculate $\boldsymbol{BA}$. Write what you observed in a sentence.

$$\boldsymbol{BA} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -3 & -6 & -9 & -12 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

- Similar to (a), but the pre-multiplication of the diagonal matrix results in the **scaling of the rows** of the original matrix.

(c) Use what you learned in (b) to create a 4×4 matrix $\boldsymbol{C}$ that would

i. zero out the first row

ii. multiply the second row by -1

iii. multiply the third row by 5

iv. leave the last row the same

when you pre-multiply a size-compatible matrix $\boldsymbol{M}$ by $\boldsymbol{C}$.

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{M} = \begin{bmatrix} a & a & a & a \\ b & b & b & b \\ c & c & c & c \\ d & d & d & d \end{bmatrix}$$

$$\boldsymbol{CM} = \begin{bmatrix} 0a & 0a & 0a & 0a \\ -1b & -1b & -1b & -1b \\ 5c & 5c & 5c & 5c \\ 1d & 1d & 1d & 1d \end{bmatrix}$$