Problem 0.1 (MTH 371 Winter 2022, Homework # 3: due February 23). *Given a $n \times n$ symmetric sparse matrix A (one from the test matrices provided).*

- *Form the following graph matrices associated with the sparsity pattern of A and store them in CSR format:*
  - *Adjacency matrix $\mathbb{A} = (\alpha_{ij})$ where $\alpha_{ij} = 1$ for any non-zero entry a_{ij} of A and zero otherwise.*
  - *The edge_vertex connectivity matrix E where in row e of E we have only two nonzero entries (equal to one) at positions (e, i) and (e, j) where e runs over the edges of the graph, i.e., the pairs $e = (i, j)$ for which $a_{ij} \neq 0$.*
  - *form the transpose of E, E^T, which is the vertex_edge relation matrix.*
  - *Form the diagonal matrix D with entries d_i on the diagonal being the degree $d_i = \sum\limits_j \alpha_{ij}$ (the rowsums of the adjacency matrix $\mathbb{A}$).*
  - *Form the edge_edge adjacency matrix $\mathbb{A}_E = EE^T$ as a product of two sparse matrices.*
  - *Form the graph Laplacian matrix $\mathcal{L} = D - \mathbb{A}$.*
- *For any given (small) number K, e.g., $K = 2, 5, ...$, $K << n$.*
  - *For a given number d, $d \geq K$ and $d < n$ ($d = 2, 5, 10$) compute d eigenvectors $\mathbf{q}_1, \ldots, \mathbf{q}_d$, corresponding to the first d minimal eigenvalues of the graph Laplacian $\mathcal{L}$. For this use any available eigensolver library for symmetric (sparse) matrices. Form the coordinate vectors $\mathbf{x}_i \in \mathbb{R}^d$, $i = 1, 2, \ldots, n$, as the ith row of the eigenvector matrix $Q = [\mathbf{q}_1, \ldots, \mathbf{q}_d]$.*
  - *Either implement the K-means algorithm, or use a library that implements it, using the coordinates $\mathbf{x}_i \in \mathbb{R}^d$, of the vertices $i = 1, 2, \ldots, n$. The K-means algorithm (used for the chosen K) generate $n_c = K$ aggregates (groups of vertices) $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{n_c}$.*
    1. *Form the relation matrix P = vertex_aggregate as a CSR matrix.*
    2. *Form the coarse matrix $A_c = P^T AP$ as a CSR matrix using products of sparse matrices P^T, A and P.*
  - *Optionally: Using graph visualization software, visualize the graph partitioned into aggregates, where vertices in a given aggregate use the same color, and desirably use different colors for different aggregates.*