Course reader: *Time series signals*

- Features in time series signals can be categorized as:
  1. *Transients*: Brief, non-reoccurring events. Could be sharp (like a spike) or smooth (like a taper).
  2. *Repeating*: Events that reoccur at more-or-less regular intervals. Could also be sharp (like a square wave) or smooth (like a sine wave).

- Sharp transients are often noteworthy events, and tend to arise from noise, sudden sensor failures, or major events like a stock market crash.

- Smooth transients include the Gaussian; Hann, Hamming, and Blackman windows; and Planck taper. They are often used as smoothing kernels in signal processing and spectral analysis.

- Repeating features of a signal are those that are present throughout or that re-occur with some regular intervals. A sine wave is a canonical example, although ongoing features need not be smooth or sinusoidal.

- Three examples are shown in the video: sine waves, square waves, triangle waves. It is useful to be familiar with sine waves, because they form the basis of the Fourier transform, which is one of the most important operations in all of applied mathematics.

- Multiple signals can be mixed together. Signal mixing happens often in real measurements (think of a microphone recording sound from multiple people, traffic, birds, etc.).

- An interesting situation is when the different sources have different frequency components. In this case, it is possible to separate the sources simply by applying temporal filters.

- "Chirps" are signals that change in frequency over time. They are called chirps because they're like the sounds that birds make.

- Dipolar chirps are defined by a start and end frequency, and the signal changes smoothly and linearly from one to the other frequency.

- Multipolar and frequency-sliding chirps are more flexible: You can define any arbitrary time series of frequencies (although they must be below the Nyquist frequency, or half of the sampling rate) and the frequency of the signal will change smoothly to follow that time series.

- There are two ways to conceptualize the frequency of a signal:
  1. "Static frequency": The number of cycles per unit time, for example the number of cycles in one second (Hertz). This is the way people typically think about frequency.
  2. "Instantaneous frequency": The rate of change of the phase (timing) of the signal. This provides an estimate of the frequency at each time point.

- Instantaneous frequency is more relevant for chirps.

Exercises

Note about the exercises: I give my answers in MATLAB code but you should feel free to use whatever program (Python, C++, Julia, html5) you feel most comfortable with.

1. Adjust the decaying exponential after the impulse to produce an exponential increase leading up to the impulse.

2. In the square transient, how does the duration of the square affect the rate of "bouncing" in the amplitude spectrum?

3. How could you make the triangle transient decay twice as slowly as it increases?

4. What is the effect on the triangle time series of adding the following line of code? Try to come up with your answer before testing it in MATLAB or Python.

   ```
   signal = 1-signal;
   ```

5. In the video and in the code, I show how to compute the empirical full-width at half-maximum (FWHM). The analytical equation is $2\sigma\sqrt{2\log 2}$, where σ is the s parameter I mentioned in the video (variable `width` in the code). Implement this equation in the code. How does the analytical FWHM compare to the empirical FWHM? Does this depend on the duration of the simulation or the sampling rate?

6. The Hann taper is based on a cosine. What happens if you change the `cos` to a `sin`? Can you do something in the sine-based function so it is identical to the Hann taper?

7. The Morlet wavelet is created by point-wise multiplying a sine wave with a Gaussian. Does the resulting function change (in the time and frequency domains) if you use a Hann or Planck taper instead of Gaussian?

8. For the ongoing-square signal, what happens to the amplitude spectrum as the duration of the "up-state" of the squares gets longer? And if they get shorter? Is there a difference between setting the threshold to be -.99 vs. +.99?

9. One of the take-home messages from the "Multicomponent oscillators" lecture was that multiple distinct sources of signal can be isolated if they have little or no overlap in the frequency domain. This spectral separability was very clear in the example given in the video. In this exercise, create a composite signal by summing a 4-Hz sine wave, a square wave using a 6-Hz sine wave, and a triangle wave using an $m = 3$. Do you think the signals are still easily separable?

Answers

1. The following two adjustments will do the trick:

```
expchange = exp((1:50)/5);
signal(tidx-length(expchange)+1:tidx) = expchange./max(expchange);
```

2. Shorter squares lead to "longer bounces." This happens because the non-stationarities in the signal are more extreme. As the square gets shorter, the transient approaches the impulse, which has a uniform power spectrum.

3. One solution:

```
y = cumsum([ ones(1,duridx) -.5*ones(1,duridx*2) ]);
```

4. In the time domain, the signal flips from being an upward-pointing triangle to a downward-pointing triangle. In the frequency domain, only the 0-Hz component changes; the rest of the spectrum is preserved. That's because spectral amplitude has nothing to do with the sign or direction of the signal (that would affect only the phases).

5. MATLAB code is
`fwhm = 2*sqrt(2*log(2))*width;`
For simulations of > 3 seconds and > 100 Hz sampling rate, the two solutions are the same to 2-3 significant digits.

6. Sine and cosine are related by a phase-shift of $\pi/2$. Therefore, changing `cos` to `sin` and adding a phase of `+pi/2` will produce a Hann taper using a half-sine wave.

7. For the Hann: no, not much, except that you no longer easily control the width of the wavelet the way you can with the Gaussian. The Planck window is more different because of the flat-top, but this depends on the parameters you select for the Planck and the Gaussian.

8. The "pulses" (harmonics resulting from rhythmic edges) change in height but not in frequency (unless you change the sine wave frequency). There is no difference in the amplitude spectrum of the two extreme thresholds except for the DC (0-Hz) component.

9. The answer is no; the signals are no longer easily separable. The non-stationarities in the signal make a clean spectral separation difficult. The signals are likely to be separable using more advanced source-space separation methods that you would learn about in an advanced linear algebra course.