

Calculus III Final Review

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Convergence: 10.3–10.5

Convergence Notes

◦ Fundamentals

- Let $\sum_{n=1}^{\infty} a_n$ be given and note for which series for which convergence is known, i.e., the geometric series and p -series:

Geometric: let $c \neq 0$, if $|r| < 1$, then **p -Series:** converges if $p > 1$.

$$\sum_{n=0}^{\infty} cr^n = \frac{c}{1-r}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- **The n^{th} Term Divergence Test:** a relatively easy test that can be used to quickly determine if a test diverges if the $\lim_{n \rightarrow \infty} a_n \neq 0$. If $\lim_{n \rightarrow \infty} a_n = 0$, then the test is inconclusive and other tests must be applied.

◦ Tests for Positive Series

- **Direct Comparison Test:** use if dropping terms from the denominator or numerator gives a series b_n wherein convergence is easily found, then compare to the original series a_n as follows:

$$\sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges} \quad \leftarrow 0 \leq a_n \leq b_n$$

$$\sum_{n=1}^{\infty} b_n \text{ diverges} \implies \sum_{n=1}^{\infty} a_n \text{ diverges} \quad \leftarrow 0 \leq b_n \leq a_n$$

- **Limit Comparison Test:** use when the direct comparison test isn't convenient or when comparing two series. One can take the dominant term in the numerator and denominator from a_n to form a new positive sequence b_n if needed.

Assuming the following limit $L = \lim_{n \rightarrow \infty} \frac{a_n}{b_n}$ exists, then:

$$L > 0 \implies \sum_{n=1}^{\infty} a_n \text{ converges} \iff \sum_{n=1}^{\infty} b_n \text{ converges}$$

$$L = 0 \text{ and } \sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

$$L = \infty \text{ and } \sum_{n=1}^{\infty} a_n \text{ converges} \implies \sum_{n=1}^{\infty} b_n \text{ converges}$$

- **Ratio Test:** often used in the presence of a factorial ($n!$) or when the are constants raised to the power of n (c^n).

Assuming the following limit $\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ exists, then

$$\rho < 1 \implies \sum a_n \text{ converges absolutely}$$

$$\rho > 1 \implies \sum a_n \text{ diverges}$$

$$\rho = 1 \implies \text{test is inconclusive}$$

- **Root Test:** used when there is a term in the form of $f(n)^{g(n)}$.

Assuming the following limit $C = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$ exists, then

$$C < 1 \implies \sum a_n \text{ converges absolutely}$$

$$C > 1 \implies \sum a_n \text{ diverges}$$

$$C = 1 \implies \text{test is inconclusive}$$

- **Integral Test:** if the other tests fail and $a_n = f(n)$ is a decreasing function, then one can use the improper integral $\int_1^{\infty} f(x) dx$ to test for convergence.

Let $a_n = f(n)$ be a positive, decreasing, and continuous function $\forall x \geq 1$, then:

$$\int_1^{\infty} f(x) dx \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

$$\int_1^{\infty} f(x) dx \text{ diverges} \implies \sum_{n=1}^{\infty} a_n \text{ diverges}$$

○ Tests for Non-Positive Series

- **Alternating Series Test:** used for series in the form $\sum_{n=0}^{\infty} (-1)^n a_n$

Converges if $|a_n|$ decreases monotonically ($|a_{n+1}| \leq |a_n|$) and if $\lim_{n \rightarrow \infty} a_n = 0$

- **Absolute Convergence:** used if the series $\sum a_n$ is not alternating; simply test if $\sum |a_n|$ converges using the test for positive series.

Convergence Problems

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Power Series: 10.6

Power Series Notes

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Power Series Problems

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Taylor Series: 10.7–10.8

Taylor Series Notes

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Taylor Series Problems

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Parametric Equations: 11.1

Parametric Notes

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Parametric Problems

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Arc Length, Polar Coordinates: 11.2–11.4

- **Polar Coordinates Notes**

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Polar Coordinate Problems

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Conic Sections

Conic Sections Notes

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Conic Section Problems

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Quiz Questions

Quiz 3

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Quiz 4

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Final Review Questions