

# Calculus III Final Review

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# Convergence: 10.3–10.5

## Convergence Notes

### ◦ Fundamentals

- Let  $\sum_{n=1}^{\infty} a_n$  be given and note for which series for which convergence is known, i.e., the geometric series and  $p$ -series:

**Geometric:** let  $c \neq 0$ , if  $|r| < 1$ , then       **$p$ -Series:** converges if  $p > 1$ .

$$\sum_{n=0}^{\infty} cr^n = \frac{c}{1-r}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- **The  $n^{\text{th}}$  Term Divergence Test:** a relatively easy test that can be used to quickly determine if a test diverges if the  $\lim_{n \rightarrow \infty} a_n \neq 0$ . If  $\lim_{n \rightarrow \infty} a_n = 0$ , then the test is inconclusive and other tests must be applied.

### ◦ Tests for Positive Series

- **Direct Comparison Test:** use if dropping terms from the denominator or numerator gives a series  $b_n$  wherein convergence is easily found, then compare to the original series  $a_n$  as follows:

$$\sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges} \quad \leftarrow 0 \leq a_n \leq b_n$$

$$\sum_{n=1}^{\infty} b_n \text{ diverges} \implies \sum_{n=1}^{\infty} a_n \text{ diverges} \quad \leftarrow 0 \leq b_n \leq a_n$$

- **Limit Comparison Test:** use when the direct comparison test isn't convenient or when comparing two series. One can take the dominant term in the numerator and denominator from  $a_n$  to form a new positive sequence  $b_n$  if needed.

Assuming the following limit  $L = \lim_{n \rightarrow \infty} \frac{a_n}{b_n}$  exists, then:

$$L > 0 \implies \sum_{n=1}^{\infty} a_n \text{ converges} \iff \sum_{n=1}^{\infty} b_n \text{ converges}$$

$$L = 0 \text{ and } \sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

$$L = \infty \text{ and } \sum_{n=1}^{\infty} a_n \text{ converges} \implies \sum_{n=1}^{\infty} b_n \text{ converges}$$

- **Ratio Test:** often used in the presence of a factorial ( $n!$ ) or when the are constants raised to the power of  $n$  ( $c^n$ ).

Assuming the following limit  $\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$  exists, then

$$\rho < 1 \implies \sum a_n \text{ converges absolutely}$$

$$\rho > 1 \implies \sum a_n \text{ diverges}$$

$$\rho = 1 \implies \text{test is inconclusive}$$

- **Root Test:** used when there is a term in the form of  $f(n)^{g(n)}$ .

Assuming the following limit  $C = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$  exists, then

$$C < 1 \implies \sum a_n \text{ converges absolutely}$$

$$C > 1 \implies \sum a_n \text{ diverges}$$

$$C = 1 \implies \text{test is inconclusive}$$

- **Integral Test:** if the other tests fail and  $a_n = f(n)$  is a decreasing function, then one can use the improper integral  $\int_1^{\infty} f(x) dx$  to test for convergence.

Let  $a_n = f(n)$  be a positive, decreasing, and continuous function  $\forall x \geq 1$ , then:

$$\int_1^{\infty} f(x) dx \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

$$\int_1^{\infty} f(x) dx \text{ diverges} \implies \sum_{n=1}^{\infty} a_n \text{ diverges}$$

## ○ Tests for Non-Positive Series

- **Alternating Series Test:** used for series in the form  $\sum_{n=0}^{\infty} (-1)^n a_n$

Converges if  $|a_n|$  decreases monotonically ( $|a_{n+1}| \leq |a_n|$ ) and if  $\lim_{n \rightarrow \infty} a_n = 0$

- **Absolute Convergence:** used if the series  $\sum a_n$  is not alternating; simply test if  $\sum |a_n|$  converges using the test for positive series.

## Convergence Problems

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# Power Series: 10.6

## Power Series Notes

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## Power Series Problems

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# Taylor Series: 10.7–10.8

## Taylor Series Notes

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## Taylor Series Problems

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# Parametric Equations: 11.1

## Parametric Notes

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## Parametric Problems

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# Arc Length, Polar Coordinates: 11.2–11.4

- **Polar Coordinates Notes**

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## **Polar Coordinate Problems**

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# Conic Sections: 11.5

## Conic Sections Notes

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## Conic Section Problems

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## Quiz Questions

### Quiz 3

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### Quiz 4

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# Final Review Questions