

1. Consider the matrix below to be the augmented matrix of a system of linear equation

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \\ 3 & 4 & 5 & 6 \end{array} \right]$$

(a) Write the associated system of linear equations.

$$\begin{cases} 1x_1 + 2x_2 + 3x_3 = 4 \\ 5x_1 + 6x_2 + 7x_3 = 8 \\ 9x_1 + 0x_2 + 1x_3 = 2 \\ 3x_1 + 4x_2 + 5x_3 = 6 \end{cases}$$

(b) Define matrix \mathbf{A} and vectors \mathbf{x} and \mathbf{b} so that the system of equations above can be represented as $\mathbf{Ax} = \mathbf{b}$

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 0 & 1 \\ 3 & 4 & 5 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 4 \\ 8 \\ 2 \\ 6 \end{bmatrix}$$

(c) Define vectors \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 and then write the above system as a single vector equation.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 5 \\ 9 \\ 3 \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} 2 \\ 6 \\ 0 \\ 4 \end{bmatrix} \quad \mathbf{a}_3 = \begin{bmatrix} 3 \\ 7 \\ 1 \\ 5 \end{bmatrix}$$
$$\mathbf{a}_1x_1 + \mathbf{a}_2x_2 + \mathbf{a}_3x_3 = \mathbf{b}$$

2. Parametrize the solution set (write the general solution) to the equation $x + 2y + 3z = 5$ as a linear combination of vectors.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$