1. **4.35** The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution:

| x | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $f(x)$ | 0.01 | 0.25 | 0.4 | 0.3 | 0.04 |

Using Theorem 4.2 $(\sigma^2 = E(X^2) - \mu^2)$, find the variance of $X$.

$$\mu = \sum_{x=2}^{6} xf(x) = 4.11, \quad E(X^2) = \sum_{x=2}^{6} x^2 f(x) = 17.63$$
$$\implies \sigma^2 = 17.63 - 4.11^2 = \boxed{0.738}$$

2. **4.36** Suppose that the probabilities are $0.4, 0.3, 0.2,$ and $0.1$, respectively, that $0, 1, 2,$ or $3$ power failures will strike a certain subdivision in any given year.

Find the mean and variance of the random variable $X$ representing the number of power failures striking this subdivision.

$$x = [0, 1, 2, 3], \quad f(x) = [0.4, 0.3, 0.2, 0.1]$$
$$\mu = \sum_{x=0}^{3} xf(x) = \boxed{1}, \quad E(X^2) = \sum_{x=0}^{3} x^2 f(x) = 2$$
$$\implies \sigma^2 = 2 - 1 = \boxed{1}$$

3. **4.37** A dealer's profit, in units of \$5000, on a new automobile is a random variable $X$ having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{else} \end{cases}$$

Find the variance of $X$.

$$\mu = \int_0^1 2x(1-x)\,dx = \frac{1}{3}$$
$$E(X^2) = \int_0^1 2x^2(1-x)\,dx = \frac{1}{6}$$
$$\sigma^2 = \frac{1}{6} - \frac{1}{3^2} = \frac{1}{18} \implies \text{var}(X) = \boxed{\frac{5000^2}{18}}$$

4. **4.38** The proportion of people who respond to a certain mail-order solicitation is a random variable $X$ having the density function

$$f(x) = \begin{cases} \frac{2}{5}(x+2), & 0 < x < 1, \\ 0, & \text{else} \end{cases}$$

Find the variance of $X$.

$$\mu = \int_0^1 \frac{2x(x+2)}{5}\,dx = \frac{8}{15}$$
$$E(X^2) = \int_0^1 \frac{2x^2(x+2)}{5}\,dx = \frac{11}{30}$$
$$\sigma^2 = \frac{11}{30} - \left(\frac{8}{15}\right)^2 = \boxed{0.082}$$

5. **4.43 (Bonus)** The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable $Y = 3X - 2$, where $X$ has the density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-\frac{x}{4}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean and variance of the random variable $Y$.

$$\mu_Y = E(3X-2) = \int_0^\infty \frac{1}{4}(3x-2)e^{-\frac{x}{4}}\,dx = 10$$
$$E(Y^2) = \int_0^\infty \frac{1}{4}(3x-2)^2 e^{-\frac{x}{4}}\,dx = 244$$
$$\sigma_Y^2 = 244 - 10^2 = \boxed{144}$$

6. **4.50** For a laboratory assignment, if the equipment is working, the density function of the observed outcome $X$ is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{elsewhere} \end{cases}$$

Find the variance and standard deviation of $X$.

$$\sigma^2 = \boxed{\frac{1}{18}} \qquad \text{by question 3 (4.37)}$$
$$\sigma = \boxed{\sqrt{\frac{1}{18}}}$$

7. **4.54** Using Theorem 4.5 and Corollary 4.6, i.e.,

$$E(aX+b) = aE(X) + b, \quad b = 0 \implies \sigma^2_{aX+c} = a^2\sigma^2_X = a^2\sigma^2,$$

find the mean and variance of the random variable $Z = 5X + 3$, where $X$ has the probability distribution of Exercise 4.36 (Problem 2, $\mu = 1, \sigma^2 = 1$).

$$\sigma^2_{5X+3} = 5^2(1) = \boxed{25}$$

8. **4.71 (Bonus)** The length of time $Y$, in minutes, required to generate a human reflex to tear gas has the density function

$$f(y) = \begin{cases} \frac{1}{4}e^{-\frac{y}{4}}, & 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

(a) What is the mean time to reflex?

$$\mu = \int_0^\infty \frac{1}{4} y e^{-\frac{y}{4}}\, dy = \boxed{4}$$

(b) Find $E(Y^2)$ and $\text{var}(Y)$.

$$E(Y^2) = \int_0^\infty \frac{1}{4} y^2 e^{-\frac{y}{4}}\, dy = \boxed{32}$$
$$\sigma^2 = 32 - 4^2 = \boxed{16}$$

9. **4.101** Consider Review Exercise 3.73 on page 108. It involved $Y$, the proportion of impurities in a batch, and the density function is given by

$$f(y) = \begin{cases} 10(1-y)^9, & 0 \le y < 1, \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the expected percentage of impurities.

$$\mu = \int_0^1 10y(1-y)^9 = \frac{1}{11} = \boxed{0.09}$$

(b) Find expected value of proportion of quality material, i.e., $E(1-Y)$.

$$E(1-Y) = 1 - 0.09 = \boxed{0.91}$$

(c) Find the variance of the random variable $Z = 1 - Y$.

$$\sigma^2_Z = \sigma^2_{1-Y} = \sigma^2_Y = \int_0^1 10y^2(1-y)^9 - \frac{1}{11^2} = \frac{1}{66} - \frac{1}{11^2} = \frac{5}{726} = \boxed{0.0068}$$

10. **4.62** If $X$ and $Y$ are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$, find the variance of the random variable $Z = -2X + 4Y - 3$.

$$\sigma_Z^2 = a_X\sigma_X^2 + a_Y\sigma_Y^2 = -2^2(5) + 4^2(3) = \boxed{68}, \quad \text{by corollary 4.11}$$

11. **4.63** Repeat Exercise 4.62 if X and Y are not independent and $\sigma_{XY} = 1$.

$$\sigma_Z^2 = a_X\sigma_X^2 + a_Y\sigma_Y^2 + 2a_Xa_Y\sigma_{XY}^2 = 68 + 2(-8)(1) = \boxed{52}$$

12. Let $X$ and $Y$ be random variables with the following information:

$$E(X) = 6, \quad E(Y) = -\frac{1}{2}, \quad \sigma_X^2 = 4, \quad \sigma_Y^2 = 6, \quad \sigma_{XY} = 2$$

(a) Compute $E(3X - 4Y)$

$$= E(3X) - E(4Y) = 18 - (-2) = \boxed{20}$$

(b) Compute $\text{var}(3X - 4Y)$

$$= 3^2(4) - 4^2(6) + 2(3)(4)(2) = \boxed{-12}$$

(c) Compute $E(2X - Y^2)$

$$\sigma_Y^2 = E(Y^2) - \mu_Y^2 \implies 6 = E(Y^2) + \frac{1}{2}$$
$$E(Y^2) = 5.5$$

$$\implies E(2X) - E(Y^2) = 2(6) - 5.5 = \boxed{6.5}$$

13. Let $X$ and $Y$ be independent random variables with the following information:

$$E(X) = -1, \quad E(Y) = 4, \quad \sigma_X^2 = 6, \quad \sigma_Y^2 = 8$$

(a) Compute $E(9X + 2Y)$

$$= E(9X) + E(2Y) = 9(-1) + 2(4) = \boxed{-1}$$

(b) Compute $\text{var}(9X + 2Y)$

$$= 9^2(6) + 2^2(8) = \boxed{518}$$

14. **6.3** The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable $X$ having a continuous uniform distribution with $A = 7$ and $B = 10$.

    Find the probability that on a given day the amount of coffee dispensed by this machine will be

    (a) at most 8.8 liters;

    $$P(X \leq 8.8) = f(x; A, B) = \frac{8.8 - 7}{10 - 7} = \boxed{0.6}$$

    (b) more than 7.4 liters but less than 9.5 liters;

    $$P(7.4 > X < 9.5) = \frac{9.5 - 7.4}{3} = \boxed{0.7}$$

    (c) at least 8.5 liters.

    $$P(X \geq 8.5) = \frac{10 - 8.5}{3} = \boxed{0.5}$$

15. **6.4** A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform

    (a) What is the probability that the individual waits more than 7 minutes?

    $$P(X > 7) = \frac{10 - 7}{10} = \boxed{0.3}$$

    (b) What is the probability that the individual waits between 2 and 7 minutes?

    $$P(2 > X < 7) = \frac{7 - 2}{10} = \boxed{0.5}$$