

Calculus



Limits and Continuity

Limits	5
Limits of a Functions and Sequences.....	5
Properties of Limits.....	5
One-Sided Limit.....	5
Continuity	6
Continuous Functions	6
Intermediate Value Theorem.....	6
Limits Involving Infinity	7
Limits at Infinity and Infinite Limits	7
Asymptotes of functions.....	7

Derivatives

Derivative Fundamentals	8
Derivative Notation	8
Differentiation Rules	9
Linear, Product, Chain, Inverse	9
Powers, Polynomials, Quotients, Reciprocals	9
Exponential, Logarithmic	9
Trigonometric, Hyperbolic	9
Differentials and Related Concepts	10
Differentials.....	10
Linearization.....	10
Implicit Differentiation	10
Related Rates.....	10

Applications of Derivatives

Stationary Point	11
Maxima and Minima.....	11
Extreme Value Theorem	11
Interior Extremum Theorem	11
Mean Value Theorem	12
Rolle's Theorem.....	12
Corollaries of the Mean Value Theorem.....	12
Monotonic Functions	12
Derivative Tests	13
First-Derivative Test.....	13

Second-Derivative Test	13
Concavity	13
Higher-Order Derivative Test	13
Differential Methods	14
Newton's Method	14
Taylor's Theorem	14
General Leibniz Rule	14
Integrals	
Integral Fundamentals	15
Terminology and Notation	15
Primer: Formal Definitions	15
Definite Integrals	16
Riemann Integral	16
Integrability	16
Properties of Definite Integrals	16
The Fundamental Theorem of Calculus	17
Fundamental Theorem, Part 1	17
Fundamental Theorem, Part 2	17
The Integral of a Rate	17
Total Area	17
Integration By Substitution	18
Indefinite Integrals	18
Definite Integrals	18
Symmetric Functions	18
Area Between Curves	18
Applications of Definite Integrals	
Solid of Revolution	19
Disc Integration	19
Shell Integration	19
Arc Length	20
Dealing with Discontinuities	20
Differential Arc Length	20
Surface of Revolution	21
Revolution about the y-Axis	21
Transcendental Functions	
Inverse Functions	22
One-to-One Functions	22
Derivative Rule for Inverses	22
Logarithmic Functions	23

Natural Logarithm	23
Properties of Logarithms	23
Trigonometric Integrals	23
Logarithmic Differentiation	23
Exponential Functions	24
Euler's Number	24
Natural Exponential Function	24
Laws of Exponents	24
General Exponential Function.....	24
Exponential Change	25
Separable Differential Equations	25
Examples of Exponential Change	25
Indeterminate Forms	26
Indeterminate Form $0/0$	26
L'Hôpital's Rule.....	26
Infinite Indeterminate Forms.....	26
Indeterminate Powers	26
Inverse Trigonometric Functions	27
Principal Trigonometric Values	27
Inverse Trigonometric Tables	27
Hyperbolic Functions	28
Hyperbolic Function Tables	28
Techniques of Integration	
Integration by Parts	29
Definite Integrals by Parts	29
Trigonometric Integral Methods	30
Trigonometric Products and Powers.....	30
Trigonometric Square Roots.....	30
Trigonometric Substitutions	30
Partial Fraction Decomposition	31
Partial Fraction Principles.....	31
General Statement.....	31
Numerical Integration	32
Trapezoidal Rule.....	32
Simpson's Rule	32
Improper Integrals	33
Indirect Evaluation	33
Infinite Sequences and Series	
First-Order Differential Equations	

Parametric Equations and Polar Coordinates

Vectors and Vector-Valued Functions

Partial Derivatives

Multiple Integrals

Vector Calculus

Second-Order Differential Equations

Limits and Continuity



Limits

🌐 Limit 📖 | Thomas (2.2–2.4) 📖

- **Limit** $\lim_{x \rightarrow c}$: the value of a function (or sequence) as the input (or index) approaches some value (note: an informal definition).
 - Limits are used to define **continuity**↓, **derivatives**↓, and **integrals**↓.

Limits of a Functions and Sequences

🌐 Limit of a function 📖 | Limit of a sequence 📖 | Essence of Calculus, E7 📺

- **Limit of a function**: a fundamental concept in calculus and analysis concerning the behavior of a function near a particular input c , i.e.,

$$\lim_{x \rightarrow c} f(x) = L$$

- Reads as “ f of x tends to L as x tends to c ”

- **ϵ, δ Limit of a function**: a formalized definition, wherein $f(x)$ is defined on an open interval \mathcal{I} , except possibly at c itself, leading to the informal definition, if and only if

$$f : \mathbb{R} \rightarrow \mathbb{R}, c, L \in \mathbb{R} \Rightarrow \lim_{x \rightarrow c} f(x) = L$$



$$\forall \epsilon > 0 (\exists \delta > 0 : \forall x \in \mathcal{I} (0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon))$$

- functions **do not have** a limit when the function:
 - has a **unit step**, i.e., it “jumps” at a point;
 - is **not bounded**, i.e., it tends towards infinity;
 - or it **oscillates**, i.e., does not stay close to any single number.
- **Limit of a sequence**: the value that the terms of a sequence $(x_n)_{n \in \mathbb{N}}$ “tends to” (and not to any other) as n approaches infinity (or some other point), i.e.,

$$\lim_{n \rightarrow \infty} x_n = x$$

- **ϵ Limit of a sequence**: for every measure of closeness ϵ , the sequence’s x_n term eventually converges to the limit, i.e.,

$$\forall \epsilon > 0 (\exists N \in \mathbb{N} (\forall n \in \mathbb{N} (n \geq N \Rightarrow |x_n - x| < \epsilon)))$$

◦

Properties of Limits

◦

One-Sided Limit

◦

Continuity

Continuous Functions

-

Intermediate Value Theorem

-

Limits Involving Infinity

Limits at Infinity and Infinite Limits

-

Asymptotes of functions

-

Derivatives



Derivative Fundamentals

Derivative Notation

- ...

Differentiation Rules

Linear, Product, Chain, Inverse

-

Powers, Polynomials, Quotients, Reciprocals

-

Exponential, Logarithmic

-

Trigonometric, Hyperbolic

-

Differentials and Related Concepts

Differentials

-

Linearization

-

Implicit Differentiation

-

Related Rates

-

Applications of Derivatives



Stationary Point

Maxima and Minima

-

Extreme Value Theorem

-

Interior Extremum Theorem

-

Mean Value Theorem

Rolle's Theorem

-

Corollaries of the Mean Value Theorem

-

Monotonic Functions

Derivative Tests

First-Derivative Test

-

Second-Derivative Test

-

Concavity

-

Higher-Order Derivative Test

-

Differential Methods

Newton's Method

-

Taylor's Theorem

-

General Leibniz Rule

-

Integrals



Integral Fundamentals

Terminology and Notation

-

Primer: Formal Definitions

-

Definite Integrals

Riemann Integral

-

Integrability

-

Properties of Definite Integrals

-

The Fundamental Theorem of Calculus

Fundamental Theorem, Part 1

-

Fundamental Theorem, Part 2

-

The Integral of a Rate

-

Total Area

-

Integration By Substitution

Indefinite Integrals

-

Definite Integrals

-

Symmetric Functions

-

Area Between Curves

-

Applications of Definite Integrals



Solid of Revolution

Disc Integration

-

Shell Integration

-

Arc Length

Dealing with Discontinuities

-

Differential Arc Length

-

Surface of Revolution

Revolution about the y-Axis

○

Transcendental Functions



Inverse Functions

One-to-One Functions

-

Derivative Rule for Inverses

-

Logarithmic Functions

Natural Logarithm

-

Properties of Logarithms

-

Trigonometric Integrals

-

Logarithmic Differentiation

-

Exponential Functions

Euler's Number

-

Natural Exponential Function

-

Laws of Exponents

-

General Exponential Function

-

Exponential Change

- Separable Differential Equations

-

Examples of Exponential Change

-

Indeterminate Forms

Indeterminate Form 0/0

-

L'Hôpital's Rule

-

Infinite Indeterminate Forms

-

Indeterminate Powers

-

Inverse Trigonometric Functions

Principal Trigonometric Values

-

Inverse Trigonometric Tables

-

Hyperbolic Functions

Hyperbolic Function Tables

◦

Techniques of Integration



Integration by Parts

Definite Integrals by Parts

○

Trigonometric Integral Methods

Trigonometric Products and Powers

-

Trigonometric Square Roots

-

Trigonometric Substitutions

-

Partial Fraction Decomposition

Partial Fraction Principles

-

General Statement

-

Numerical Integration

Trapezoidal Rule

-

Simpson's Rule

-

Improper Integrals

Indirect Evaluation

-

Infinite Sequences and Series



First-Order Differential Equations



•

Parametric Equations and Polar Coordinates



Vectors and Vector-Valued Functions



•

Partial Derivatives



Multiple Integrals



Vector Calculus



Second-Order Differential Equations

