Part 1: Evaluate

(a) $7! = 7 \cdot 6 \cdot 5 \ldots \cdot 1 = 5040$

(b) $\displaystyle\sum_{x=1}^{20} x = 1 + 2 + 3 + \ldots + 20 = 210$

(c) $\displaystyle\sum_{i=1}^{20} w = 20w$

(d) $\displaystyle\sum_{x=1}^{3} \left[cx^3 + 1\right] = (c+1) + (c8+1) + (c27+1) = 36c + 3$

(e) Expand $(x+4)^2 \to (x^2 + 8x + 16)$

(f) Expand $(x-4)^2 \to (x^2 - 8x + 16)$

(g) If $f(x) = \begin{cases} \frac{1}{8} : x = 0, 3 \\ \frac{3}{8} : x = 1, 2 \\ 0 : \text{ otherwise} \end{cases}$, then compute the following:

(i) $\displaystyle\sum_{\forall x} [xf(x)] = 3 \cdot \frac{1}{8} + 2 \cdot \frac{3}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$

(ii) $\displaystyle\sum_{\forall x} \left[(x-1.5)^2 f(x)\right] = (-1.5)^2\frac{1}{8} + (-0.5)^2\frac{3}{8} + (0.5)^2\frac{3}{8} + (1.5)^2\frac{1}{8} = \frac{3}{4}$

(h) $\displaystyle\int_0^1 x\, dx = \left.\frac{x^2}{2}\right|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

(i) $\displaystyle\int_1^3 x^2\, dx = \left.\frac{x^3}{3}\right|_1^3 = 9 - \frac{1}{3} = \frac{26}{3}$

(j) $\displaystyle\int_0^1 (x^3+1)dx = \left.\frac{x^4}{4}\right|_0^1 + \left.x\right|_0^1 = \frac{1}{4} + 1 = \frac{5}{4}$

(k) $\displaystyle\int_0^\infty \left[ke^{-\frac{x}{3}}\right] dx = k\int_0^\infty e^{-\frac{x}{3}} dx = \left.-3ke^{-\frac{x}{3}}\right|_0^\infty = 0 - (-3k) = 3k$

(l) If $f(x) = \begin{cases} \frac{x^2}{3} : -1 < x < 2 \\ 0 : \text{ otherwise} \end{cases}$, then compute the following:

(i) $\displaystyle\int_{-\infty}^{\infty} [xf(x)]\, dx = \int_{-1}^{2} \frac{x^3}{3} dx = \frac{1}{3}\int_{-1}^{2} x^3 dx = \frac{1}{3} \cdot \left.\frac{x^4}{4}\right|_{-1}^{2} = \frac{1}{3} \cdot \frac{15}{4} = \frac{5}{4}$

(ii) $\displaystyle\int_{-\infty}^{\infty} \left[x^2 f(x)\right] dx = \int_{-1}^{2} \frac{x^4}{3} dx = \frac{1}{3}\int_{-1}^{2} x^4 dx = \frac{1}{3} \cdot \left.\frac{x^5}{5}\right|_{-1}^{2} = \frac{1}{3} \cdot \frac{33}{5} = \frac{11}{5}$

Part 2: Sketch

(a)

(b)

(c)

(d)