

Part 1: Evaluate

(a) $7! = 7 \cdot 6 \cdot 5 \dots \cdot 1 = \boxed{5040}$

(b) $\sum_{x=1}^{20} x = 1 + 2 + 3 + \dots + 20 = \boxed{210}$

(c) $\sum_{i=1}^{20} w = \boxed{20w}$

(d) $\sum_{x=1}^3 [cx^3 + 1] = (c + 1) + (c8 + 1) + (c27 + 1) = \boxed{36c + 3}$

(e) Expand $(x + 4)^2 \rightarrow \boxed{(x^2 + 8x + 16)}$

(f) Expand $(x - 4)^2 \rightarrow \boxed{(x^2 - 8x + 16)}$

(g) If $f(x) = \begin{cases} \frac{1}{8} & : x = 0, 3 \\ \frac{3}{8} & : x = 1, 2 \\ 0 & : \text{otherwise} \end{cases}$, then compute the following:

(i) $\sum_{\forall x} [xf(x)] = 3 \cdot \frac{1}{8} + 2 \cdot \frac{3}{8} + \frac{3}{8} = \frac{12}{8} = \boxed{\frac{3}{2}}$

(ii) $\sum_{\forall x} [(x - 1.5)^2 f(x)] = (-1.5)^2 \frac{1}{8} + (-0.5)^2 \frac{3}{8} + (0.5)^2 \frac{3}{8} + (1.5)^2 \frac{1}{8} = \boxed{\frac{3}{4}}$

(h) $\int_0^1 x \, dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$

(i) $\int_1^3 x^2 \, dx = \frac{x^3}{3} \Big|_1^3 = 9 - \frac{1}{3} = \boxed{\frac{26}{3}}$

(j) $\int_0^1 (x^3 + 1) \, dx = \frac{x^4}{4} \Big|_0^1 + x \Big|_0^1 = \frac{1}{4} + 1 = \boxed{\frac{5}{4}}$

$$(k) \int_0^{\infty} \left[k e^{-\frac{x}{3}} \right] dx = k \int_0^{\infty} e^{-\frac{x}{3}} dx = -3k e^{-\frac{x}{3}} \Big|_0^{\infty} = 0 - (-3k) = \boxed{3k}$$

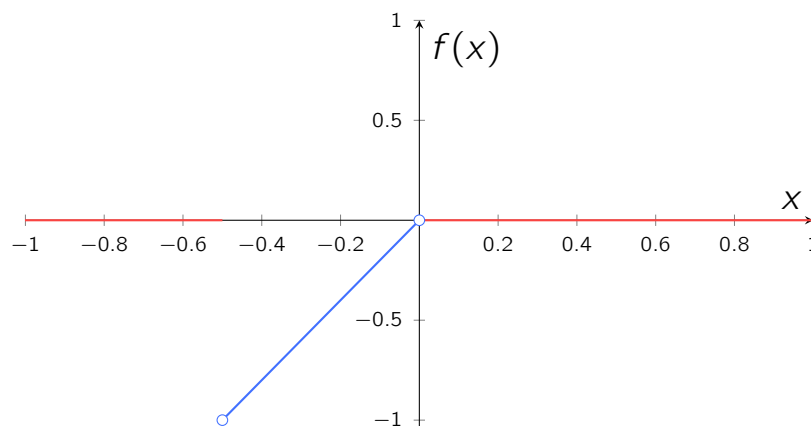
(l) If $f(x) = \begin{cases} \frac{x^2}{3} & : -1 < x < 2 \\ 0 & : \text{otherwise} \end{cases}$, then compute the following:

$$(i) \int_{-\infty}^{\infty} [x f(x)] dx = \int_{-1}^2 \frac{x^3}{3} dx = \frac{1}{3} \int_{-1}^2 x^3 dx = \frac{1}{3} \cdot \frac{x^4}{4} \Big|_{-1}^2 = \frac{1}{3} \cdot \frac{15}{4} = \boxed{\frac{5}{4}}$$

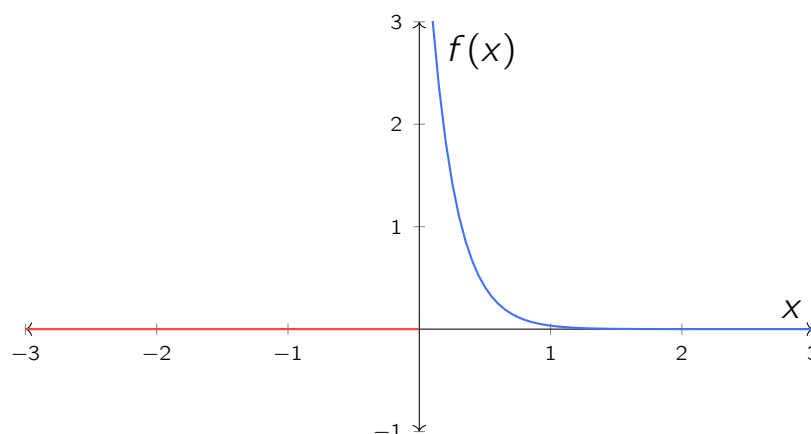
$$(ii) \int_{-\infty}^{\infty} [x^2 f(x)] dx = \int_{-1}^2 \frac{x^4}{3} dx = \frac{1}{3} \int_{-1}^2 x^4 dx = \frac{1}{3} \cdot \frac{x^5}{5} \Big|_{-1}^2 = \frac{1}{3} \cdot \frac{33}{5} = \boxed{\frac{11}{5}}$$

Part 2: Sketch

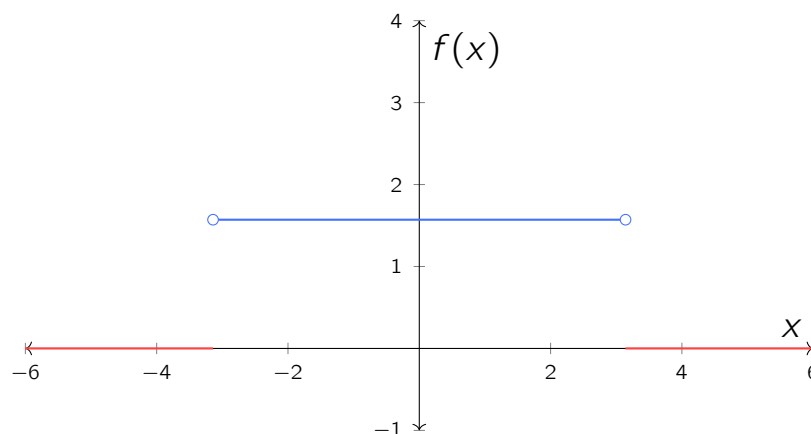
$$(a) f(x) = \begin{cases} 2x & : -0.5 < x < 0 \\ 0 & : \text{otherwise} \end{cases}$$



$$(b) f(x) = \begin{cases} 5e^{-5x} & : x > 0 \\ 0 & : \text{otherwise} \end{cases}$$



$$(c) f(x) = \begin{cases} \frac{1}{2\pi} & : -\pi < x < \pi \\ 0 & : \text{otherwise} \end{cases}$$



$$(d) f(x) = \begin{cases} 0 & : x < 0 \\ \frac{x^2}{4} & : 0 \leq x < 1 \\ \frac{x+1}{4} & : 1 \leq x < 2 \\ 1 & : x \geq 2 \end{cases}$$

