Calculus III Final Review

Convergence: 10.3–10.5

Convergence Notes

◦ Fundamentals

- Let $\sum_{n=1}^{\infty} a_n$ be given and note for which series for which convergence is known, i.e., the geometric series and p-series:

  Geometric: let $c \neq 0$, if $|r| < 1$, then

  $$\sum_{n=0}^{\infty} cr^n = \frac{c}{1-r}$$

  ***p*-Series**: converges if $p > 1$.

  $$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- **The n^{th} Term Divergence Test**: a relatively easy test that can be used to quickly determine if a test diverges if the $\lim_{n\to\infty} a_n \neq 0$. If $\lim_{n\to\infty} a_n = 0$, then the test is inconclusive and other tests must be applied.

◦ Tests for Positive Series

- **Direct Comparison Test**: use if dropping terms from the denominator or numerator gives a series b_n wherein convergence is easily found, then compare to the original series a_n as follows:

  $$\sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges} \quad \leftarrow 0 \leq a_n \leq b_n$$

  $$\sum_{n=1}^{\infty} b_n \text{ diverges} \implies \sum_{n=1}^{\infty} a_n \text{ diverges} \quad \leftarrow 0 \leq b_n \leq a_n$$

- **Limit Comparison Test**: use when the direct comparison test isn't convenient or when comparing two series. One can to take the dominant term in the numerator and denominator from a_n to form a new positive sequence b_n if needed.

  Assuming the following limit $L = \lim_{n\to\infty} \frac{a_n}{b_n}$ exists, then:

  $$L > 0 \implies \sum_{n=1}^{\infty} a_n \text{ converges} \iff \sum_{n=1}^{\infty} b_n \text{ converges}$$

  $$L = 0 \text{ and } \sum_{n=1}^{\infty} b_n \text{ converges} \implies \sum_{n=1}^{\infty} a_n \text{ converges}$$

  $$L = \infty \text{ and } \sum_{n=1}^{\infty} a_n \text{ converges} \implies \sum_{n=1}^{\infty} b_n \text{ converges}$$

- **Ratio Test**: often used in the presence of a factorial $(\boldsymbol{n!})$ or when the are constants raised to the power of n $(\boldsymbol{c^n})$.

  Assuming the following limit $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ exists, then

$$\begin{aligned} \rho < 1 &\implies \sum a_n \text{ converges absolutely} \\ \rho > 1 &\implies \sum a_n \text{ diverges} \\ \rho = 1 &\implies \text{test is inconclusive} \end{aligned}$$

- **Root Test**: used when there is a term in the form of $\boldsymbol{f(n)^{g(n)}}$.

  Assuming the following limit $C = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$ exists, then

$$\begin{aligned} C < 1 &\implies \sum a_n \text{ converges absolutely} \\ C > 1 &\implies \sum a_n \text{ diverges} \\ C = 1 &\implies \text{test is inconclusive} \end{aligned}$$

- **Integral Test**: if the other tests fail and $a_n = f(n)$ is a decreasing function, then one can use the improper integral $\int_1^\infty f(x)dx$ to test for convergence.

  Let $a_n = f(n)$ be a positive, decreasing, and continuous function $\forall x \geq 1$, then:

$$\begin{aligned} \int_1^\infty f(x)dx \text{ converges} &\implies \sum_{n=1}^\infty a_n \text{ converges} \\ \int_1^\infty f(x)dx \text{ diverges} &\implies \sum_{n=1}^\infty a_n \text{ diverges} \end{aligned}$$

Tests for Non-Positive Series

- **Alternating Series Test**: used for series in the form $\sum_{n=0}^\infty (-1)^n a_n$

  Converges if $|a_n|$ decreases monotonically $(|a_n + 1| \leq |a_n|)$ and if $\lim_{n\to\infty} a_n = 0$

- **Absolute Convergence**: used if the series $\sum a_n$ is not alternating; simply test if $\sum |a_n|$ converges using the test for positive series.

Convergence Problems

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Power Series: 10.6

Power Series Notes

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Power Series Problems

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Taylor Series: 10.7–10.8

Taylor Series Notes

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Taylor Series Problems

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Parametric Equations: 11.1

Parametric Notes

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Parametric Problems

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Arc Length, Polar Coordinates: 11.2–11.4

- **Polar Coordinates Notes**
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  Polar Coordinate Problems
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Conic Sections

Conic Sections Notes

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Conic Section Problems

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Quiz Questions

Quiz 3

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Quiz 4

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Final Review Questions