1. **6.10** According to Chebyshev's theorem, the probability that any random variable assumes a value within 3 standard deviations of the mean is at least $\frac{8}{9}$. If it is known that the probability distribution of a random variable X is normal with mean μ and variance σ^2, what is the exact value of $P(\mu - 3\sigma < X < \mu + 3\sigma)$?

$$
\begin{aligned}
Z &= \frac{X-\mu}{\sigma} \\
&\implies P(-3 < Z < 3) = 0.9987 - 0.0013 \qquad \text{by table 3A} \\
&= \boxed{0.9974}
\end{aligned}
$$

2. **6.12** The loaves of rye bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assuming that the lengths are normally distributed, what percentage of the loaves are

   (a) longer than 31.7 centimeters?

   $$\text{normalcdf}(31.7, \infty, 30, 2) = \boxed{19.77\%}$$

   (b) between 29.3 and 33.5 centimeters in length?

   $$\text{normalcdf}(-29.3, 33.5, 30, 2) = \boxed{59.67\%}$$

   (c) shorter than 25.5 centimeters?

   $$\text{normalcdf}(-\infty, 25.5, 30, 2) = \boxed{1.22\%}$$

3. **6.14** The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeter.

   (a) What proportion of rings will have inside diameters exceeding 10.075 centimeters?

   $$\text{normalcdf}(10.075, \infty, 10, 0.03) = \boxed{0.62\%}$$

   (b) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimeters?

   $$\text{normalcdf}(9.97, 10.03, 10, 0.03) = \boxed{0.6826}$$

   (c) Below what value of inside diameter will 15% of the piston rings fall?

   $$\text{invNorm}(0.15, 10, 0.03) = \boxed{9.969 \text{ cm}}$$

4. **6.17** The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. The manufacturer replaces free all motors that fail while under guarantee. If she is willing to replace only 3% of the motors that fail, how long a guarantee should be offered? Assume that the lifetime of a motor follows a normal distribution.

$$\text{invNorm}(0.03, 10, 2) = \boxed{6.24 \text{ yrs}}$$

5. **6.19** A company pays its employees an average wage of \$15.90 an hour with a standard deviation of \$1.50. If the wages are approximately normally distributed and paid to the nearest cent,

    (a) what percentage of the workers receive wages between \$13.75 and \$16.22 an hour inclusive?

    $$\text{normalcdf}(13.75, 16.22, 15.9, 1.5) = \boxed{50.86\%}$$

    (b) the highest 5% of the employee hourly wages is greater than what amount?

    $$\text{invNorm}(0.95, 15.9, 1.5) = \boxed{\$18.37}$$

6. **6.22** If a set of observations is normally distributed, what percent of these differ from the mean by

    (a) more than 1.3σ?

    $$2P(Z > 1.3) \implies 2 \cdot \text{normalcdf}(1.3, \infty, 0, 1) = \boxed{19.36\%}$$

    (b) less than 0.52σ?

    $$P(-0.52 < Z < 0.52) \implies \text{normalcdf}(-0.52, 0.52, 0, 1) = \boxed{39.69\%}$$

7. **6.80** In a human factor experimental project, it has been determined that the reaction time of a pilot to a visual stimulus is normally distributed with a mean of 0.5 seconds and standard deviation of 0.4 seconds.

   (a) What is the probability that a reaction from the pilot takes more than 0.3 second?

   $$\text{normalcdf}(0.3, \infty, 0.5, 0.4) = \boxed{0.6915}$$

   (b) What reaction time is that which is exceeded 95% of the time?

   $$\text{invNorm}(1 - 0.95, 0.5, 0.4) = \boxed{-0.1579 \text{ sec}}$$ some pilots must be psychic, or model is wrong

8. There are two machines available for cutting corks intended for use in wine bottles. The first machine produces corks with diameters that have a normal distribution with mean 3 cm and standard deviation 0.1 cm.
   The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation 0.02 cm.

   Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

   $$\text{machine}_1 = \text{normalcdf}(2.9, 3.1, 3, 0.1) = 68.26\%$$
   $$\text{machine}_2 = \text{normalcdf}(2.9, 3.1, 3.04, 0.02) = 99.86\%$$

   $\boxed{\text{Machine 2, clearly}}$