1. **8.4** The number of tickets issued for traffic violations by 8 state troopers during the Memorial Day weekend are 5, 4, 7, 7, 6, 3, 8, and 6.

   (a) If these values represent the number of tickets issued by a random sample of 8 state troopers from Montgomery County in Virginia, define a suitable population.

   Tickets given by all state troopers in Montgomery County over Memorial Day weekend.

   (b) If the values represent the number of tickets issued by a random sample of 8 state troopers from South Carolina, define a suitable population.

   Tickets given by all state troopers in South Carolina over Memorial Day weekend.

2. Let $X_1, X_2, \ldots, X_n$ are a random sample of size $n$. Classify each of the following as a statistics or not a statistic.

   (a) $\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2$

   **Statistic**: this is a function of random variables that provides information about the random sample.

   (b) $\sum_{i=1}^{n}\left|X_i-\bar{X}\right|$

   **Not a statistic**: this function sums the difference of random samples from the median, but the summation itself does not say about the sample; it would need to be divided by the sample size to become a statistic (mean absolute difference).

3. **8.2** The lengths of time, in minutes, that 10 patients waited in a doctor's office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Treating the data as a random sample, find

   (a) the mean;

   $$\bar{x} = \frac{1}{10}\sum_{i=1}^{n}[\text{data}]_i = \boxed{8.6 \text{ min}}$$

   (b) the median;

   $$\tilde{x} = [5, 6, 9, 10, 11, 15] = \boxed{9.5 \text{ min}}$$

   (c) the mode.

   $$\text{mode}(\text{data}) = \boxed{5, 10 \text{ min}}$$

4. **8.3** The reaction times for a random sample of 9 subjects to a stimulant were recorded as 2.5, 3.6, 3.1, 4.3, 2.9, 2.3, 2.6, 4.1, and 3.4 seconds. Calculate:

   (a) then;

$$\bar{x} = \frac{1}{9}\sum_{i=1}^{9}[\text{data}]_i = \boxed{3.2 \text{ sec}}$$

   (b) the median;

$$\tilde{x} = \boxed{3.1 \text{ sec}}$$

5. **8.10** For the sample of reaction times in Exercise 8.3,

   (a) the range;

$$4.3 - 2.3 = \boxed{2 \text{ sec}}$$

   (b) the standard deviation.

$$S^2 = \frac{1}{8}\sum_{i=1}^{8}(X_i - 3.2)^2 = \boxed{0.4975}$$

6. **8.11** For the data of Exercise 8.5, calculate the variance using the formula

$$\text{data} = [2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 5, 2, 1, 4, 2], n = 15, \bar{x} = 2.4$$

   (a) using form (8.2.1),

$$S^2 = \frac{1}{14}\sum_{i=1}^{15}(\text{data}_i - 2.4)^2 = \boxed{2.971}$$

   (b) using theorem 8.1,

$$S^2 = \frac{1}{15(14)}\left[15\sum_{i=1}^{15}\text{data}_i^2 - \left(\sum_{i=1}^{15}\text{data}_i\right)^2\right] = \boxed{2.971}$$

7. **8.23** The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

| x | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| P(X = x) | 0.2 | 0.4 | 0.3 | 0.1 |

(a) Find the mean $\mu$ and the variance $\sigma^2$ of $X$.

$$\mu = \sum xf(x) = \boxed{5.3}$$
$$\sigma^2 = \sum (x-\mu)^2 f(x) = \boxed{0.81}$$

(b) Find the mean $\mu_{\bar{X}}$ and the variance $\sigma^2_{\bar{X}}$ of the mean $\bar{X}$ for random samples of 36 cherry puffs.

$$\mu_{\bar{X}} = \mu = \boxed{5.3}, \quad \sigma^2_{\bar{X}} = \frac{\sigma^2}{36} = \boxed{0.0225}$$

(c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

$$\text{normalcdf}(-\infty, 5.5, 5.3, \sqrt{\tfrac{0.81}{36}}) = \boxed{0.9087}$$

8. **8.25** The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

(a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;

$$Z_l = \frac{6.4-7}{\frac{1}{\sqrt{9}}} = -1.8, \quad Z_u = \frac{7.2-7}{\frac{1}{\sqrt{9}}} = 0.6$$
$$\text{normalcdf}(-1.8, 0.6, 0, 1) = \boxed{0.9087}$$

(b) the value of $x$ to the right of which 15% of the means computed from random samples of size 9

$$Z = \text{invNorm}(1-0.15, 0, 1) = 1.04$$
$$\bar{x} = \frac{z}{\frac{\sigma}{\sqrt{n}}} + \mu = 1.04(0.3) + 7 = \boxed{7.31}$$

9. **8.27** In a chemical process, the amount of a certain type of impurity in the output is difficult to control and is thus a random variable. Speculation is that the population mean amount of the impurity is 0.20 gram per gram of output. It is known that the standard deviation is 0.1 gram per gram. An experiment is conducted to gain more insight regarding the speculation that $\mu = 0.2$

   The process is run on a lab scale 50 times and the sample average $\bar{x}$ turns out to be 0.23 gram per gram. Comment on the speculation that the mean amount of impurity is 0.20 gram per gram. Make use of the Central Limit Theorem in your work.

$$\mu = 0.2, \quad \sigma = 0.1, n = 50, \bar{x} = 0.23$$

$$Z = \frac{0.23 - 0.2}{\frac{0.1}{\sqrt{50}}} = 2.12$$

$$\text{normalcdf}(2.12, \infty, 0, 1) = 0.017 \rightarrow \boxed{\text{very unlikely, 1.7\% chance}}$$

10. **8.33** The chemical benzene is highly toxic to humans. However, it is used in the manufacture of many medicine dyes, leather, and coverings. Government regulations dictate that for any production process involving benzene, the water in the output of the process must not exceed 7950 parts per million (ppm) of benzene. For a particular process of concern, the water sample was collected by a manufacturer 25 times randomly and the sample average $\bar{x}$ was 7960 ppm. It is known from historical data that the standard deviation $\sigma$ is 100 ppm.

    (a) What is the probability that the sample average in this experiment would exceed the government limit if the population mean is equal to the limit? Use the Central Limit Theorem.

    If $\mu \approx \bar{X}$, then $Z \approx 0$, implying $P(Z \geq 0) = \boxed{\frac{1}{2}}$

    (b) Is an observed $\bar{x}$ = 7960 in this experiment firm evidence that the population mean for the process exceeds the government limit? Answer your question by computing

$$P(\bar{X} \geq 7960 : \mu = 7950)$$

    Assume that the distribution of benzene concentration is normal.

$$Z = \frac{7960 - 7950}{\frac{100}{\sqrt{25}}} = 0.5$$

$$\text{normalcdf}(0.5, \infty, 0, 1) = 0.3085 \rightarrow \boxed{\text{somewhat unlikely, 30.9\% chance}}$$

11. **8.49** A normal population with unknown variance has a mean of 20. Is one likely to obtain a random sample of size 9 from this population with a mean of 24 and a standard deviation of 4.1? If not, what conclusion would you draw?

$$T = \frac{24 - 20}{\frac{4.1}{3}} = 2.926$$

$$t_{0.01} = \text{invT}(.99, 9 - 1) = 2.896$$

$$T > t_{0.01} \implies \boxed{\text{not likely, under strong confidence interval}}$$

12. **8.50** A maker of a certain brand of low-fat cereal bars claims that the average saturated fat content is 0.5 gram. In a random sample of 8 cereal bars of this brand, the saturated fat content was 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4, and 0.2. Would you agree with the claim? Assume a normal distribution.

$$\mu = 0.5$$

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8}[\text{data}]_i = 0.475$$

$$S^2 = \frac{1}{7}\sum_{i=1}^{8}(\text{data}_i - 0.475)^2 = 0.0336$$

$$Z = \frac{4.75 - 0.5}{\sqrt{\frac{0.0336}{8}}} = -0.39$$

$$\text{normalcdf}(-\infty, -0.39, 0, 1) = 0.3482 \rightarrow \boxed{\text{somewhat unlikely, 34.8\% chance}}$$