

# Calculus III Midterm Review



This review document contains questions from quiz 1 & 2, as well as preliminary questions from sections 9.1–9.5 & 10.1. Questions are listed in no particular order.

1. True or false? (with justification)

(a) If  $a_n$  is bounded, then it converges.

(b) If  $a_n$  converges, then it must be bounded.

(c) If  $a_n$  is not bounded, then it diverges.

(d) If  $a_n$  diverges, then it is not bounded.

2. A hot anvil with cooling constant  $k = 0.02 \text{ s}^{-1}$  is submerged in a large pool of water whose temperature is  $10^\circ\text{C}$ . Let  $y(t)$  be the anvil's temperature  $t$  seconds later.

(a) What is the differential equation satisfied by  $y(t)$ ?

(b) Find a formula for  $y(t)$ , assuming the object initial temperature is  $100^\circ\text{C}$ .

3. As an object cools, its rate of cooling slows. Explain how this follows from Newton's Law of Cooling.

4. In Yellowstone park there were approximately 500 bison in 1970 and 3,000 bison in 1990.

(a) Using the model that the rate of change of the population is proportional to the population itself, set up (do not solve) an initial value problem to model this situation.

(b) Now find the particular solution satisfying your initial value problem.

5. Which of the following are first-order linear equations?

(a)  $y' + x^2y = 1$

(c)  $x^5y' + y = e^x$

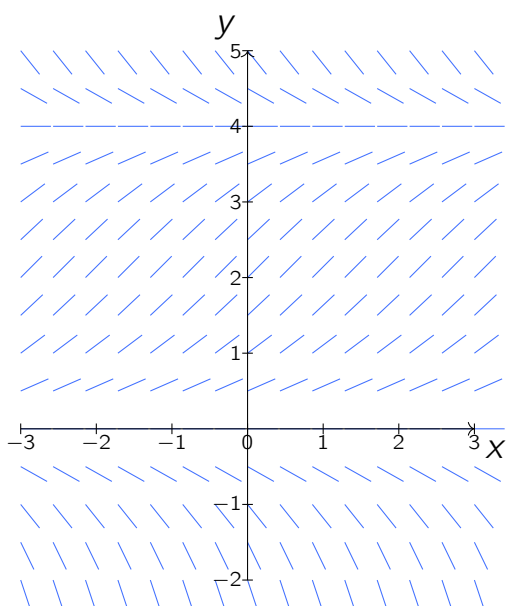
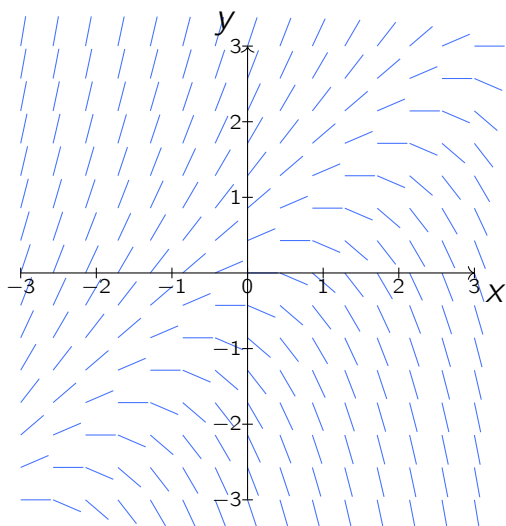
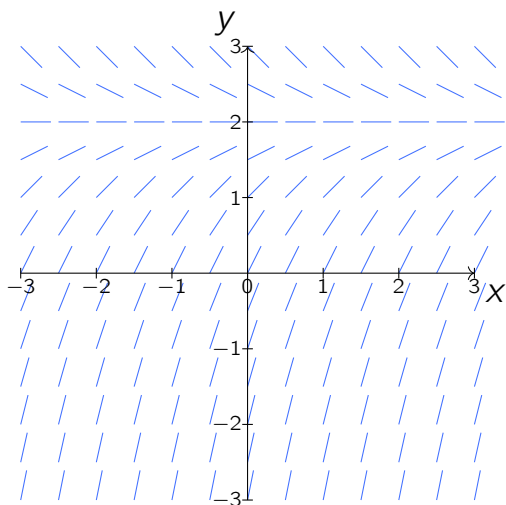
(b)  $y' + xy^2 = 1$

(d)  $x^5y' + y = e^y$

6. For what function  $P$  is the integrating factor  $\alpha(x)$  equal to  $x$ ? What about  $e^x$ ?

7. Match the differential equation that corresponds to the slope fields below. Then give a brief justification explaining how you arrived at your choice.

$$\frac{dy}{dx} = -\frac{x}{y} \quad \frac{dy}{dx} = y - x \quad \frac{dy}{dx} = 2 - y \quad \frac{dy}{dx} = 0.25y(4 - y)$$



8. Find the limit of the sequence  $a_n = \frac{n+1}{3n+2}$ . Be sure to justify your answer.

9. What is  $a_4$  for the sequence  $a_n = n^2 - n$ ?

10. Which of the following sequences converge to zero?

$$\frac{n^2}{n^2+1} \qquad 2^n \qquad \left(-\frac{1}{2}\right)^n$$

11. Which of the following sequences is defined recursively?

$$a_n = \sqrt{4+n} \qquad b_n = \sqrt{4+b_{n-1}}$$

12. Let  $a_n$  be the  $n^{th}$  decimal approximation to  $\sqrt{2}$ . I.e.,  $a_1 = 1$ ,  $a_2 = 1.4$ ,  $a_3 = 1.41$  and so on. What is  $\lim_{n \rightarrow \infty} a_n$ ?

13. Biologists stocked a lake with 400 fish and estimated the carrying capacity to be 10,000. The number of fish tripled in the first year. Assuming the size of the fish population satisfies the logistic equation, find an expression for the size of the population after  $t$  years.

14. Find the general term,  $a_n$ , for the sequence given below. Assume that we start our sequence at  $n = 1$ .

$$1, \frac{4}{2}, \frac{9}{4}, \frac{16}{8}, \frac{25}{16}, \frac{36}{32} \dots$$

15. Use Euler's method with  $h = 0.2$  to estimate  $y(0.4)$ , where  $y(x)$  is the solution to the following initial value problem:

$$y' = 2xy^2, y(0) = 1$$

16. What is the slope of the segment in the slope field for  $\frac{dy}{dt} = ty + 1$  at point (2,3)?

17. What is the equation of the isocline of slope  $c = 1$  for  $\frac{dy}{dt} = y^2 - t$ ?

18. Let  $y(t)$  be the solution to  $\frac{dy}{dt} = F(t, y)$  with  $y(1) = 3$ . How many iterations of Euler's Method are required to approximate  $y(3)$  if the time step is  $h = 0.1$ ?

19. True or false? If  $k > 0$ , then all solutions of  $y' = -k(y - b)$  approach the same limit as  $t \rightarrow \infty$ .

20. Write a solution to  $y' = 4(y - 5)$  that tends to  $-\infty$  as  $t \rightarrow \infty$ .

21. Does  $y' = -4(y - 5)$  have a solution that tends to  $\infty$  as  $t \rightarrow \infty$ ?

22. Find the general solution  $y' = xe^{-\sin x} - y \cos x$ .



23. True or false?

(a)  $t \frac{dy}{dt} = 3\sqrt{1+y}$  is a separable differential equation.

(b)  $yy' + x + y = 0$  is a first-order linear differential equation.

24. Determine the order of the following differential equations:

(a)  $x^5 y' = 1$

(c)  $y''' + x^4 y' = 2$

(b)  $(y')^3 + x = 1$

(d)  $\sin(y'') + x = y$

25. Which of the following differential equations are directly integrable?

(a)  $y' = x + y$

(d)  $\frac{dw}{dt} = \frac{2t}{1+4t}$

(b)  $x \frac{dy}{dx} = 3$

(e)  $\frac{dx}{dt} = t^2 e^{-3t}$

(c)  $\frac{dP}{dt} = 4P + 1$

(f)  $t^2 \frac{dx}{dt} = x - 1$

26. Which of the following differential equations are separable?

(a)  $\frac{dy}{dx} = x - 2y$

(d)  $y' = 1 - y^2$

(b)  $xy' + 8ye^x = 0$

(e)  $t \frac{dy}{dt} = 3\sqrt{1+y}$

(c)  $y' = x^2y^2$

(f)  $\frac{dP}{dt} = \frac{P+t}{t}$

27. Which of the following equations are first-order?

(a)  $y' = x^2$

(d)  $x^2y' - e^xy = \sin y$

(b)  $y'' = y^2$

(e)  $y'' + 3y' = \frac{y}{x}$

(c)  $(y')^3 + yy' = \sin x$

(f)  $yy' + x + y = 0$