Relations
ความสัมพันธ์

Relationships

- Relationships between elements of sets occur in many contexts.
    - Employee and his/her salary.
    - A person and a relative.
- In maths, relationships between a positive integer and
    - An integer that it divides.
    - An integer that is congruent to modulo 5.
- Relationships between a real number and another number that is larger than it.
- A real number x and the value $f(x)$ where f is a function.

Relationships

- Relationships between
  - A program and a variable it uses
  - A computer language and a valid statement in this language
- Binary relation: Relationships between elements of 2 sets.
  - A subset of the Cartesian product of the sets.
- Used to solve problems such as determining which pairs of cities are linked by airline flights in a network.
- Relationships between elements of more than 2 sets (*n*-ary).

Chapter Summary

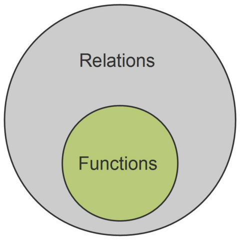
1. Relations and Their Properties
2. n-ary Relations and Their Applications
3. Representing Relations

1. Relations and Their Properties

An **ordered pair** is a set of inputs and outputs and represents a relationship between the two values.

A **relation** is a set of inputs and outputs, and a **function** is a relation with one output for each input.

Some relationships make sense and others don't. Functions are relationships that make sense. **All functions are relations**, but not all relations are functions.

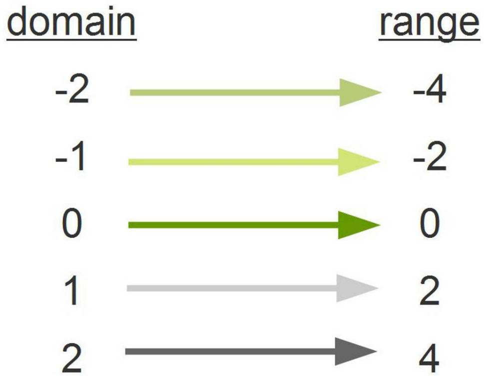


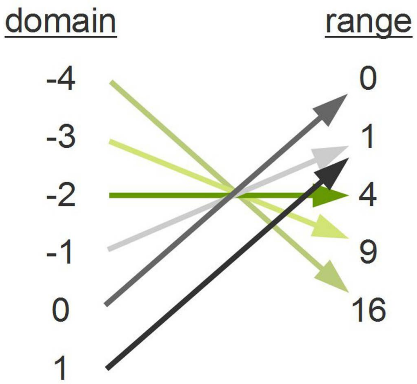
Functions as Relations

A function is a relation that for each input, there is only one output.

The **domain** is the **input** or the x-value, and the **range** is the **output**, or the y-value.

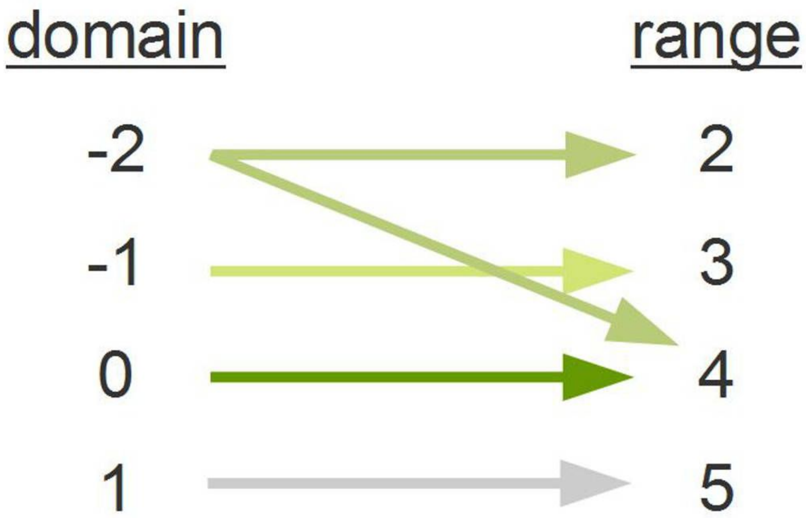
Each x-value is related to only one y-value.





Functions as Relations

This mapping is **not a function**. The input for -2 has more than one output.



Ordered n-tuples

The *ordered n-tuple* (a_1, a_2, ..., a_n) is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.

- Two *n*-tuples are equal if and only if their corresponding elements are equal.
- 2-tuples are called *ordered pairs*.

The ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

Cartesian Product

Definition: The Cartesian Product of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example 1:

$A = \{a, b\}$ $B = \{1, 2, 3\}$

$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Definition: A subset R of the Cartesian product $A \times B$ is called a **relation** from the set A to the set B.

Cartesian Product

Example 2:

$A = \{1, 3, 5\}$ $B = \{2, 4\}$

$A \times B =$

$B \times A =$

$B \times B =$

Binary Relations

Definition: A binary relation R from a set A to a set B is a subset $R \subseteq A \times B$.

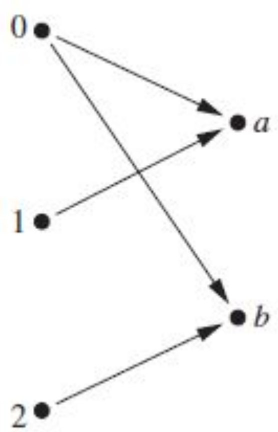
In other words, a binary relation from A to B is a set R of ordered pairs, where the first element of each ordered pair comes from A and the second element comes from B.

Example 1: Let A be the set of cities in the USA, and let B be the set of the 50 states in the USA. Define the relation R by specifying that (a, b) belongs to R if a city with name a is in the state b.

Binary Relations

Example 2: Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$.

- $\{(0, a), (0, b), (1, a), (2, b)\}$ is a relation from A to B.



R	a	b
0	×	×
1	×	
2		×

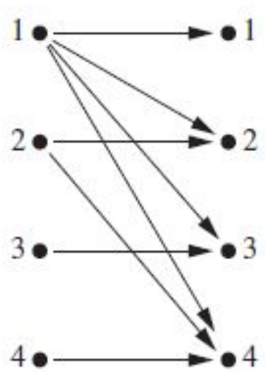
Binary Relations on a Set

Definition: A binary relation R on a set A is a subset of $A \times A$ or a relation from A to A.

- Example 1: Suppose that $A = \{a, b, c\}$.
  - Then $R = \{(a, a), (a, b), (a, c)\}$ is a relation on A.

Binary Relations on a Set

- Example 2: Let $A = \{1, 2, 3, 4\}$, which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?
  - Because (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b.



R	1	2	3	4
1	×	×	×	×
2		×		×
3			×	
4				×

Binary Relations on a Set

- Example 3: Let $A = \{2, 4\}$ and Let $B = \{1, 3, 5\}$, Find
  - $R_1 = \{(a, b) \mid a \in A \;\wedge\; b \in B \;\wedge\; b = 1\}$
  - $R_2 = \{(a, b) \mid a \in A \;\wedge\; b \in B \;\wedge\; a = 2b\}$

Binary Relations on a Set

- Example 4: Consider these relations on the set of integers:
  - $R_1 = \{(a, b) \mid a \le b\}$,
  - $R_2 = \{(a, b) \mid a > b\}$,
  - $R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,
  - $R_4 = \{(a, b) \mid a = b\}$,
  - $R_5 = \{(a, b) \mid a = b + 1\}$,
  - $R_6 = \{(a, b) \mid a + b \le 3\}$.
  - Which of these relations contain each of the pairs (1, 1), (2, 1), (1, -1)?

Binary Relations on a Set

- Example 5: How many relations are there on a set *A* ?

Properties of Relations

1. Reflexive Relations

In some relations, an element is always related to itself.

Definition: R is reflexive if $(a, a) \in R$ for every element $a \in A$.

- Written symbolically, R on the set A is reflexive if and only if

  $\forall a[(a, a) \in R]$.

- A relation on A is reflexive if every element of A is related to itself.

Reflexive Relations

Example 1: Consider the following relations on {1, 2, 3, 4}:

R_1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},

R_2 = {(1, 1), (1, 2), (2, 1)},

R_3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},

R_4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},

R_5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},

R_6 = {(3, 4)}.

Which of these relations are reflexive?

Reflexive Relations

Example 2: The following relations on the integers are reflexive:

$R_1 = \{(a, b) \mid a \leq b\}$

$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$

$R_4 = \{(a, b) \mid a = b\}$

The following relations are not reflexive:

$R_2 = \{(a, b) \mid a > b\}$

$R_5 = \{(a, b) \mid a = b + 1\}$

$R_6 = \{(a, b) \mid a + b \leq 3\}$

Properties of Relations

2. Symmetric Relations

In some relations, an element is related to a 2nd element if and only if the 2nd element is also related to the first element.

Definition: R is symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

- Written symbolically, R on the set A is symmetric if and only if

  $\forall a \forall b[(a, b) \in R \longrightarrow (b, a) \in R]$.

- A relation on A is symmetric if and only if a is related to b always implies that b is related to a.

Symmetric Relations

Example 3: Consider the following relations on {1, 2, 3, 4}:

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,

$R_2 = \{(1, 1), (1, 2), (2, 1)\}$,

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,

$R_6 = \{(3, 4)\}$.

Which of these relations are symmetric?

Symmetric Relations

Example 4: The following relations on the integers are symmetric:

$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$

$R_4 = \{(a, b) \mid a = b\}$

$R_6 = \{(a, b) \mid a + b \leq 3\}$

The following relations are not symmetric:

$R_1 = \{(a, b) \mid a \leq b\}$

$R_2 = \{(a, b) \mid a > b\}$

$R_5 = \{(a, b) \mid a = b + 1\}$

Properties of Relations

3. Antisymmetric Relations

In some relations, an element is related to a 2nd element, then this 2nd element is not related to the first.

Definition: R is antisymmetric if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ for all a, $b \in A$.

- Written symbolically, R on the set A is antisymmetric if and only if

  $$\forall a \forall b[(a, b) \in R \wedge (b, a) \in R \longrightarrow (a = b)].$$

- A relation on A is antisymmetric if and only if there are no pairs of distinct elements a and b with a related to b and b related to a.

Antisymmetric Relations

Example 5: Consider the following relations on {1, 2, 3, 4}:

R_1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},

R_2 = {(1, 1), (1, 2), (2, 1)},

R_3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},

R_4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},

R_5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},

R_6 = {(3, 4)}.

Which of these relations are antisymmetric?

Antisymmetric Relations

Example 6: The following relations on the integers are antisymmetric:

$R_1 = \{(a, b) \mid a \leq b\}$

$R_2 = \{(a, b) \mid a > b\}$

$R_4 = \{(a, b) \mid a = b\}$

$R_5 = \{(a, b) \mid a = b + 1\}$

The following relations are not antisymmetric:

$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$

$R_6 = \{(a, b) \mid a + b \leq 3\}$

Properties of Relations

4. Transitive Relations

In some relations, an element is related to a 2nd element, then this 2nd element is related to the 3rd element. Therefore, the first element is also related to the 3rd element.

Definition: R is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$ for all $a, b, c \in A$.

- Written symbolically, R on the set A is transitive if and only if

$$\forall a \forall b \forall c[(a, b) \in R \land (b, c) \in R \longrightarrow (a, c) \in R].$$

Transitive Relations

Example 7: Consider the following relations on {1, 2, 3, 4}:

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,

$R_2 = \{(1, 1), (1, 2), (2, 1)\}$,

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,

$R_6 = \{(3, 4)\}$.

Which of these relations are transitive?

Transitive Relations

Example 8: The following relations on the integers are transitive:

$R_1 = \{(a, b) \mid a \leq b\}$

$R_2 = \{(a, b) \mid a > b\}$

$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$

$R_4 = \{(a, b) \mid a = b\}$

The following relations are not transitive:

$R_5 = \{(a, b) \mid a = b + 1\}$

$R_6 = \{(a, b) \mid a + b \leq 3\}$

Combining Relations

- Because relations from *A* to *B* are subsets of *A* x *B*, 2 relations from *A* to *B* can be combined.
- Given two relations R_1 and R_2, we can combine them using basic set operations to form new relations such as $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, and $R_2 - R_1$.

Example: Let A = {1, 2, 3} and B = {1, 2, 3, 4}.

- The relations $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ and $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$ can be combined using basic set operations to form new relations:

Composition

- There is another way that relations are combined similar to composition of functions.
- Let R be a relation from a set A to a set B and S a relation from B to a set C.
- The **composite** of R and S is the relation consisting of ordered pairs (a, c), where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.
- We denote the composite of R and S by $S \circ R$.

Composition

Example: What is the composite of the relations R and S, where

R is the relation from {1, 2, 3} to {1, 2, 3, 4} with R = {(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)} and

S is the relation from {1, 2, 3, 4} to {0, 1, 2} with S = {(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)}?

Composition

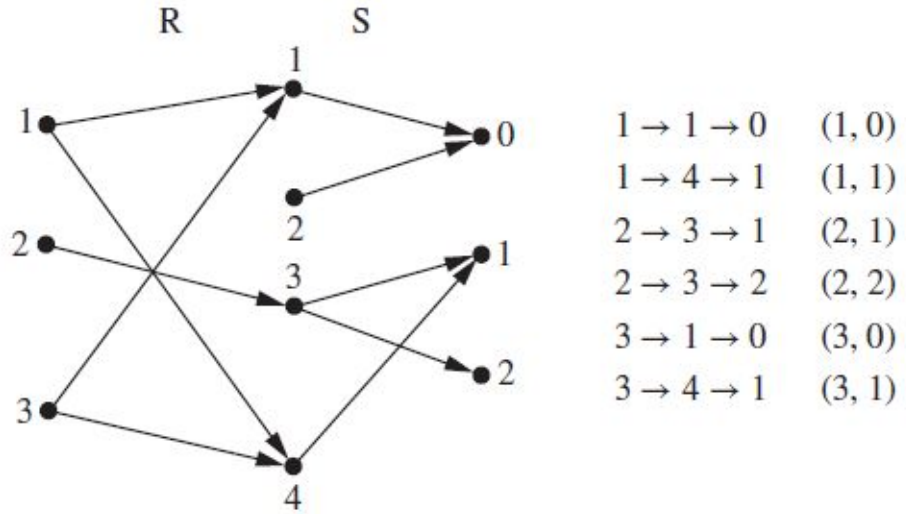


FIGURE 3 **Constructing $S \circ R$.**

Powers of a Relation

The powers of a relation R can be recursively defined from the definition of a composite of two relations.

Definition: Let R be a binary relation on the set A. Then the powers R^n of the relation R can be defined recursively by:

Basis Step: $R^1 = R$

Inductive Step: $R^{n+1} = R^n \circ R$

Powers of a Relation

Example: Let $R = \{(1, 1), (2, 1), (3, 2), (4, 3)\}$.

Find the powers R^n, $n = 2, 3, 4, \ldots$.

Powers of a Relation

- The powers of a transitive relation are subsets of the relation.

Theorem 1: The relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1,2,3 \ldots$.

2. *n*-ary Relations

There are many relationships among elements of more than 2 sets.

- For instance, there is a relationship involving the name of a student, the student's major, and the student's grade point average (GPA).
- Similarly, there is a relationship involving the airline, flight number, departure port, destination, departure time, and arrival time of a flight.
- ***n*-ary relations** are used to represent databases.

n-ary Relations

Definition: Let $A_1, A_2, \ldots, A_n$ be sets.

An n-ary relation over these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$.

The sets $A_1, A_2, \ldots, A_n$ are called the *domains* of the relation, and n is called its *degree*.

n-ary Relations

Example 1: Let R be the relation on $N \times N \times N$ consisting of triples (a, b, c), where a, b, and c are integers with $a < b < c$.

Then $(1, 2, 3) \in R$, but $(2, 4, 3) \notin R$.

The degree of this relation is 3.

Its domains are all equal to the set of natural numbers (N).

n-ary Relations

Example 2: Let R be the relation on Z × Z × Z consisting of all triples of integers (a, b, c) in which a, b, and c form an arithmetic progression.

That is, $(a, b, c) \in R$ if and only if there is an integer k such that $b = a + k$ and $c = a + 2k$, or equivalently, such that $b - a = k$ and $c - b = \mathrm{k}$.

For instance, $(1, 3, 5) \in \mathrm{R}$ because $3 = 1 + 2$ and $5 = 1 + 2 * 2$,

but $(2, 5, 9) \notin \mathrm{R}$ because $5 - 2 = 3$ while $9 - 5 = 4$.

This relation has degree 3 and its domains are all equal to the set of

Integers (Z).

n-ary Relations

Example 3: Let R be the relation consisting of 5-tuples (A, N, S, D, T) representing airplane flights, where

A is the airline,
N is the flight number,
S is the starting point,
D is the destination, and
T is the departure time.

For instance, if Thai Airways has flight 924 from BKK to MUC at 15:00.

The degree of this relation is 5, and its domains are the set of all airlines, the set of flight numbers, the set of cities, the set of cities (again), and the set of times.

Databases and Relations

A database consists of **records**, which are n-tuples, made up of **fields**.

The fields are the entries of the n-tuples.

- The **relational data model** represents a database of records as an n-ary relation.
- Relations used to represent databases are called **tables**.
- A domain of an n-ary relation is called a **primary key**.
    - No 2 n-tuples in the relation have the same value from this domain.

Databases and Relations

Name	ID	Major
Ash	231455	IT
Blue	888323	IT
Green	102147	DSBA

R = {(Ash, 231455, IT), (Blue, 888323, IT), (Green, 102147, DSBA)}

Databases and Relations

Name	ID	Major
Ash	231455	IT
Blue	888323	IT
Green	102147	DSBA

Table: Name of relation

Attribute/Column: Name of set

Domain: Set boundary

Tuple/Row: Relations

Cardinality: Number of rows

Degree: Number of columns

Operations on *n*-ary Relations

The most basic operation on an *n*-ary relation can answer queries on databases that ask for all *n*-tuples in the *n*-ary relation that satisfy certain conditions.

- For instance, find all the records of all DSBA students in a database of student records.
- To perform such tasks, the **selection** operator is used.

Relational Algebra

Definition: Let R be an *n*-ary relation and C a condition that elements in R may satisfy.

Then the **selection** operator $\boldsymbol{s_C}$ maps the *n*-ary relation R_1 to the *n*-ary relation of all *n*-tuples from R_2 that satisfy the condition C.

- Condition can be in the forms of =, <, >, <=, >=, != or Boolean operators (^, v, ~).
- Written symbolically, $R_1 = s_c(R_2)$.

Selection on *n*-ary Relations

Name	ID	Major
Ash	231455	IT
Blue	888323	IT
Green	102147	DSBA

R = {(Ash, 231455, IT), (Blue, 888323, IT), (Green, 102147, DSBA)}

To find the records of IT major in the *n*-ary relation R, the operator s_{C1} where C_1 is the condition Major = “IT”.

The result is two 3-tuples (Ash, 231455, IT) and (Blue, 888323, IT).

Selection on *n*-ary Relations

Relation Sells:

Shop	Product	Price
Paul	Book	50
Paul	Pen	20
Alice	Pen	15
Alice	Book	30
Lynda	Pencil	5

$R_1 = s_{\text{Shop="Paul"}}(\text{Sells})$

Shop	Product	Price
Paul	Book	50
Paul	Pen	20

Selection on *n*-ary Relations

Relation Sells:

Shop	Product	Price
Paul	Book	50
Paul	Pen	20
Alice	Pen	15
Alice	Book	30
Lynda	Pencil	5

$R_1 = s_{\text{Shop="Alice"}}(s_{\text{Product="Pen"}}(\text{Sells}))$

$R_1 = s_{\text{Product="Pen"}}(s_{\text{Shop="Alice"}}(\text{Sells}))$

Relational Algebra

Projections are used to form new *n*-ary relations by deleting the same fields in every record of the relation.

Definition: The projection $P_{i1,i2,\ldots,im}$ where $i_1 < i_2 < \cdots < i_m$, maps the *n*-tuple $(a_1, a_2, \ldots, a_n)$ to the *m*-tuple $(a_{i1}, a_{i2}, \ldots, a_{im})$, where $m \le n$.

- In other words, the projection $P_{i1,i2,\ldots,im}$ deletes $n - m$ of the components of an *n*-tuple, leaving the i_1th, i_2th,…, and i_mth components.
- Written symbolically, $R_1 = P_{i1,i2,\ldots,im}(R_2)$.

Projection on *n*-ary Relations

Name	ID	Major
Ash	231455	IT
Blue	888323	IT
Green	102147	DSBA

R = {(Ash, 231455, IT), (Blue, 888323, IT), (Green, 102147, DSBA)}

What is the table obtained when the projection $P_{1,2}$ is applied?

Projection on *n*-ary Relations

Relation Sells:

Shop	Product	Price
Paul	Book	50
Paul	Pen	20
Alice	Pen	15
Alice	Book	30
Lynda	Pencil	5

$R_1 = P_{\text{Product}}(\text{Sells})$

Product
Book
Pen
Pencil

$R_1 = P_{\text{Shop}}(\text{Sells})$

Shop
Paul
Alice

Projection on *n*-ary Relations

Relation Sells:

Shop	Product	Price
Paul	Book	50
Paul	Pen	20
Alice	Pen	15
Alice	Book	30
Lynda	Pencil	5

$R_1 = P_{\text{Product, Price}}(\text{Sells})$

$R_1 = s_{\text{Product=”Pen”}}(P_{\text{Product, Price}}(\text{Sells}))$

$R_1 = P_{\text{Product, Price}}(s_{\text{Product=”Pen”}}(\text{Sells}))$

Relational Algebra

The **Join** operation is used to combine 2 tables into one when these tables share some identical fields.

Definition: Let R be a relation of degree m and S a relation of degree n.

The **join** $J_p(R, S)$, where $p \leq m$ and $p \leq n$, is a relation of degree $m + n - p$ that consists of all $(m + n - p)$-tuples $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$, where the m-tuple $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p)$ belongs to R and the n-tuple $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$ belongs to S.

- In other words, the join operator J_p produces a new relation from 2 relations by combining all m-tuples of the 1st relation with all n-tuples of the 2nd relation, where the last p components of the m-tuples agree with the 1st p components of the n-tuples.

(Natural) Join on *n*-ary Relations

r_1

<u>Employee</u>	Department
Smith	sales
Black	production
White	production

r_2

<u>Department</u>	Head
production	Mori
sales	Brown

$r_1 \bowtie r_2$

Employee	Department	Head
Smith	sales	Brown
Black	production	Mori
White	production	Mori

(Natural) Join on *n*-ary Relations

Joins can be Incomplete

r_1

Employee	Department
Smith	sales
Black	production
White	production

r_2

Department	Head
production	Mori
purchasing	Brown

$r_1 \bowtie r_2$

Employee	Department	Head
Black	production	Mori
White	production	Mori

Joins can be Empty

r_1

Employee	Department
Smith	sales
Black	production
White	production

r_2

Department	Head
marketing	Mori
purchasing	Brown

$r_1 \bowtie r_2$

Employee	Department	Head

(Natural) Join on n-ary Relations

TABLE 5 Teaching_assignments.

Professor	*Department*	*Course_ number*
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE 6 Class_schedule.

Department	*Course_ number*	*Room*	*Time*
Computer Science	518	N521	2:00 P.M.
Mathematics	575	N502	3:00 P.M.
Mathematics	611	N521	4:00 P.M.
Physics	544	B505	4:00 P.M.
Psychology	501	A100	3:00 P.M.
Psychology	617	A110	11:00 A.M.
Zoology	335	A100	9:00 A.M.
Zoology	412	A100	8:00 A.M.

What relation results when the join operator J_2 is used to combine the relation displayed in Tables 5 and 6?

(Natural) Join on n-ary Relations

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

Result = $P_{\text{sname}}(s_{\text{bid}=101}(\text{S1} \bowtie \text{R1}))$

(Natural) Join on n-ary Relations

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

$S1 \bowtie R1 =$

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

Result = $P_{sname}(s_{bid=101}(S1 \bowtie R1))$

(Natural) Join on n-ary Relations

There are other join operations such as outer join, equi join, and full join (covered in Database module).

Also, relations can be combined using set operations such as union, intersect, and difference (covered in Database module).

(Natural) Join on n-ary Relations Exercise

A

l-name	*f-name*	*age*
Bouvier	Selma	40
Bouvier	Patty	40
Smith	Maggie	2

B

l-name	*f-name*	*ID*
Bouvier	Selma	1232
Smith	Selma	4423

Find A⋈B

(Natural) Join on n-ary Relations Exercise

Employee

Name	EmpId	DeptName
Harry	3415	Finance
Sally	2241	Sales
George	3401	Finance
Harriet	2202	Sales

Dept

DeptName	Manager
Finance	George
Sales	Harriet
Production	Charles

Find Employee⋈Dept

Relational Algebra Ex

SALESPERSON

S-ID	NAME	StartYear	Dept_No
101	P.Jackson	1981	1
102	S.J.Rock	1982	1
112	J.A.Ash	1990	2
150	K.Paul	1992	2
201	B.White	1995	3
314	P.R.Brown	1998	3

TRIP

TripID	S-ID	FromCity	ToCity	DepartDate	ReturnDate
T2014	314	New York	Houston	1/2/00	5/2/00
T2101	101	London	New York	21/2/00	23/2/00
T3025	112	New York	Houston	30/10/00	10/11/00
T7541	150	Bangkok	London	14/5/00	17/5/00
T6213	102	Rome	Paris	25/8/00	12/9/00
T4214	314	New York	Bangkok	25/8/99	31/8/99

Relational Algebra Exercise

Given relations Salesperson, Trip, and Expenses, write the relational algebra for the following relations.

a) Name of salesperson who has worked more than 25 years.

b) Name and Department number of salesperson who has traveled to New York.

3. Representing Relations

Apart from table, there are 2 alternative methods for representing relations.

- Zero-one Matrices
- Directed graphs

Representing Relations Using Matrices

A relation between finite sets can be represented using a zero-one matrix.

Suppose *R* is a relation from $A = \{a_1, a_2, \ldots, a_m\}$ to $B = \{b_1, b_2, \ldots, b_n\}$.

- The elements of the two sets can be listed in any particular arbitrary order. When A = B, we use the same ordering.

The relation *R* is represented by the matrix $M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

The matrix representing *R* has a 1 as its (i, j) entry when a_i is related to b_j and a 0 if a_i is not related to b_j.

Representing Relations Using Matrices

Example 1: Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$.

Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$, and $a > b$. What is the matrix representing R (assuming the ordering of elements is the same as the increasing numerical order)?

Representing Relations Using Matrices

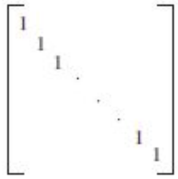
Example 2: Suppose that $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$.

Which ordered pairs are in the relation R represented by the matrix

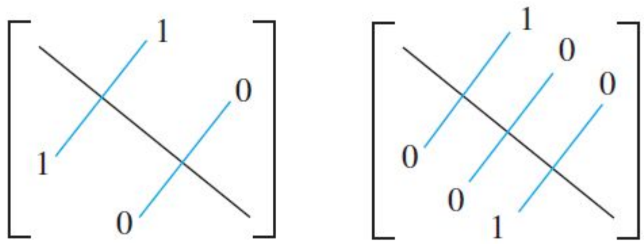
$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}?$$

Matrices of Relations on Sets

- If R is a <u>reflexive</u> relation, all the elements on the main diagonal of M_R are equal to 1.



- R is a <u>symmetric</u> relation, if and only if $m_{ij} = 1$ whenever $m_{ji} = 1$.
- R is an <u>antisymmetric</u> relation, if and only if $m_{ij} = 0$ or $m_{ji} = 0$ when $i \neq j$.



Matrices of Relations on Sets

Example 3: Suppose that the relation R on a set is represented by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Is R reflexive, symmetric, and/or antisymmetric?

Matrices of Relations on Sets

Example 4: Suppose that the relation R_1 and R_2 on a set A are represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

What are the matrices representing $R_1 \cup R_2$ *and* $R_1 \cap R_2$?

Representing Relations Using Digraphs

Definition: A *directed graph*, or *digraph*, consists of a set *V* of *vertices* (or *nodes*) together with a set *E* of ordered pairs of elements of *V* called *edges* (or *arcs*).

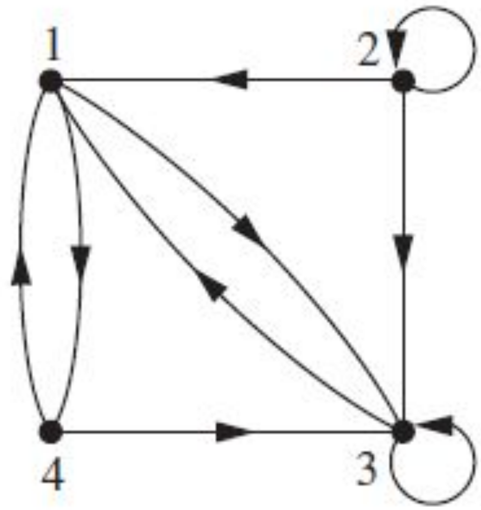
The vertex *a* is called the *initial vertex* of the edge (*a*, *b*), and the vertex *b* is called the *terminal vertex* of this edge.

An edge of the form (*a*, *a*) is called a *loop*.

Example 7: A drawing of the directed graph with vertices *a*, *b*, *c*, and *d*, and edges (*a*, *b*), (*a*, *d*), (*b*, *b*), (*b*, *d*), (*c*, a), (*c*, *b*), and (*d*, *b*).

Representing Relations Using Digraphs

Example 8: What are the ordered pairs in the relation represented by this directed graph?



Relations Properties Using Digraphs

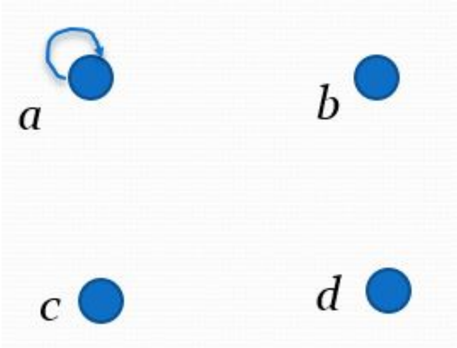
Reflexivity: A loop must be present at all vertices in the graph.

Symmetry: If (x, y) is an edge, then so is (y, x).

Antisymmetry: If (x, y) with $x \neq y$ is an edge, then (y, x) is not an edge.

Transitivity: If (x, y) and (y, z) are edges, then so is (x, z).

Relations Properties Using Digraphs



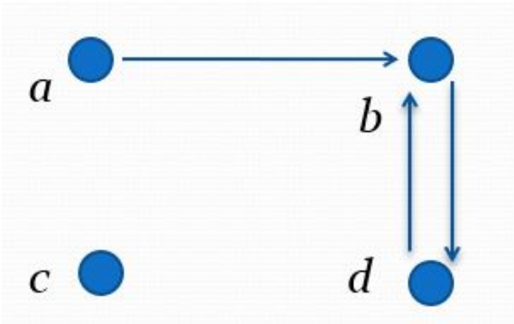
Reflexivity?

Symmetry?

Antisymmetry?

Transitivity?

Relations Properties Using Digraphs



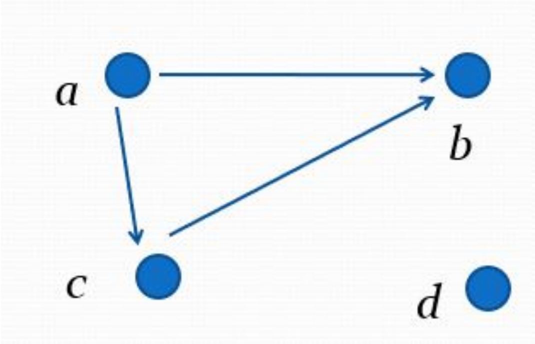
Reflexivity?

Symmetry?

Antisymmetry?

Transitivity?

Relations Properties Using Digraphs



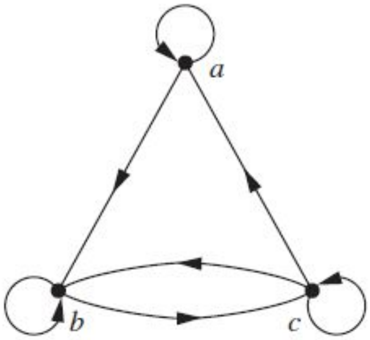
Reflexivity?

Symmetry?

Antisymmetry?

Transitivity?

Relations Properties Using Digraphs



Reflexivity?

Symmetry?

Antisymmetry?

Transitivity?