Chapter 8: Search Trees (Part 2)

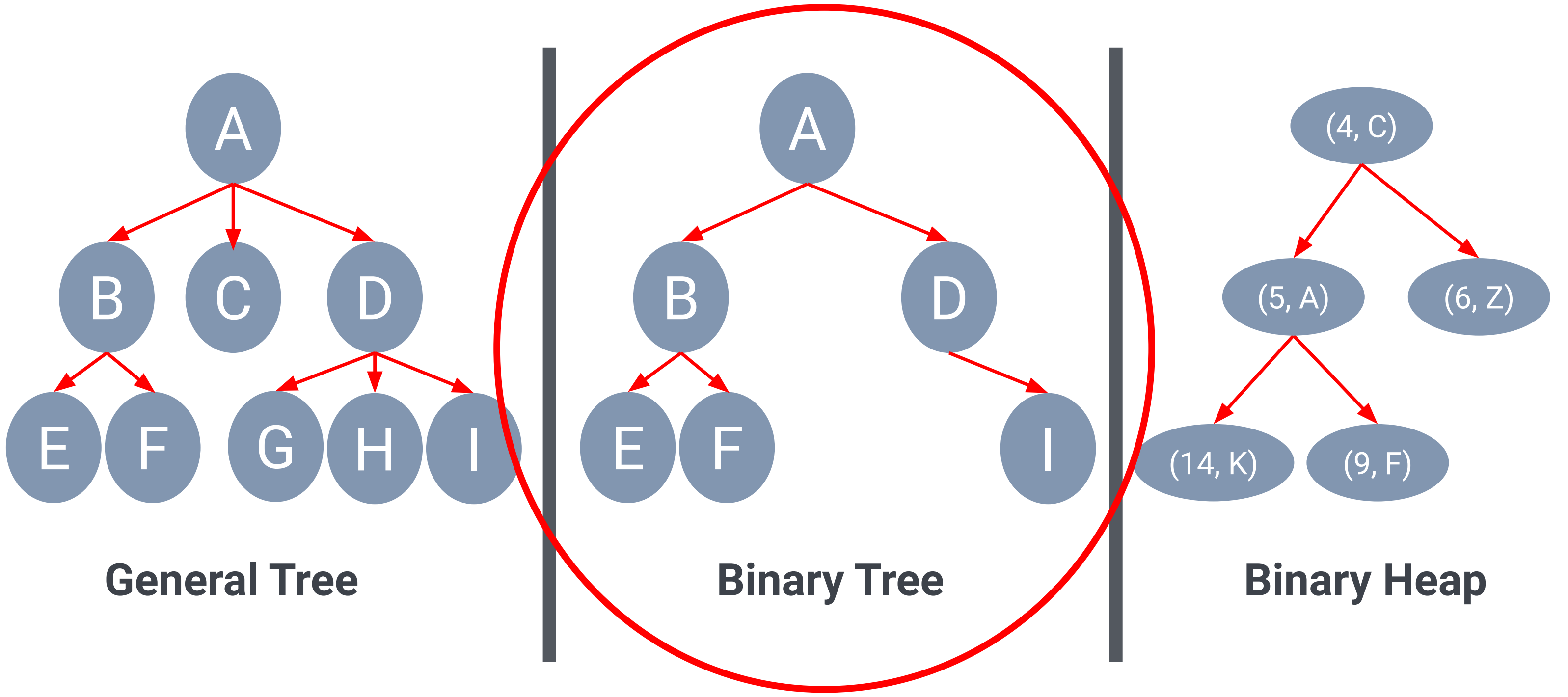
Dr. Sirasit Lochanachit

Outline

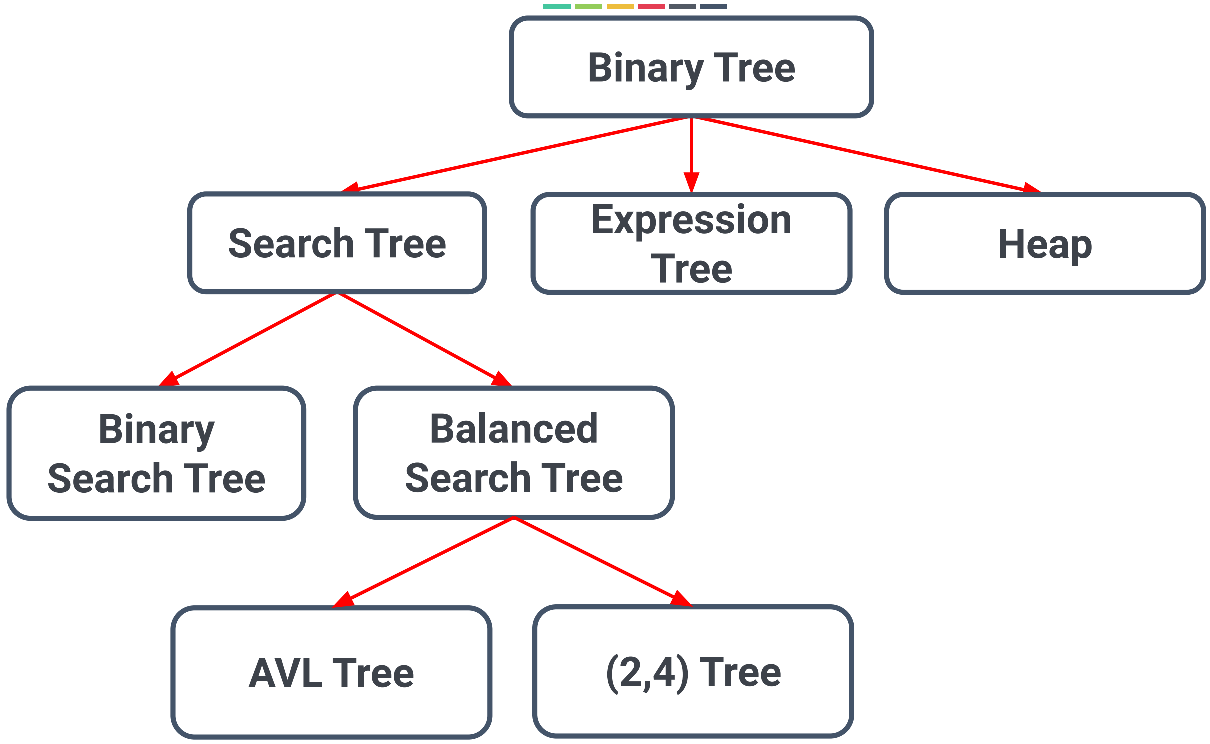
AVL Trees:

- Definition, properties and methods (Insert and rotation)
- Balancing Algorithms and Operation examples

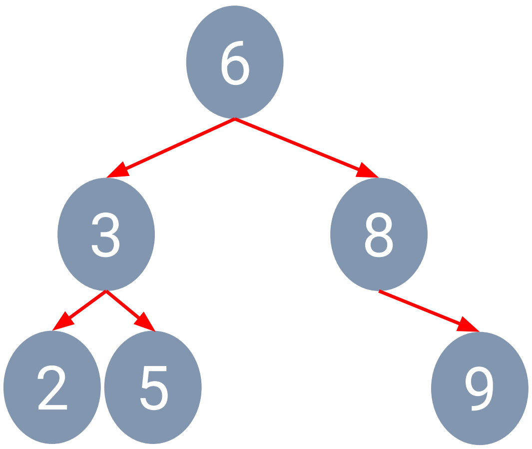
Types of Trees (Revisited)



Types of Binary Trees (Revisited)



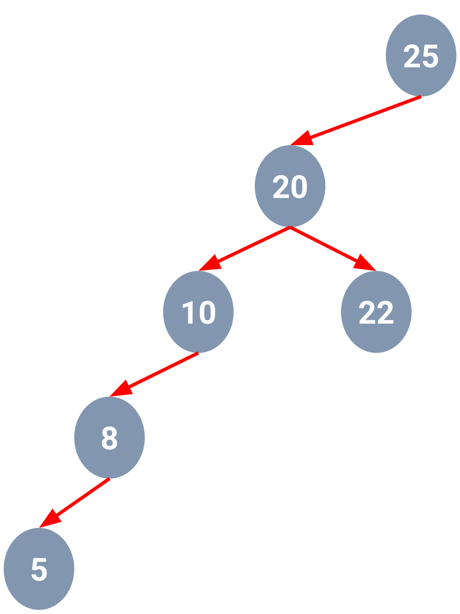
Binary Search Tree (Revisited)



- A **binary search tree (BST)** is a binary tree that stores an ordered sequence of elements or pairs of keys and values and has the following properties [1]:
  - All keys/elements in the ***left subtree*** are **less than** their ***root***.
  - All keys/items in the **right subtree** are **greater than** or **equal to** their ***root***.
  - Each subtree itself is a binary search tree.
- The example uses BST for storing a set of integers.

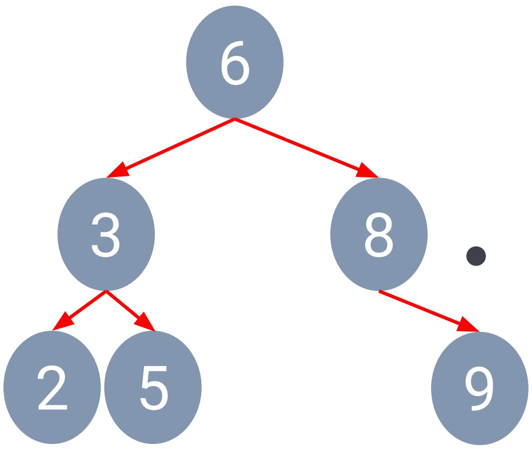
[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

Binary Search Tree (Revisited)



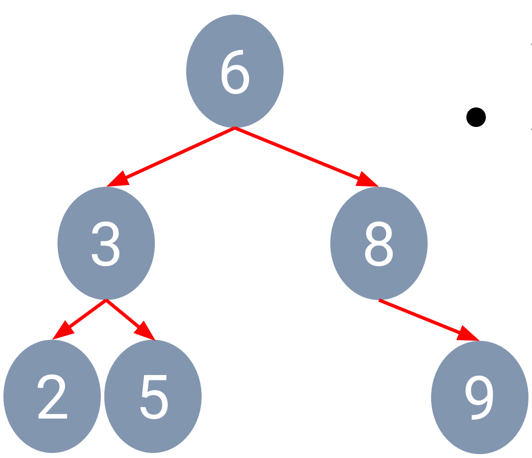
- Running time of inserting node is also proportional to the **height of tree** (i.e. $\log_2 n$ or n) == O(h).
- A **balanced search tree** has the same number of nodes in both left and right subtree.
  - Worst-case performance is $O(\log_2 n)$.
- Inserting keys in sorted order would construct an **imbalanced tree**.
  - Provides poor performance of $O(n)$.
- Other operations' performances are also limited by the height of the tree.

Balanced Binary Search Tree



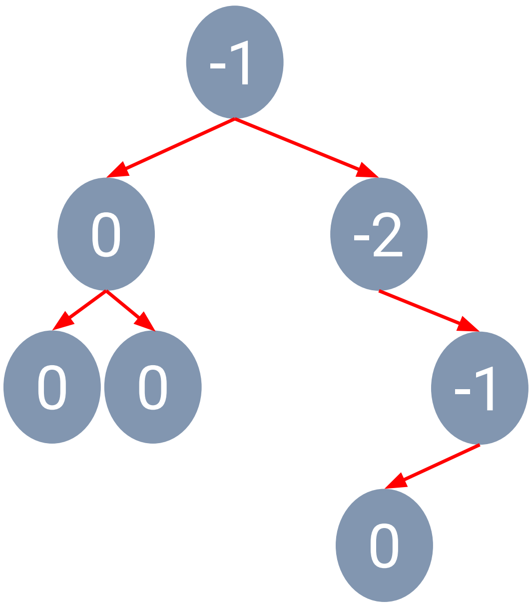
- A **balanced binary search tree (BST)** maintains the balance through a rotation operation which consequently provides a better performance.
    - A child is rotated to be above its parent.
- Several types of binary tree that automatically ensure balance
    - AVL tree
    - Splay tree
    - Red-black tree

AVL Tree



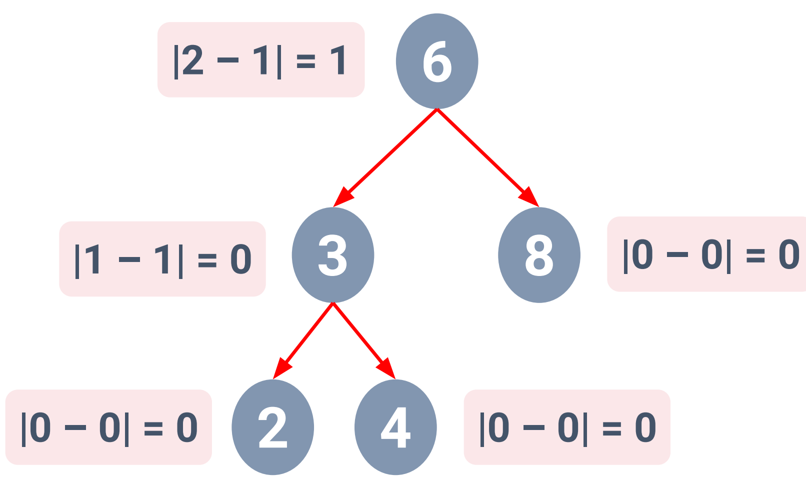
- AVL tree is named after its inventors: G.M. Adelson-Velskii and E.M. Landis.
- AVL tree introduces a **balance factor** for each node in the tree.
    - The height difference between the left and right subtree (H_{left} - H_{right})
    - If a subtree is left heavy, then the factor is > 0.
    - If a subtree is right heavy, then the factor is < 0.
    - If the factor is 0, the tree is perfectly balanced.

Balance Factor in AVL Tree

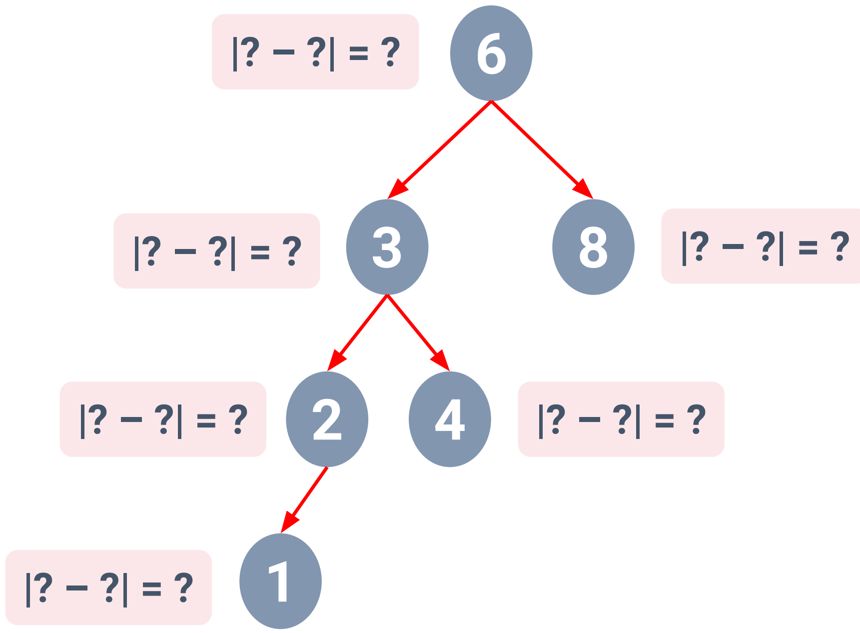


- AVL tree is considered to be <u>balanced</u> when the balance factor is -1, 0, or 1.
  - $|H_{left} - H_{right}| <= 1$
- AVL tree uses **trinode restructuring**, involving reconfigurations of three nodes.
- When new node is inserted into the tree,
  - Balance factor of a new leaf is zero.
  - Balance factor of its parent (and possibly every ancestors) has to be updated (+1 or -1 depends on left or right child).

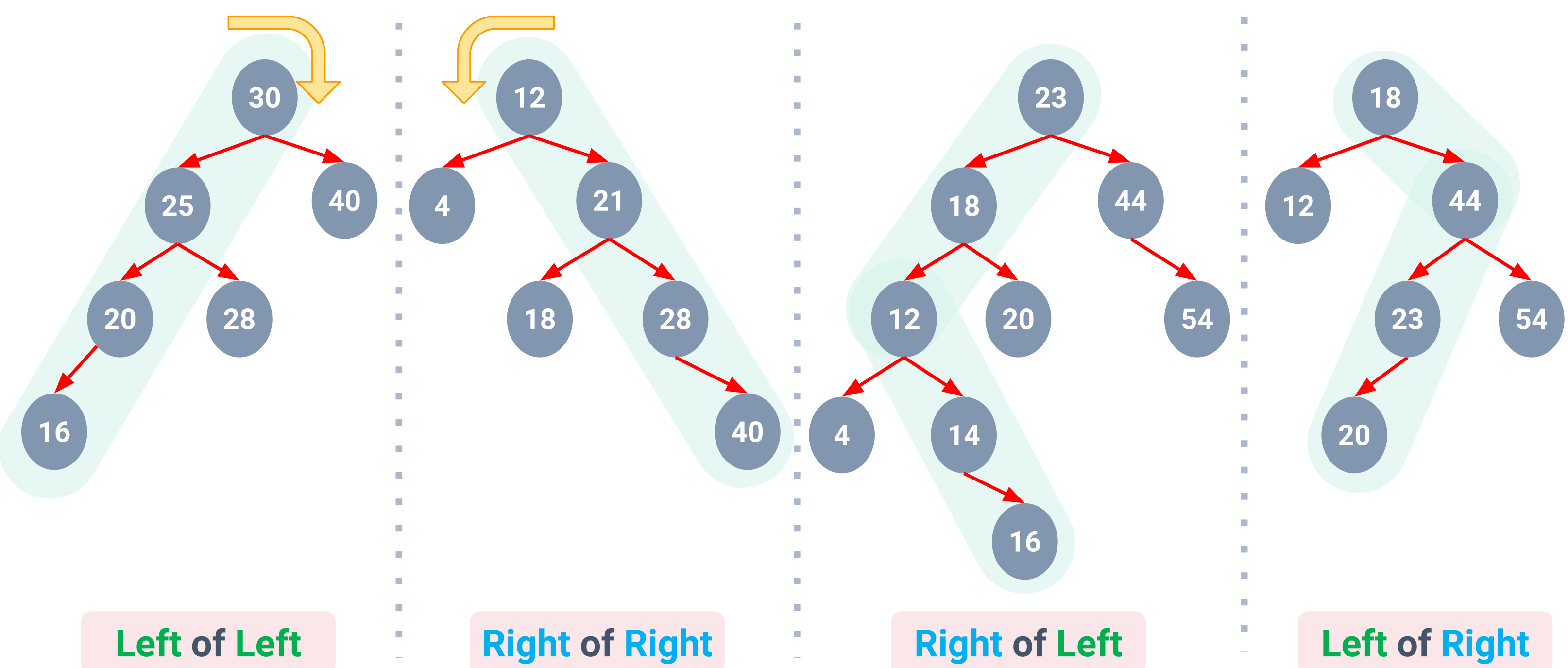
Balance Factor in AVL Tree



Balance Factor in AVL Tree



Balancing AVL Tree



Balancing AVL Tree

Left of Left

Start → (X) = (Y).left → (Y).left = (X).right → (X).Right = (Y) → Stop

Right of Right

Start → (X) = (Y).right → (Y).right = (X).left → (X).left = (Y) → Stop

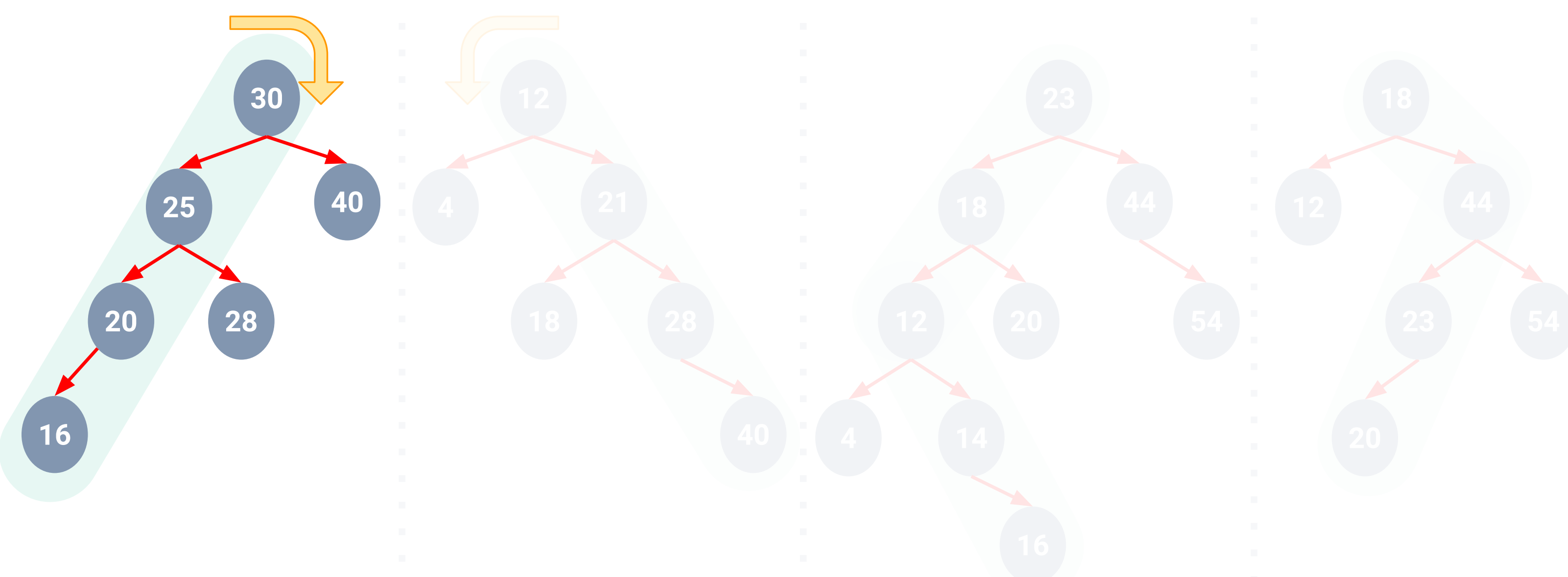
Right of Left

Start → Right-Of-Right (Y1) → Left-Of-Left(Y) → Stop

Left of Right

Start → Left-Of-Left(Y1) → Right-Of-Right(Y) → Stop

Balancing AVL Tree (LoL)



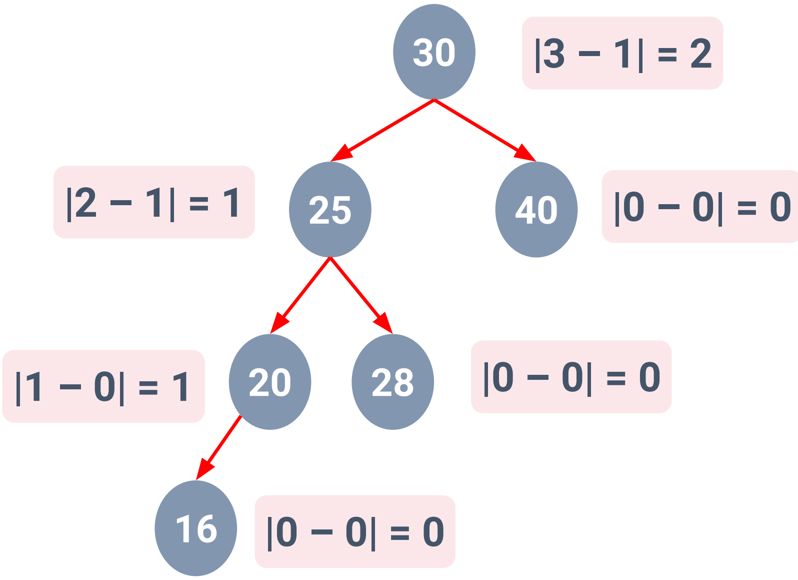
Left of Left

Right of Right

Right of Left

Left of Right

Balancing AVL Tree (LoL)

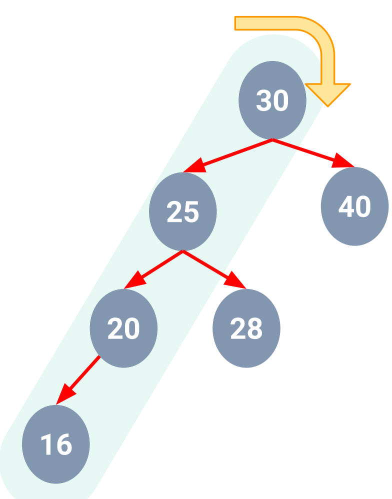


Balance Factor Condition is $\left|H_{left} - H_{right}\right| \leq 1$

A tree is **left-heavy** with a balance factor of 2 at the root.

Require a **right rotation**.

Right Rotation

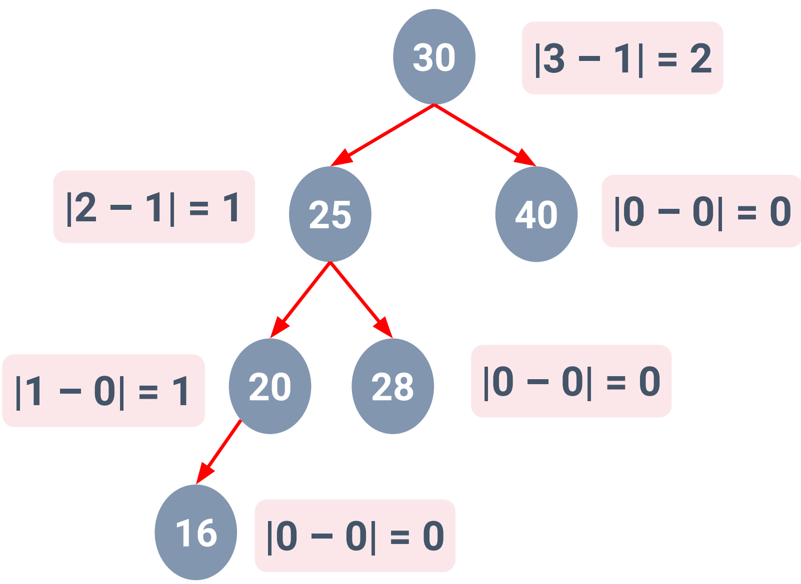


Left of Left

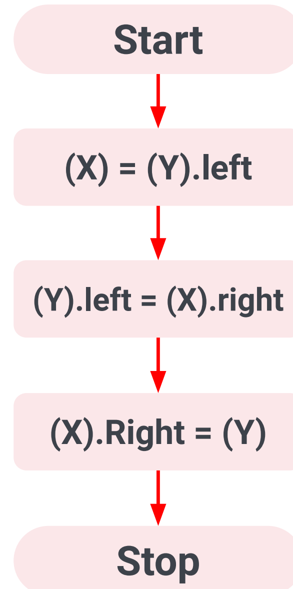
To perform a right rotation (at node 30), do **4 steps** below:

- Promote the left child (25) to be the root of the subtree.
- Move the old root (30) to be the right child of the new root.
- If the new root (25) already had a right child (28),
  - The right child (28) become the left child of the new right child (30).
- Update parents pointers of old root node (if exist).

Balancing AVL Tree (LoL)

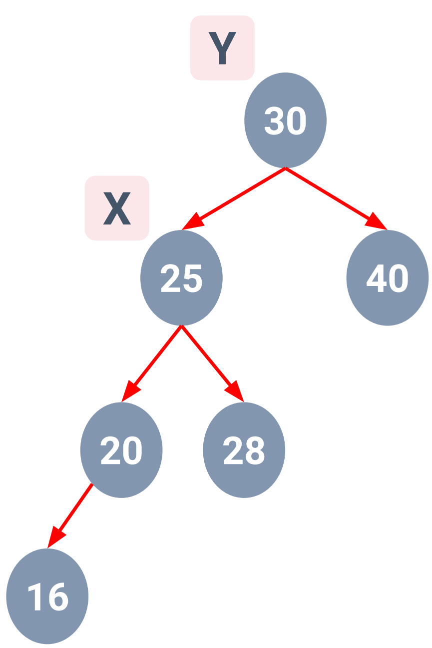


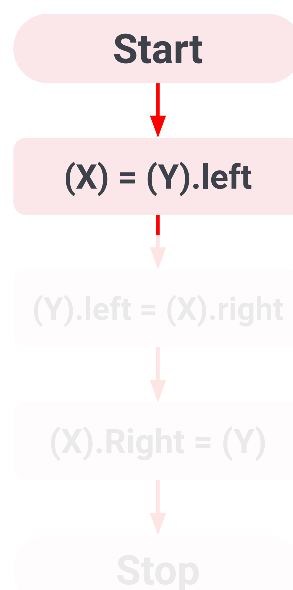
Balance Factor Condition is $|H_{left} - H_{right}| \leq 1$



where (Y) is a node which is a rotated node

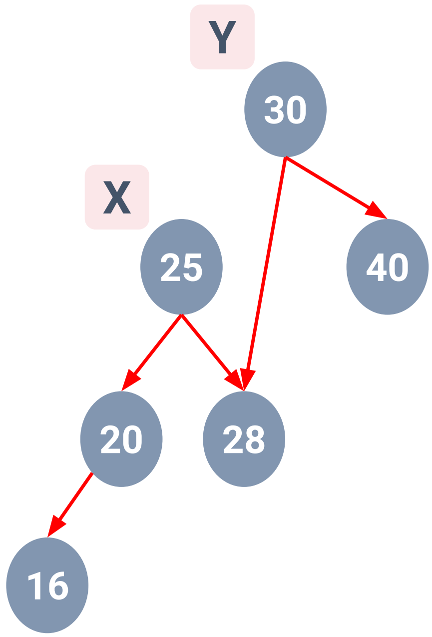
Balancing AVL Tree (LoL)





where (Y) is a node which is a rotated node

Balancing AVL Tree (LoL)



Start

(X) = (Y).left

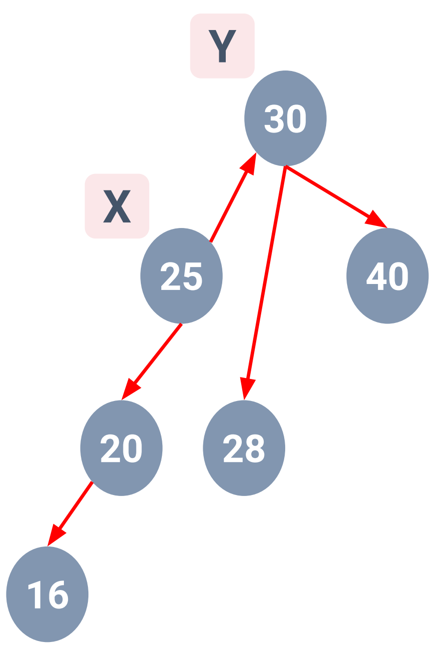
(Y).left = (X).right

(X).Right = (Y)

Stop

where (Y) is a node which is a rotated node

Balancing AVL Tree (LoL)



Start

(X) = (Y).left

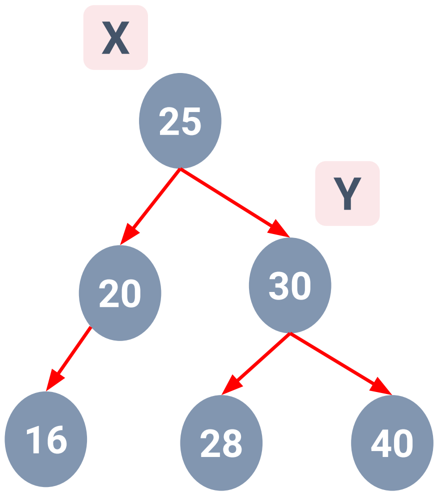
(Y).left = (X).right

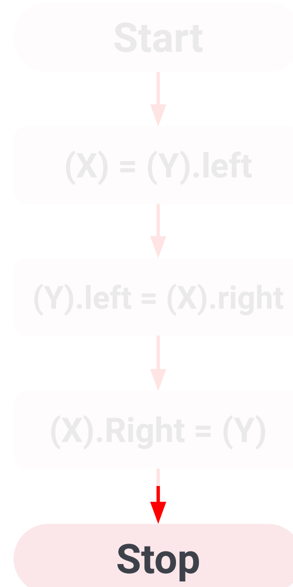
(X).Right = (Y)

Stop

where (Y) is a node which is a rotated node

Balancing AVL Tree (LoL)





where (Y) is a node which is a rotated node

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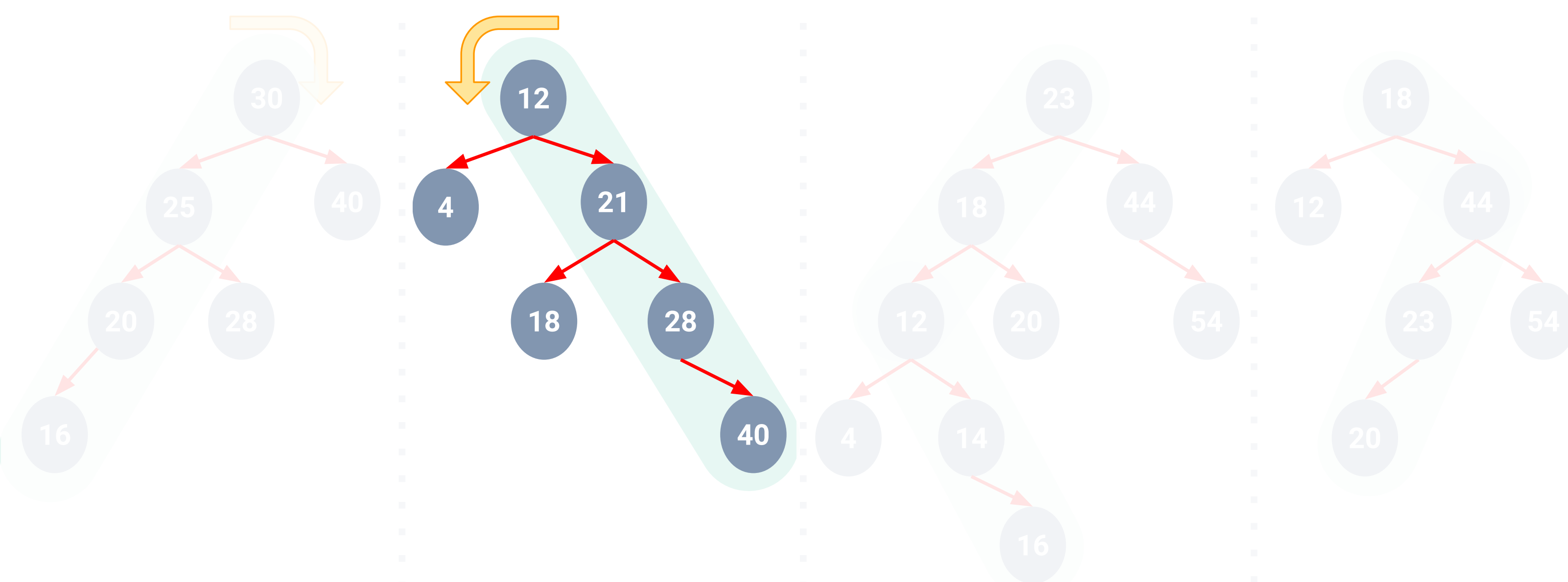
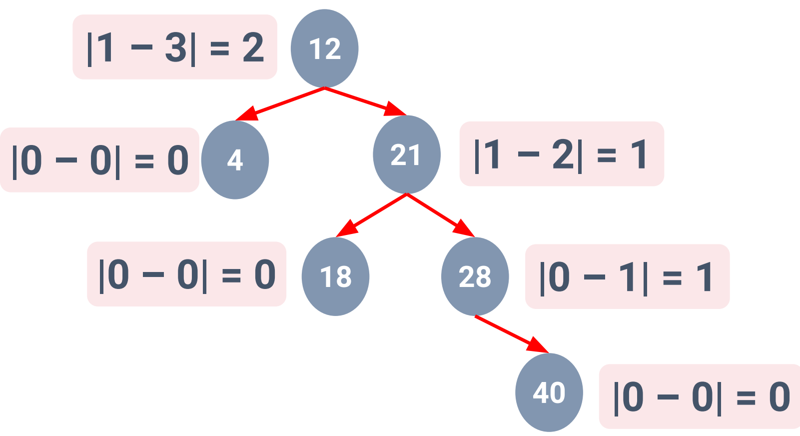
Balancing AVL Tree



Left of Left | **Right of Right** | Right of Left | Left of Right

Balancing AVL Tree (RoR)

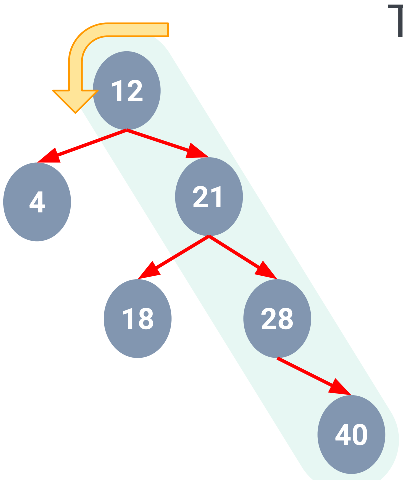


A tree is **right-heavy** with a balance factor of 2 at the root.

Require a **left rotation**.

Balance Factor Condition is $|H_{left} - H_{right}| \leq 1$

Left Rotation

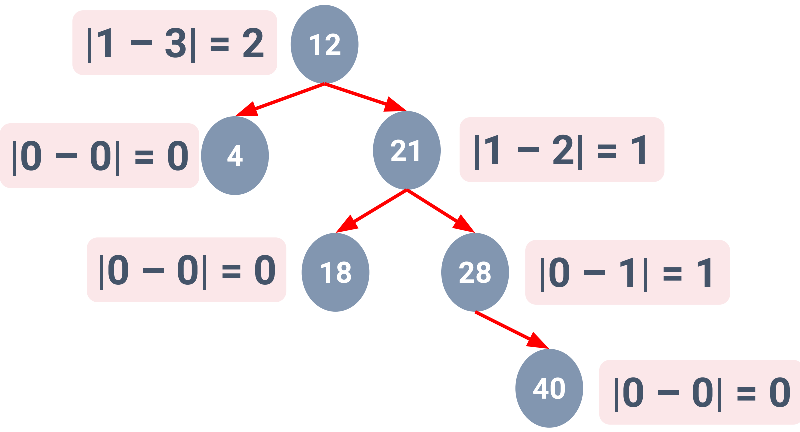


Right of Right

To perform a left rotation (at node 12), do **4 steps** below:

- Promote the right child (21) to be the root of the subtree.
- Move the old root (12) to be the left child of the new root.
- If the new root (21) already had a left child (18),
  - The left child (18) become the right child of the new left child (12).
- Update parents pointers of old root node (if exist).

Balancing AVL Tree (RoR)

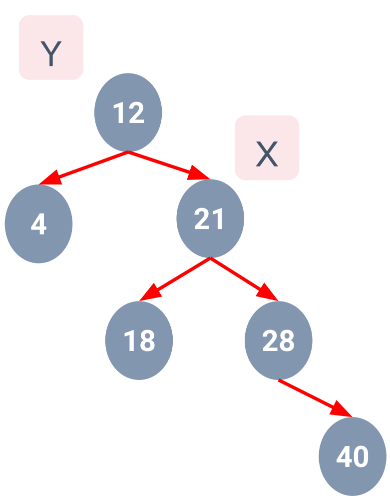


Balance Factor Condition is $|H_{left} - H_{right}| \leq 1$

Start
↓
(X) = (Y).right
↓
(Y).right = (X).left
↓
(X).left = (Y)
↓
Stop

where (Y) is a node which is a rotated node

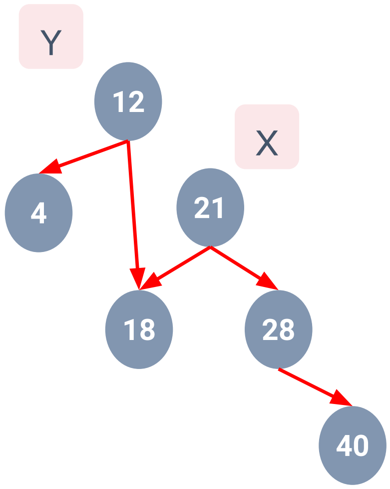
Balancing AVL Tree (RoR)



Start
↓
(X) = (Y).right
↓
(Y).right = (X).left
↓
(X).left = (Y)
↓
Stop

where (Y) is a node which is a rotated node

Balancing AVL Tree (RoR)



Start

(X) = (Y).right

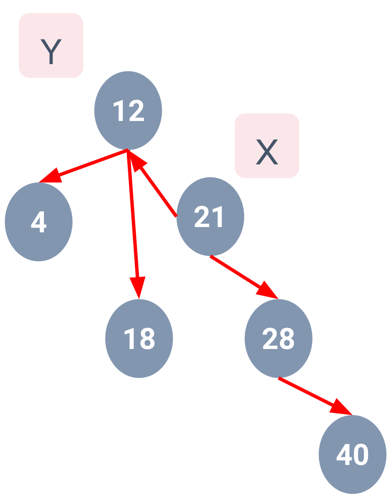
(Y).right = (X).left

(X).left = (Y)

Stop

where (Y) is a node which is a rotated node

Balancing AVL Tree (RoR)



Start

(X) = (Y).right

(Y).right = (X).left

(X).left = (Y)

Stop

where (Y) is a node which is a rotated node

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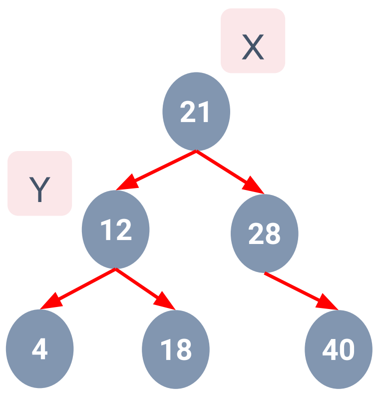
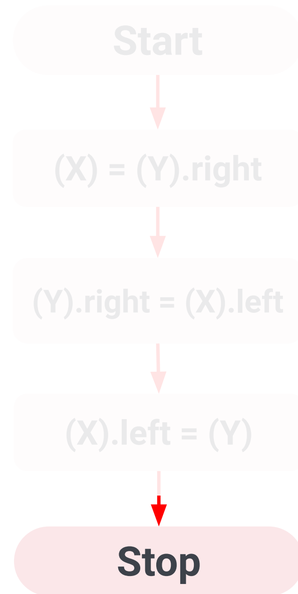
Balancing AVL Tree (RoR)





where (Y) is a node which is a rotated node

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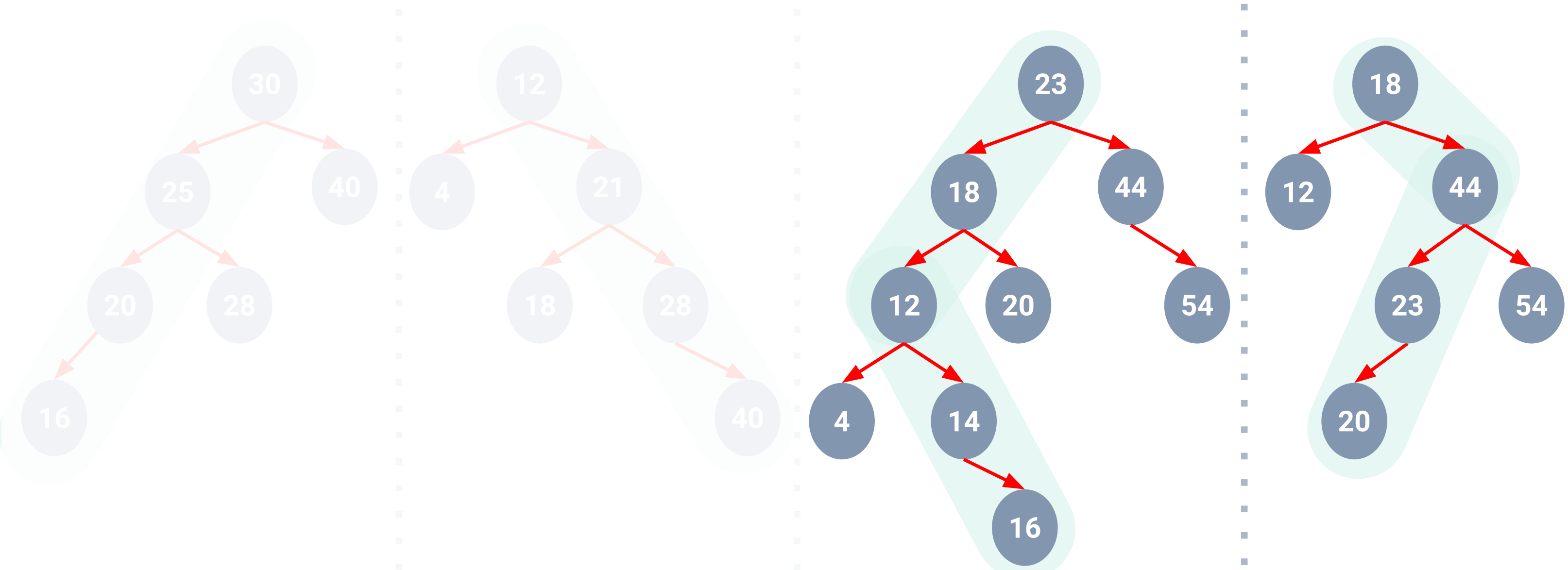
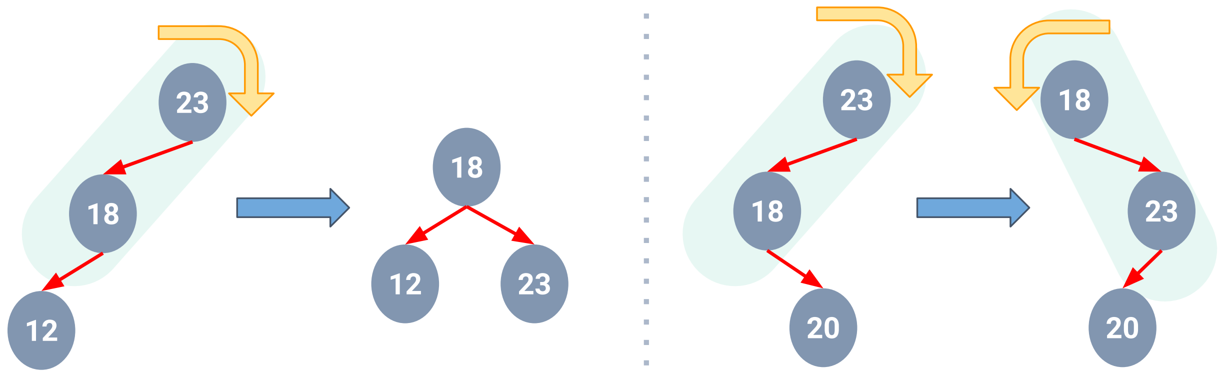
Balancing AVL Tree



Left of Left | Right of Right | Right of Left | Left of Right

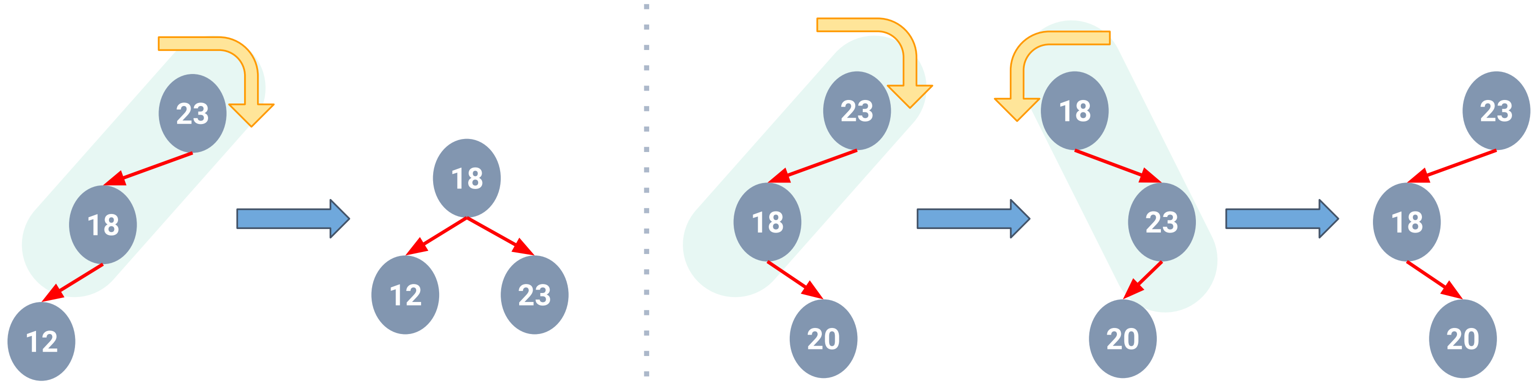
Right Rotation



Left Heavy | Balanced | Left Heavy | Right Heavy

Right Rotation | Right Rotation

Right Rotation



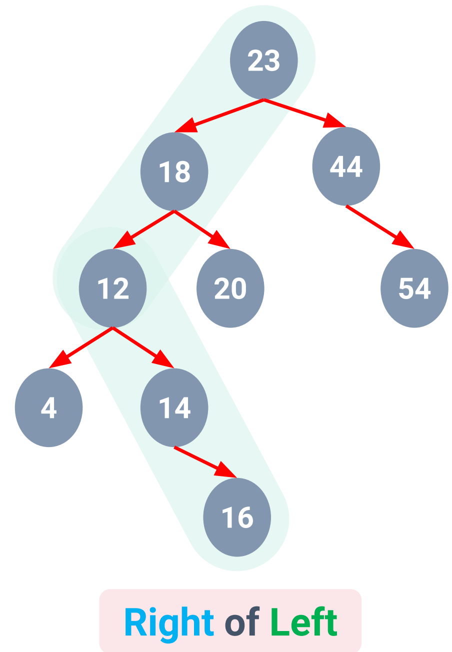
Left Heavy | Balanced | Left Heavy | Right Heavy | Left Heavy

Right Rotation | Left Rotation

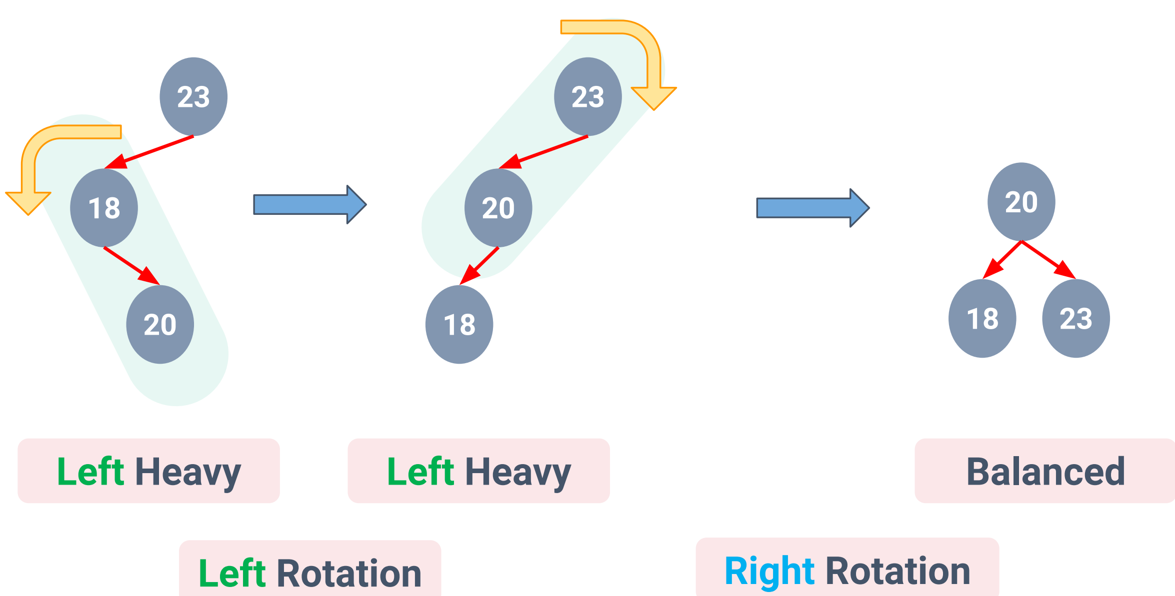
Left then Right Rotation

To solve this problem, there are additional Rules:

- If a subtree needs a **right rotation**,
  - Check the balance factor of the left child.
  - If the left child is right-heavy, then do **left rotation on the left child**.
  - Then do right rotation on the subtree.



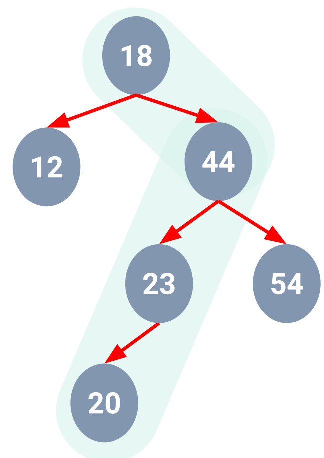
Left then Right Rotation



Right then Left Rotation

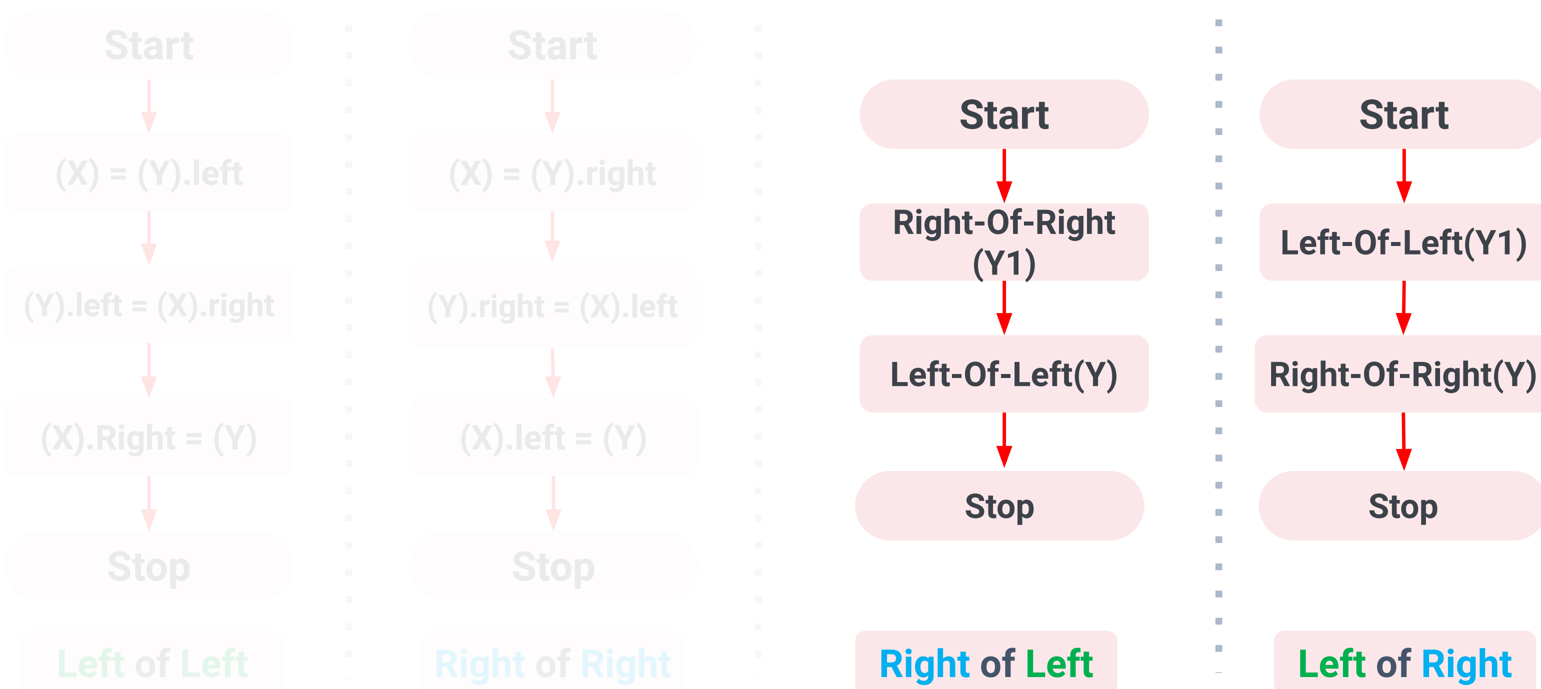
Additional Rules:

- If a subtree needs a **left rotation**,
  - Check the balance factor of the right child.
  - If the right child is left-heavy, then do **right rotation on the right child**.
  - Then do left rotation on the subtree.



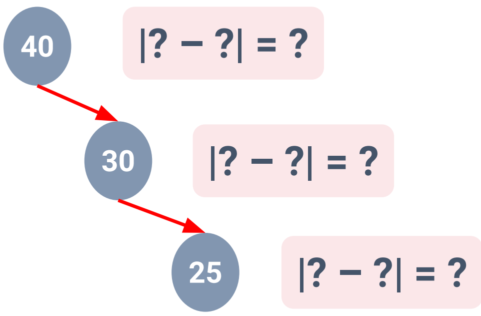
Left of Right

Balancing AVL Tree



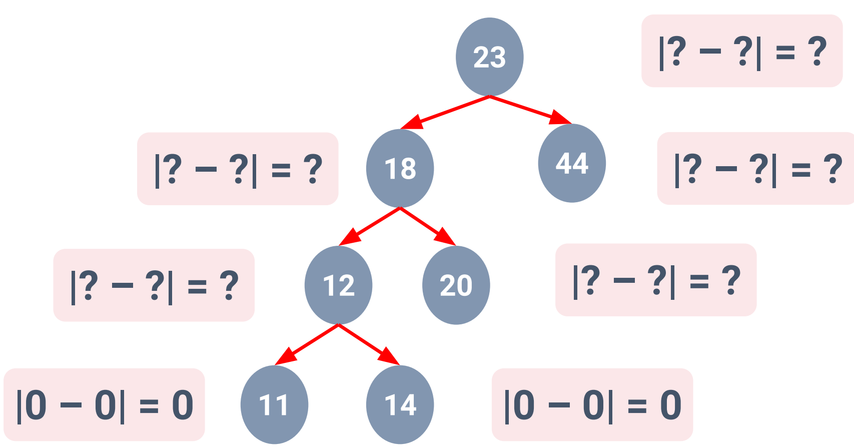
Balancing AVL Tree Exercise 1

Exercise 1: Rebalance the given AVL tree.

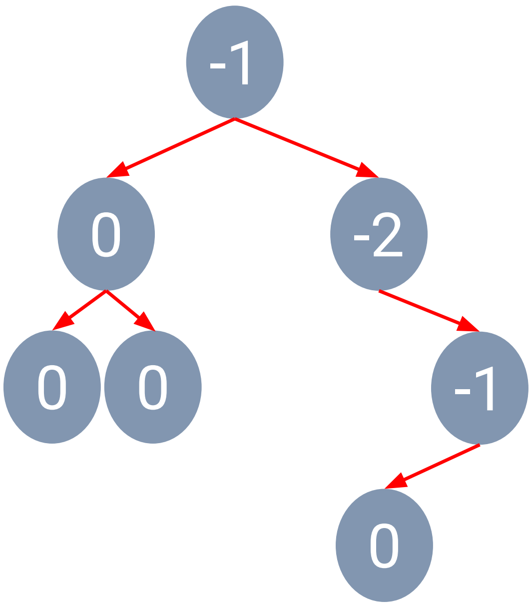


Balancing AVL Tree Exercise 2

Exercise 2: Rebalance the given AVL tree.



Balance Factor in AVL Tree



- AVL tree is considered to be <u>balanced</u> when the balance factor is -1, 0, or 1.
  - $|H_{left} - H_{right}| <= 1$
- AVL tree ensure that accessing the node costs only $\boldsymbol{O(\log_2 n)}$ time.

Individual Assignment

- Assignment#6: BST and AVL trees
- Due 09.00 am, Tuesday 06/10/2020.
- Submission
  - Email: sirasit@it.kmitl.ac.th
  - Paper: in classroom next week
- Can be either written by hand or typing.
- Make sure to submit on time!!
  - Late submission has penalty on the score.
- If unable to submit on time for reasonable reasons, let me know asap.