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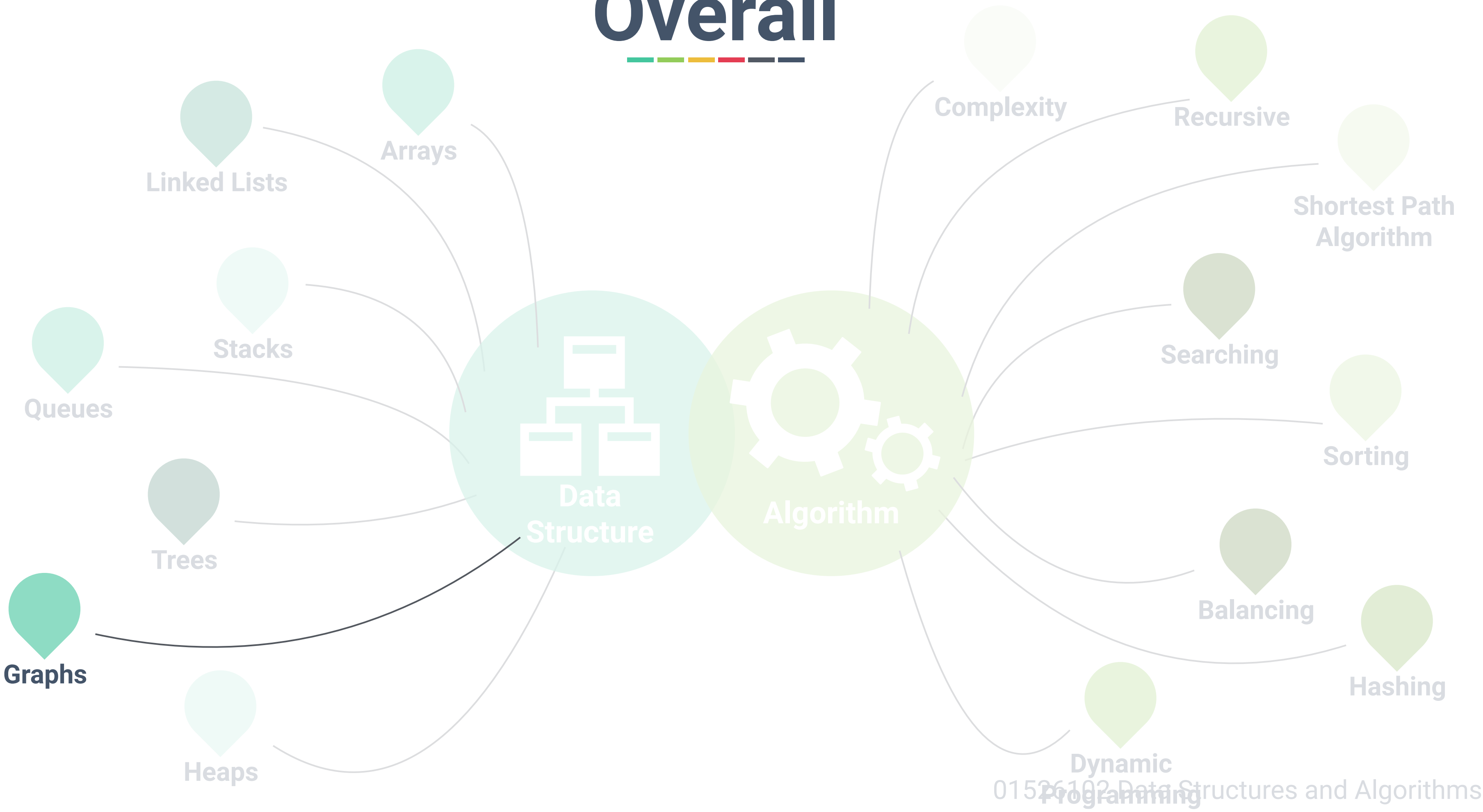
ENGINEERING PROGRAMS
King Mongkut's Institute of Technology Ladkrabang

# Chapter 11: Graphs

Dr. Sirasit Lochanachit

# Overall


Arrays
Linked Lists
Stacks
Queues
Trees
Graphs
Heaps
Data Structure
Algorithm
Complexity
Recursive
Shortest Path Algorithm
Searching
Sorting
Balancing
Hashing
Dynamic Programming

# Outline

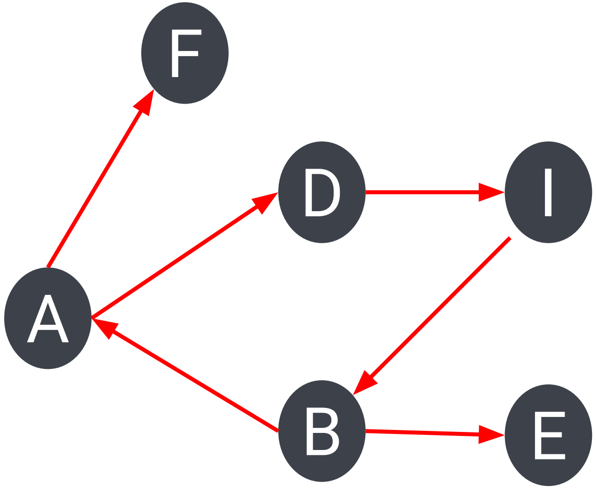
Graphs

- Definition, elements and types
- Graph Representation

Graph Algorithms

- Traversal
  - Depth-first
  - Breadth-first
- Shortest Path

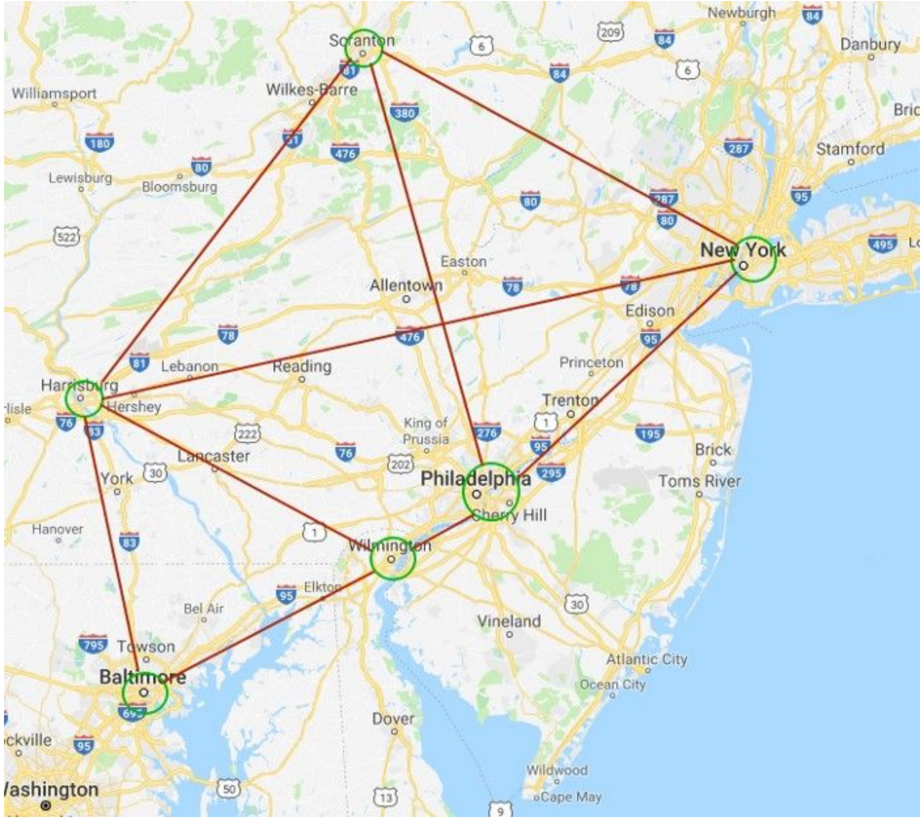
# What is a Graph?

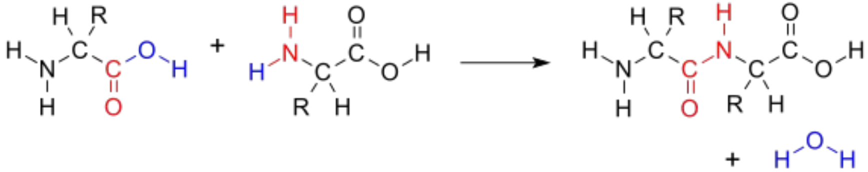



- A **graph** is a set of objects, called vertices or **nodes**, where the actual data is stored and a collection of connections between them, called **edges** or arcs[1].
- A graph can be used to represent relationships between pairs of objects.

[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

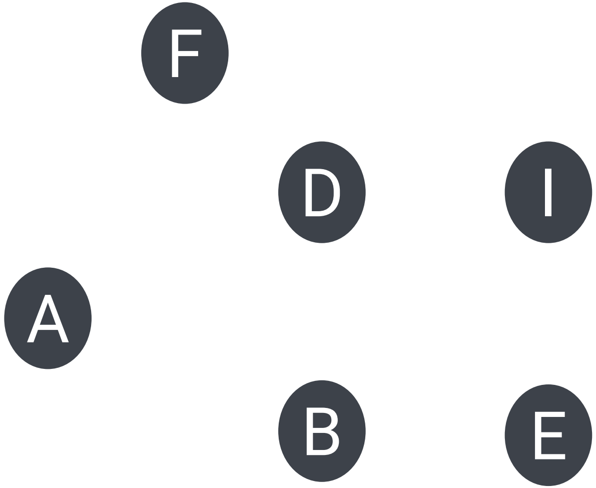
# Graphs



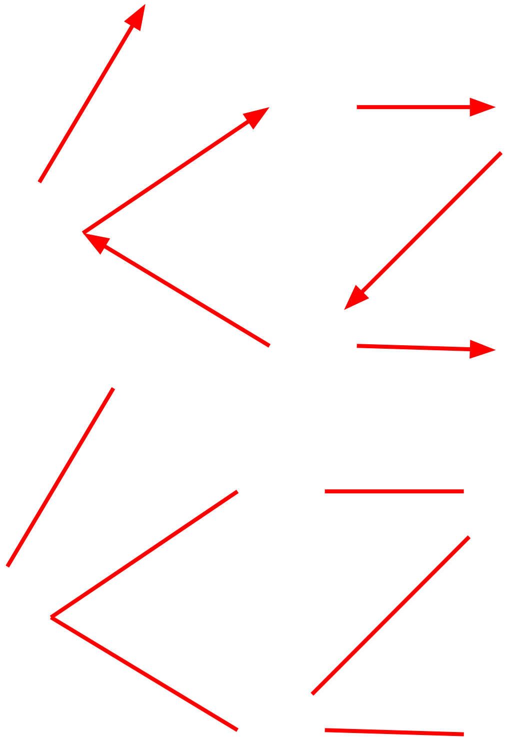



Ref: https://miro.medium.com/max/700/1*2jL0bKbT8ttskAlEVDv1yg.jpeg

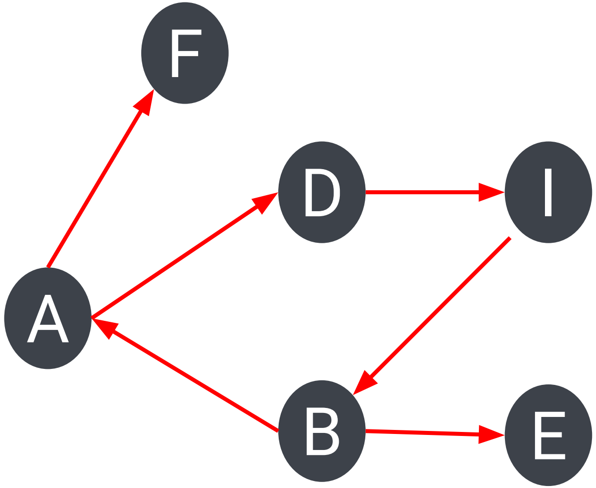
# The Basic Elements of Graph



# The Basic Elements of Graph



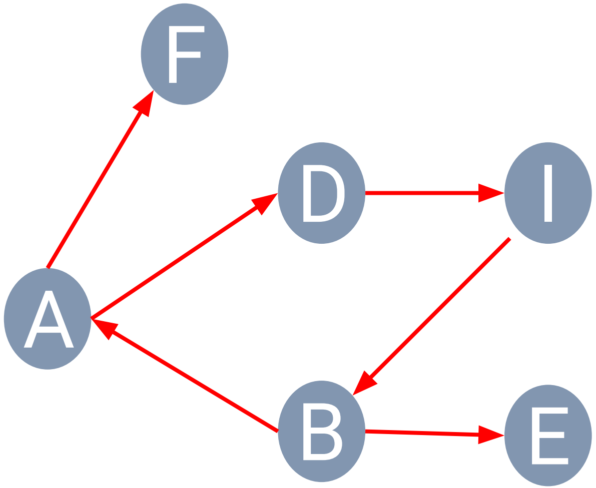
# The Basic Elements of Graph



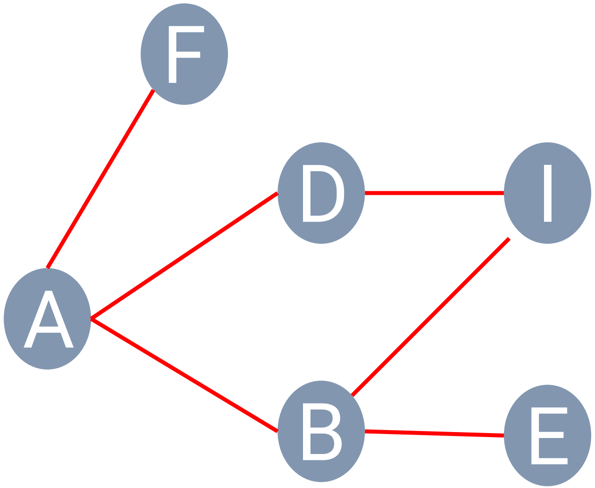

- Formally, a graph *G* is a set *V* of vertices and a collection *E* of pairs of vertices, called edges[1].

[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

# Graph Types



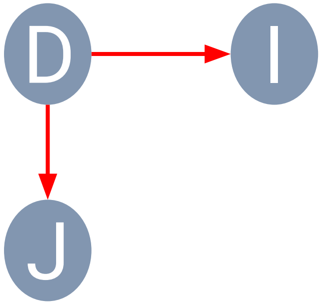

Directed




Undirected

- An edge $(u, v)$ is directed from $u$ to $v$ if the pair $(u, v)$ is ordered.
- An edge $(u, v)$ is undirected if the pair $(u, v)$ is not ordered.

# Graph Terminology

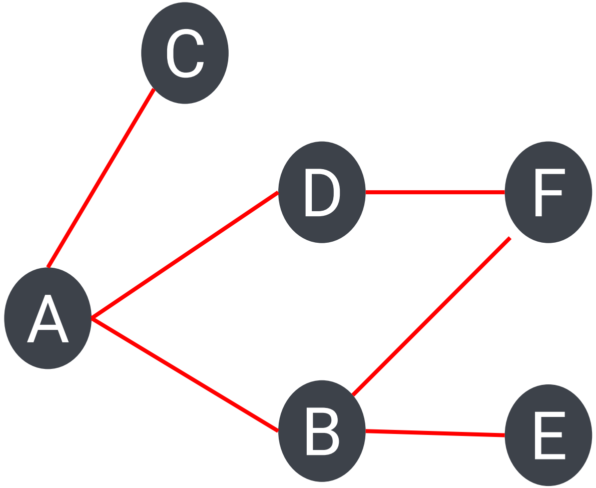



V = {D, I, J}

E = {(D, I), (D, J)}

- **Endpoints**: Two nodes (*u*, *v*) that are joined by an edge.
    - These two nodes are **adjacent**.
- **Origin**: First endpoint (*u*) on a directed edge.
- **Destination**: Second endpoint (*v*) on a directed edge.

# Graph Representation

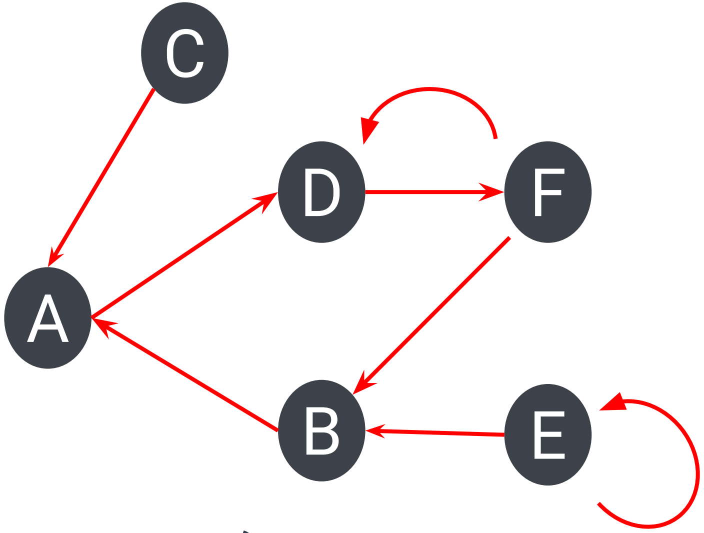



|  | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A |  |  |  |  |  |  |
| B |  |  |  |  |  |  |
| C |  |  |  |  |  |  |
| D |  |  |  |  |  |  |
| E |  |  |  |  |  |  |
| F |  |  |  |  |  |  |

- V = {A, B, C, D, E, F}
- E = {(A, C), (A, D), (A, B), (D, F), (B, E), (B, F)}

Adjacency Matrix

# Graph Representation

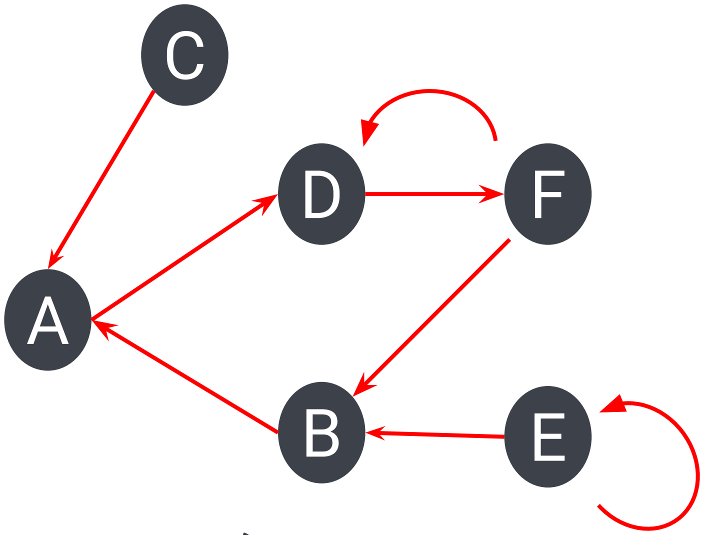


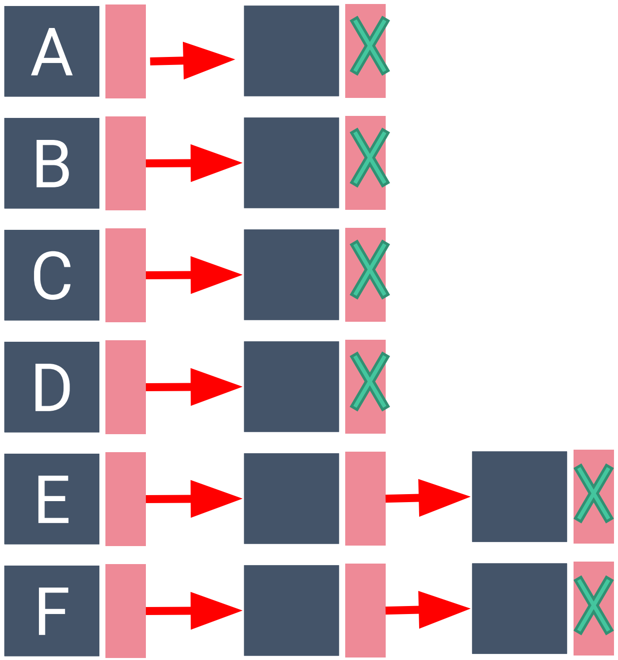

|   | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 1 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 | 0 |
| C | 1 | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 0 | 0 | 1 |
| E | 0 | 1 | 0 | 0 | 1 | 0 |
| F | 0 | 1 | 0 | 1 | 0 | 0 |

Adjacency Matrix

- V = {A, B, C, D, E, F}
- E = {(C, A), (A, D), (B, A), (D, F), (F, D), (E, B), (F, B), (E, E)}

# Graph Representation




- V = {A, B, C, D, E, F}
- E = {(C, A), (A, D), (B, A), (D, F), (F, D), (E, B), (F, B), (E, E)}

Adjacency List

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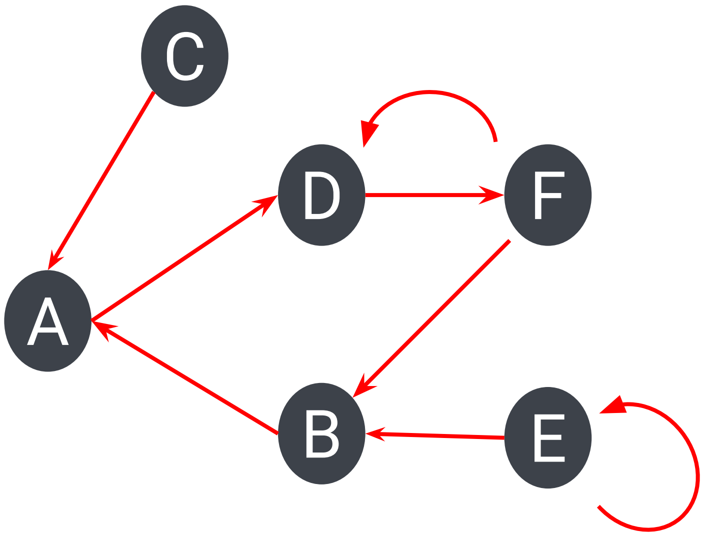
# Graph Representation

## Node Representation

| Node | Name | Phone |
|---|---|---|
| A | Able | |
| B | Baker | |
| C | Charlie | |
| D | Denver | |
| E | Ethan | |
| F | Fred | |

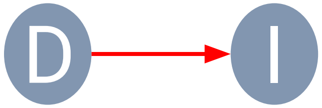



## Edge Representation

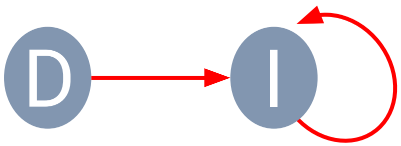
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | 1 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 | 0 |
| C | 1 | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 0 | 0 | 1 |
| E | 0 | 1 | 0 | 0 | 1 | 0 |
| F | 0 | 1 | 0 | 1 | 0 | 0 |

Adjacency Matrix

# Graph Terminology

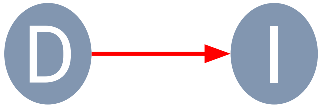



- A **path** is a sequence of nodes and edges that starts at a node and ends at a node such that each node is adjacent to the next one[2].
- Formally, a path is a sequence of nodes $V_1, V_2, V_3, \ldots, V_n$ where $(V_1, V_2), (V_2, V_3), \ldots, (V_{n-1}, V_n) \in E$.
- A **loop** is a special case of path where two endpoints are the same.
  - An edge that starts and ends with the same node.




[2] Bradley N. Miller, David L. Ranum, Problem Solving with Algorithms and Data Structures using Python, 2011

# Graph Properties




- A **cycle** is a path that starts and ends at the same node, having at least one edge.
- A **simple path** is a path that does not contain the same edge more than once.
- A **simple cycle** is a simple path that starts and ends at the same node.

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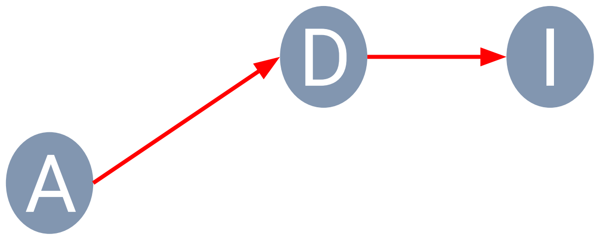
# Graph Properties Example

Simple?

Cyclic?

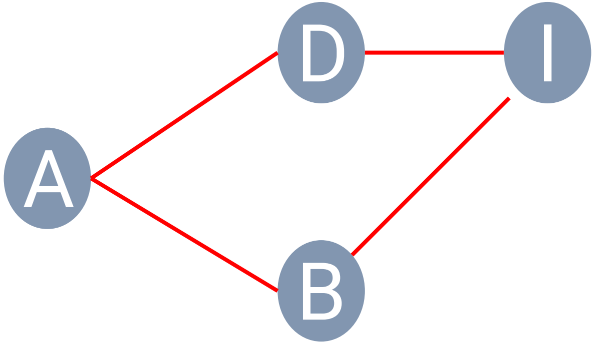



A, D, I

# Graph Properties Example

Simple?

Cyclic?

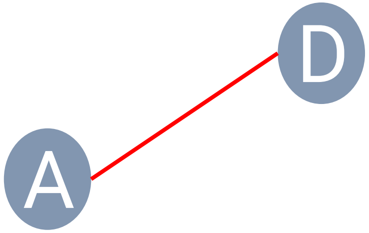



A, D, I, B, A

# Graph Properties Example

Simple?

Cyclic?




A, B, A

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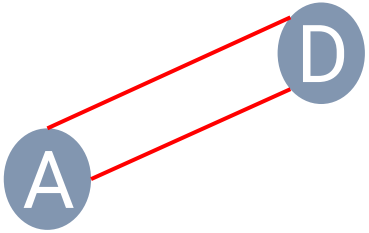
# Graph Properties Example

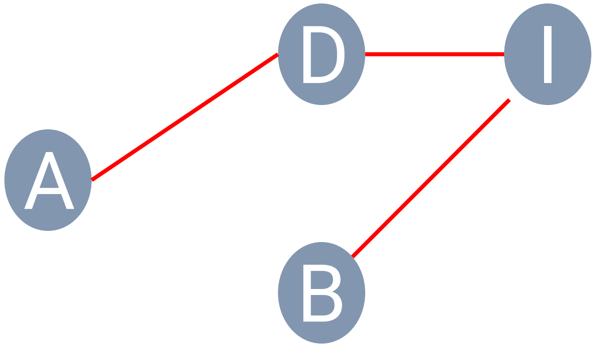
Simple?

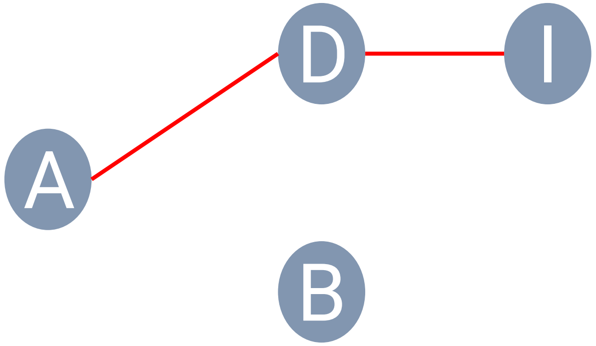
Cyclic?




A, D, A

# Graph Notations




- A graph is **connected** if, for any two nodes, there is a path between them.
- The **in-degree** of a node *v* is the number of the incoming edges of *v*.
- The **out-degree** of a node *v* is the number of the outgoing edges of *v*.

# Graph Algorithms

- Traversals
  - Depth-first traversal
  - Breadth-first traversal
- Minimum Spanning Tree
  - Prim-Jarnik Algorithm
  - Kruskal's Algorithm
- Shortest Path
  - Dijkstra Algorithm
- Etc.

# Graph Traversals

- A **traversal** is a systematic procedure for exploring a graph by examining all of its nodes and edges.
- Graph traversal algorithms are key to answering many fundamental questions about graphs involving the notion of **reachability**, that is, in determining how to travel from one node to another while following paths of a graph[1].
- Two efficient graph traversal algorithms: **depth-first traversal** and **breadth-first traversal**.

[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

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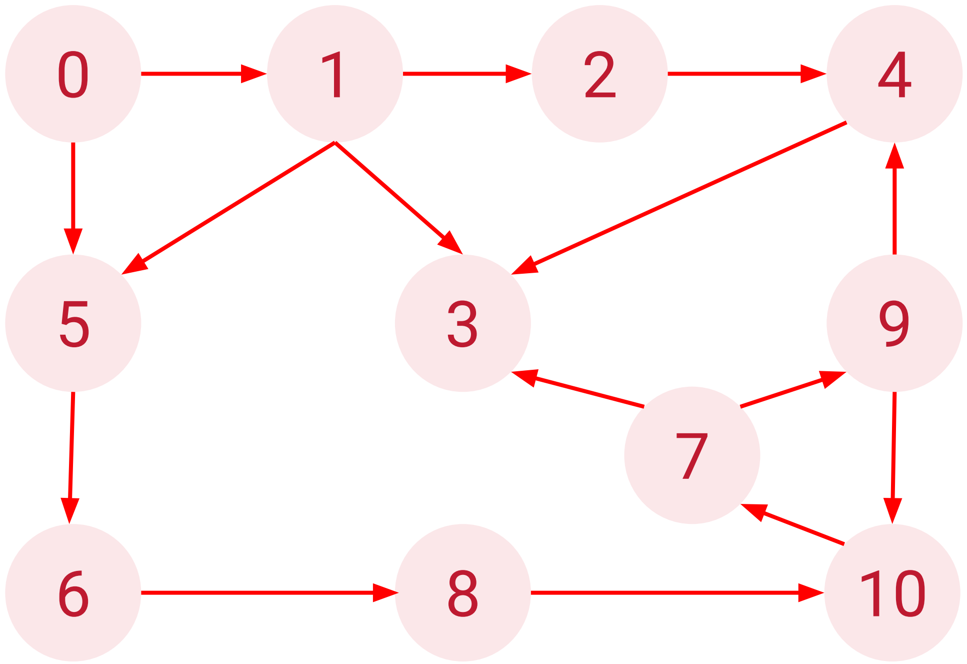
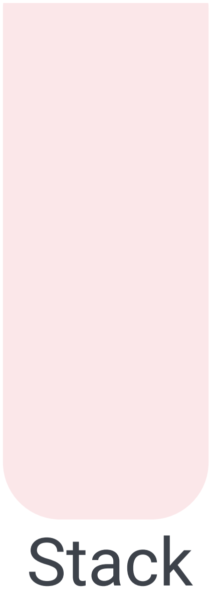
# Depth-First Traversal with Stack




Output :

# Depth-First Traversal with Stack

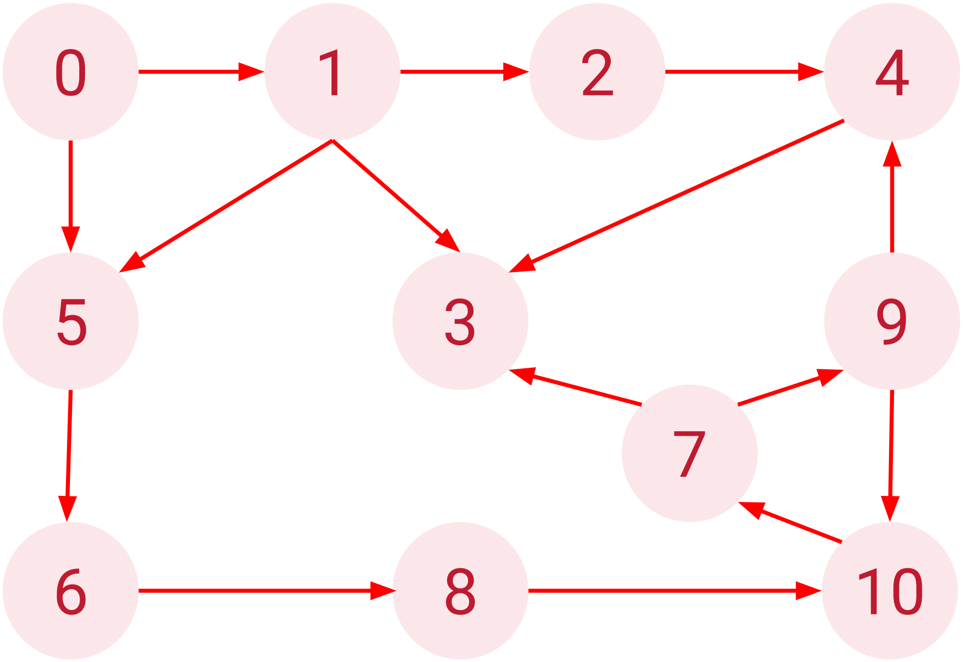
0

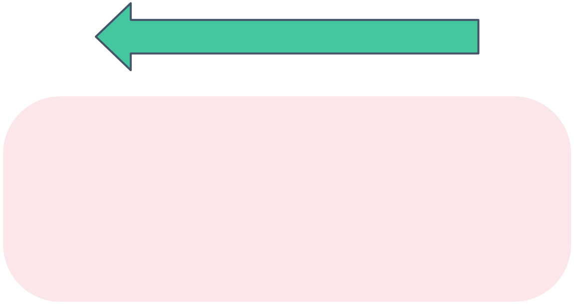
0

Stack

Output : 0

# Breadth-First Traversal with Queue



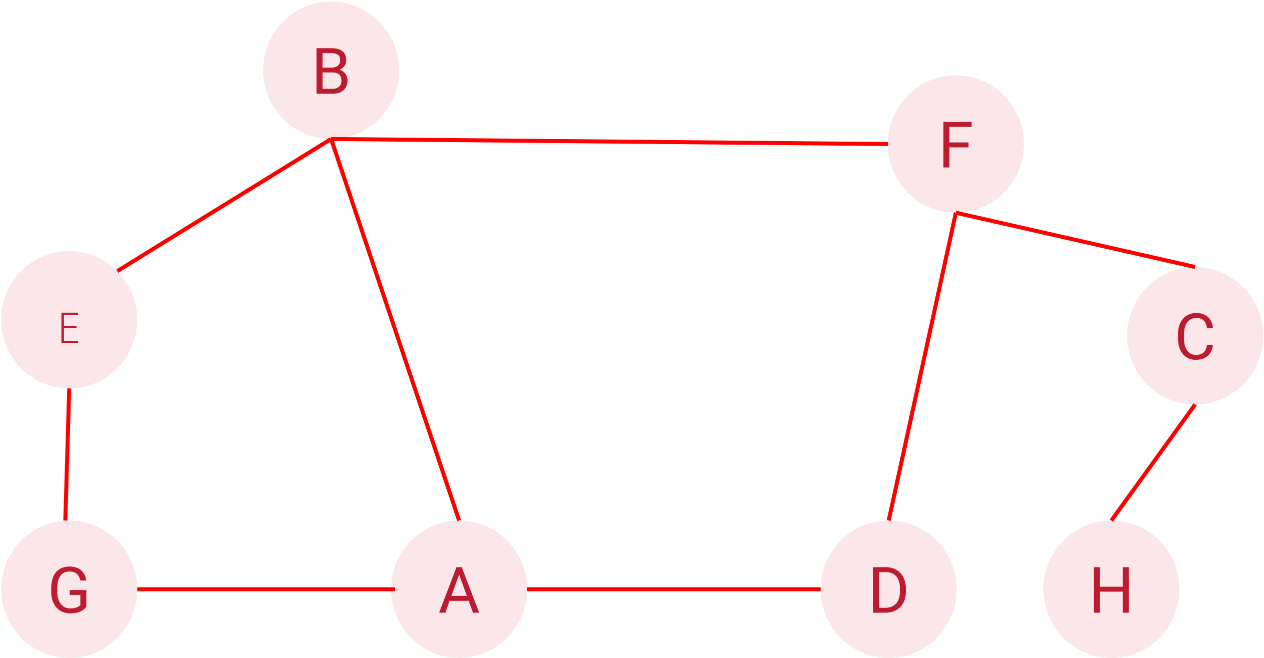
Queue

Output :

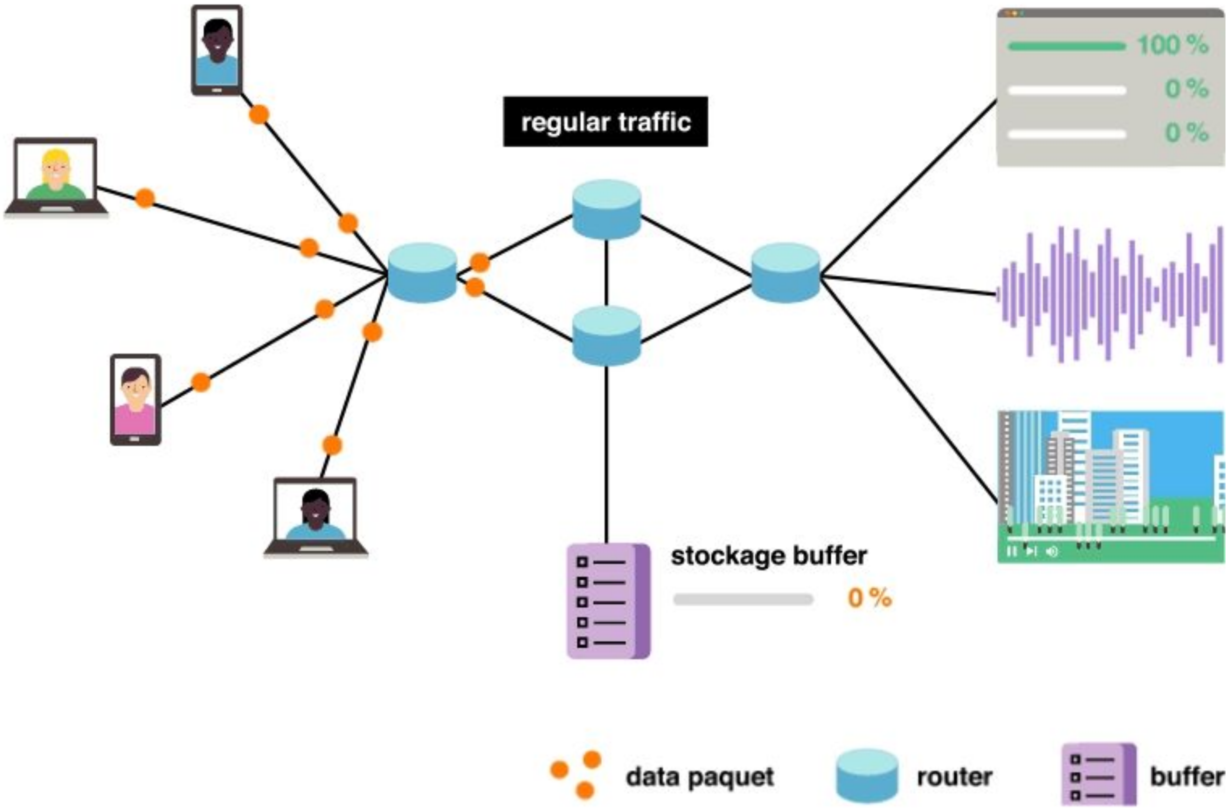
# Breadth-First Traversal with Queue


Output :

# Depth-First and Breadth-First Exercise



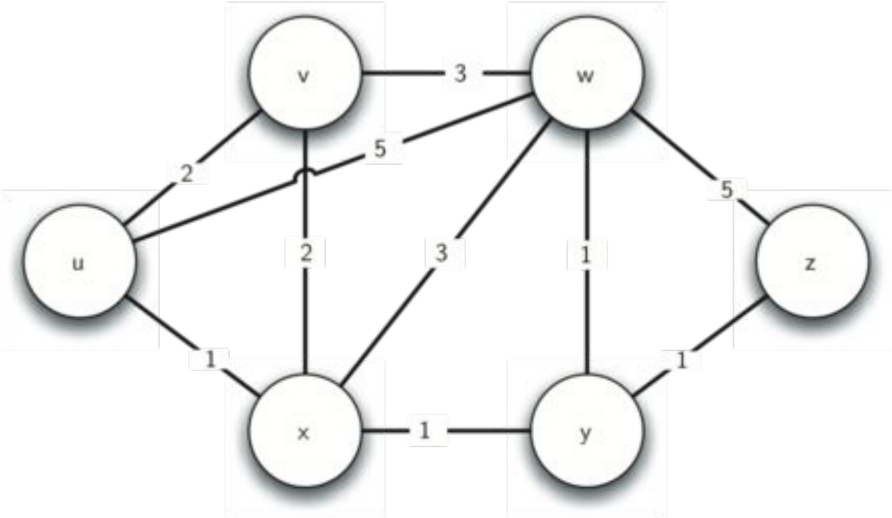
# Shortest Path Problem




Ref: https://hellofuture.orange.com/en/telecoms-networks-well-equipped-to-deal-with-traffic-peaks/

# Shortest Path Problem

- The network of routers can be represented as a graph with weighted edges.

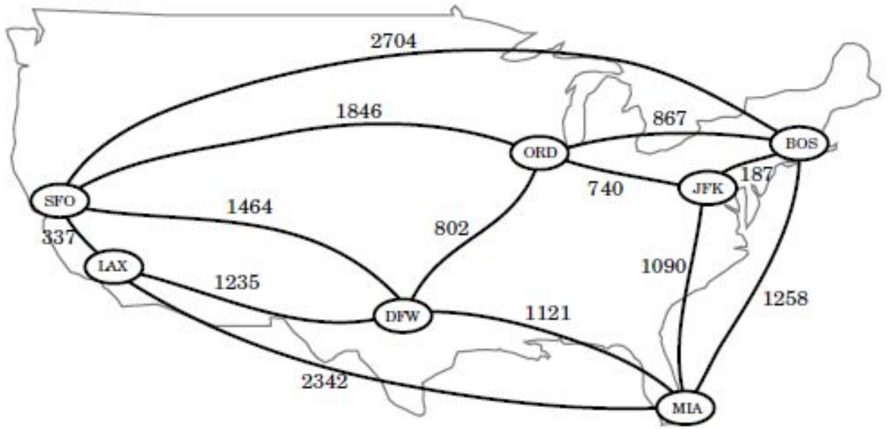


# Shortest Path Problem

- The breadth first strategy can be used to find a **shortest path** from some starting node to every other node in a connected graph.
  - This approach is suitable in cases where each edge is equal to others.
  - However, for other situations, this approach is not efficient.
- It is natural, therefore, to consider graphs whose edges are <u>not weighted equally</u>.

# Weighted Graphs

- A **weight graph** is a graph that has a numeric label *w*(*e*) associated with each edge *e*, called the **weight** of edge *e*.
- For $e = (u, v)$, $w(u, v) = w(e)$.
- Such weights might represent:
  - Costs
  - Lengths
  - Capacities
  - etc.



# Defining Shortest Paths in a Weighted Graph

- Let *G* be a weighted graph.
- The **length** (or **weight**) of a **path** is the sum of the weights of the edges of *P*.
  - P = (($v_0$, $v_1$), ($v_1$, $v_2$), …, ($v_{k-1}$, $v_k$))
  - Length of P, denoted w(P) is defined as $$w(P) = \sum_{i=0}^{k-1} w(v_i, v_{i+1}).$$
- The distance from a node *u* to a node *v* in *G*, denoted *d*(*u*, *v*) is the length of a minimum-length path (also called **shortest path**) from *u* to *v*.

# Shortest Paths Algorithms

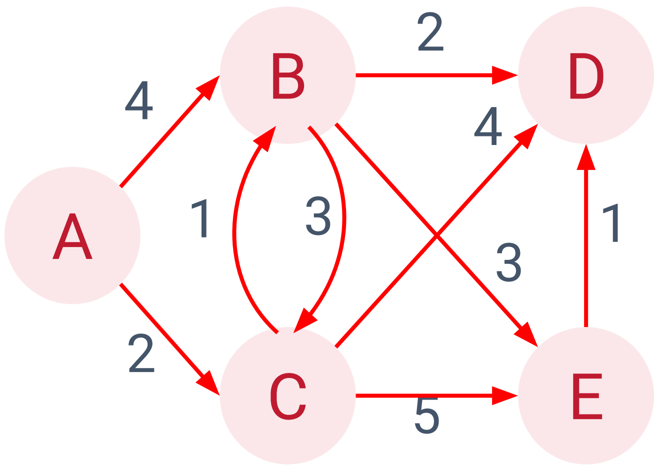
- Shortest path in a graph with all equal weights can be solved with breadth-first traversal algorithm.
- Distance cannot be arbitrarily low negative numbers.
  - For instance, the weight of edges represent the cost to travel between cities. If someone pay you to go between the cities, the cost would be negative.
  - Edge weights in *G* should be nonnegative (that is, $w(e) >= 0$) for each edge.

# Dijkstra's Algorithm

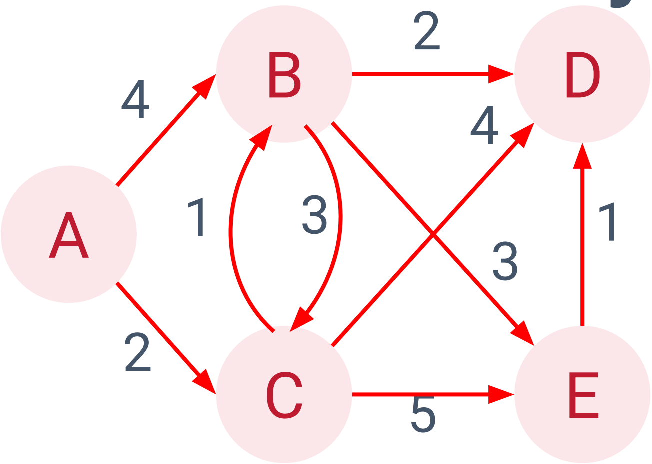
- An iterative algorithm that provides the shortest path from one starting node to all other nodes in the graph[2].
- Apply **greedy method** to solve the problem by repeatedly selecting the best choice from among those available in each iteration.
  - Useful for optimising cost function over a collection of objects.
- "Weight" breadth-first search starting at the source node *s*.
- Used in link-state routing protocols in computer network.

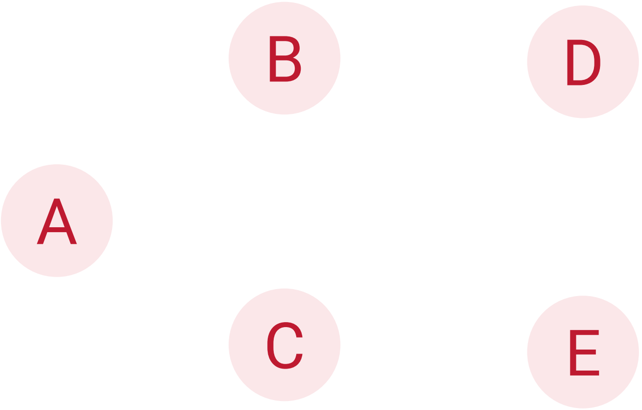
[2] Bradley N. Miller, David L. Ranum, Problem Solving with Algorithms and Data Structures using Python, 2011

# Dijkstra’s Algorithm



# Dijkstra's Algorithm

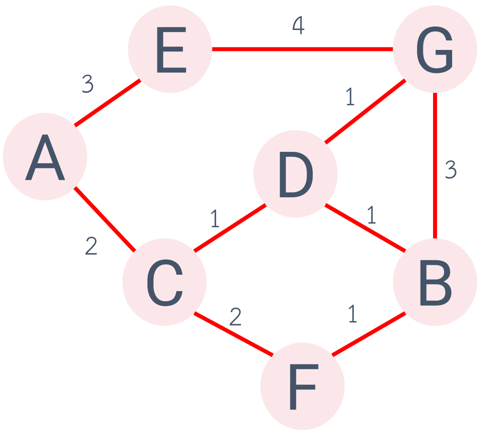



Q = {'A', 'B', 'C', 'D', 'E'}

| Node | Cumulative weight #1 | Cumulative weight #2 | Cumulative weight #3 | Cumulative weight #4 | Route |
|---|---|---|---|---|---|
| A | | | | | |
| B | | | | | |
| C | | | | | |
| D | | | | | |
| E | | | | | |

# Dijkstra's Algorithm Exercise




Find the shortest path for each node from node A

Find the distance from A to F

| Node | Cumulative weight #1 | Cumulative weight #2 | Cumulative weight #3 | Cumulative weight #4 | Cumulative weight #5 | Cumulative weight #6 | Route |
|---|---|---|---|---|---|---|---|
| A | | | | | | | |
| B | | | | | | | |
| C | | | | | | | |
| D | | | | | | | |
| E | | | | | | | |
| F | | | | | | | |
| G | | | | | | | |

# Dijkstra's Algorithm

- Dijkstra's algorithm works only when the weights are all positive.
  - If there is a negative weight on one of the edges in the graph, the algorithm would never exit.
- Another problem is a complete representation of the graph must be presented for the algorithm to run.
  - Every router has a complete map of all the routers: Not practical.
- Other algorithms allow each router to discover the graph as they go.
  - For instance, distance vector routing algorithm (Computer Networks).
  - Each node computes best path without full view of graph, exchanging link information as they go.