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Chapter 5: Linked Lists

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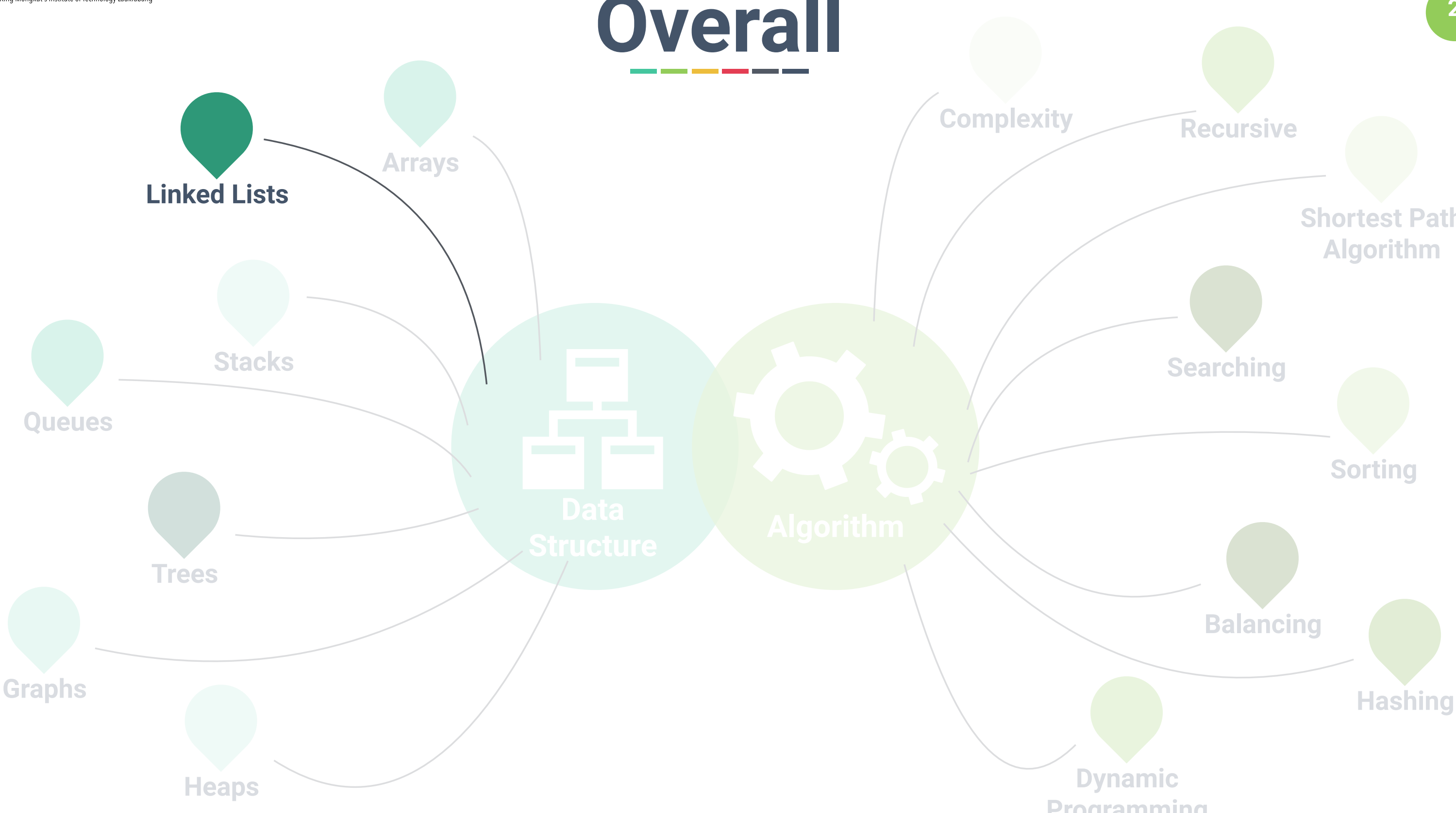
Overall



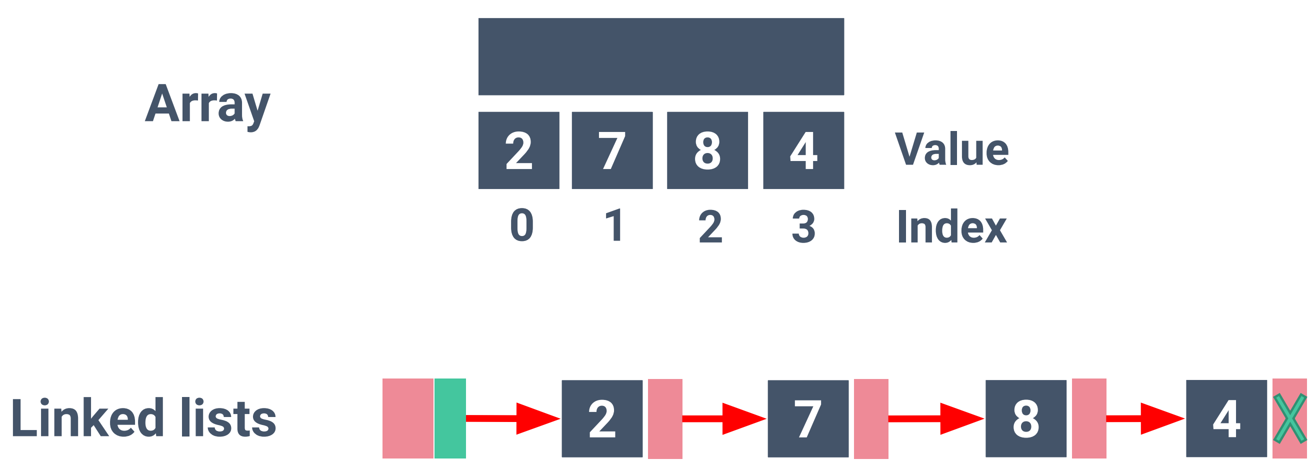
Linked Lists

Disadvantages of array:

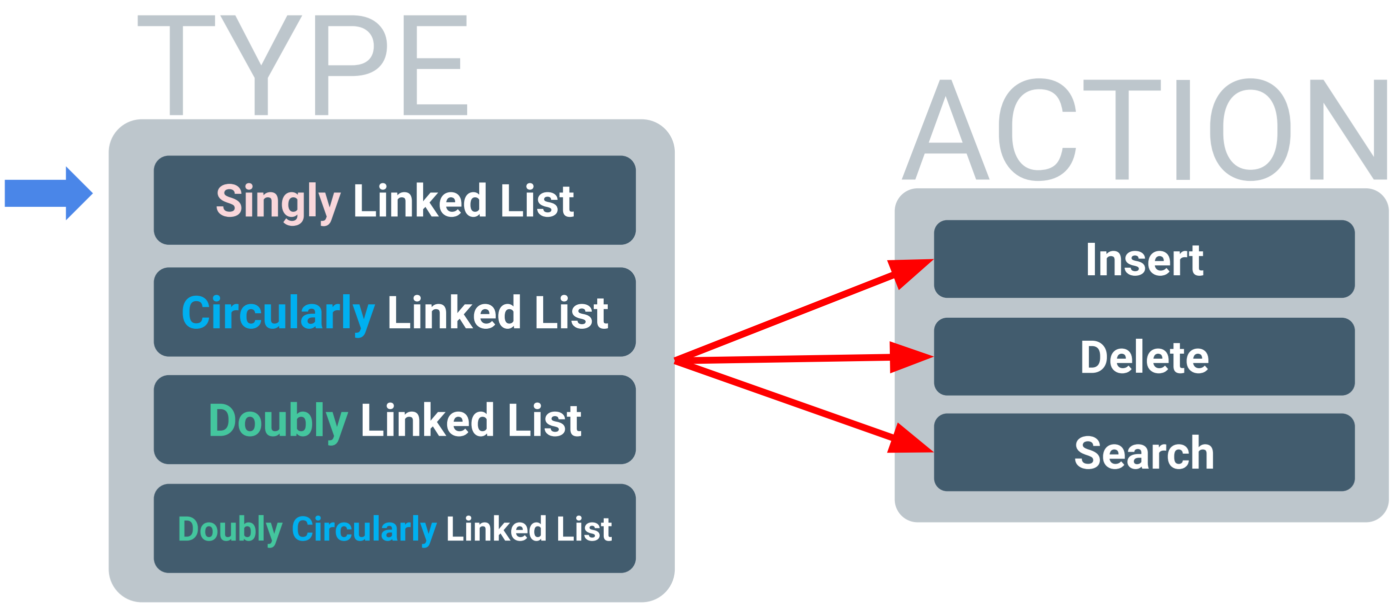
- Length of array has to be pre-allocated, empty space wasted.
- Adding or removing elements between values in the array is expensive - $O(n)$

Linked Lists

To avoid these limitations, an alternative to array is **linked list**.



Linked Lists



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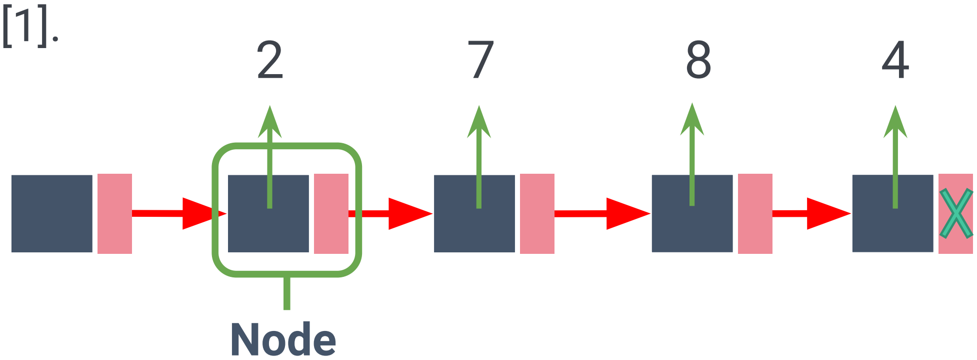
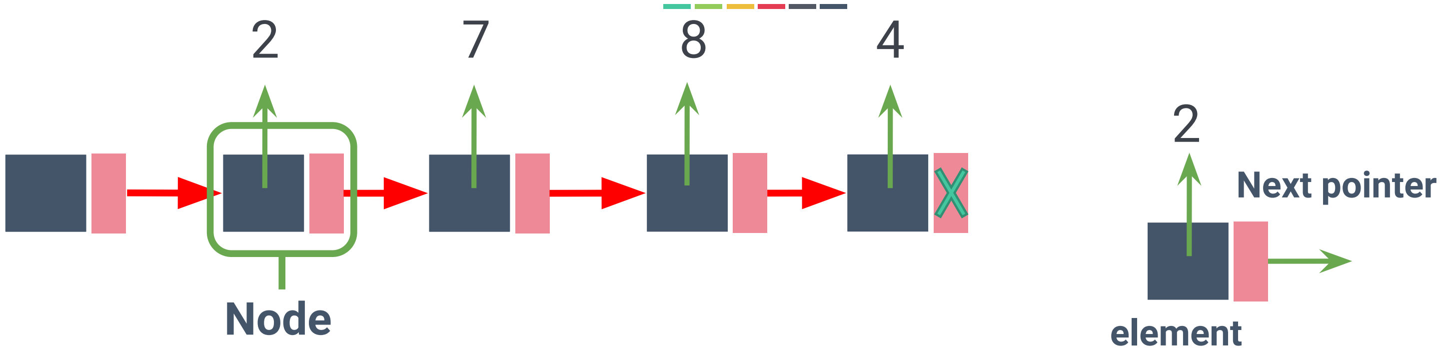
Singly Linked Lists

A singly **linked list** is a collection of nodes that form a linear order of a sequence [1].



[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

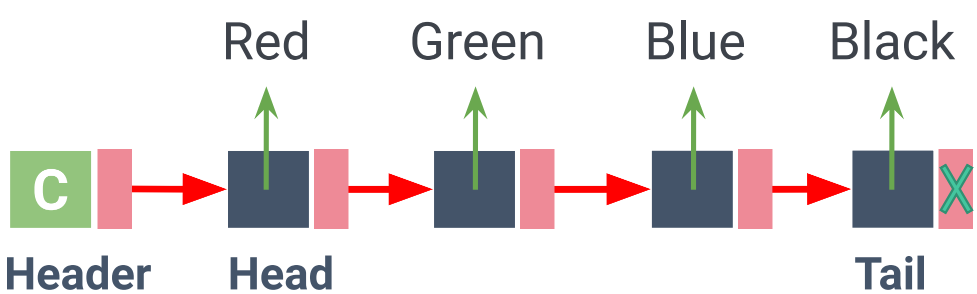
Singly Linked Lists



Each node keeps:

- A reference to an object/value which is its element.
- **Link/Pointer**: One or more references to adjacent nodes or subsequent nodes.
  - Reference to **None** if there is no further node.

Singly Linked Lists



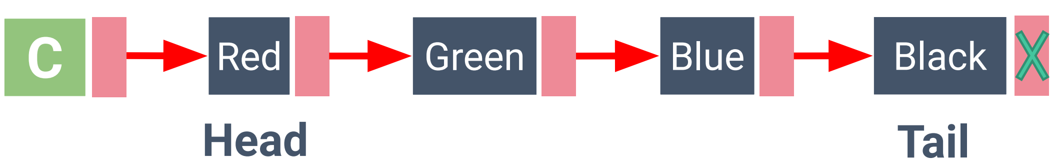
- **Head** and **tail** identify the first and last node, respectively.
- **Header** node can contain a counter to keep track the number of nodes that form a list.

Linked Lists

Real-life examples of Linked Lists:

Retrieved from https://live.staticflickr.com/5610/15429943089_edc7011843_o_d.jpg CC BY 2.0
https://live.staticflickr.com/23/26472155_8cc5066b66_o_d.jpg CC BY-SA 2.0

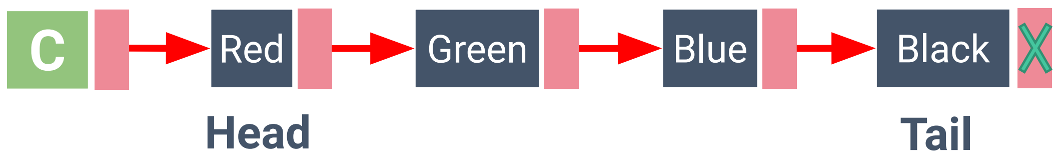
Singly Linked Lists



For simplicity, the linked list illustration will embed element within the node.

Note that each node still contains a reference to the element, not the element itself directly.

Singly Linked Lists



- **Traversing** or **link hopping** is the process of moving from one node to another according to each node's subsequent pointer.
- Linked Lists provides **sequential access** only.
  - Locating the element in a linked list requires *O*(n) time to traverse the list from the beginning.

Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None

Suppose that it takes 1 byte to store an integer.

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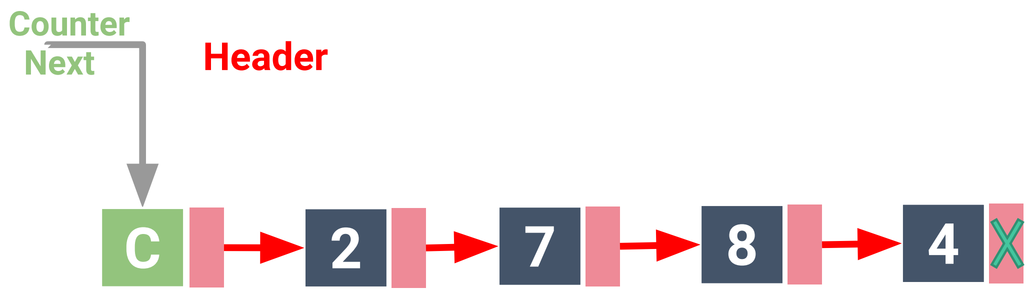
Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None



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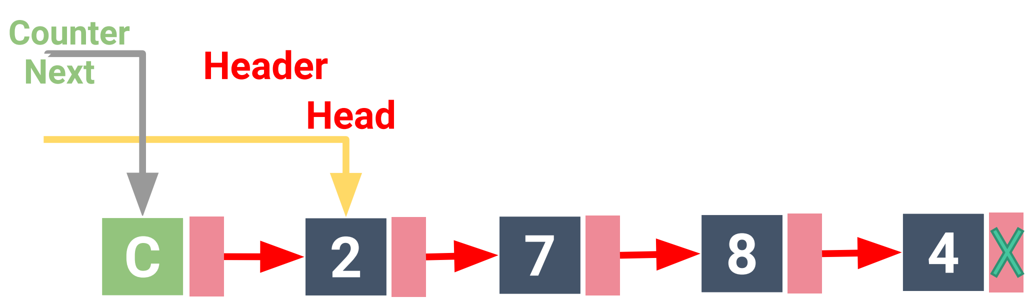
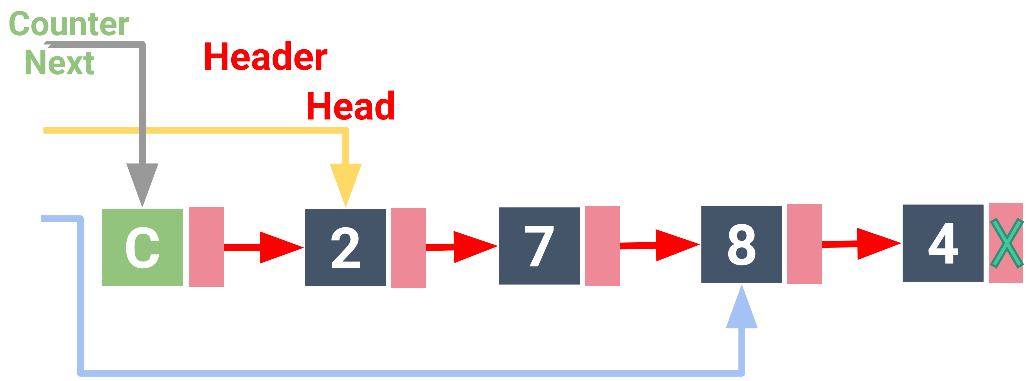
Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None



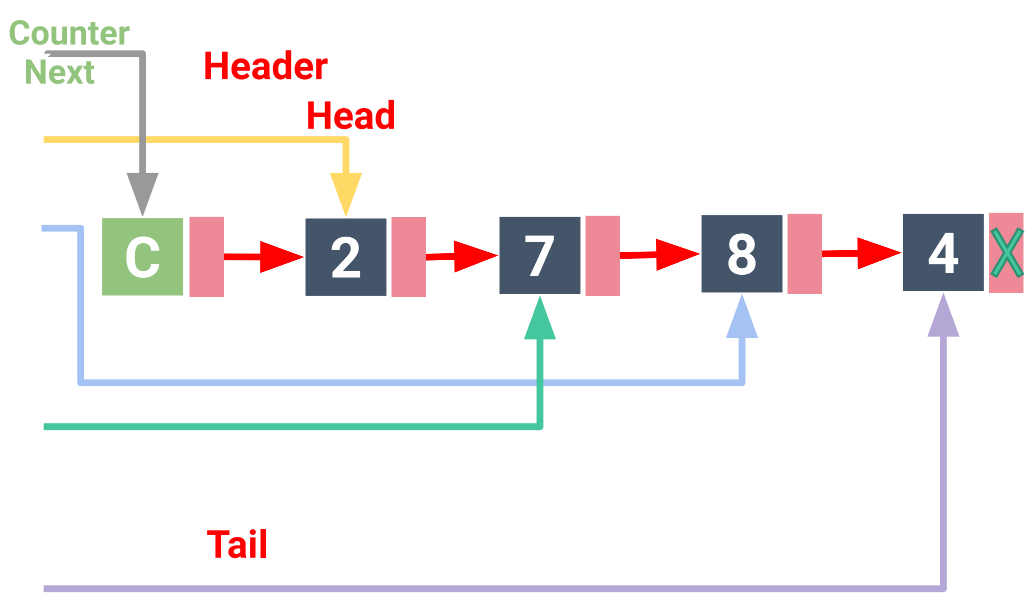
Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None



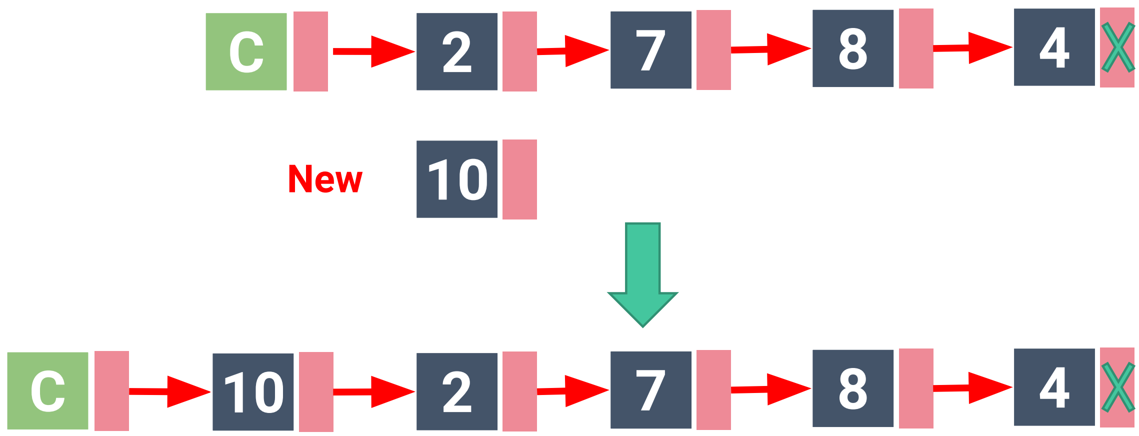
Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None



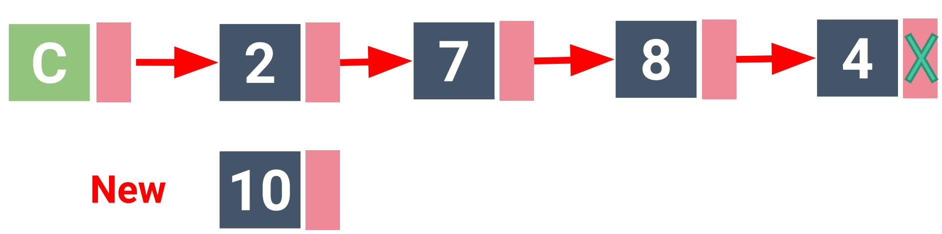
Singly Linked Lists

Insert at the first node



Singly Linked Lists

Insert at the first node



Step 1: Create a new node storing reference to an element.

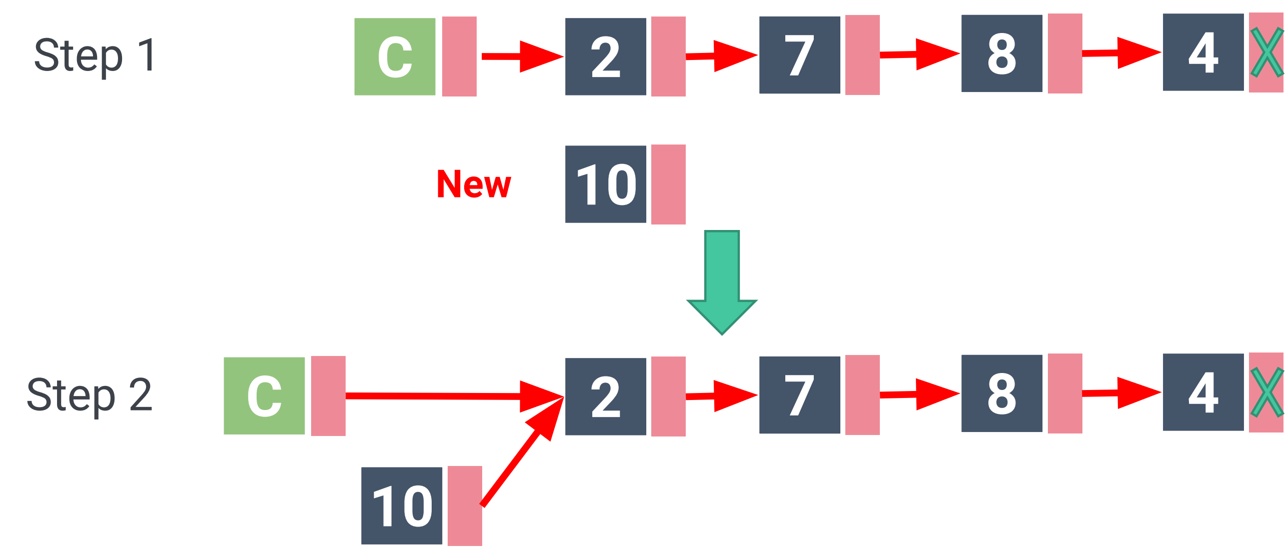
Step 2: Set new node's next pointer to the current/old head.

Step 3: Set the list's head to reference the new node.

Step 4: Increment the node count.

Singly Linked Lists

Insert at the first node



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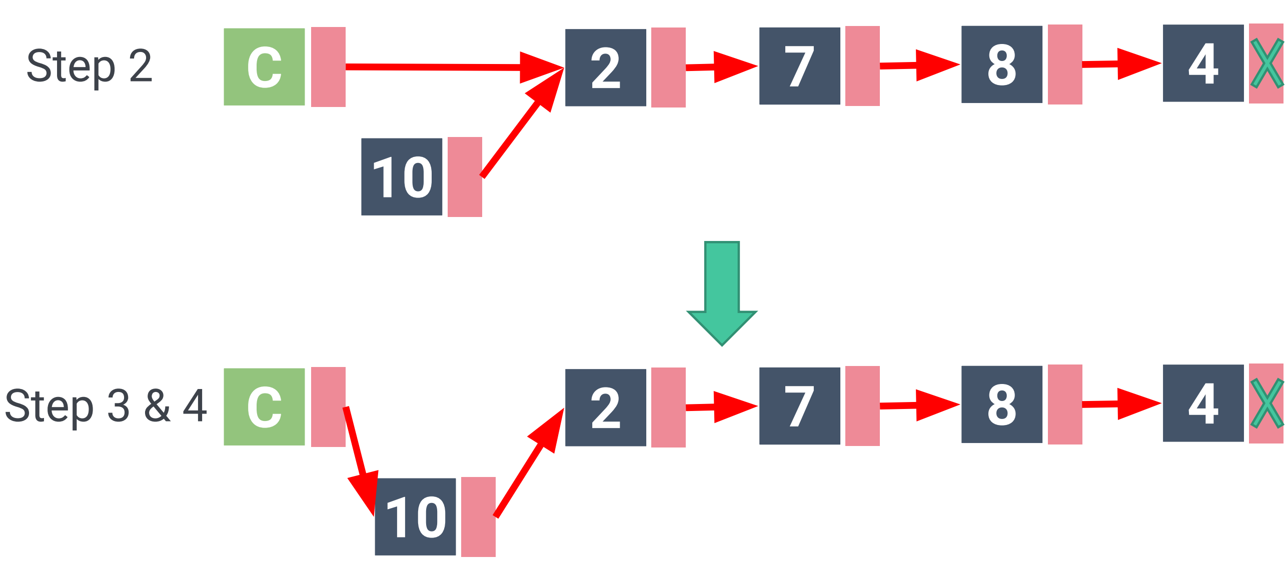
Singly Linked Lists

Insert at the first node



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Singly Linked Lists

Address/ Byte#	Value	Value	Value
6000	4	4	5
6001	6002	6002	6006
6002	2	2	2
6003	6008	6008	6008
6004	8	8	8
6005	6012	6012	6012
6006		10	10
6007			6002
6008	7	7	7
6009	6004	6004	6004
6010			
6011			
6012	4	4	4
6013	None	None	None

Step 1 — Create a new node

Step 2, 3 & 4 — Set Pointers

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Singly Linked Lists

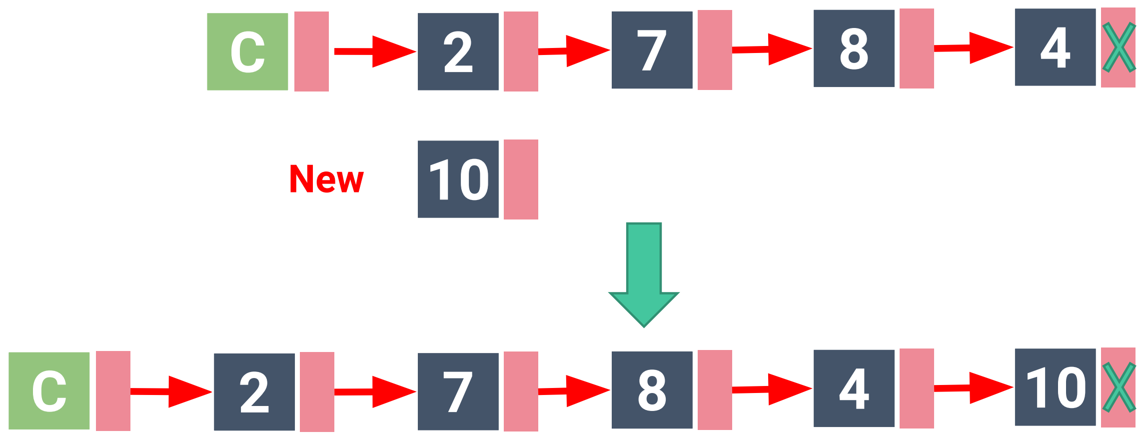
Insert at the first node

```
Algorithm add_front(L, e):
    new_node = Node(e)          # Create new node instance
    new_node.next = L.head      # Set new node's next pointer to the old head
    L.head = new_node           # Update the list's head to reference the new node
    L.size = L.size + 1         # Increment the node count
    if L.tail == None:          # List was empty
        L.tail = L.head
```

$O(1)$

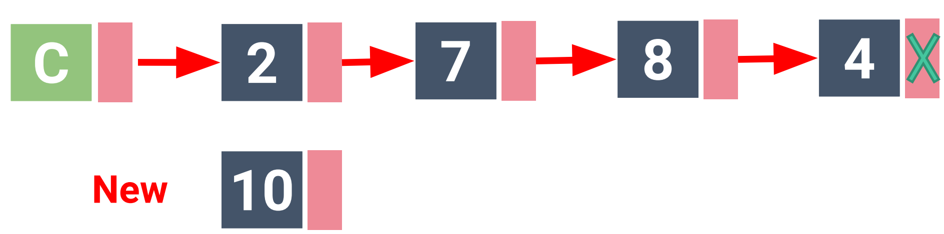
Singly Linked Lists

Insert at the last node



Singly Linked Lists

Insert at the last node



Step 1: Create a new node storing reference to an element.

Step 2: Set new node’s next pointer to <u>None</u>.

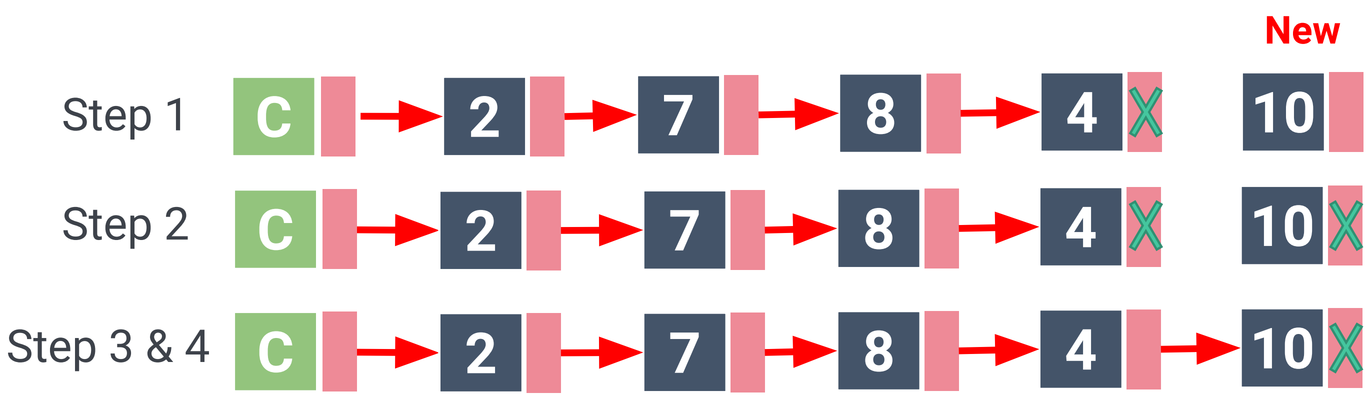
Step 3: <u>Update the list’s tail</u> to reference the new node.

Step 4: Increment the node count.

$O(1)$

Singly Linked Lists

Insert at the last node



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Singly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None

Step 1

Create a new node

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None

Step 2, 3 & 4

Set Pointers

Address/ Byte#	Value
6000	5
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	6006

Singly Linked Lists

Insert at the last node

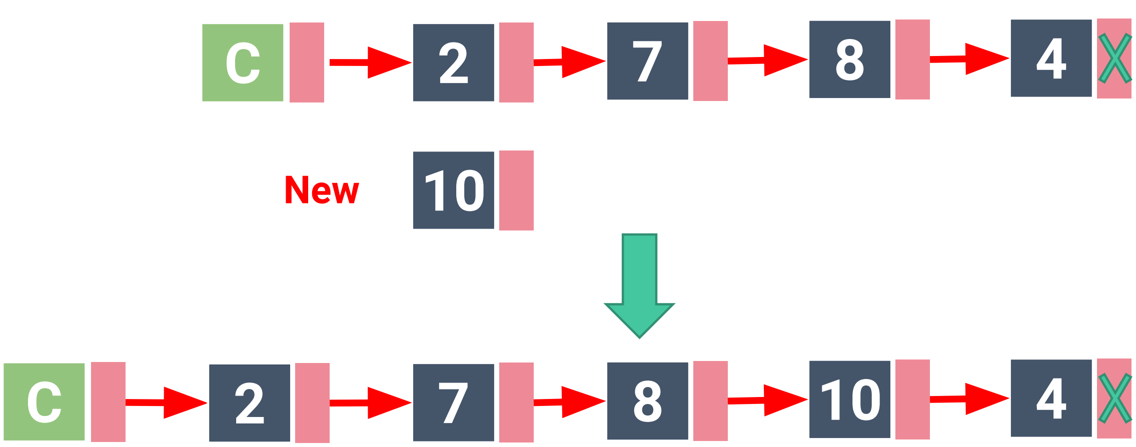
```
Algorithm add_last(L, e):
    new_node = Node(e)             # Create new node instance
    new_node.next = None           # Set new node's next pointer to None
    if L.tail == None:             # List was empty
        L.head & L.tail = new_node
    else:
        L.tail.next = new_node     # Make old tail point to new node
        L.tail = new_node          # Update the list's tail to reference the new node
    L.size = L.size + 1            # Increment the node count
```

$O(1)$

Singly Linked Lists

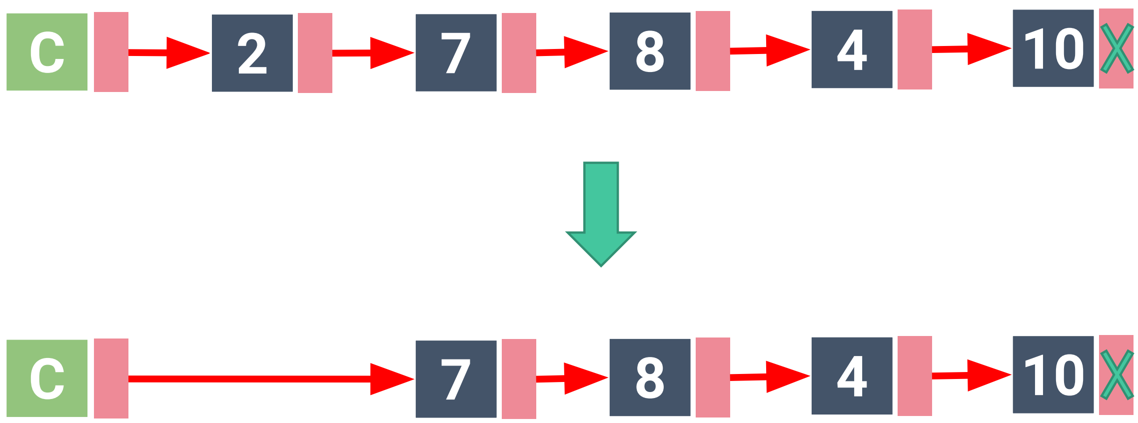
Insert between nodes

Only update the pointers of neighbouring nodes.



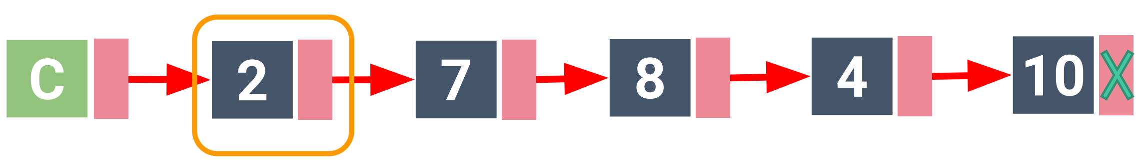
Singly Linked Lists

Delete at the first node



Singly Linked Lists

Delete at the first node



Step 1: Set head node’s next pointer to the subsequent node.

- If head is **None**, then the list is empty, return error.

Step 2: Decrement the node count.

$O(1)$

Singly Linked Lists

Address/ Byte#	Value
6000	5
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	6006

Step 1

Set Pointers

Address/ Byte#	Value
6000	5
6001	6008
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	6006

Step 2

Update counter

Address/ Byte#	Value
6000	4
6001	6008
6002	
6003	
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	6006

Singly Linked Lists

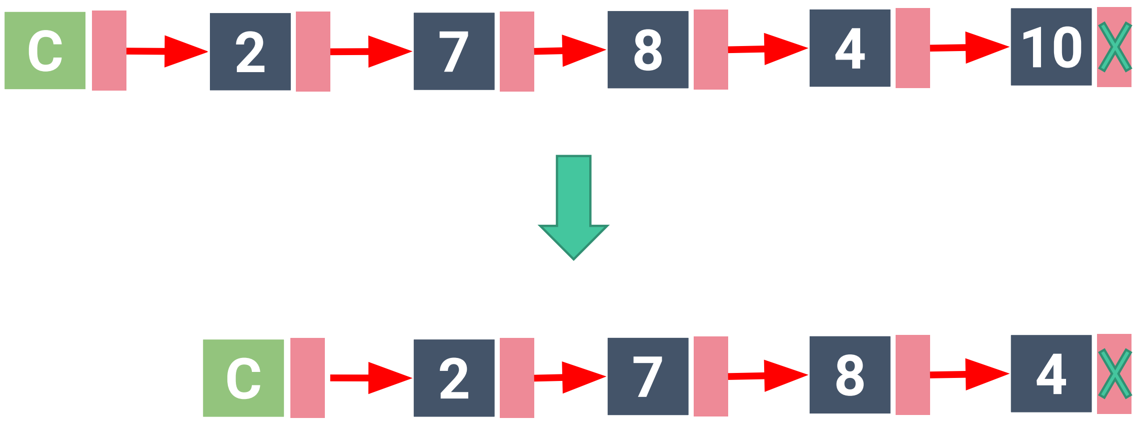
Delete at the first node

```
Algorithm remove_first(L):
    if L.head == None:        # List is empty
        return Error
    L.head = L.head.next      # Make head point to next node or None if empty
    L.size = L.size - 1       # Decrement the node count
    if L.head == None:        # List is empty after first node is removed
        L.tail = None
```

$O(1)$

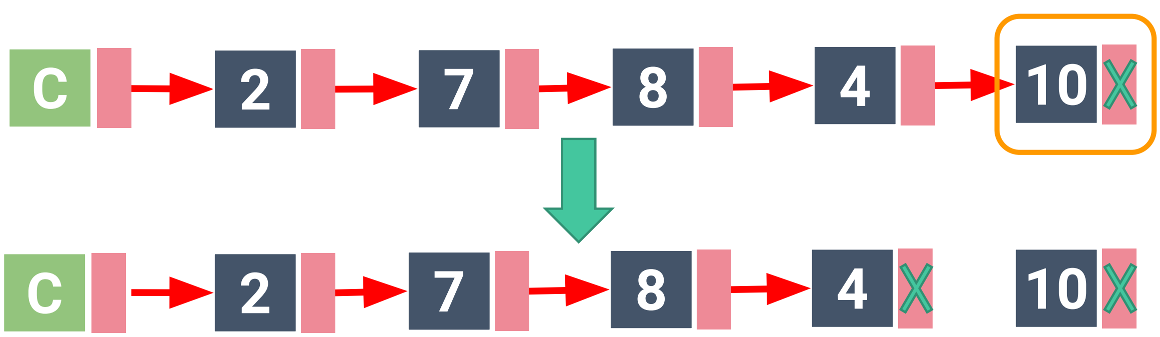
Singly Linked Lists

Delete at the last node



Singly Linked Lists

Delete at the last node



Step 1: Find the next to last node, then update the next pointer to **None**.

Step 2: Decrement the node count.

O(n) - why?

Singly Linked Lists

Deletion of the last node in Singly Linked Lists:

- No direct link from the tail node to the node before the tail.
  - There is only a link from the node before the tail to the tail node.
- To access the node before the tail, need to start from the head node and search through the list - *O*(n).
- To address this problem, **doubly linked list** is proposed as an alternative to singly linked list. also keeps links in backward direction.

Singly Linked Lists

Address/ Byte#	Value
6000	5
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	6006

Step 1

Set Pointers

Address/ Byte#	Value
6000	5
6001	6008
6002	2
6003	6008
6004	8
6005	6012
6006	10
6007	None
6008	7
6009	6004
6010	
6011	
6012	4
6013	None

Step 2

Delete Node

Address/ Byte#	Value
6000	4
6001	6008
6002	
6003	
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	None

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Singly Linked Lists

Delete at the last node

```
Algorithm remove_last(L):
    if L.head == None:  return Error        # List is empty
    if L.head == L.tail:                    # List has one node
        L.head & L.tail = None
    else:
        p = L.head                          # Initialise pointer to traverse the list
        while p.next.next != None:          # Traverse until next to last node is found
            p = p.next
        p.next = None, L.tail = p           # Update tail pointer
    L.size = L.size - 1                     # Decrement the node count
```

$O(n)$

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Singly Linked Lists: Stacks

How to Implement a Stack?

Array!!

and

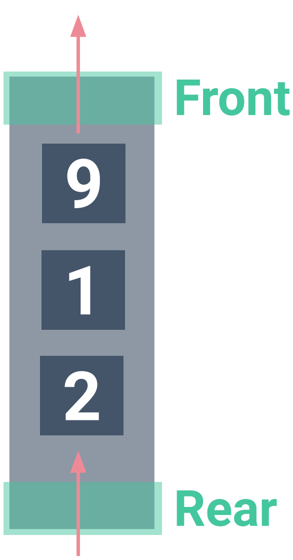
Linked Lists!!

- **Singly Linked Lists**

Asymptotic Performance

Operation	Running Time
S.push(element)	$O(1)$
S.pop()	$O(1)$
S.top()	$O(1)$
S.is_empty()	$O(1)$
len(S)	$O(1)$

Singly Linked Lists: Queues



How to Implement a Queue?

Array!!

and

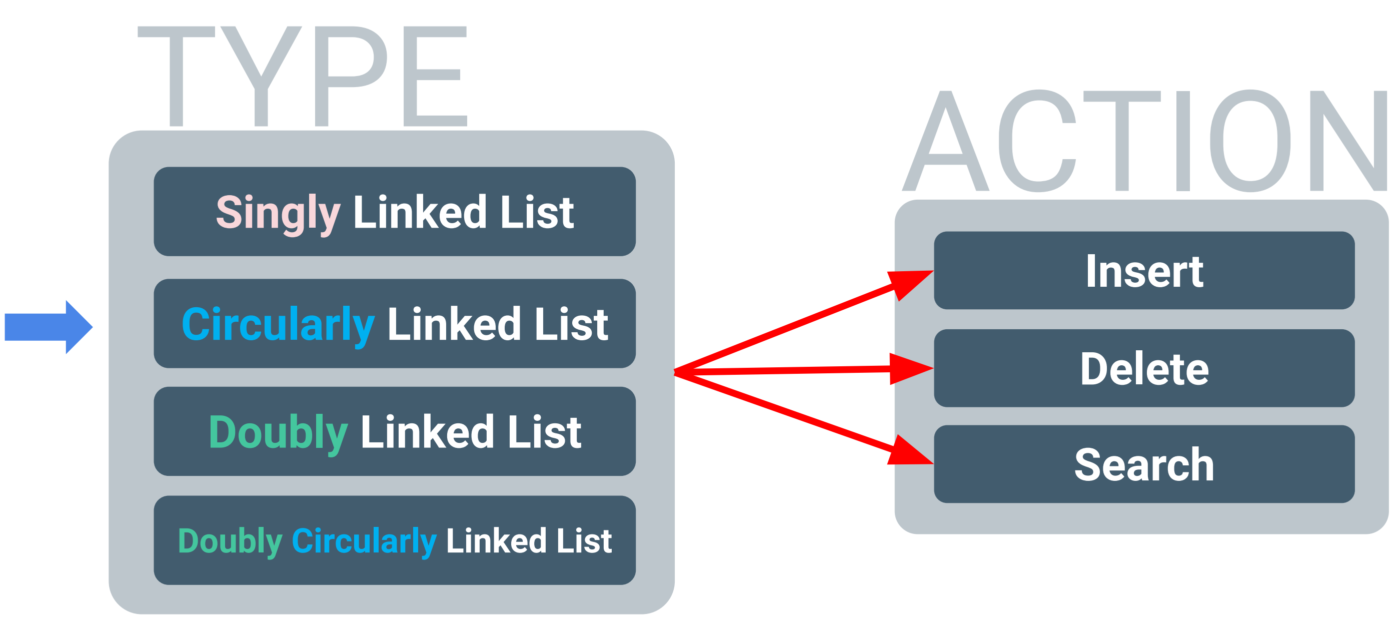
Linked Lists!!

- **Singly Linked Lists**

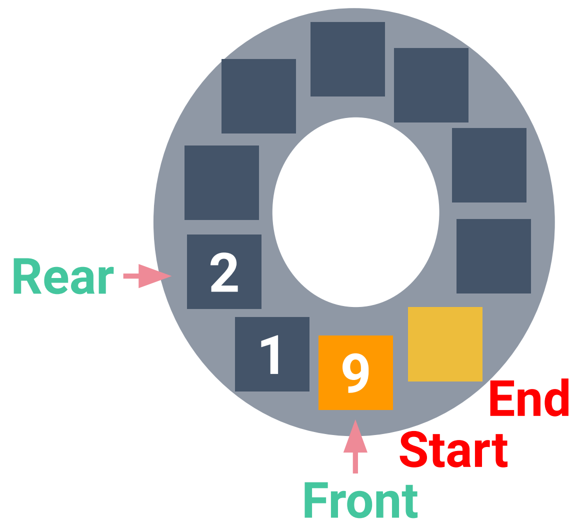
Asymptotic Performance

Operation	Running Time
Q.enqueue(e)	$O(1)$
Q.dequeue()	$O(1)$
Q.first()	$O(1)$
Q.is_empty()	$O(1)$
len(Q)	$O(1)$

Linked Lists



Circular Queue

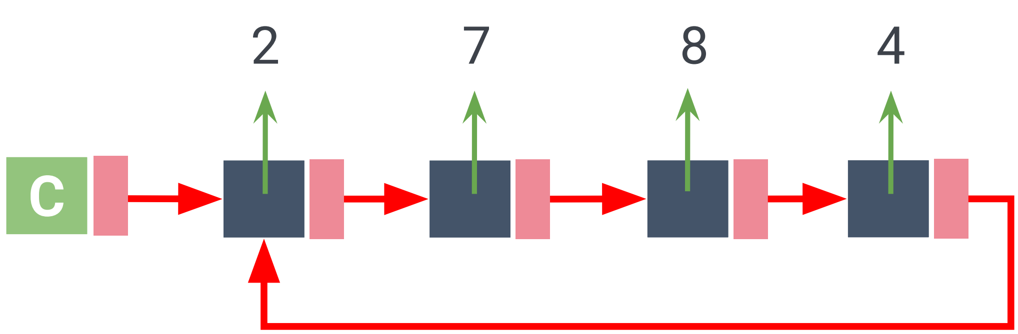


Circular is artificial for array

- Modular arithmetic (%)

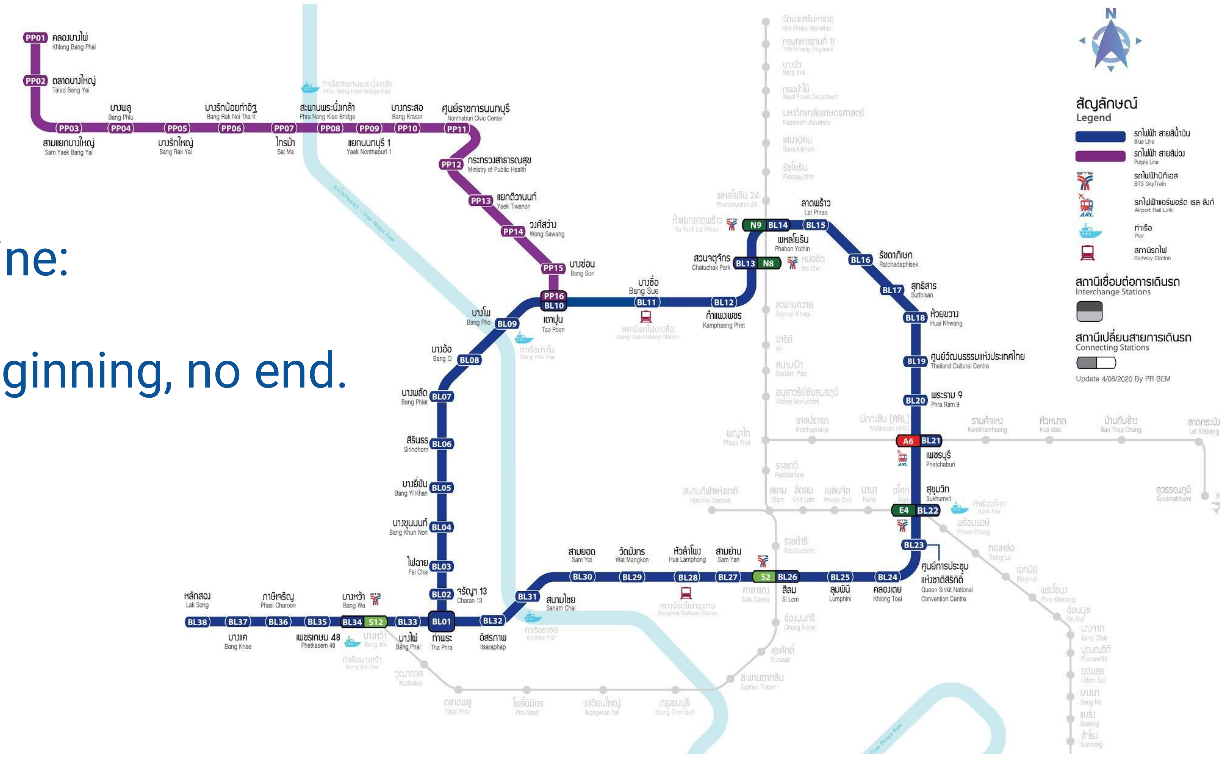
Circularly Linked Lists

A **circularly linked list** adds the notion of having the tail of the list to point back to the head of the list as the next node.



[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

Circularly Linked Lists



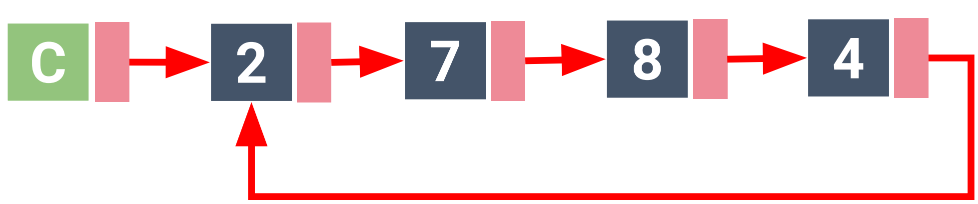
Blue line:

No beginning, no end.

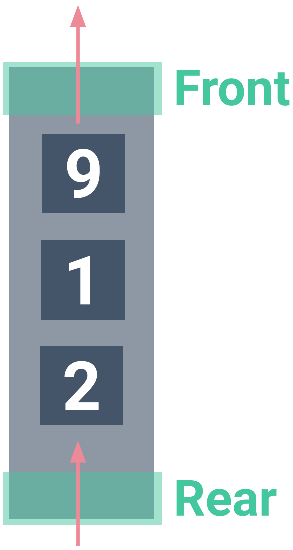
Circularly Linked Lists

Address/ Byte#	Value
6000	4
6001	6002
6002	2
6003	6008
6004	8
6005	6012
6006	
6007	
6008	7
6009	6004
6010	
6011	
6012	4
6013	6002

Suppose that it takes 1 byte to store an integer.



Circularly Linked Lists: Queues



How to Implement a Queue?

Array!!

and

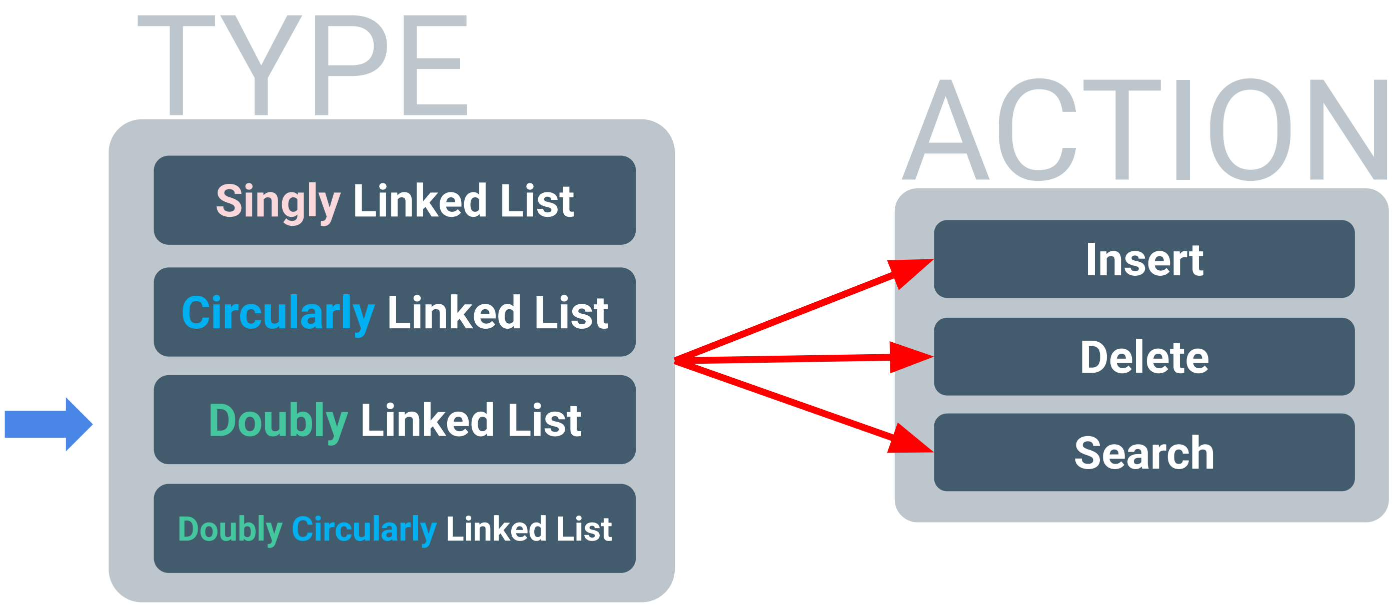
Linked Lists!!

- Singly Linked Lists
- **Circularly Linked Lists**

Asymptotic Performance

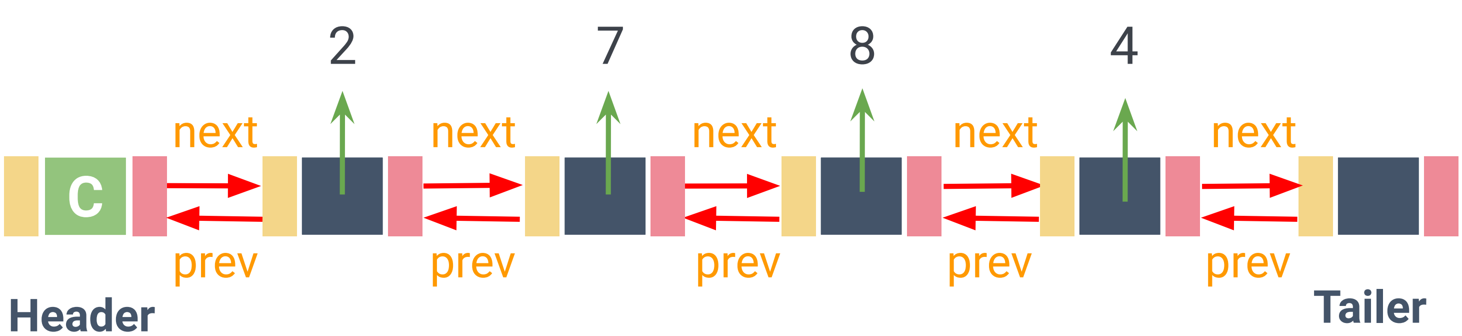
Operation	**Running Time**
Q.enqueue(e)	$O(1)$
Q.dequeue()	$O(1)$
Q.first()	$O(1)$
Q.is_empty()	$O(1)$
len(Q)	$O(1)$

Linked Lists



Doubly Linked Lists

To add more symmetry to the list, ***doubly linked lists*** allow each node to keep a reference to the node before it and a pointer to the node after it.

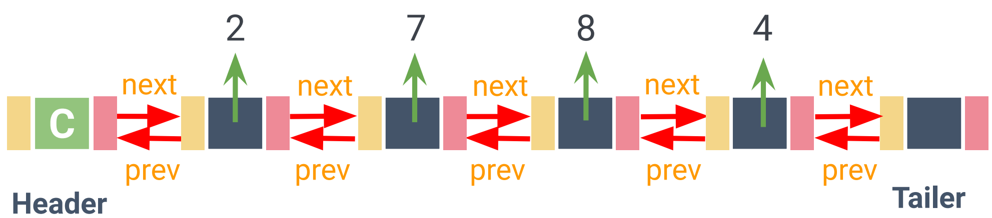


[1] Michael T. Goodrich et al., Data Structures and Algorithms in Python, 2013

Doubly Linked Lists

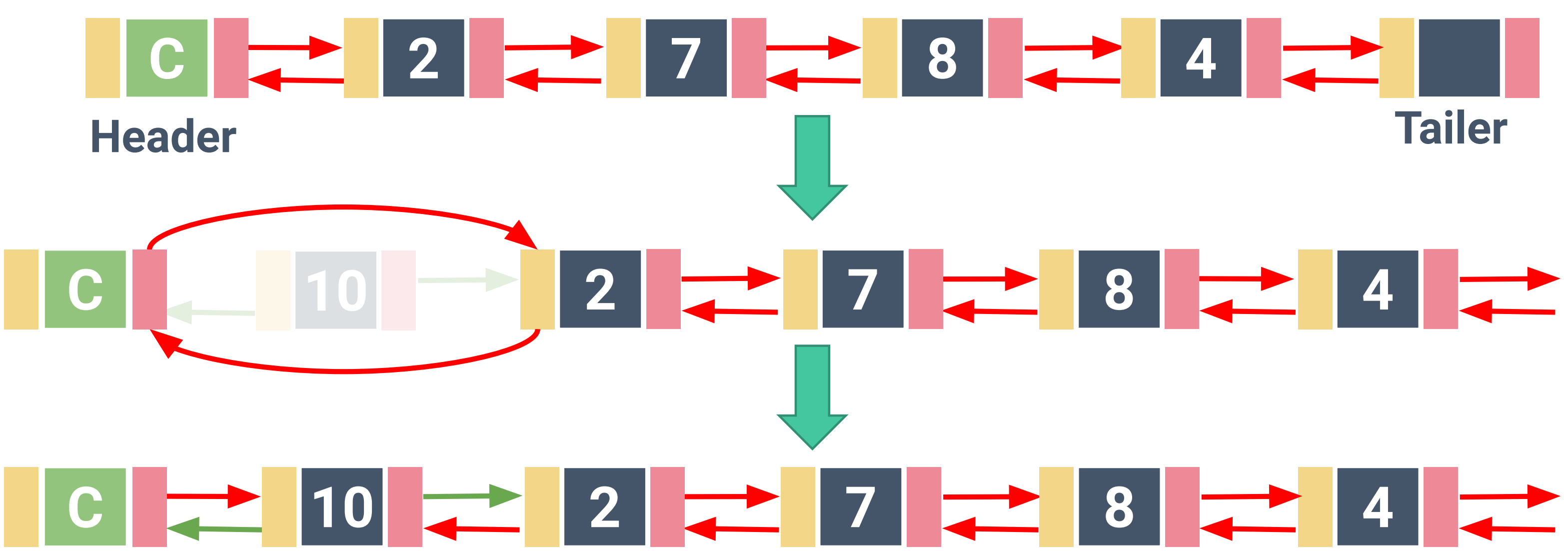
Address/ Byte#	Value		
6000	4		Header
6001	6002	next	
6002	2		
6003	6008	next	
6004	6000	prev	
6005	8		
6006	6012	next	
6007	6008	prev	
6008	7		
6009	6005	next	
6010	6002	prev	
6011			
6012	4		
6013	6015	next	
6014	6005	prev	
6015	6012	prev	Tailer
6016			

Suppose that it takes 1 byte to store an integer.



Doubly Linked Lists

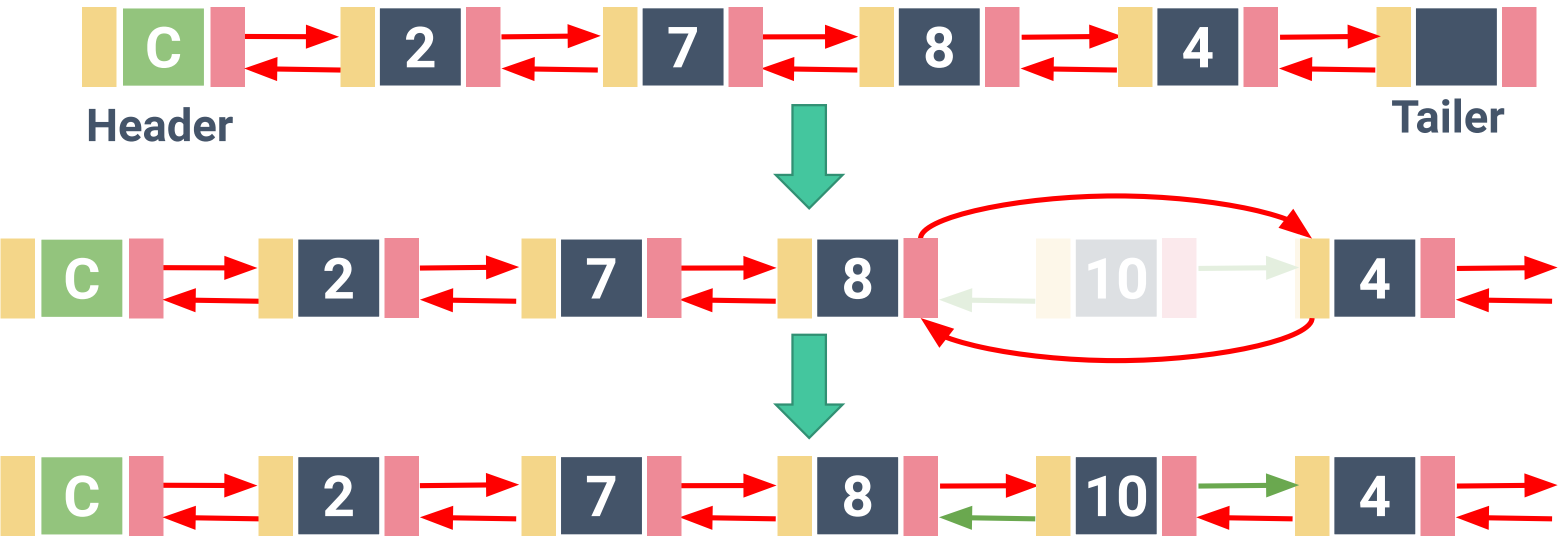
Insert at the first node



Doubly Linked Lists

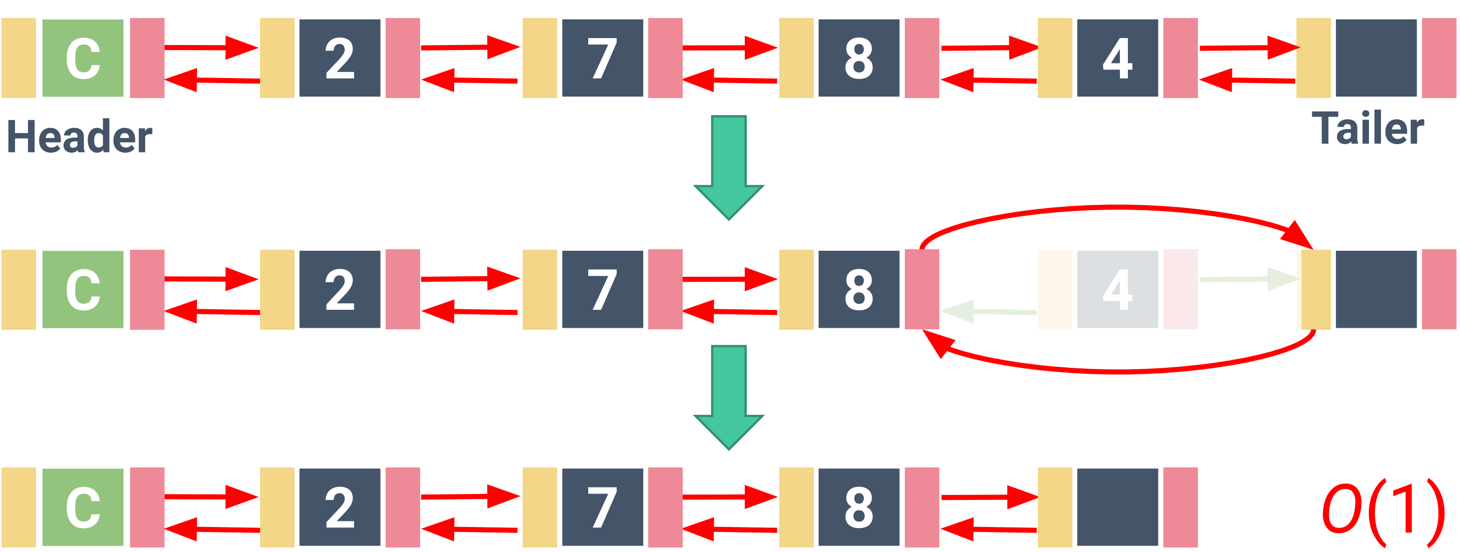
Insert between nodes

Only update the pointers of neighbouring nodes.



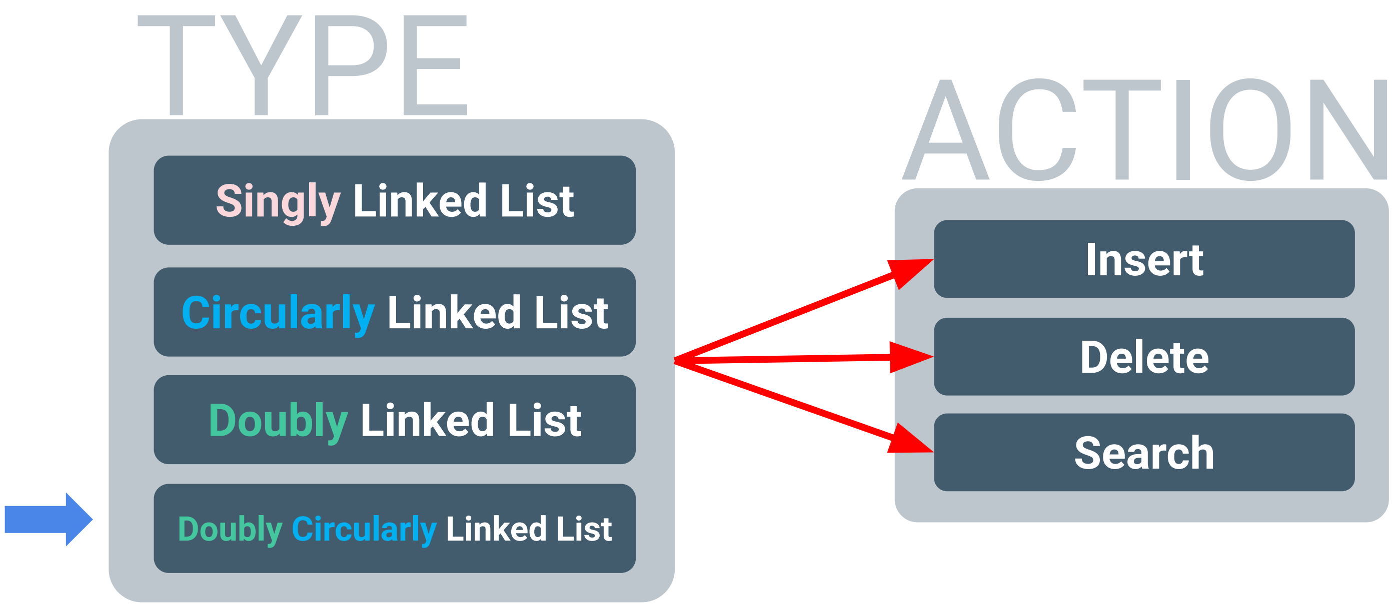
Doubly Linked Lists

Delete at the last node

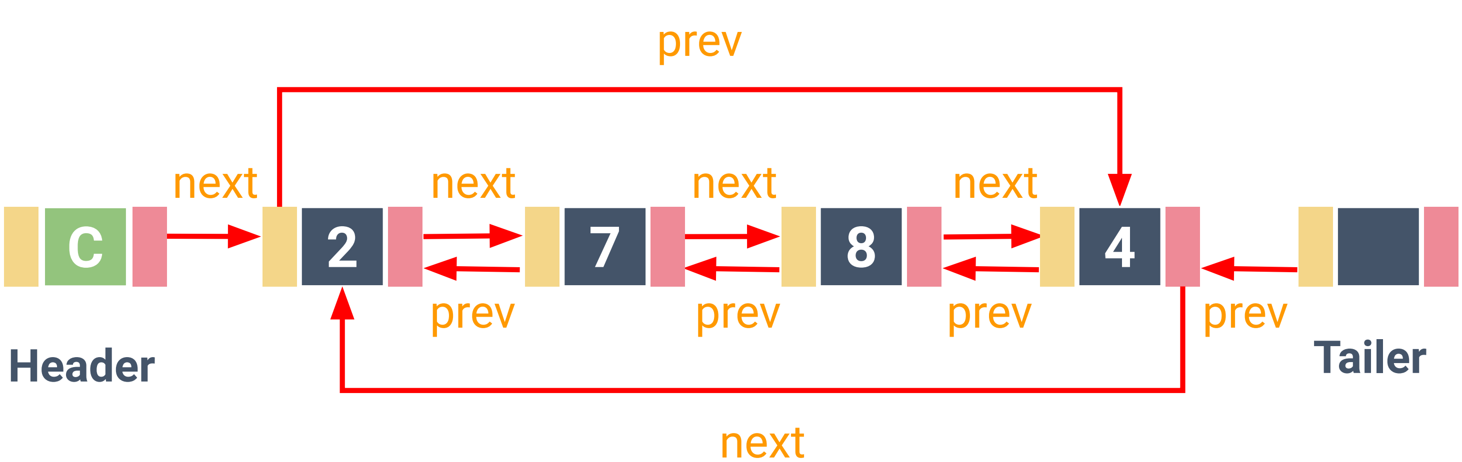


$O(1)$

Linked Lists



Doubly Circularly Linked Lists



Linked Lists

Linked list properties:

- Each node contains an element and a pointer(s) to the next node (and previous node).
- **Sequential access** only: nodes are read from the beginning.
  - Not convenient to have an index, unlike array-based sequences.
- No pre-allocated fixed size of memory, resizeable.
- Insertion and deletion operations are more efficient compared to array.
  - Take $O(1)$ - constant time to add and remove elements at any part in linked lists.

Faculty of Information Technology
King Mongkut's Institute of Technology Ladkrabang

Linked Lists

Linked list's limitations:

- Accessing the data/node in lists takes linear time - $O(n)$
  - To find the item or node at certain location, linked list has to start from the first node and traversing until the target is found.
  - For example, find the 10th node, has to traversing 10 times.
  - Unable to perform binary search.
- Use extra storage than the array to keep next pointers/references.
  - Impractical for storing small data such as characters.

Linked Lists vs Arrays

Operations	Array (Dynamic size)	Linked List
Indexing/searching	$O(1)$	$O(n)$
Add/remove at beginning	$O(n)$	$O(1)$
Add/remove at end	$O(1)$	$O(1)$ when last element is known $O(n)$ when last element is unknown
Add/remove in between	$O(n)$	$O(1)$
Wasted memory space (average)	$O(2n)$	$O(2n)$ - Singly linked list or O(3n) - Doubly linked list

Individual Assignment

- Assignment#3: Queues
- Due 09.00 am, Tuesday 01/09/2020.
- Submission
    - Email: sirasit@it.kmitl.ac.th
    - Paper: in classroom next week
- Can be either written by hand or typing.