4. Numerical Example

Consider the following FFLS (taken from [5]) and solve it by proposed method.

$(6, 1, 4) \otimes (x_1, y_1, z_1) \oplus (5, 2, 2) \otimes (x_2, y_2, z_2) \oplus (3, 2, 1) \otimes (x_3, y_3, z_3) = (58, 30, 60)$

$(12, 8, 20) \otimes (x_1, y_1, z_1) \oplus (14, 12, 15) \otimes (x_2, y_2, z_2) \oplus (8, 8, 10) \otimes (x_3, y_3, z_3) = (142, 139, 257)$

$(24, 10, 34) \otimes (x_1, y_1, z_1) \oplus (32, 30, 30) \otimes (x_2, y_2, z_2) \oplus (20, 19, 24) \otimes (x_3, y_3, z_3) = (316, 297, 514)$

Solution

The given FFLS may be written as

$$\begin{pmatrix} (6,1,4) & (5,2,2) & (3,2,1) \\ (12,8,20) & (14,12,15) & (8,8,10) \\ (24,10,34) & (32,30,30) & (20,19,24) \end{pmatrix} \begin{pmatrix} (x_1,y_1,z_1) \\ (x_2,y_2,z_2) \\ (x_3,y_3,z_3) \end{pmatrix} = \begin{pmatrix} (58,30,60) \\ (142,139,257) \\ (316,297,514) \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & 5 & 3 \\ 12 & 14 & 8 \\ 24 & 32 & 20 \end{pmatrix}, M = \begin{pmatrix} 1 & 2 & 2 \\ 8 & 12 & 8 \\ 10 & 30 & 19 \end{pmatrix}, N = \begin{pmatrix} 4 & 2 & 1 \\ 20 & 15 & 10 \\ 34 & 30 & 24 \end{pmatrix}$$

$$b = \begin{pmatrix} 58 \\ 142 \\ 316 \end{pmatrix}, h = \begin{pmatrix} 30 \\ 139 \\ 297 \end{pmatrix}, g = \begin{pmatrix} 60 \\ 257 \\ 514 \end{pmatrix}$$

The augmented matrix

$$(A, b) = \begin{pmatrix} 6 & 5 & 3 & 58 \\ 12 & 14 & 8 & 142 \\ 24 & 32 & 20 & 316 \end{pmatrix}$$

Applying elementary row operations on matrix (A, b)

First $R_1 \to \frac{R_1}{6}$, we get $\begin{pmatrix} 1 & \frac{5}{6} & \frac{3}{6} & \frac{58}{6} \\ 12 & 14 & 8 & 142 \\ 24 & 32 & 20 & 316 \end{pmatrix}$

Again we apply elementary operations in sequence

$R_2 \to R_2 - 12 R_1$, $R_3 \to R_3 - 24 R_1$, $R_2 \to \frac{R_2}{4}$, $R_3 \to R_3 - 12 R_2$,

$R_3 \to \frac{R_3}{2}$, $R_2 \to R_2 - \frac{1}{2} R_3$, $R_1 \to R_1 - \frac{5}{6} R_2$, $R_1 \to R_1 - \frac{1}{2} R_3$

Finally, we get $\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

From this row reduced form of augmented Matrix (A, b),

we have $x_1 = 4, x_2 = 5, x_3 = 3.$

The augmented matrix

$$(A, h - Mx) = \begin{pmatrix} 6 & 5 & 3 & 10 \\ 12 & 14 & 8 & 23 \\ 24 & 32 & 20 & 50 \end{pmatrix}$$

Similarly, applying elementary row operations on Matrix (A, h – Mx) in sequence

$R_1 \to \frac{R_1}{6}$, $R_2 \to R_2 - 12 R_1$, $R_3 \to R_3 - 24 R_1$, $R_2 \to \frac{R_2}{4}$, $R_3 \to R_3 - 12 R_2$,

$R_3 \to \frac{R_3}{2}$, $R_2 \to R_2 - \frac{R_3}{2}$, $R_1 \to R_1 - \frac{5}{6} R_2$, $R_1 \to R_1 - \frac{R_3}{2}$

Finally, we get $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{pmatrix}$

From this row reduced form of augmented matrix (A, h – Mx),

we have $y_1 = 1, y_2 = \frac{1}{2}, y_3 = \frac{1}{2}.$

Similarly, applying elementary row operations on augmented matrix

$$(A, g - Nx) = \begin{pmatrix} 6 & 5 & 3 & 31 \\ 12 & 14 & 8 & 72 \\ 24 & 32 & 20 & 156 \end{pmatrix}$$

Finally, we get $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

From this row reduced form of augmented matrix (A, g – Nx),

we have $z_1 = 3, z_2 = 2, z_3 = 1.$

Substituting the values of x_i, y_i, z_i where $i = 1, 2, ..., n$ in the FFLS solution

$\tilde{x}_i = (x_i, y_i, z_i)$, for all $i = 1, 2, ..., n$

we get

$\tilde{x}_1 = (x_1, y_1, z_1) = (4, 1, 3)$

$\tilde{x}_2 = (x_2, y_2, z_2) = (5, 1/2, 2)$

and $\tilde{x}_3 = (x_3, y_3, z_3) = (3, 1/2, 1)$

We have the same solution with this method as the system given in [5].